

Effect of the cross-section on the energy absorption and bending behavior of bent tubes

Quirino Estrada^{1*} , José Manuel Delgado², Eladio Martínez Rayón²,
Elva Reynoso¹, Alejandro Rodríguez-Mendez³, Aztlán Bastarrachea⁴,
Jesús Silva¹, Manuel Nandayapa¹

¹ Instituto de Ingeniería y Tecnología, Universidad Autónoma de Ciudad Juárez (UACJ), Ciudad Juárez, Chihuahua, México

² Departamento de Ingeniería Mecánica, TecNM/CENIDET, Cuernavaca, Morelos, Mexico

³ Departamento de Ingeniería Mecánica, Tecnológico Nacional de México campus Ciudad Guzmán, Ciudad Guzmán, Jalisco, México

⁴ Departamento de Ciencias Básicas, Tecnológico Nacional de México campus Ciudad Juárez, Ciudad Juárez, Chihuahua, México

* Corresponding author's e-mail: quirino.estrada@uacj.mx

ABSTRACT

When a bent tube is subjected to a transverse load, it has the capacity to resolve this load into both axial and bending components, allowing the structure to achieve a combined stress state. This confers higher structural efficiency compared with straight tubes. The crashworthiness performance of bent tubes depends on multiple factors such as material, configuration, and geometric parameters. However, studies on the cross-section of bent tubes have rarely been reported, despite their superior performance compared to their straight counterparts under bending loads. The current article first evaluated the mechanical response of a straight tube versus a bent tube. Subsequently, a second numerical analysis evaluated the effect of the cross-section on the bending resistance and energy absorption of bent tubes. For this purpose, five bent tubes with different cross-sections, including circular, triangular, square, and rectangular shapes, were numerically evaluated using three-point bending tests. In all cases, the bent tubes had the same geometric characteristics and mass. During the simulations, the tubes were assigned an elastoplastic material model for aluminum alloy 6063-T5. The reliability of the numerical results was confirmed through the experimental validation of a circular straight tube subjected to a three-point bending test. According to the results, the effectiveness of the bent tube with respect to the straight tube was confirmed, with an E_a improvement of 22.84%. Furthermore, the importance of the cross-section in the crashworthiness performance of bent tubes was demonstrated. The best crashworthiness performance was with the circular cross-section, which achieved a CFE value of 0.87, representing a 13.14% improvement with respect to the lowest CFE obtained for the rectangular bent profile (BR-01).

Keywords: bent tubes, structural strength, energy absorption, thin-walled tubes, finite element analysis.

INTRODUCTION

Enhancing passenger safety is the main objective of crashworthiness design since global statistics report more than 1.35 million fatalities due to traffic accidents [1–3]. Passive energy absorption mechanisms, such as thin-walled structures, are widely used due to their excellent

relationship between energy absorption performance and weight [4–6]. Thin-walled structures absorb energy through plastic deformation, which is maximized when a controlled deformation is achieved [7]. Regardless of the loading direction, material, and cross-section, single profiles can be classified according to the trajectory of their longitudinal axis. Two configurations can be

identified. The first one corresponds to structures with a rectilinear trajectory, such as straight profiles, whereas curved, arched, and S-shaped profiles belong to the category of non-linear trajectories. A curved profile is defined as a thin-walled structure with an initial and intentional curvature. Due to the eccentricity introduced by the curvature, the structure experiences a smooth transition from axial compression to controlled bending and stretching [8]. Meanwhile, a hybrid arched tube or bent tube, which is the subject of this study, refers to an arched tube with straight tangents. This type of structure is defined by its centerline radius of curvature (ρ) and slope angle (β). Although the effectiveness of arched tubes has been reported through numerical [9], experimental [10], and theoretical studies [11], their behavior can still be further explored. Parameters such as curvature angle, geometry, material, and loading angle can be further studied [12–14]. Zhang et al. [15] performed a numerical and experimental study to evaluate the energy absorption of arched tubes subjected to transverse loads. During the study, both straight and arched circular tubes were evaluated through quasi-static and dynamic three-point bending tests. The parametric study focused on the effects of punch diameter, loading velocity, thickness, and boundary conditions at the tube ends. In addition, an optimization process was performed, and the use of foam-filled tubes was investigated. The results demonstrated the effectiveness of arched tubes with respect to straight ones, achieving an improvement in P_m of 2.6 times. Similarly, Fu et al. [16] studied the bending resistance and energy absorption of arched square beams using quasi-static three-point bending tests. The arched tubes were produced from straight tubes by a three-point bending process. Once the beams were arched, they were inverted and evaluated under both quasi-static and dynamic three-point bending tests. The study included an analysis of the influence of loading conditions and geometric parameters. Additionally, thickness variation in the structures was investigated to further improve the crashworthiness performance of arched tubes. According to the results, regardless of the loading velocity, the superiority of arched tubes with respect to straight tubes was confirmed, with an increase of crush force efficiency ranging from 8.8% to 32.7%. Likewise, Zheng et al. [17] investigated the influence of fully fixed end constraints on curved tubes subjected to transverse loads. The curved tubes

were fabricated with different curvature radii and evaluated experimentally under both quasi-static and dynamic loading conditions. For comparison, the study also presents the assessment of tubes with free ends. The results revealed an increase in energy absorption of 2.8–4.1 times for fully constrained curved tubes with respect to unconstrained structures, which was attributed to the contribution of axial forces. Additionally, the authors presented a theoretical model to predict the energy absorption of curved tubes. Zhang et al. [18] also conducted a numerical and experimental investigation of the crashworthiness performance of single- and multi-cell arched tubes. The structures were fabricated using a simple three-point bending process and evaluated under both dynamic and quasi-static loading conditions. The study confirmed the superior crashworthiness performance of multi-cell arched tubes compared with single-cell structures.

According to the aforementioned state of the art, the use of arched or curved tubes for controlling dynamic loads is both feasible and effective when compared with straight tubes. However, the available studies are limited and focus mainly on circular and square curved tubes, as well as on the effects of the manufacturing process and boundary conditions on the energy absorption of bent tubes. Consequently, a systematic understanding of the influence of alternative cross-sections remains limited. To bridge this gap, the current article investigated the effect of the cross-section on the mechanical response and energy absorption performance of bent tubes with free-end constraints. For this purpose, five alternative cross-sections—circular, triangular, square, and rectangular—with identical mass constraints were designed and evaluated using finite element simulations of quasi-static three-point bending tests. In all cases, the structures were modeled with the properties of aluminum alloy 6063-T5.

CRASHWORTHINESS INDICES

The bent tubes are evaluated by analyzing the force-displacement curves, in addition to several dimensional and dimensionless crashworthiness indices presented in Table 1. The most important indices are the peak load (P_{max}), defined as the maximum load reached at the beginning of the bending process; the mean force (P_m); and the energy absorbed (E_a) through plastic deformation,

which is obtained by calculating the area under the force-displacement curve. Furthermore, the crush force efficiency (CFE) is defined as the ratio of P_m to P_{max} . A value close to unity indicates optimal force performance.

EXPERIMENTAL VALIDATION OF A NUMERICAL MODEL OF A THREE-POINT BENDING TEST

The current article is based on numerical simulations. Therefore, an initial finite element model was developed and experimentally validated. The model, designated SC-00, represents a three-point bending test of a straight circular tube performed at a loading rate of 5 mm/min. This model serves a dual purpose. First, it allows the calibration and validation of computational techniques, including contact interaction, material behavior, and mesh quality. Second, it serves as a baseline for assessing the relative effectiveness of the bent circular tube. With respect to the

finite element model, the tube has a length (l) of 400 mm, a diameter (d) of 25.4 mm, and a thickness (t) of 1.18 mm. The structure was modeled using S4R elements and assigned elastoplastic material properties for aluminum alloy 6063-T5, with a Young's modulus of 67,000 MPa, a Poisson's ratio of 0.33, a yield stress (σ_y) of 158.79 MPa, and a density of 2700 kg/m³ [21]. Regarding the contact interaction, a friction coefficient of $\mu_1 = 0.3$ [23] was used. In addition, a self-contact friction coefficient of $\mu_2 = 0.15$ [22] was considered. Finally, details of the finite element model are shown in Figure 1. To determine the appropriate element size, a mesh convergence analysis was performed using two mesh sizes: a global mesh for the end regions and a local mesh for the middle region of the structure.

The convergence analysis evaluated three mesh configurations: coarse (6 mm global/3 mm local), medium (3 mm global/1.5 mm local), and fine (1mm global/0.5 mm local). The medium mesh with global and local element sizes of 3 mm and 1.5 mm, respectively, provided the best

Table 1. Crashworthiness indices, where δ is the displacement and F the bending force [19, 20]

P_{max} [kN]	P_m [kN]	E_a [kJ]	SEA [J/g]	CFE
Obtained from the curve	$P_m = \frac{E_a}{\delta}$	$E_a = \int_0^{\delta} F \cdot d\delta$	$SEA = \frac{E_a}{m}$	$CFE = \frac{P_m}{P_{max}}$

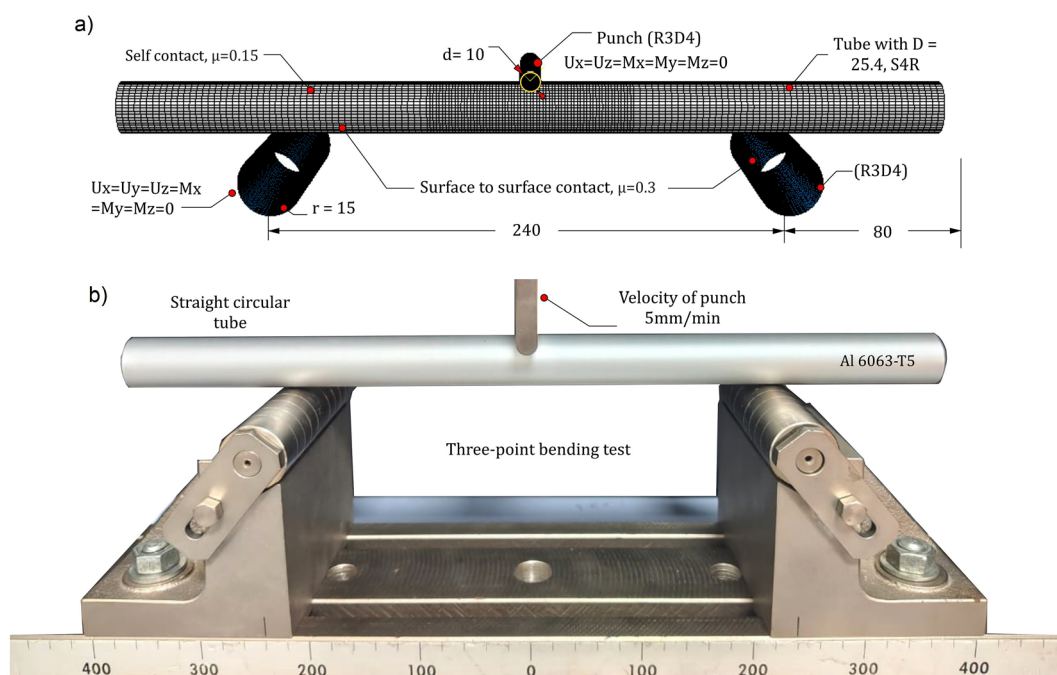


Figure 1. a) Discrete model and b) experimental setup of the three-point bending test for the straight circular tube ST-00

balance between accuracy and computational cost (Figure 2a). The comparison between the numerical and experimental results is presented in Figure 2. Figure 2a compares the force-displacement curves obtained from both models, including the results from the mesh convergence analysis. The medium mesh exhibited excellent agreement with the experimental result. In this sense, at the beginning of the bending test, the force increases rapidly until reaching the maximum peak load (P_{max}) of approximately ~ 1.41 kN. Subsequently, the force decreases smoothly to approximately 0.75 kN. The final deformation mode obtained from both the numerical and experimental models is illustrated in Figure 2b, where a local indentation beneath the punch can be observed. As a result, a central plastic hinge line is formed, and the cross-section undergoes a transition from a circular to an oval shape. Considering the small differences of less than 3% between the force-displacement curves and the accurate prediction of the deformation mode, the finite element model is successfully validated. Therefore, numerical techniques such as mesh discretization, contact interaction, and material models, which are independent of the overall geometry of the structures, were also validated. Consequently, the effect of the

cross-section of the bent tubes will be evaluated exclusively through numerical simulations.

NUMERICAL SIMULATIONS

Feasibility of bent tubes vs straight tubes

The feasibility of using bent tubes for crashworthiness applications is highly promising because they exhibit superior energy absorption compared with straight tubes. Unlike straight tubes, bent tubes experience not only bending but also combined axial and bending forces that are transmitted along the tube toward the supports. Consequently, these structures can achieve improved structural performance. With this in mind, a numerical study comparing straight and bent circular tubes under three-point bending tests was carried out. The first model consists of a straight tube with a length of 400 mm, a diameter (d) of 25.4 mm, and a thickness (t) of 1.18 mm. The second model has the same circular cross-section and length of 400 mm. In both cases, the structures were evaluated using three-point bending tests at a loading rate of 5 mm/min. Details of the finite element models are presented in Figure 3.

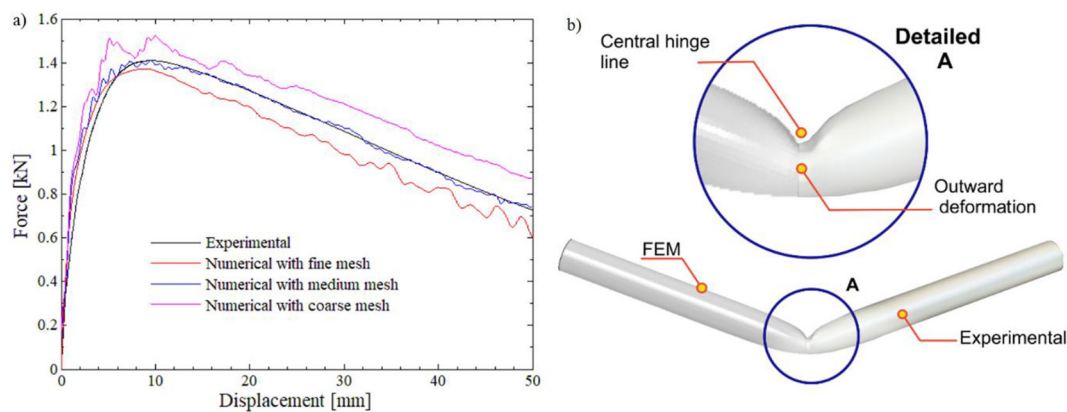


Figure 2. a) Force-displacement curves and b) final deformation modes obtained from the numerical and experimental models

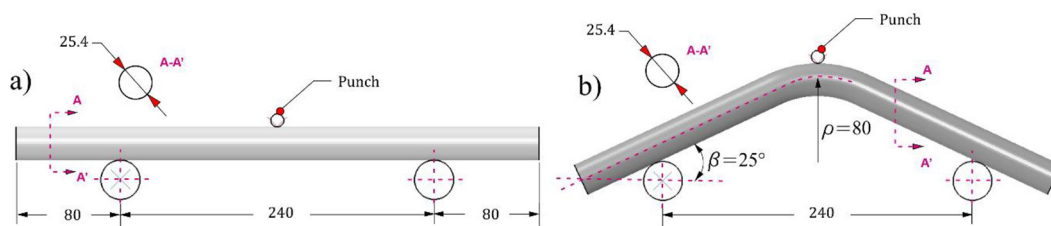


Figure 3. Discrete model of a) a straight and b) a bent tube for the three-point bending test

The numerical force-displacement curves are presented in Figure 4a. Significant quantitative and qualitative differences in the mechanical response were observed. At the beginning of the test, within the elastic regime, both structures exhibited similar stiffness up to approximately 0.8 kN. Beyond this point, the bent tube exhibited greater structural strength and stability, reaching a higher peak load (P_{max}) of approximately 1.555 kN. Once the P_{max} value was reached, the bent tube maintained an almost constant load, resulting in improved crush force efficiency. Physically, localized wrinkles formed along the compressed face of the bent geometry, promoting greater cross-sectional stability. In contrast, the straight tube (ST-00) reached a peak load of approximately 1.4 kN, after which its load-carrying capacity gradually decreased. This progressive softening is attributed to the sudden ovalization of the cross-section, which considerably reduces the section modulus beneath the punch. The superiority of the bent tube (BC-01) is confirmed by the energy absorption results shown in Figure 4b. At the end of the 50 mm displacement, the bent tube absorbed 67.66 J of energy, representing an increase in E_a of 22.84% with respect to the straight tube. This improvement is attributed to the intentional curvature of the bent tube, which stabilizes the plastic hinge during the bending process, minimizes cross-sectional ovalization, and promotes energy absorption through plastic deformation. Therefore, the effectiveness of the bent tube with respect to the straight tube is confirmed, consistent with the findings reported in [15, 16], where curved tubes with end-constraint and straight tubes were studied.

Effect of the cross-section of bent tubes on crashworthiness performance

The main objective of this section was to evaluate the effect of the cross-section on the crashworthiness performance of AL6063-T5 bent tubes, the geometry of which is defined by the centerline bend radius (ρ) and slope angle (β). For this purpose, circular, triangular, square, and rectangular cross-sections were investigated. It should be noted that, in order to isolate the pure influence of the cross-section, this study considered only the structural response of idealized geometries generated from the CAE model, without accounting for manufacturing-induced effects such as residual stresses, cross-sectional ovalization, or thickness variations. This assumption is useful because it prevents the effects of cross-sectional geometry from being coupled with manufacturing factors, thereby establishing clear and fundamental insights for the design of bent tube cross-sections. Under this assumption, the bent tubes were numerically evaluated using quasi-static three-point bending tests at a loading rate of 5 mm/min.

The validated numerical procedures described in Section Experimental validation of a numerical model of a three-point bending test, including contact interaction, mesh discretization, and material modeling, were employed throughout the study. The geometric specifications of the numerical models are summarized in Figure 5 and Table 2. At this point, a special case is analyzed for the rectangular cross-section, for which two variants are presented. In both cases, the geometric dimensions are the same, with the only difference being their orientation with respect to the applied load. Hence, BR-01 is loaded transversely along the major axis, whereas BR-02 is loaded along the minor axis.

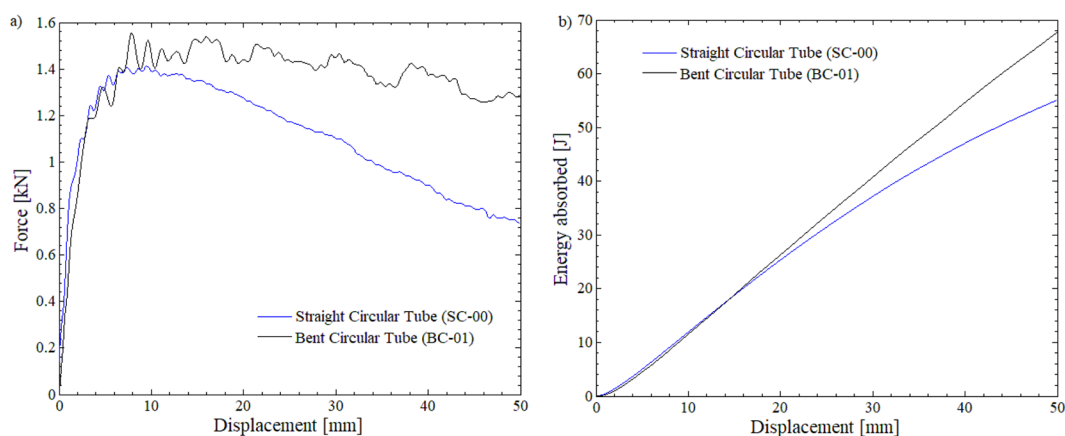


Figure 4. a) Force-displacement and b) energy absorption responses for the straight and bent circular tubes

Table 2. Geometrical specifications for bent tubes, units in mm

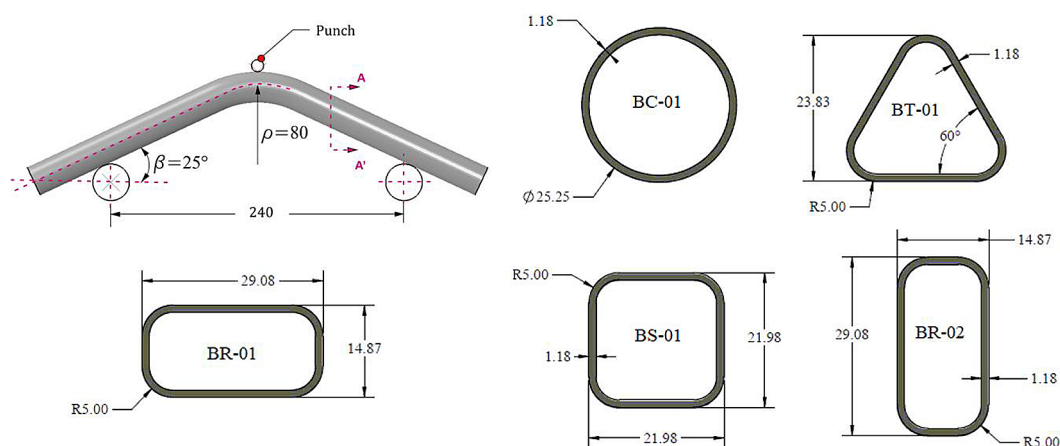
Specimen code	Base	β ($^{\circ}$), ρ (mm)	Length (mm)	Thickness (mm)	Mass (g)
BC-01	Circular	25, 80	400	1.18	101
BT-01	Triangular	25, 80	400	1.18	101
BS-01	Square	25, 80	400	1.18	101
BR-01	Rectangular	25, 80	400	1.18	101
BR-02	Rectangular	25, 80	400	1.18	101

RESULTS AND DISCUSSION

The mechanical response of the bent tubes was evaluated by analyzing the force-displacement curves shown in Figure 6. As can be seen, all bent tubes exhibited very similar qualitative behavior, particularly in their ability to maintain structural strength after yielding, unlike the straight tube analyzed previously. The bent tubes presented better post-yielding load-carrying capacity without sudden failure, which is in accordance with the results reported in [16] for curved tubes with end-constraint. In general, the bent tubes exhibited P_{max} values ranging from 1.22 kN to 1.95 kN. After reaching the initial P_{max} , the force did not decrease suddenly. In some cases, such as BT-01 and BR-02, the structures had a slight increase in structural strength, which is attributed to improved geometric stabilization during deformation. The highest load-carrying capacity was obtained for BR-02, which reached a P_{max} of approximately 1.95 kN. This behavior is attributable to two factors. First, the rectangular cross-section maximizes bending resistance. Second, it provides a larger second moment of inertia. Similar behavior was observed for the triangular bent tube (BT-01), which reached a P_{max} of approximately 1.916 kN. This response is associated with the geometric efficiency of the

triangular cross-section, where the inclined walls act as diagonal braces when the external load is applied to the apex, allowing a more uniform distribution of the bending load and delaying local buckling. In contrast, the poorest mechanical performance was obtained for BR-01, where the load was applied along the major axis of the rectangular section. Under this loading condition, the second moment of inertia is significantly reduced, leading to a continuous decrease in load to approximately 0.8 kN, as a result of premature local flattening.

The stiffness and stability of the structures depend strongly on the cross-section; therefore, the study of the collapse mode becomes essential [24]. The initial and final deformation modes of the bent tubes are presented in Figures 7 and 8. In addition, top and cross-sectional views of the region beneath the punch are provided. Regardless of the tube geometry, all structures developed a central plastic hinge line beneath the punch (see the top views), promoting the formation of a flattened region at the middle of the cross-section. As can be seen, the formation of secondary hinge lines was governed by the cross-sectional geometry. For example, the triangular bent tube (BT-01) developed a larger number of plastic hinge lines, resulting in higher energy absorption than the circular bent tube (BC-01). Furthermore, the


Figure 5. Geometrical specifications for bent tubes, units in mm

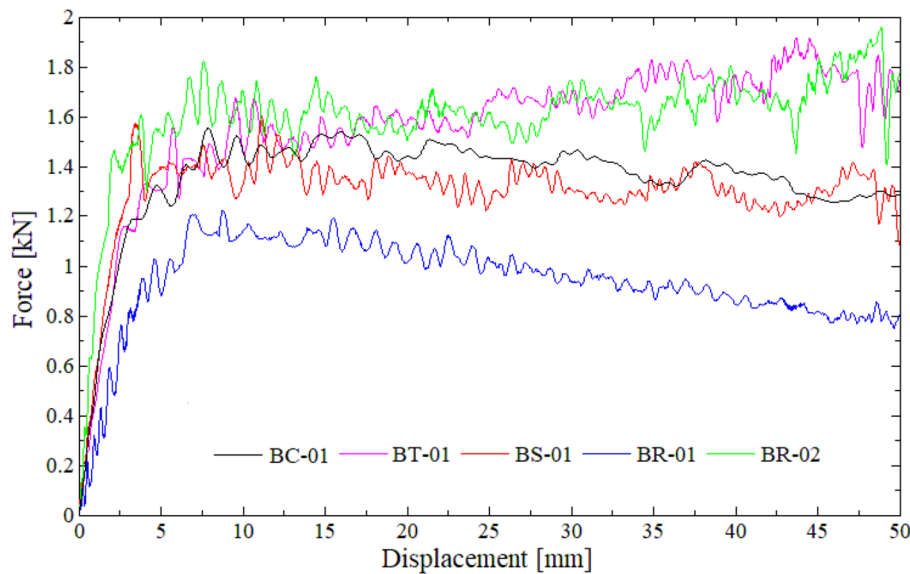


Figure 6. Comparison of force vs displacement curves of bent tubes with different cross-section geometries

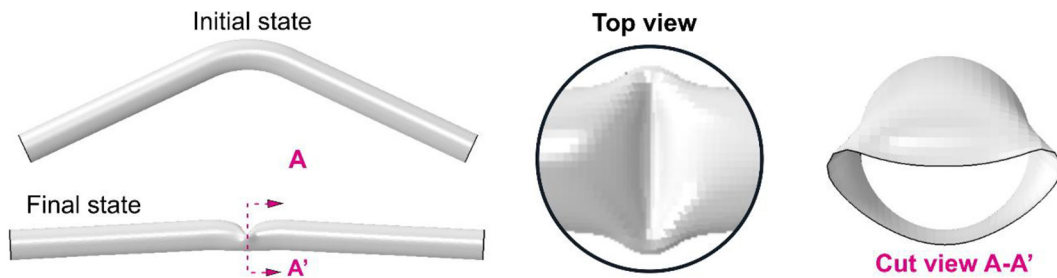


Figure 7. State of deformation of the circular bent tube (BC-01)

lateral views show that the indentation produced by the punch promoted the formation of a principal outward bulge in all cases.

A summary of the crashworthiness indices is presented in Table 3. In general, all structures exhibited acceptable energy absorption performance; however, noticeable differences were observed due to the cross-sectional topology and loading orientation. Regarding P_{max} , the values ranged from 1.223 kN to 1.959 kN. The highest value was obtained for the rectangular bent tube BR-02, where the load was applied along the minor axis, whereas the lowest value corresponded to BR-01. The only difference between these two structures is the loading orientation, which in BR-01 was applied along the major axis. Mechanically, loading along the minor axis (BR-02) increases both the second moment of area and the elastic section modulus, thereby improving the bending resistance of the tube.

A similar trend was observed for the mean force (P_m), where a value close to 0.94 was achieved. The shape of the cross-section plays

a fundamental role in determining the structural stability of the tubes and, consequently, influences E_a , SEA , and CFE . The highest E_a value of 80.065 J was achieved by structure BR-02, representing an improvement of 70.23% compared with the lowest E_a computed for BR-01. The same trend was observed for the specific energy absorption (SEA). Since all specimens had the same mass (101 g), the best SEA performance reached a maximum value of 0.793 J/g. This behavior reflects the greater stability of the rectangular cross-section when the load is applied along its minor axis, which provides a larger second moment of area. Consequently, a larger portion of the structure surfaces undergoes stable and progressive plastic deformation. The vertically oriented rectangular cross-section allows a favorable stress distribution, suppressing premature cross-sectional flattening and localized instabilities, thereby enhancing the energy absorption capacity.

The crush force efficiency (CFE) is considered one of the most important crashworthiness

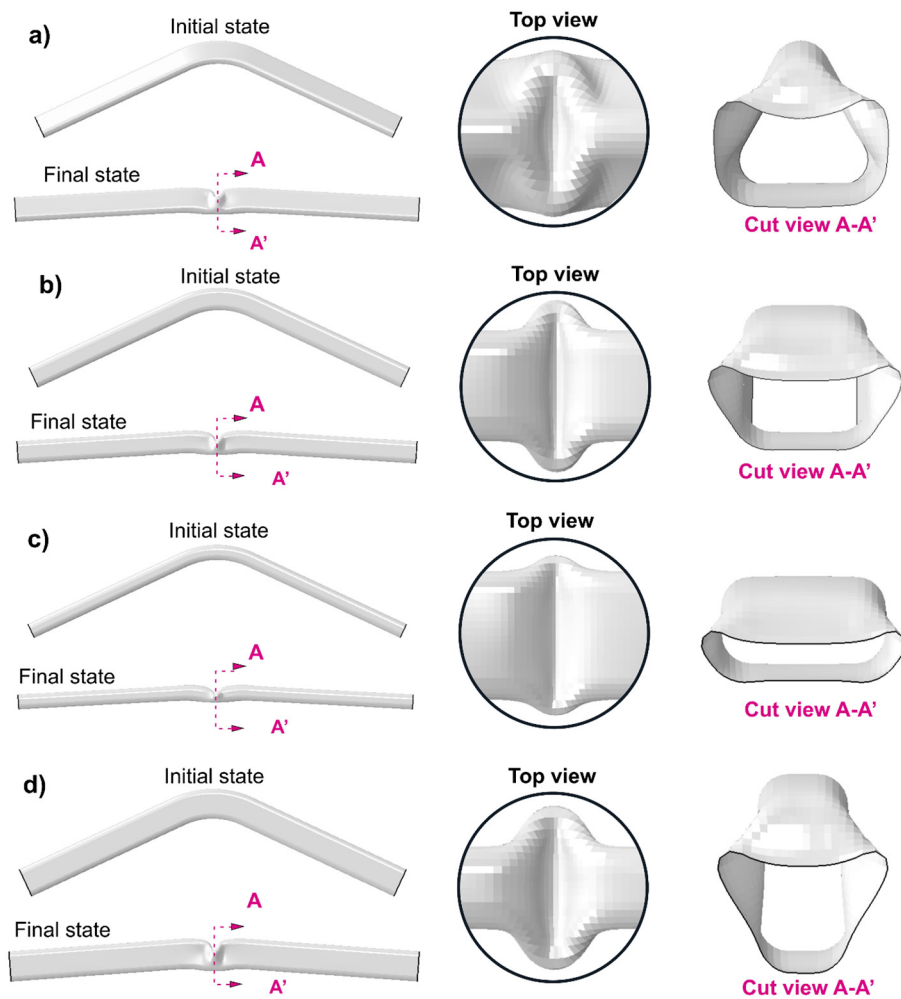


Figure 8. State of deformation of the bent tubes: a) BT-01, b) BS-01, c) BR-01, and d) BR-02

Table 3. Summary of numerical results for bent tubes with different cross-section shapes

Specimen code	Peak load P_{max} (kN)	Mean crushing force P_m (kN)	Energy absorption E_a (J)	Specific energy absorption SEA (J/gr)	Crushing force Efficiency CFE
BC-01	1.555	1.353	67.661	0.670	0.870
BT-01	1.916	1.566	78.323	0.775	0.818
BS-01	1.602	1.303	65.153	0.645	0.813
BR-01	1.223	0.941	47.031	0.466	0.769
BR-02	1.959	1.601	80.065	0.793	0.817

parameters, since it involves the effectiveness of the force during the bending process. In this context, and for the current analysis, CFE is prioritized over E_a and SEA because it is directly related to passenger safety through the uniformity of the force during the bending process. Furthermore, because all structures have the same mass, CFE becomes a relevant parameter for the present study. The bent tubes presented three trends: high, intermediate, and low performance. The lowest value of CFE was achieved

by the rectangular tube BR-01 with a CFE of 0.769. Meanwhile, the triangular, square, and rectangular (BR-02) bent tubes exhibited similar values of approximately 0.81, corresponding to an intermediate performance and representing an improvement of 6.22% with respect to BR-01. However, the circular cross-section achieved the highest CFE value of 0.87, representing an improvement of 13.14% over the lowest value. Although the bent circular tube did not exhibit the highest E_a or P_{max} , it showed the

smallest difference between P_{max} and P_m , resulting in a better balance between strength, stability, and plastic deformation and better deceleration control. This behavior can be explained by the combined effect of the bent geometry and the circular cross-section, which promotes a more efficient transmission of the transverse load into axial and bending components, minimizing stress concentrations and promoting a continuous and progressive plastic folding mechanism rather than the rapid localized buckling and geometric flattening observed in polygonal sections, which was also reported in [16] for circular and square curved tubes. Consequently, a greater amount of controlled plastic deformation is achieved. Therefore, the circular bent tube (BC-01) is recommended for the crashworthy design of bent tubes and could be used as guidelines for engineers and designers. Finally, although manufacturing imperfections may reduce the overall crashworthiness performance, the relative trends and structural performance among different bent tubes remain unaltered. Likewise, the structural mechanism that determines the effectiveness of the bent tubes is mainly associated with the cross-sectional properties, including their capacity to distribute stress, their bending resistance, and their tendency to form premature localized buckling. Therefore, the current study using idealized geometries serves as an essential baseline for understanding the effect of cross-sectional geometry. The results indicate that the circular cross-section provides superior stress distribution and stability, offering useful guidelines before introducing manufacturing-related complexities in future studies.

CONCLUSIONS

In the current article, the effect of the cross-section on the mechanical response of bent tubes was successfully evaluated through three-point bending tests. For this purpose, different cross-sections, such as circular, triangular, square, and rectangular, were numerically studied. The main objective of this work was to evaluate the geometry; therefore, manufacturing-induced stress states or imperfections were not considered. In all cases, the structures presented the same length, mass, thickness, centerline bend radius (ρ), and slope angle (β). Additionally, they were modeled with elastoplastic properties for aluminum

alloy 6063-T5. The main findings of this study are listed below.

1. The intentional curve of the bent tubes enables a more uniform distribution of the transverse load. Consequently, greater structural stabilization is achieved by maximizing the efficiency of the rolling deformation and delaying the loss of load-carrying capacity.
2. With respect to the straight circular tube, the bent circular tube presented an increase of energy absorption (E_a) and crush force efficiency (CFE) up to 22.84% and 11.54%, respectively. In this case, the geometry of the bent tube allowed for better control of the transverse load, since it resolves into both axial and bending loads.
3. The effect of cross-section becomes fundamental since it determines the structural strength and collapse mode of the curved tubes. The lowest load-carrying capacity was computed for the curved rectangular tube (BR-01) with a mean force (P_m) of 0.94 kN and specific energy absorption (SEA) of 0.466 J/g. For this case, the load is applied along the major axis of the rectangle. Meanwhile, the highest strength was computed for BR-02 with a P_m of 1.601 kN and SEA of 0.793 J/g. This represented an improvement of 70.23% in both cases.
4. Since the crush force efficiency (CFE) is one of the most important crashworthiness indicators, all bent structures presented an acceptable performance with values within 0.76 and 0.87. In this case, the poorest CFE was computed for the curved rectangular tube (BR-01), which is attributed to its low resistance to support the transverse load. This condition is explained by the second moment of area. Meanwhile, the highest CFE value was exhibited by the curved circular tube, which yielded an improvement of 13.14%.
5. Since the highest CFE of 0.87 was obtained by the bent circular tube (BC-01), this geometry is recommended and could be considered as a baseline for engineers and designers during the crashworthy design of the structures. Its superiority over the other geometries lies in its better stress distribution and crush stability.
6. As future work, the effect of the manufacturing process on the cross-section will be evaluated, considering factors such as residual stress, wrinkling phenomena, and progressive damage in order to identify the best manufacturing conditions for producing bent tubes with complex cross-sections.

REFERENCES

1. He, J. Y., Xiao, W. X., Schwebel, D. C., Zhu, M. T., Ning, P. S., Li, L.,..., Hu, G. Q. Road traffic injury mortality and morbidity by country development status, 2011–2017. *Chinese Journal of Traumatology*. 2021; 24(2): 88–93.
2. Ahmed, S. K., Mohammed, M. G., Abdulqadir, S. O., El-Kader, R. G. A., El-Shall, N. A., Chandran, D.,..., Dhama, K. Road traffic accidental injuries and deaths: A neglected global health issue. *Health Science Reports*. 2023; 6(5): e1240.
3. Al-Madani, H. M. Global road fatality trends' estimations based on country-wise micro level data. *Accident Analysis & Prevention*. 2018; 111: 297–310.
4. Dang, V., Rogala, M., Topa, A., Nguyen, P., Tran, T. Optimization of non-closed multi-cell hexagonal columns generated through topology optimization for enhanced crashworthiness under multiple load angles. *Structural and Multidisciplinary Optimization*. 2025; 68(12): 254.
5. Rogala, M., Gajewski, J. The effect of energy components modifications on the crashworthiness of thin-walled structures. *Advances in Science and Technology Research Journal*. 2024; 18(6): 110–120.
6. Szklarek, K., Kotelko, M. (2025). Multi-objective optimization of thin-walled hybrid frusta acting as collision energy absorbers. *Advances in Science and Technology Research Journal*. 2025; 19(12): 341–360.
7. Teng, L., Qingtian, D., Xinbo, L. Energy absorption and deformation modes of several thin-walled tubes under dynamic compression. *Structures*. 2023; 54: 890–897.
8. Gao, Y., Cheng, Q., Xiao, W., Guo, Y., Wen, Y. Non-linear dynamic response of bistable curved tubes under impact loading. *Thin-Walled Structures*. 2025; 217 A: 113757.
9. Fu, X., Zhang, X., Yu, Q. Bending collapse of optimal arched thin-walled structures. *Thin-Walled Structures*. 2022; 180: 109828.
10. Abdulqadir, S. F. Design a new energy absorber longitudinal member and compare with S-shaped design to enhance the energy absorption capability. *Alexandria Engineering Journal*. 2018; 57(4): 3405–3418.
11. Ai, S., Wei, J., Xie, Z., Tan, H. Analysis of negative stiffness structures with B-spline curved beams. *Thin-Walled Structures*. 2024; 195: 111418.
12. Wang, X., Yin, W., Zou, L., Cai, C., Li, P., Zou, H., Yang, M., Li, J., Li, Y. Curved and straight combined cross-section structure for energy absorption. *International Journal of Mechanical Sciences*. 2022; 224, 107333.
13. Cheng, Z., Li, S., Cheng, C., Guo, X., Shi, Y., Shu, S. Investigation on variation characteristics of bent tube axis and determination of bending die motion trajectory in free bending process. *The International Journal of Advanced Manufacturing Technology*. 2023; 124(10): 3371–3389.
14. Hao, W., Ge, D., Ma, Y., Yao, X., Shi, Y. Experimental investigation on deformation and strength of carbon/epoxy laminated curved beams. *Polymer Testing*. 2012; 31(4): 520–526.
15. Zhang, X., Fu, X., Yu, Q. Energy absorption of arched thin-walled structures under transverse loading. *International Journal of Impact Engineering*. 2021; 157: 103992.
16. Fu, X., Zhang, X. (2023). Three-point bending of thin-walled arched beams with square sections. *Thin-Walled Structures*. 2023; 182: 110201.
17. Zheng, D., Zhang, J., Lu, B., Zhang, T., Xie, J. Energy absorption performance of fully clamped curved tubes under transverse loading. *Thin-Walled Structures*. 2022; 175: 109240.
18. Zhang, X., Xiong, J., Fu, X. On the Crashworthiness of single-and multi-cell arched beams fabricated by three-point bending method. *International Journal of Structural Stability and Dynamics*. 2025; 25(14): 2550154.
19. Zhou, B., Zhang, H., Han, S., Ji, X. Crashworthiness analysis and optimization of a novel thin-walled multi-cell structure inspired by bamboo. *Structures*. 2024; 59: 10582.
20. Tran, T. Study on the crashworthiness of windowed multi-cell square tubes under axial and oblique impact. *Thin-Walled Structures*. 2020; 155: 106907.
21. Estrada, Q., Vergara-Vazquez, J., Rodriguez-Mendez, A., Tran, T., Gierz, L., Elias-Espinosa, M.,..., Baltazar-Tadeo, L. A. Bending crashworthiness performance of square beams with holes and metallic foam filler. *The International Journal of Advanced Manufacturing Technology*. 2025; 139(9): 4917–4938.
22. Zhu, H., Qin, C., Wang, J. Q., Qi, F. J. Characterization and simulation of mechanical behavior of 6063 aluminum alloy thin-walled tubes. *Advanced Materials Research*. 2011; 197: 1500–1508.
23. Zhang, X., Zhang, H., Wang, Z. Bending collapse of square tubes with variable thickness. *International Journal of Mechanical Sciences*. 2016; 106: 107–116.
24. Estrada, Q., Szwedowicz, D., Gutierrez-Wing, E., Silva-Aceves, J., Rodriguez-Mendez, A., Elias-Espinosa, M.,..., Bedolla-Hernandez, J. Energy absorption of single and multi-cell profiles under bending load considering damage evolution. *Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering*. 2019; 233(8): 2120–2138.