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Group-specific probabilistic modeling of passenger service dwell time at bus stops under mixed bus operations

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ABSTRACT

Variability in passenger service dwell time at bus stops affects berth occupancy, stop capacity, vehicle queue formation, and adjacent traffic conditions, particularly at stops shared by urban buses, suburban buses, taxis, and other passenger-serving vehicles. Existing approaches generally represent dwell time at an aggregated bus level, while a methodological gap remains in the probabilistic parameterization of its individual components for different user groups sharing the same stop. This study develops and verifies a group-specific probabilistic parameterization of passenger service time components under mixed service conditions. The novelty lies in combining group-specific probabilistic models of passenger demand and post-service technical time with an event-based representation of boarding and alighting within a unified simulation framework. The analysis used 5,953 observations of boarding and alighting time collected at bus stops in Kraków and Kielce, Poland. Boarding and alighting passenger counts, boarding and alighting time, and post-service technical time were analyzed separately by user group. Passenger counts were modeled using discrete probability distributions, while post-service technical time was modeled using continuous distributions. Parameters were estimated using maximum likelihood estimation and maximum goodness-of-fit estimation based on the Anderson–Darling statistic, with uncertainty assessed by bootstrap resampling. For urban buses, the negative binomial distribution provided the best fit for boarding passenger counts at 15 of 18 stops and for alighting passenger counts at 14 of 18 stops. For suburban buses, acceptable fits were obtained at four stops for both boarding and alighting. The negative binomial distribution was adopted as a practical passenger-count model because it frequently occurred among the best-fitting distributions and its parameters could be derived from the mean and standard deviation. In 69 of 78 complete fits, parameters calculated from these statistics fell within the uncertainty range of the maximum likelihood estimates. Post-service technical time was represented by a gamma distribution for urban buses and by a logistic approximation for other users. Simulation-based verification yielded mean absolute errors of 0.81 s for urban buses, 1.91 s for suburban buses, and 2.04 s for taxis and other vehicles. The results support separate parameterization of passenger service time for individual user groups at shared-use bus stops.

Keywords: bus stops, passenger service time, boarding and alighting, technical time, probabilistic modeling, simulation, mixed bus service.

INTRODUCTION

Passenger service dwell time at a bus stop is a key parameter in evaluating public transport operations, bus stop capacity, service reliability, and the effects of buses on adjacent street infrastructure. Rashidi, Ataeian, and Ranjitkar [1] identified bus dwell time as a multifactorial variable affected by passenger demand as well as organizational, technical, and operational conditions. Tirachini [2] further demonstrated the influence of fare collection systems, bus floor height, and passenger characteristics. Consequently, a single average dwell-time value may be insufficient, particularly when a bus stop is used by vehicles with different passenger service arrangements.

Boarding and alighting passenger counts are generally recognized as primary determinants of bus dwell time. Sun et al. [3] showed that service time depends not only on passenger counts but also on the organization of passenger movements at vehicle doors. Fernández et al. [4] investigated the effects of platform height, door width, and fare collection method, whereas Fernández, Valencia, and Seriani [5] examined passenger flows through public transport vehicle doors. Fletcher and El-Geneidy [6] demonstrated the effects of fare payment method and crowding, while Egge et al. [7] examined the influence of fare policies on dwell time. Glick and Figliozzi [8] showed that different portions of the dwell-time distribution may require different interpretations. Recent analyses based on automated passenger count data have also confirmed substantial differences in dwell-time magnitude and variability among stops and observation periods [9, 10]. These studies confirm that passenger counts are fundamental but insufficient for a comprehensive representation of the passenger service process.

Probabilistic and regression-based models of bus dwell time and service time constitute an important research area. Meng and Qu [11] proposed a probabilistic approach for estimating bus dwell time at bus bays, whereas Bian et al. [12] developed a service-time estimation model for curbside bus stops. An earlier probabilistic representation of passenger service processes at bus stops was proposed by Adamski [13]. These approaches demonstrate that passenger service time is inherently variable and should be represented by its distribution rather than only by its mean value.

A related issue concerns passenger arrivals and their relationship with bus arrivals. Jolliffe and Hutchinson [14] showed that passenger arrivals cannot always be interpreted as a fully random process. More recently, Chen et al. [15] demonstrated that passenger arrival patterns depend partly on how passengers use schedule information. Kieu, Bhaskar, and Chung [16] also emphasized the importance of consistently defining and monitoring public transport travel-time variability. These issues are particularly relevant to shared-use bus stops, where the distribution of passenger demand among routes and operators is often unknown and urban, suburban, regional, intercity, and long-distance buses may use the same stop together with other passenger-serving vehicles.

Studies of bus stop capacity and delay emphasize that vehicle service time affects not only bus dwell time but also vehicle queues and adjacent traffic conditions. Gu et al. [17] analyzed the capacity of isolated curbside bus stops, whereas Gu, Cassidy, and Li [18] developed models of bus queueing at curbside stops. Gu et al. [19] investigated the effects of stops near signalized intersections on car and bus delays, and Shen et al. [20] developed capacity approximations for near- and far-side stops in dedicated bus lanes. More recent studies have considered blocking, overtaking, berth allocation, and interactions among successive buses. Luo, Liu, and Jin [21] developed a queue-time estimation model accounting for the blocking effect, while Wang et al. [22] combined dwell-time estimation with overtaking analysis. Shen et al. [23] proposed heuristic methods for berth allocation at multiline, multiberth curbside stops, and Wang et al. [24] distinguished bus dwell time from additional time lost while serving a stop. These studies describe operational mechanisms associated with berth occupancy and vehicle interactions but do not focus on group-specific parameterization of the passenger service process itself.

Recent research has also emphasized microscopic representations of passenger boarding and alighting. Xue et al. [25] applied a cellular automaton model to represent local interactions and the spatial organization of passenger movements. Huo et al. [26] investigated bus delay at stops, while Luo and Yang [27] developed a probabilistic capacity model for curbside stops with multiple berths. Hu, Shen, and Gu [28] demonstrated that overtaking policies can substantially affect bus stop capacity. These approaches confirm the importance of vehicle interactions and stop configuration but generally represent passenger service at an aggregated bus level.

The problem of shared bus stop use was examined by Ezquerro, Alonso, and Moura [29], who considered bus stop sharing as a means of improving the use of public space. Although shared use may increase infrastructure utilization efficiency, it also increases service-process complexity when vehicles differ in length, door configuration, fare collection system, and stopping pattern. Passengers waiting for different vehicle groups may be distributed across different parts of the platform, while vehicles may stop at less predictable positions. A limitation of many existing models is therefore the treatment of dwell time as a single aggregated variable without separately representing passenger counts, boarding and alighting time, and post-service technical time.

In the Polish literature, bus stop operations were investigated by Rudnicki [30], who analyzed bus stop performance using simulation, and by Bąk [31], who represented a bus stop as a system characterized by an incoming bus flow and a service time corresponding to bus dwell time. Bauer [32] addressed the modeling of boarding and alighting time at bus stops. These studies provide an important methodological background but primarily concern urban bus operations and do not distinguish passenger service time components among different user groups sharing the same infrastructure.

A detailed microscopic model of boarding and alighting at shared-use bus stops, including waiting passenger positions, vehicle stopping positions, walking distances to doors, and individual service times, was presented in [33]. Traffic-related interactions after passenger service, including berth occupancy, blocked departures, vehicle queueing, and re-entry into traffic, were analyzed separately in [34]. Distinguishing between these processes is necessary because passenger service time and traffic-related delays result from different operational mechanisms.

Existing bus stop capacity and queueing models generally assume a fixed stop configuration or a specified number of berths [17, 20]. Other approaches introduce predefined berth-allocation or overtaking rules [23, 28]. Such assumptions may be insufficient for bus stops shared by different user groups. Under mixed service conditions, the number of effectively used service positions and the spacing between successive vehicles may vary over time because of irregular arrivals of suburban, regional, intercity, and long-distance operators and other passenger-serving vehicles. Despite the extensive literature on bus dwell time, passenger service processes, and bus stop capacity, a methodological gap remains in the separate probabilistic parameterization of passenger service time components for vehicle groups with different service characteristics using the same bus stop. In particular, existing approaches generally do not jointly distinguish group-specific boarding and alighting passenger counts, boarding and alighting time, and post-service technical time within a common probabilistic framework.

The present study addresses this gap by combining group-specific parameterization of these passenger service components with a distribution-fitting procedure based on two complementary estimation criteria: maximum likelihood estimation and maximum goodness-of-fit estimation based on the Anderson–Darling statistic. The procedure additionally evaluates parameter uncertainty through bootstrap resampling and identifies statistically homogeneous groups of fitted distributions with respect to the analyzed goodness-of-fit measures. The resulting probabilistic parameterization is integrated with an event-based representation of boarding and alighting and subsequently verified within a simulation framework. This integrated approach constitutes the methodological novelty and original scientific contribution of the study.

Compared with existing studies, which generally represent dwell time at an aggregated bus level or focus on individual determinants of the passenger service process, the proposed approach simultaneously accounts for differences among user groups, passenger-count distributions, boarding and alighting processes, and post-service technical time. This allows the variability of passenger service at shared-use bus stops to be represented without assuming a single common parameter set for all vehicles.

The objective of this study is to develop and verify a group-specific probabilistic parameterization of passenger service time components at bus stops operating under mixed bus service conditions. The analysis considers boarding and alighting passenger counts, boarding and alighting time, and post-service technical time. The methodological framework includes fitting discrete and continuous probability distributions, estimating their parameters using maximum likelihood estimation and a maximum goodness-of-fit method based on the Anderson–Darling statistic, assessing parameter uncertainty through bootstrap resampling, and verifying the group-specific parameterization within a simulation model.

The remainder of the paper is organized as follows. The Data and Methods section describes the study sites, analyzed variables, statistical procedures, simulation model, and verification measures. The Results section presents the group-specific parameterization of passenger counts, boarding and alighting time, and post-service technical time, followed by model verification. The Discussion interprets the results in relation to previous studies, practical applications, and model limitations. The Conclusions section summarizes the main findings and scientific contribution of the study.

DATA AND METHODS

Dataset and scope of analysis

The analysis was based on a dataset of passenger service events recorded at bus stops used by different vehicle groups. A single vehicle-stop event constituted the basic unit of observation. For each event, the vehicle group, boarding and alighting passenger counts, characteristic times associated with the passenger service process, and the positions of the vehicle and passengers within the bus stop area were identified.

The selection of the main study sites was preceded by reconnaissance observations of shared-use bus stops in six Polish cities: Kraków, Kielce, Łódź, Olsztyn, Lublin, and Rzeszów, as well as by pilot measurements at 26 bus stops in Kielce. The final sites were purposively selected to represent stops jointly used by vehicle groups with different passenger service characteristics and to provide variation in passenger demand, service organization, and stop configuration. The purpose of the site selection was therefore not to obtain a random sample of all bus stops, but to capture a broad range of operating conditions characteristic of shared-use stops.

The study included bus stops located in Kraków, denoted as KRAK, and Kielce, denoted as KIEL, along divided urban streets with two or three traffic lanes in each direction. The main surveys were conducted at 18 bus stops during 22 observation periods and covered 33 adjacent signalized and unsignalized intersections. A total of 5,953 observations of boarding and alighting time were recorded, including 2,171 for the urban operator, 3,399 for suburban operators, and 383 for taxis and other vehicles providing passenger service. Supplementary measurements used for model verification were conducted at four additional bus stops in Kielce. Data from these locations were not used to develop the simulation model. The total observation period was 97 h. Video data were recorded using three cameras, and individual events were extracted with a temporal accuracy of ±0.1 s.

The adequacy of the sample size was assessed using two complementary approaches based on pilot measurements conducted in Kielce. Boarding and alighting time was used as the reference variable because it exhibited the greatest variability among the analyzed passenger-service parameters. Assuming an approximately normal distribution, a 95% confidence level, and a maximum error of 1.0 s in estimating the mean, the required minimum samples were 154 observations for urban buses, 1110 for suburban buses, and 138 for taxis and other vehicles. The final samples of 2171, 3399, and 383 observations, respectively, exceeded these requirements. For variables whose distributions could substantially depart from normality, including boarding and alighting passenger counts and post-service technical time, sample adequacy was additionally assessed using Wilks' distribution-free method [35]. At individual stops, smaller samples for boarding or alighting mainly reflected the actual directional structure of suburban passenger demand, with some stops functioning predominantly as boarding or alighting locations. This imbalance was therefore treated as a characteristic of the observed demand rather than as insufficient survey coverage, while parameter uncertainty was additionally evaluated using bootstrap resampling.

The selected locations represented different traffic arrangements and positions relative to adjacent intersections. The dataset included stops located in bus bays (KIEL-01/02, KIEL-05, KIEL-07), along bus lanes (KIEL-08, KRAK-01, KRAK-02, KRAK-04, KRAK-05, KRAK-08, KRAK-09), in bus bays adjacent to bus lanes (KRAK-03, KRAK-06, KRAK-07), and within general-purpose traffic lanes (KIEL-03, KIEL-04, KIEL-06, KIEL-09, KIEL-10). Stops located on intersection approaches, downstream of intersections, and at midblock locations were considered. Formally, the analyzed facilities included two- and three-berth bus stops. However, under mixed service conditions, the actual number of service positions used effectively could vary over time and exceed the number of formally designated berths.

Vehicle groups and analyzed variables

Three groups of passenger-serving vehicles were distinguished:

- urban buses, denoted by *u*;
- suburban, regional, intercity, and long-distance buses, denoted by *s*;
- taxis and other vehicles, denoted by *to*.

Group *u* included high-, medium-, and low-capacity vehicles operating within the urban public transport system. Group *s* included private and regional operators serving passengers with minibuses, midibuses, regional buses, and long-distance coaches. Group *to* comprised taxis

and other vehicles stopping within the bus stop area to board or alight passengers. The term *other users*, denoted by *ou*, was additionally introduced to represent the combined group of suburban operators, taxis, and other vehicles. Figure 1 presents the analyzed passenger service process at a bus stop shared by different user groups.

The following variables were used in the analysis:

- $n_{b,g}$ – boarding passenger count for a vehicle from group g [pass/bus, pass/veh];
- $n_{a,g}$ – alighting passenger count for a vehicle from group g [pass/bus, pass/veh];
- $t_{ba,g}$ – boarding and alighting time, measured from the beginning of actual passenger service to the completion of service for the last boarding or alighting passenger [s];
- $t_{sba,g}$ – time from vehicle stop to the completion of boarding and alighting [s];
- $t_{tech,g}$ – post-service technical time following the completion of boarding and alighting [s].

The boarding passenger count, $n_{b,g}$, and alighting passenger count, $n_{a,g}$, were determined separately for each group and provided the basis for the subsequent parameterization of the passenger service process. The relationship between the time from vehicle stop to the completion of boarding and alighting and the boarding and alighting time was expressed as:

$$t_{\mathrm{sba},g} = t_{r,g} + t_{\mathrm{ba},g} \tag{1}$$

where: $t_{r,g}$ is the time from vehicle stop to the beginning of actual passenger service for group g [s].

Passenger service dwell time was expressed as:

$$t_{d,g} = t_{\mathrm{sba},g} + t_{\mathrm{tech},g} \tag{2}$$

Delays caused by vehicle queueing, berth occupancy, blocked departure, traffic signal operations, or difficulties re-entering the traffic stream were not treated as components of passenger service time. These effects constitute a separate level of operational bus stop analysis, as described in [34].

Statistical methods for variable parameterization

Basic descriptive statistics were calculated for the analyzed variables, including the mean, median, standard deviation, minimum, and maximum values. The analyses were conducted separately for the identified vehicle groups. To characterize passenger demand structure, the total mean hourly passenger demand served at each bus stop, the share of passengers served by suburban buses, and the directional suburban demand indicator were determined.

Theoretical probability distributions were fitted to the boarding passenger count, n_b, the alighting passenger count, n_a, and the post-service technical time, t_{tech}. Passenger counts were treated as discrete variables, whereas post-service technical time was treated as a continuous variable. The Poisson, Bernoulli, geometric, and negative binomial distributions were considered for the passenger counts. The gamma, lognormal, logistic, Weibull, normal, and exponential distributions were considered for the post-service technical time. Distribution fitting was performed

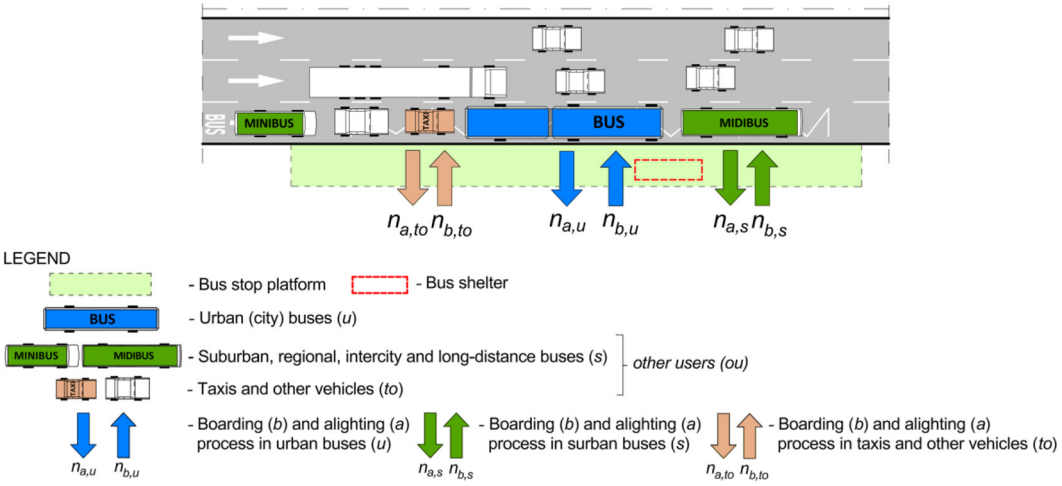


Figure 1. Schematic representation of the analyzed passenger service process at a bus stop shared by different user groups

using the *fitdistrplus* package [36]. A nonhomogeneous Poisson process was also considered as a supplementary approach for describing the temporal variability of passenger counts. These analyses were performed using the *NHPoisson* package [37].

Distribution parameters were estimated using maximum likelihood estimation (MLE). For the post-service technical time, maximum goodness-of-fit estimation based on the Anderson–Darling statistic, hereafter denoted as MGF(AD), was also applied. The objective function was minimized using the Nelder–Mead algorithm [38]. Agreement between the empirical and theoretical distributions was evaluated using the Anderson–Darling statistic, log-likelihood, and the goodness-of-fit test p-value at a significance level of $\alpha = 0.05$ [39]. The stability of the estimated parameters was assessed using bootstrap resampling. For each sample, 2,001 bootstrap replications were performed, and the mean values and standard errors of the distribution parameters and goodness-of-fit measures were calculated [40, 41]. For practical determination of passenger-count distribution parameters, the relationships between the mean and standard deviation of the passenger count per vehicle-stop event were also analyzed, and empirical regression equations were developed. These relationships were used to determine the parameters of the negative binomial distribution when aggregated passenger demand characteristics were available but the complete empirical distribution of the passenger count per vehicle was unavailable.

Statistical calculations were performed in R 4.1.3 [42]. The simulation procedures were developed in Python 3.9.13 [43]. Tables and figures were prepared using Microsoft Excel 2019.

Simulation model and verification measures

The estimated probability distribution parameters were used as inputs to the simulation model developed for representing passenger service processes at bus stops. The model reproduced individual vehicle stop events at the microscopic level and accounted for the user group, boarding and alighting passenger counts, boarding and alighting time, and post-service technical time. A detailed description of the microsimulation algorithm was provided in [33]. In the present study, the model was used to verify the group-specific parameterization of passenger service time components.

The main assumptions of the simulation model were derived from the passenger service mechanisms identified in the field observations. Passenger service was represented at the level of individual vehicle-stop and passenger events rather than by assigning a single random dwell-time value to each vehicle. Boarding and alighting time was determined from passenger counts, passenger distribution along the platform, vehicle stopping position, passenger access to the service points, and individual boarding and alighting times. These components were represented separately for the individual user groups because the field observations indicated differences in passenger demand, vehicle configuration, stopping behavior, and service organization. Post-service technical time was modeled as a separate random variable occurring after completion of boarding and alighting. Traffic-related delays were excluded from this component because queueing, berth blockage, traffic signal operations, and re-entry into the traffic stream arise from different operational mechanisms. These assumptions preserve the physical interpretation of the individual passenger service components and avoid representing the entire process by a single aggregated dwell-time variable.

The passenger service time was verified by comparing field measurements with the results of 100 repeated simulation runs. Basic descriptive statistics of the simulation results were calculated for each analyzed case. Agreement between the simulated and observed values was evaluated using mean error, mean absolute error, root mean square error, and the percentile-based PROC measure. Mean error was used to determine the direction of the deviation between the simulation results and the measurements, whereas mean absolute error (MAE) and root mean square error (RMSE) were used as the principal measures of error magnitude. The validation measures were selected to provide complementary information on model performance. Mean error identifies systematic overestimation or underestimation. MAE expresses the typical magnitude of the deviation directly in seconds and therefore provides an easily interpretable measure of operational accuracy. RMSE gives greater weight to larger deviations and is consequently more sensitive to individual cases in which the simulated time differs substantially from the observed

value. Because the proposed model generates a distribution of results through repeated simulation runs, point-error measures alone are insufficient to determine whether an observed value is consistent with the variability reproduced by the model. The percentile-based PROC measure was therefore used to determine the position of the observed value within the empirical distribution obtained from the simulation runs. If the model adequately reproduces the variability of the passenger service process, PROC values should span the interval from 0 to 1 without systematic concentration near either extreme. Together, the adopted measures assess bias, typical error magnitude, sensitivity to larger deviations, and consistency between field observations and the simulated distribution.

RESULTS

Framework for group specific parameterization of passenger service time

The fundamental assumption of the proposed approach is the group-specific parameterization of the passenger service process. For each user group, the following parameter set was defined:

$$\theta_g = \{\theta_{n_b,g}, \theta_{n_a,g}, \theta_{t_{\text{ba}},g}, \theta_{t_{\text{tech}},g}\} \quad (3)$$

where: θ_g – set of model parameters for user group g, $\theta n_{\text{b,g}}$ – parameters of the boarding passenger count, $\theta n_{a,g}$ – parameters of the alighting passenger count, $\theta_{tba,g}$ – parameters of boarding and alighting time, $\theta_{tech,g}$ – parameters of post-service technical time.

This formulation preserves the distinction among urban buses, suburban buses, and taxis and other vehicles. The individual components of the parameter set were analyzed separately, allowing the effect of the user group on the structure of passenger service time at the bus stop to be evaluated. Statistical significance is reported for analyses supported by formal inferential procedures, including goodness-of-fit tests, regression models, and comparisons based on bootstrap-derived goodness-of-fit measures. Differences presented in descriptive figures are interpreted as empirical patterns and are not referred to as statistically significant unless supported by a corresponding statistical test.

To characterize passenger demand structure at the analyzed bus stops, the suburban passenger share and the directional suburban demand indicator were determined (Figure 2). The total mean hourly passenger demand served at the bus stop was calculated as:

$$N_{\text{tot}} = N_{b,u} + N_{a,u} + N_{b,s} + N_{a,s} + N_{b,ot} + N_{a,ot} \quad (4)$$

where: N_{tot} – total mean hourly passenger demand served at the bus stop [pass/h], $N_{b,g}$ – boarding passenger count for group g [pass/h], $N_{a,g}$ – alighting passenger count for group g [pass/h].

The share of passengers served by suburban buses was calculated as:

$$S_s = \frac{N_{b,s}+N_{a,s}}{N_{\text{tot}}} \cdot 100\% \quad (5)$$

where: S_S – suburban passenger share [%], $N_{b,s}$ – boarding passenger count for suburban buses [pass/h], $N_{a,s}$ – alighting passenger count for suburban buses [pass/h], N_{tot} – total mean hourly passenger demand served at the bus stop [pass/h].

The directionality of suburban passenger demand was described using the following indicator:

$$D_s = \frac{N_{b,s}-N_{a,s}}{N_{b,s}+N_{a,s}} \quad (6)$$

where: D_S – directional suburban demand indicator [–], $N_{b,s}$ – boarding passenger count for suburban buses [pass/h], $N_{a,s}$ – alighting passenger count for suburban buses [pass/h].

Positive values indicate a predominance of boarding, whereas negative values indicate a predominance of alighting. The use of S_S and D_S allowed the bus stops to be characterized not only by total passenger demand but also by the distribution of passenger service among the user groups. This distinction is important because bus stops with similar total passenger demand may differ substantially in the suburban passenger share and the directionality of suburban demand.

Figure 2 shows substantial differences in passenger demand structure among the analyzed bus stops. Stops with similar total passenger demand differed in both the suburban passenger share and the directionality of suburban demand. High values of S_S, as well as strongly positive or negative values of D_S, indicate that suburban buses did not merely

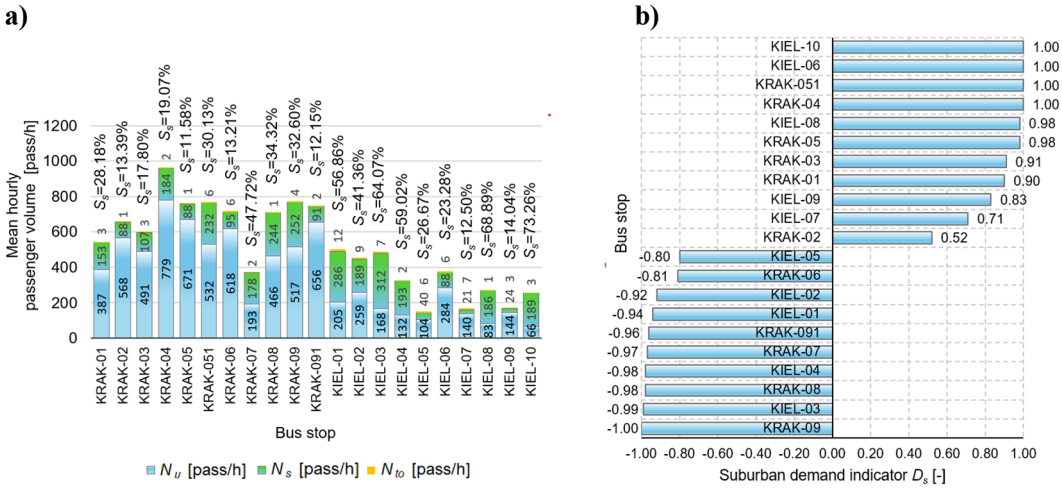


Figure 2. Passenger demand structure at the analyzed bus stops: a) mean hourly passenger demand by service group and suburban passenger share S_S, b) directional suburban demand indicator D_S

provide a supplementary component of passenger service at the bus stop. At many locations, they accounted for a substantial and strongly directional component of passenger demand. This finding supports the separate parameterization of boarding and alighting passenger counts for the individual user groups because both the magnitude and direction of passenger demand affect passenger service time and bus stop berth occupancy. Consequently, two stops with similar total passenger demand may require different passenger-service parameter sets when the suburban passenger share or the dominant direction of passenger movement differs. This is particularly relevant to stops at which suburban services predominantly collect passengers traveling away from the city or discharge passengers arriving from suburban areas.

Probabilistic parameterization of boarding and alighting passenger counts

Boarding and alighting time depends directly on the boarding passenger count, n_b, and the alighting passenger count, n_a, recorded during a single vehicle stop event. Therefore, the first stage of the analysis examined variations in the mean and standard deviation of passenger counts across the analyzed bus stops and user groups. The results for urban and suburban buses are presented in Figure 3. Urban buses generally served higher mean passenger counts. However, at selected bus stops, the mean passenger counts for suburban buses were comparable or higher, particularly for one direction of passenger movement. The suburban bus results also more frequently indicated a

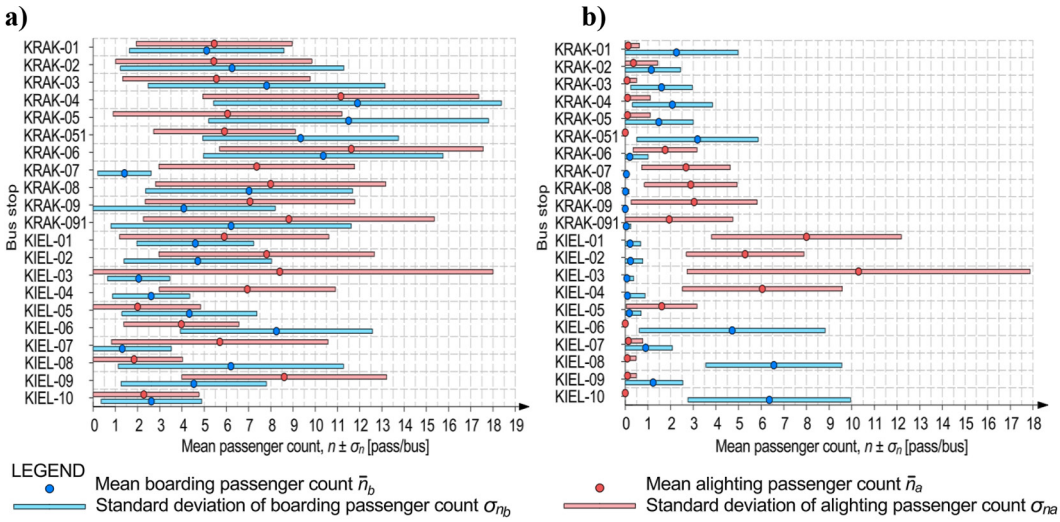


Figure 3. Mean and standard deviation of boarding and alighting passenger counts per vehicle by bus stop and service group: a) urban buses, b) suburban buses

clear predominance of either boarding or alighting, confirming substantial variation in the local passenger demand structure.

The standard deviations shown in Figure 3 indicate that similar mean passenger counts do not necessarily correspond to similar variability. This difference is particularly relevant for simulation because two stops with comparable mean demand may generate different ranges of passenger counts per vehicle-stop event. The result supports parameterizing not only the mean passenger count but also its dispersion separately for boarding, alighting, and user group.

In the conventional approach, passenger demand at bus stops is analyzed as a passenger arrival process associated with timetables, vehicle headways, bus punctuality, and passenger waiting behavior [13, 14]. However, this approach requires detailed data on the assignment of passengers to individual routes and operators, vehicle occupancy, and timetable variability. Obtaining this level of information is particularly difficult at bus stops served simultaneously by urban and suburban operators. Because the available field dataset contained passenger counts recorded during individual vehicle stop events, the demand analysis was reduced to fitting discrete probability distributions to boarding and alighting passenger counts. The Poisson, Bernoulli, negative binomial, and geometric distributions were examined.

Examples of the goodness-of-fit measures obtained for bus stop KIEL-01 are presented in Table 1. The same analyses were performed for the remaining bus stops. Because of the small sample sizes at individual locations, the results for taxis and other vehicles were analyzed jointly for all bus stops. Only stop events involving passenger service were included for this group. The tables identify cases in which the null hypothesis of agreement between the empirical and theoretical distributions was rejected at the significance level of $\alpha = 0.05$. Convergent parameter estimates for the Bernoulli distribution were not obtained for the analyzed datasets. The negative binomial and Poisson distributions occurred most frequently among the best-fitting distributions. For urban buses, the negative binomial distribution provided the best fit to the boarding passenger count at 15 of the 18 analyzed bus stops and to the alighting passenger count at 14 of the 18 stops. Distribution fitting was less conclusive for suburban buses, particularly at stops with a high proportion of vehicle stop events without passenger service. For this group, acceptable fits of the negative binomial distribution were obtained at four bus stops for the boarding passenger count and at four bus stops for the alighting passenger count.

The KIEL-01 results also illustrate that the statistical evidence differed between user groups and passenger movements. For urban buses,

Table 1. Goodness-of-fit measures for theoretical probability distributions fitted to boarding and alighting passenger counts at bus stop KIEL-01

Passenger movement	Distribution	AD statistic [-]	Log-likelihood [–]	*p*-value [-]
Urban operator				
Boarding	Negative binomial	1.33 ± 0.44	-49.0 ± 2.5	0.22
	Poisson	2.10 ± 1.10	-50.4 ± 4.0	0.08
	Geometric	3.71 ± 0.79	-49.0 ± 1.8	0.01*
Alighting	Negative binomial	0.72 ± 0.36	-55.6 ± 3.5	0.54
	Geometric	1.89 ± 0.60	-55.2 ± 3.1	0.11
	Poisson	5.30 ± 2.40	-69.3 ± 8.6	<0.001*
Suburban operators				
Boarding	Geometric	3.27 ± 0.50	-8.97 ± 0.6	0.02*
	Poisson	31.00 ± 11.00	-18.8 ± 4.8	<0.001*
	Negative binomial	32.00 ± 11.00	-18.7 ± 4.8	<0.001*
Alighting	Negative binomial	0.58 ± 0.23	-95.7 ± 3.7	0.67
	Poisson	2.70 ± 1.60	-102.9 ± 7.3	0.04*
	Geometric	4.88 ± 0.98	-106.7 ± 2.9	<0.001*

Note: AD – Anderson-Darling statistic; p-value – probability value for the goodness-of-fit test; * indicates rejection of the null hypothesis that the empirical data follow the fitted theoretical distribution at the significance level of $\alpha = 0.05$.

the negative binomial distribution could not be rejected for either boarding (p-value = 0.22) or alighting (p-value = 0.54), whereas the geometric distribution was rejected for boarding and the Poisson distribution for alighting at $\alpha = 0.05$. For suburban buses, none of the tested distributions provided an acceptable fit for boarding, whereas the negative binomial distribution provided an acceptable fit for alighting (p-value = 0.67). This example illustrates why distribution selection was performed separately by user group and passenger movement rather than by adopting one distribution on the basis of pooled data. The negative binomial distribution is a discrete distribution characterized by two parameters, r and p. In this study, it was used to describe the passenger count recorded during a single vehicle stop event. Its probability mass function was expressed as:

$$P(X = k) = \binom{r+k-1}{r-1} p^r (1-p)^k \tag{7}$$

where: k ∈ {0, 1, 2, ...} – passenger count, r, p – distribution parameters, $r > 0$, $0 < p < 1$.

The mean and variance of the negative binomial distribution are given by:

$$\bar{x} = \frac{r(1-p)}{p} \tag{8}$$

$$\sigma^2 = r\frac{1-p}{p^2} \tag{9}$$

The negative binomial distribution was adopted as the basis for generating passenger counts in the simulation model. This decision was supported by its frequent occurrence among the best-fitting distributions and by the possibility of determining its parameters from the mean and standard deviation of the passenger count. Examples of the r and p parameter values estimated using MLE and calculated from the corresponding means and standard deviations are presented in Table 2.

The complete parameter results showed that, in 69 of the 78 complete fits, the r and p values calculated from the mean and standard deviation fell within the uncertainty range of the MLE estimates defined using the bootstrap standard errors. Differences extending beyond this range occurred mainly for small samples, datasets with a high proportion of zero values, or cases characterized by high estimation uncertainty. This result confirms the usefulness of the simplified procedure for determining negative binomial distribution parameters when only aggregated passenger count characteristics are available.

For practical application of the model, the relationship between the mean passenger count and its standard deviation was also examined. Empirical regression equations were developed separately for boarding and alighting passenger counts and for the individual user groups. Examples of the results are presented in Figure 4. The

Table 2. Example parameters of the fitted negative binomial distributions for boarding and alighting passenger counts at bus stops KRAK-01 and KIEL-05

Passenger movement	Bus stop	Fitted negative binomial distribution parameters			
		MLE estimate		Calculated using Equations 8 and 9	
		r [-]	p [-]	r [-]	p [-]
Urban operator					
Boarding	KRAK-01	4.10 ± 1.20	0.44 ± 0.07	3.73	0.42
	KIEL-05	3.00 ± 12.00	0.40 ± 0.15	3.82	0.47
Alighting	KRAK-01	3.80 ± 1.10	0.41 ± 0.06	4.32	0.44
	KIEL-05	0.36 ± 0.66	0.15 ± 0.07	0.66	0.25**
Suburban operators					
Boarding	KRAK-01	1.05 ± 0.15	0.32 ± 0.04	0.99	0.31
	KIEL-05	0.20 ± 1.30	0.54 ± 0.15	0.36	0.66
Alighting	KRAK-01	0.11 ± 0.52	0.48 ± 0.11	0.15	0.54
	KIEL-05	10.00 ± 190.00	0.77 ± 0.16	3.14	0.66
Taxis and other vehicles					
Boarding	All bus stops	9.00 ± 26.00	0.98 ± 0.07	6.61	0.97
Alighting	All bus stops	0.60 ± 26.00	0.53 ± 0.05	0.61	0.52

Note: **indicates cases in which the approximate parameters calculated from the mean and standard deviation differed substantially from the MLE estimates, that is, by more than the standard error obtained from the bootstrap analysis.

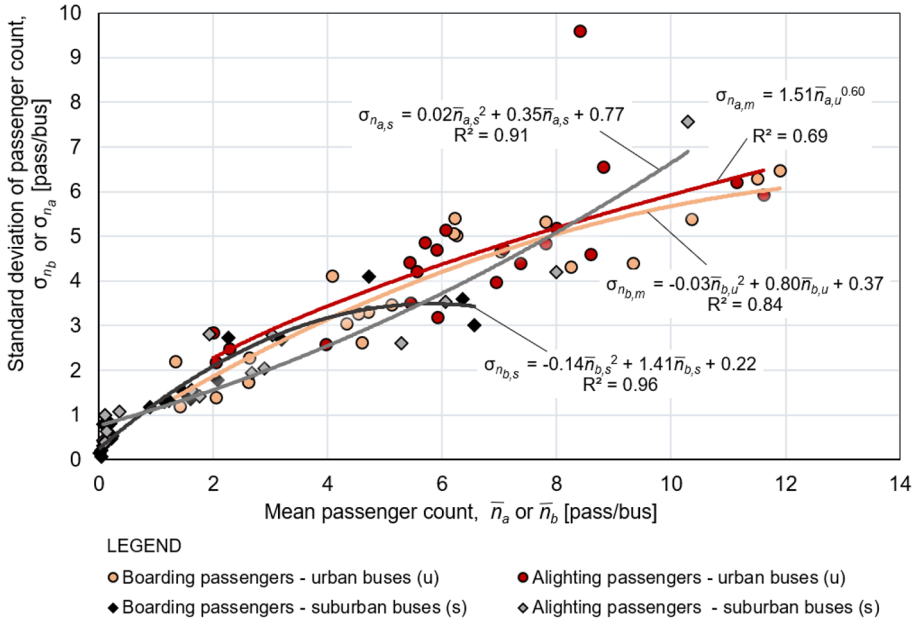


Figure 4. Relationship between the mean and standard deviation of boarding and alighting passenger counts by service group

resulting regression models and their coefficients were statistically significant at $\alpha = 0.05$.

The estimated parameters were also used to demonstrate how the shape of the negative binomial distribution changes with the mean passenger count. Examples for the alighting passenger counts of urban and suburban buses are presented in Figure 5. The illustrated range of mean values corresponded to those observed in the field measurements.

Figure 5 shows that, as the mean alighting passenger count increases, the distribution shifts toward higher values and spans a wider range of passenger counts, while the probability mass at zero and very low values decreases. The different patterns of change in the r and p parameters for urban and suburban buses reflect differences in the relationship between the mean and variability of passenger counts in the two groups.

As an exploratory approach, boarding and alighting passenger counts were also represented using a nonhomogeneous Poisson process in which the intensity parameter, $\lambda(t)$, varies over time. This approach corresponds to the interpretation of passenger demand as a passenger arrival process at a bus stop. However, the available field dataset contained passenger counts recorded during individual vehicle stop events rather than the actual arrival times of individual passengers. Therefore, individual passenger events were reconstructed by assuming that passenger service was distributed uniformly over the duration of each stop event.

Three forms of the intensity function $\lambda(t)$ were tested:

$$\lambda_s(t) = \exp(\beta_1 + \beta_2 \cdot t + \beta_3 \cdot t^2) \quad (10)$$

$$\lambda_s(t) = \exp(\beta_1 + \beta_2 \cdot t) \quad (11)$$

$$\lambda_s(t) = exp(\beta_1) \quad (12)$$

where: $\lambda_s(t)$ – nonhomogeneous Poisson process intensity at time t, expressed in passengers per second [pass/s], t – elapsed time from the beginning of the observation period [s], β_1 – model intercept [–], β_2 – model coefficient [s^{-1}], β_3 – model coefficient [s^{-2}].

The models were estimated using time expressed in seconds. For presentation in Figure 6, the estimated intensity was converted to passengers per minute according to $\lambda_{min} = 60\lambda_s(t)$. The best fit, based on the log-likelihood, was obtained for the relationship defined by Equation 11. Figure 6 presents an example of the variation in passenger-event intensity, $\lambda_{min}(t)$, associated with alighting passenger counts, whereas Table 3 provides the corresponding model coefficients for boarding and alighting passenger counts estimated using time expressed in seconds.

Because individual passenger events had to be reconstructed indirectly, the results obtained using the nonhomogeneous Poisson process were

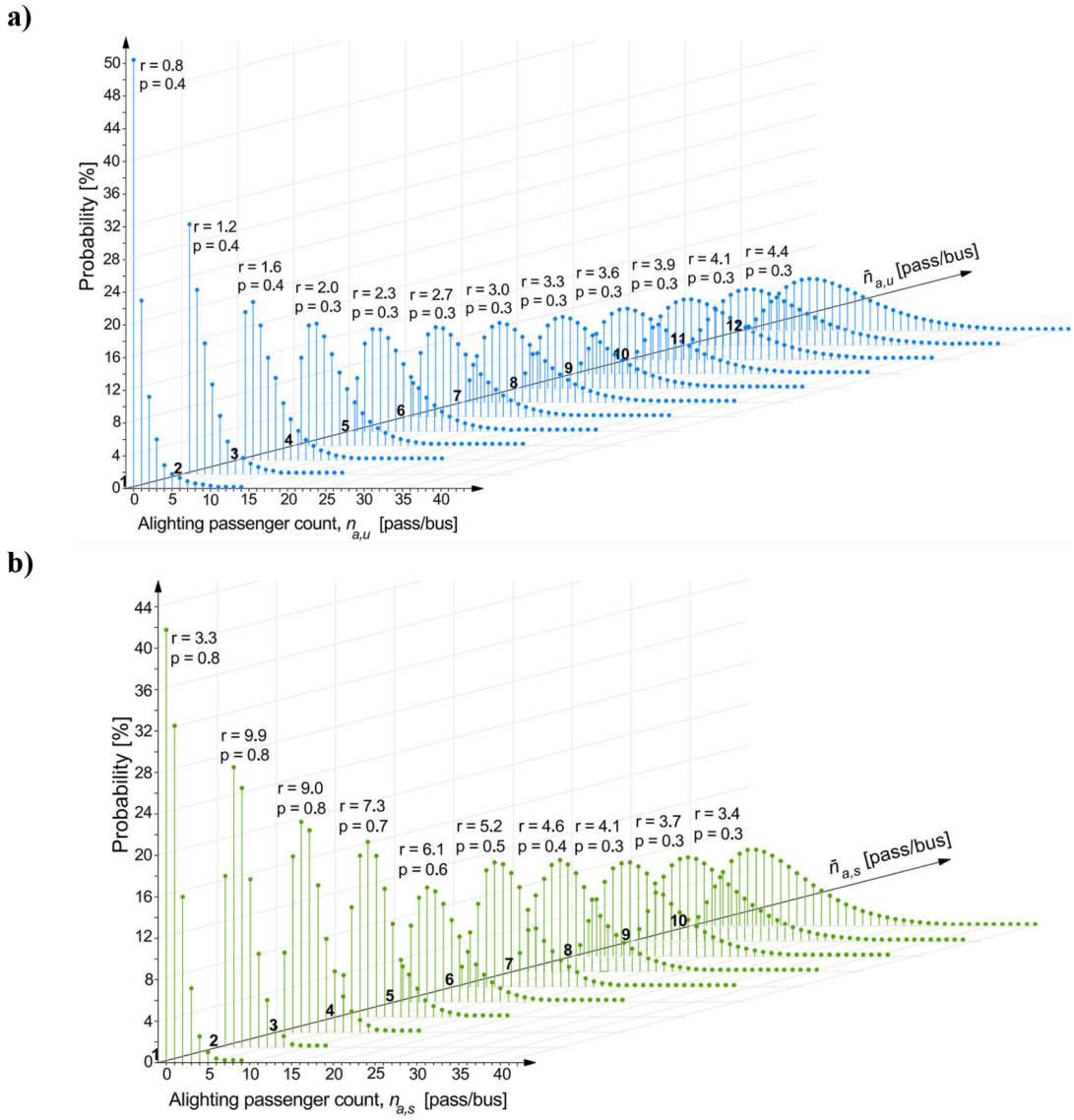


Figure 5. Negative binomial probability distributions for alighting passenger counts as a function of the mean alighting passenger count: a) urban buses, b) suburban buses

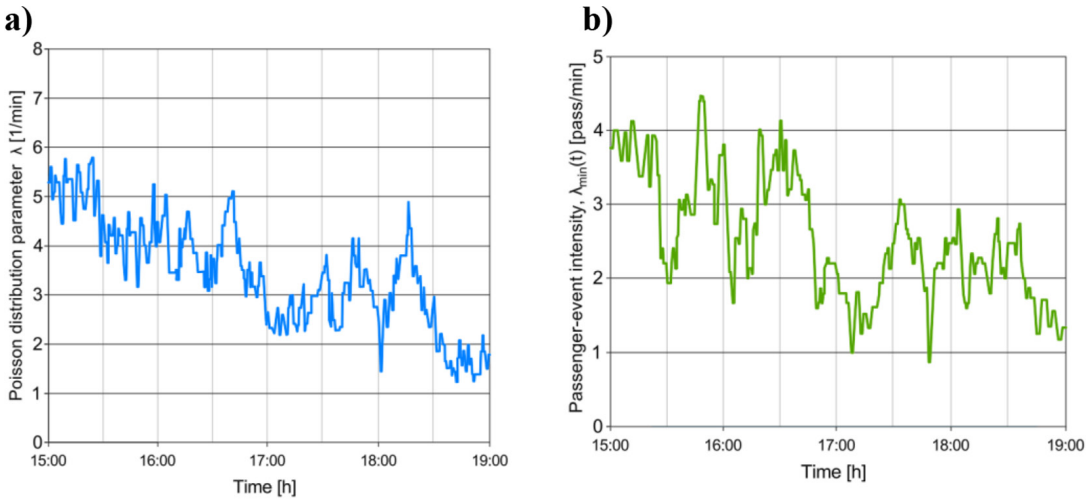


Figure 6. Variation in passenger-event intensity, $\lambda_{min}(t)$, expressed in passengers per minute for alighting passenger counts observed at bus stop KRAK-01 between 15:00 and 19:00: a) urban buses, b) other users

Table 3. Fitted coefficients in the nonhomogeneous Poisson process model for boarding and alighting passenger counts

Passenger movement	Model coefficients		Log-likelihood [–]
	β_1 [-]	β_2 [s^{-1}]	
Urban operator			
Boarding	-2.496 ± 0.067	-0.0000723 ± 0.0000096	-2,817.13
Alighting	-2.494 ± 0.066	-0.0000615 ± 0.0000092	-2,963.75
Other users			
Boarding	-2.848 ± 0.078	-0.000055 ± 0.000011	-2,336.26
Alighting	-5.870 ± 0.330	-0.000002 ± 0.000044	-248.04

treated as supplementary and were not incorporated into the subsequent simulation model. A group-specific parameterization of the negative binomial distribution was adopted to generate passenger counts, whereas the nonhomogeneous passenger arrival process was retained as a direction for further investigation.

Group-specific parameterization of time from vehicle stop to completion of boarding and alighting

In the next stage of the analysis, differences in boarding and alighting time among the identified bus stop user groups were evaluated. The results presented in Figure 7 indicate a more compact time distribution for urban buses and greater variability for suburban buses. The mean boarding and alighting time was similar for urban and suburban buses, at 15.6 s and 16.8 s, respectively, whereas the corresponding standard deviations were 8.9 s and 17.9 s. Thus, the difference between these two groups was expressed more strongly in the variability of passenger service time than in its mean value. Taxis and other vehicles formed the most irregular group, with a mean boarding and alighting time of 32.0 s and a standard deviation of 14.5 s.

The relationship between mean boarding and alighting time and the boarding and alighting

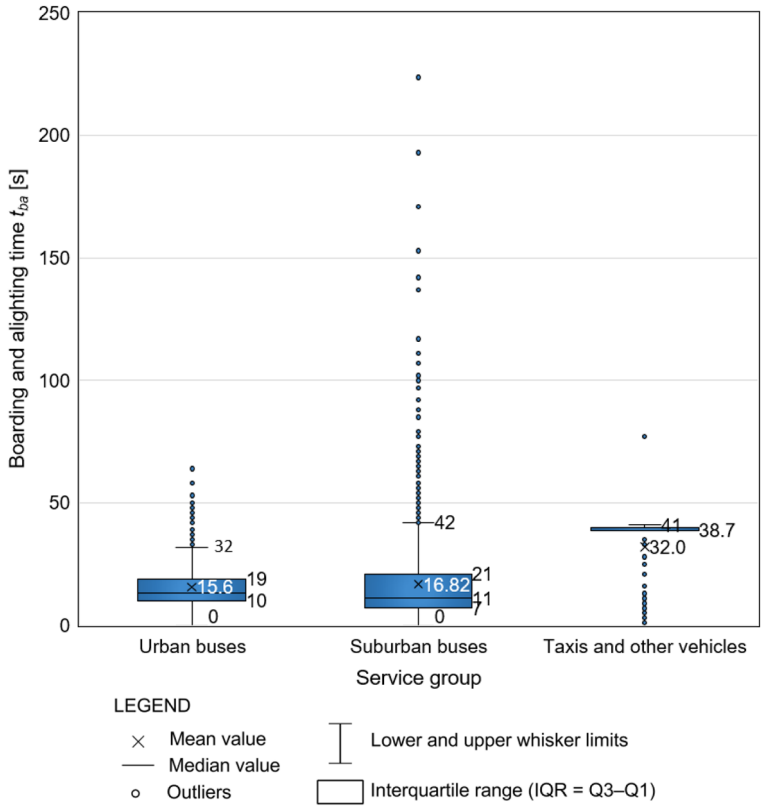


Figure 7. Distribution of boarding and alighting time t_{ba} by service group

passenger counts is presented in Figure 8. The heat maps show that service time depends not only on the total passenger count but also on the relationship between the boarding and alighting passenger counts. For suburban buses, the mean times were higher than those for urban buses for comparable combinations of passenger counts and were accompanied by larger standard deviations. For example, for events involving 1–2 boarding and 1–2 alighting passengers, the mean boarding and alighting time was 8.1 s for urban buses and 16.4 s for suburban buses, with corresponding standard deviations of 7.6 s and 10.7 s. For events with no boarding passengers and 3–4 alighting passengers, the corresponding mean values were 6.4 s and 11.6 s. This indicates that passenger service for this group was not only longer but also less predictable. The examples also show that comparable passenger-count combinations may produce substantially different service times depending on the user group. Greater variability in boarding and alighting time may adversely affect bus stop operations by increasing uncertainty in berth occupancy and disrupting the smooth approach of successive vehicles. For urban buses, boarding and alighting time increased more consistently with passenger counts, whereas suburban buses exhibited greater variability, particularly for events dominated by either boarding or alighting.

Based on the empirical results, boarding and alighting time was treated as a variable requiring group-specific parameterization. It was not modeled as a single random value assigned to a vehicle stop event but as the outcome of passenger counts, passenger distribution along the platform, vehicle stopping position, door configuration, and individual service times. A detailed description of the microsimulation algorithm was provided in [33], and the main components of the module are summarized in Table 4. The module included both demand-related parameters, represented by the boarding and alighting passenger counts, and spatial and temporal parameters associated with the positions of waiting passengers, vehicle stopping position, passenger access time to the doors, and individual service times.

Before actual passenger service began, the component $t_{r,g}$, representing the time from vehicle stop to the beginning of boarding or alighting, was included. This component depends on the minimum time required to open the doors and, when no passengers are alighting, on the time required for the first boarding passenger to reach the vehicle:

a)

$n_a \setminus n_b$	0	1–2	3–4	5–6	7–8	9–10	11–12	13–15	16–20	21–30	31–40
0	4.6 ± 2.1	7.9 ± 9.8	9.0 ± 5.7	9.7 ± 2.6	10.5 ± 3.7	13.2 ± 2.8	12.6 ± 2.9	15.7 ± 3.1	23.0 ± 5.6	21.0 ± 4.1	
1–2	5.2 ± 2.5	8.1 ± 7.6	11.7 ± 9.1	13.3 ± 8.5	13.1 ± 4.5	17.1 ± 4.5	14.5 ± 3.1	13.6 ± 1.8	17.5 ± 7.8	23.0 ± 3.2	
3–4	6.4 ± 2.6	12.3 ± 9.9	10.2 ± 2.9	11.7 ± 4.9	14.0 ± 6.1	17.4 ± 9.3	17.0 ± 3.6	19.8 ± 1.3	16.3 ± 6.7	24.2 ± 10.6	
5–6	10.7 ± 8.5	13.0 ± 10.5	11.9 ± 6.9	12.5 ± 2.7	14.7 ± 4.1	16.2 ± 3.2	15.8 ± 7.0	18.3 ± 5.5	24.5 ± 14.3		
7–8	9.9 ± 1.5	10.9 ± 3.3	12.6 ± 3.1	17.1 ± 7.7	14.6 ± 4.1	18.3 ± 4.8	17.7 ± 6.8	24.1 ± 6.8	28.6 ± 12.9	37.7 ± 8.1	
9–10	9.8 ± 1.8	17.8 ± 10.1	13.5 ± 2.5	16.4 ± 6.4	15.6 ± 5.7	22.1 ± 9.5	21.7 ± 4.9	19.0 ± 7.3	29.8 ± 6.8	33.0 ± 13.1	
11–12	15.7 ± 10.8	14.3 ± 3.8	19.1 ± 10.3	16.9 ± 3.1	19.4 ± 8.3	17.6 ± 7.3	28.5 ± 9.3	16.8 ± 10.1	20.9 ± 8.2		
13–15	16.2 ± 9.7	16.4 ± 9.0	20.6 ± 11.9	18.5 ± 10.5	22.0 ± 5.7	23.3 ± 4.8	26.9 ± 7.0	22.6 ± 15.9	28.5 ± 11.7	32.0 ± 7.2	
16–20		23.0 ± 3.7	17.7 ± 3.1	22.8 ± 4.1	24.1 ± 12.7	18.9 ± 6.1	26.6 ± 2.2	25.3 ± 7.9	33.0 ± 3.8	50.7 ± 13.0	
21–30	16.0 ± 2.9	11.1 ± 2.9		22.0 ± 5.2	25.5 ± 7.8	25.0 ± 3.3	31.6 ± 7.4			42.0 ± 10.2	
31–40		30.0 ± 4.2	30.0 ± 5.5								58.0 ± 15.8

b)

$n_a \setminus n_b$	0	1–2	3–4	5–6	7–8	9–10	11–12	13–15	16–20
0	31.0 ± 17.4	11.6 ± 7.0	22.5 ± 12.7	30.8 ± 18.5	34.8 ± 31.2	43.1 ± 36.3	57.4 ± 62.7	47.5 ± 49.8	142.0 ± 50.2
1–2	8.1 ± 14.0	16.4 ± 10.7	20.3 ± 5.0	12.5 ± 4.9	43.5 ± 3.5	153.0 ± 55.6		6.0 ± 5.2	
3–4	11.6 ± 9.6	25.2 ± 20.4							
5–6	14.9 ± 11.1	27.9 ± 17.9							
7–8	21.2 ± 12.0	28.8 ± 7.0							
9–10	20.0 ± 13.2	35.7 ± 17.9							
11–12	32.3 ± 16.1								
13–15	34.2 ± 15.0	56.0 ± 25.2							
16–20	45.9 ± 21.4								
21–30	41.2 ± 19.4								
31–40	39.0 ± 22.5								

LEGEND

Boarding and alighting time t_{ba} ± standard deviation *SD* (s)

0–10 ± *SD*	10–20 ± *SD*	20–30 ± *SD*	30–45 ± *SD*	45–60 ± *SD*	60–90 ± *SD*	90–120 ± *SD*	120–160 ± *SD*

Figure 8. Mean boarding and alighting time t_{ba} as a function of boarding n_b and alighting n_a passenger counts: a) urban buses, b) suburban buses

Table 4. Group-specific parameters used in the boarding and alighting time module

Model component	Symbol / variable	Probabilistic representation	Role in group-specific parameterization
Boarding passenger count	$n_{b,g}$	Discrete probability distribution, primarily the negative binomial distribution	Estimated separately for each service group *g* and determines the boarding passenger count for the vehicle.
Alighting passenger count	$n_{a,g}$	Discrete probability distribution, primarily the negative binomial distribution	Estimated separately for each service group *g* and determines the alighting passenger count for the vehicle.
Waiting passenger position	$x_{p,g}$	Bounded beta distribution	Represents the spatial distribution of waiting passengers along the bus stop platform and may vary by bus stop and service group.
Vehicle stopping position	$P_{z,g}$	Empirical or modeled stopping-position distribution	Determines the relative distance between waiting passengers and the vehicle doors and depends on the service group and bus stop layout.
Effective service point	$S_{g,d}$	Derived from stopping position and door configuration	Represents the actual boarding and alighting location and differentiates multi-door urban buses from predominantly single-door suburban vehicles.
Passenger walking speed	V_{pas}	Positive normal distribution	Represents the walking-speed component used to determine passenger access time to the vehicle.
Individual alighting time	$t_{a,g,d}$	Lognormal distribution	Represents the unit alighting time specific to the service group and door configuration.
Individual boarding time	$t_{b,g,d}$	Lognormal distribution	Represents the unit boarding time specific to the service group and door configuration.
Door/service configuration	d	Categorical parameter	Describes the number and use of service doors or service points and differentiates urban buses, suburban buses, and taxis and other vehicles.

$$t_{r,g} = \begin{cases} t_{\mathrm{door},g}, & n_{a,g} > 0 \\ \max\left(t_{\mathrm{door},g}, t_{\mathrm{acc},(1)}\right), & n_{a,g} = 0 \text{ and } n_{b,g} > 0 \\ 0, & n_{a,g} = 0 \text{ and } n_{b,g} = 0 \end{cases} \quad (13)$$

where: $t_{r,g}$ – time from vehicle stop to the beginning of actual passenger service for group g[s], $t_{door,g}$ – minimum time associated with door opening or vehicle preparation for passenger service [s], $t_{acc,(1)}$ – shortest access time of a boarding passenger to the vehicle [s].

For vehicles with automatically operated doors, $t_{door,g}$ = 2 s was adopted, whereas $t_{door,g}$ = 0 s was used for the remaining vehicles. This parameter represents situations in which passengers wait immediately adjacent to the doors or the vehicle waits for the first boarding passenger to reach it, particularly when the vehicle stops away from the main group of waiting passengers.

The measurement and analysis results [33] confirm that boarding and alighting time should be parameterized separately for individual user groups. Differences among urban buses, suburban buses, and taxis and other vehicles result not only from passenger counts but also from the organization of passenger service, vehicle stopping position, and the manner in which passengers access the vehicle doors.

Post-service technical time distributions

Post-service technical time, denoted by t_{tech}, included activities performed after the last passenger had been served but before the vehicle began to move. Cases in which a vehicle remained stopped because the berth ahead was occupied, vehicles were queued, re-entry into the traffic stream was not possible, or traffic signal operations directly prevented departure were excluded from the analysis. These situations were treated as traffic-related delays rather than as components of post-service technical time. This distinction is consistent with the model of traffic interactions within the bus stop area presented in [34]. Post-service technical time was analyzed separately for urban buses and *other users*. This distinction reflected differences in service completion procedures, vehicle types, door-closing mechanisms, and driver activities performed after boarding and alighting had been completed. Passenger service was considered complete when the last passenger finished boarding or alighting. When a passenger purchased a ticket from the driver, service was considered

complete when the ticket transaction ended. Selected continuous probability distributions were fitted to the empirical distributions of post-service technical time: gamma, normal, lognormal, exponential, logistic, and Weibull distributions. The analyses were conducted using the pooled datasets without stratification by individual bus stop. Distribution parameters were estimated using MLE and MGF(AD), following the procedure described in the methodological section. Examples of the goodness-of-fit measures obtained for *other users* are presented in Table 5.

The analysis showed that, for urban buses, the lognormal distribution provided the best fit under MGF(AD), whereas the gamma distribution provided the best fit under MLE. For *other users*, the logistic distribution yielded the lowest Anderson–Darling statistic under both estimation procedures. Under MGF(AD), it also formed a separate highest-ranked homogeneous group, whereas the remaining distributions belonged to groups characterized by significantly higher Anderson–Darling statistics. Although the null hypothesis of agreement between the empirical and theoretical distributions was rejected at α = 0.05 for all distributions examined, the consistent performance of the logistic distribution under both estimation procedures supported its use as a practical approximation in the simulation model. Histograms of post-service technical time with the fitted probability density functions are presented in Figure 9. The gamma distribution was adopted for urban buses, whereas the logistic distribution was adopted for *other users*. The distribution parameters estimated using MGF(AD) and MLE are presented in Tables 6 and 7. Values following the ± symbol represent standard errors obtained from the bootstrap analysis.

The gamma distribution adopted to represent post-service technical time for urban buses was expressed as:

$$f(t_{\text{tech},u}) = \frac{\beta^{\alpha}}{\Gamma(\alpha)} t_{\text{tech},u}^{\alpha-1} e^{-\beta t_{\text{tech},u}} \quad (14)$$

where: $t_{tech,u}$ – post-service technical time for urban buses [s], α, β – gamma distribution parameters, $\Gamma(\alpha)$ – gamma function.

The MLE estimates of the gamma distribution parameters α and β were adopted for the subsequent simulation analyses.

The logistic distribution adopted to represent post-service technical time for *other users* was expressed as:

$$f(t_{tech,ou}) = \frac{\exp\left[-(t_{\text{tech},ou}-\mu)/s\right]}{s\{1+\exp[-(t_{\text{tech},ou}-\mu)/s]\}^2} \quad (15)$$

where: $t_{tech,ou}$ – post-service technical time for *other users* [s], μ – location parameter of the logistic distribution [s], s – scale parameter of the logistic distribution [s].

Table 5. Goodness-of-fit measures for theoretical probability distributions fitted to post-service technical time for *other users*

Optimization criterion	Distribution	AD Statistic [-]	Log-likelihood [-]	*p*-value [-]
MGF(AD)	Logistic**	25.0 ± 2.4**	-614 ± 30	0.01*
	Exponential**	63.5 ± 8.4	-485 ± 21**	0.00*
	Weibull	84.3 ± 9.7	-543 ± 29	0.00*
	Gamma	101.0 ± 25.0	-589 ± 73	0.00*
	Lognormal	387.0 ± 88.0	-1610 ± 250	0.00*
	Normal	—	-733 ± 61	0.00*
MLE	Logistic**	24.2 ± 2.1**	-613 ± 28	0.01*
	Normal	31.7 ± 3.3	-666 ± 32	0.01*
	Gamma**	37.5 ± 2.6	-432 ± 26**	0.00*
	Weibull	43.8 ± 2.5	-448 ± 27	0.00*
	Lognormal	55.7 ± 2.4	-518 ± 33	0.00*
	Exponential	62.6 ± 8.0	-483 ± 20	0.00*

Note: * indicates rejection of the null hypothesis that the data follow the fitted distribution at the significance level of α = 0.05; ** indicates the best-fitting distribution according to the adopted goodness-of-fit measures, i.e., the highest log-likelihood or the lowest Anderson–Darling statistic; the horizontal red lines separate statistically homogeneous groups with respect to the analyzed statistic.

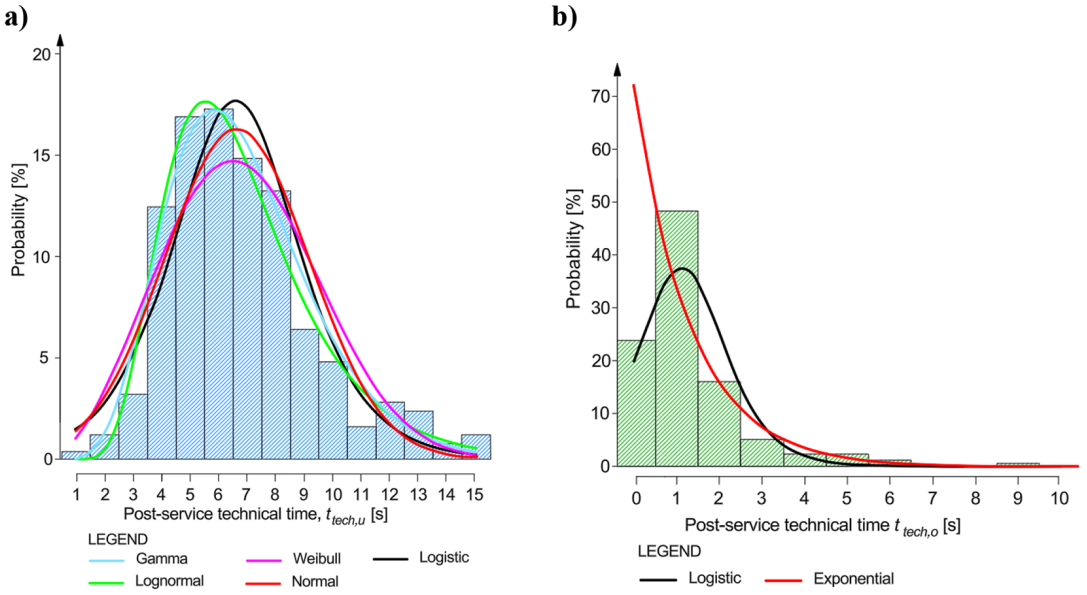


Figure 9. Histograms of post-service technical time t_{tech} with fitted probability density functions: a) urban buses, b) *other users*

Table 6. Parameters of the fitted gamma distribution for the post-service technical time of urban buses

Parameters of the gamma distribution fitted to post-service technical time			
MGF(AD) estimates		MLE estimates	
α [-]	β [s^{-1}]	α [-]	β [s^{-1}]
7.14 ± 0.81	1.05 ± 0.13	7.01 ± 0.71	1.03 ± 0.11

Table 7. Parameters of the fitted logistic distribution for the post-service technical time of *other users*

Parameters of the logistic distribution fitted to post-service technical time			
MGF(AD) estimates		MLE estimates	
μ [s]	s [s]	μ [s]	s [s]
1.12 ± 0.05	0.67 ± 0.03	1.11 ± 0.05	0.65 ± 0.05

Because post-service technical time cannot assume negative values, values were generated from the logistic distribution conditional on $t_{tech,ou} \geq 0$.

The MLE estimates of the logistic distribution parameters μ and s were adopted for the subsequent simulation analyses. The results confirm that post-service technical time should be treated as a user-group-dependent component. For urban buses, this component exhibited a more regular pattern and could be represented by the gamma distribution. For *other users*, it was characterized by a greater concentration of short values and higher irregularity, supporting the use of the logistic distribution as an approximation for simulation purposes.

Verification of group-specific parameterization of passenger service time at bus stops

The group-specific parameterization was verified by comparing field measurements with the results of repeated simulation runs. The analyzed variable was $t_{sba,g}$, representing the time from vehicle stop to the completion of passenger boarding and alighting. Simulations were performed separately for each analyzed bus stop and user group. A total of 100 simulation runs were conducted for each case. The observed mean was considered consistent with the simulation results when it fell within the range of mean values obtained across the simulation runs.

Mean error, mean absolute error (*MAE*), and root mean square error (*RMSE*) were also calculated. Mean error was used to determine the direction of deviation between the simulation results and the field measurements, whereas MAE and RMSE were used as the primary measures of error magnitude:

$$MAE_g = \frac{1}{K}\sum_{k=1}^{K} \mid \bar{t}_{\mathrm{sba,sim},k,g} - \bar{t}_{\mathrm{sba,obs},k,g} \mid \quad (16)$$

$$RMSE_g = \sqrt{\frac{1}{K}\sum_{k=1}^{K}\left(\bar{t}_{\mathrm{sba,sim},k,g} - \bar{t}_{\mathrm{sba,obs},k,g}\right)^2} \quad (17)$$

where: $\bar{t}_{\mathrm{sba,sim},k,g}$ – mean simulated time for bus stop k and user group g[s], $\bar{t}_{\mathrm{sba,obs},k,g}$ – mean observed time for bus stop k and user group g[s], K – number of analyzed bus stops [–].

The verification results are presented in Table 8. The highest agreement was obtained for urban buses. For this group, the observed mean fell within the range of simulated results in 21 of 22 cases, and the *MAE* was 0.81 s. For suburban buses, agreement was obtained in 19 of 22 cases, with an *MAE* of 1.91 s. For taxis and other vehicles, the observed values fell within the range of simulated results in 18 of 22 cases, and the *MAE* was 2.04 s. Agreement between the observed mean values and the range of simulated mean values was also obtained for four additional bus stops that had not been used in model development. The higher error values for suburban buses and taxis and other vehicles were primarily attributable to greater passenger demand variability, smaller sample sizes, and isolated long passenger service events, including cases in which passengers purchased tickets from the driver.

As a supplementary verification measure, the percentile-based PROC measure was used to determine the position of the observed value within the empirical distribution of results obtained from the simulation runs:

$$PROC_{t_{\mathrm{sba}},i,g} = \frac{1}{R}\sum_{r=1}^{R} I\left(t_{\mathrm{sba,sim},i,g}^{(r)} \le t_{\mathrm{sba,obs},i,g}\right) \quad (18)$$

where: $PROC_{tsba,i,g}$ – empirical percentile position of the observed value relative to the simulation results [–], R – number of simulation runs [–], $t_{\mathrm{sba,obs},i,g}$ – time obtained in simulation run r for vehicle i and user group g[s], $t_{sba,obs,i,g}$ – time recorded during field measurements for vehicle i and user group g[s], $I[\cdot]$ – indicator function.

The *PROC* values range from 0 to 1. Values close to 0 indicate that the observed time is lower than most of the simulated values, whereas values close to 1 indicate that it is higher than most of the simulation results. When the model adequately reproduces the variability of the passenger service process, the *PROC* values should be distributed throughout the entire interval without pronounced concentration near the extreme values.

Histograms of $PROC_{tsba}$ are presented in Figure 10. The values are distributed throughout the interval from 0 to 1, and the vehicle shares in the individual classes are close to those expected under a uniform distribution. This indicates that the observed values are not systematically concentrated below or above the simulation results and that the model does not exhibit a pronounced tendency to overestimate or underestimate *tsba*.

The verification results confirm that the group-specific parameterization of passenger service time reproduces both the more regular passenger service process of urban buses and the greater variability observed for suburban buses and taxis and other vehicles. The results also indicate that passenger service time should not be treated as a single random variable common to all vehicles but as a component dependent on the user group and the characteristics of the passenger service process.

The obtained numerical parameters primarily represent shared-use bus stops operating under conditions comparable to those analyzed in Kraków and Kielce. Their transfer to substantially different

Table 8. Agreement measures for observed and simulated mean passenger boarding and alighting time

User group	Observed mean [s]	Simulated mean [s]	Mean error [s]	*MAE* [s]	*RMSE* [s]
Urban buses	14.65	15.21	0.56	0.81	0.88
Suburban buses	20.84	19.06	-1.77	1.91	3.23
Taxis and other vehicles	8.31	6.71	-1.60	2.04	3.18

Note: Mean error was calculated as the difference between the simulated and observed values; *MAE* – mean absolute error; *RMSE* – root mean square error.

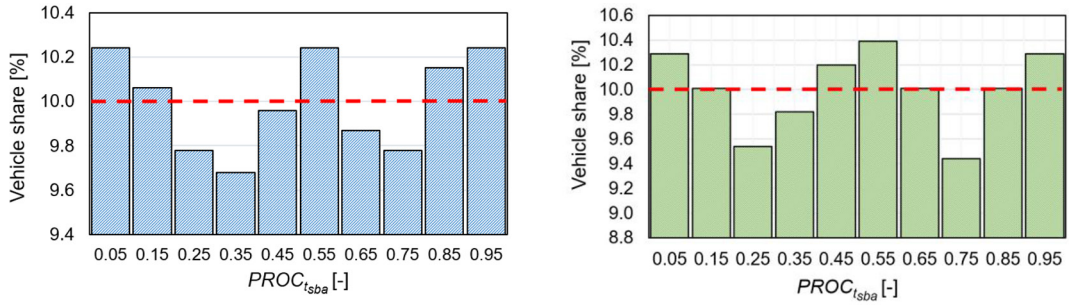


Figure 10. Histograms of $PROC_{tsba}$ values for the time from vehicle stop to completion of boarding and alighting: a) urban buses, b) *other users*

operating environments should therefore be interpreted with caution, whereas the proposed parameterization procedure has broader applicability.

From a practical perspective, the resulting group-specific parameters can be used as input data in microscopic simulations of shared-use bus stops, avoiding the use of a single common passenger service time for all vehicle groups.

The results support the use of a group-specific probabilistic parameterization of passenger service time components under mixed bus service conditions. The resulting parameterization was subsequently confirmed through simulation-based verification.

DISCUSSION

The results indicate that passenger service time at bus stops shared by different user groups should not be represented using a single common parameter set. In the literature, bus dwell time is most frequently related to boarding and alighting passenger counts, fare collection method, door configuration, and vehicle characteristics. These effects were demonstrated by Tirachini [2] and Sun et al. [3], while Fernández, Valencia, and Seriani [5] examined passenger flow through public transport vehicle doors. The present results show that, under mixed bus service conditions, the user group itself is also an important factor because it affects passenger demand structure, vehicle stopping behavior, the boarding and alighting process, and post-service technical time.

Recent studies have likewise emphasized the operational importance of dwell-time variability and bus stop organization. Wang et al. [22] incorporated dwell-time variability into the analysis of bus stop operations and overtaking. Shen et al. [23] and Hu et al. [28] demonstrated the importance of berth allocation and overtaking rules for bus stop capacity. Ezquerro, Alonso, and Moura [29] further showed the operational relevance of shared bus stop use. The present study complements these approaches by focusing on the passenger service process itself and by showing that its components require different parameter sets for urban buses, suburban buses, and taxis and other vehicles. The proposed parameterization therefore provides group-specific passenger service inputs that can subsequently be incorporated into broader models of berth occupancy, queueing, and bus stop capacity.

The negative binomial distribution was adopted as a practical basis for modeling boarding and alighting passenger counts. It did not provide the best fit in every case, particularly for suburban buses, which were characterized by a higher frequency of stops without passenger service and greater irregularity in passenger counts. Its selection should therefore not be interpreted as identifying a universally optimal distribution for every bus stop. Instead, it represents a compromise among goodness of fit, parameter stability, and applicability within the simulation model. An important advantage of this approach is that the distribution parameters can be related to the mean and standard deviation of the passenger count. This increases the practical applicability of the method when detailed data on the distribution of passenger flows among bus routes and operators are unavailable.

Passenger-arrival modeling has also been addressed using probabilistic approaches. Kieu and Cai [44] represented passenger arrivals using a nonhomogeneous Poisson process, while Kieu et al. [45] considered passenger arrivals and separate operational components of bus dwell time. These approaches provide a useful reference for the passenger-count and post-service technical-time components analyzed in the present study. The

procedure used to select distributions and estimate their parameters was also important. Probabilistic approaches to the estimation of bus dwell time or service time were applied by Meng and Qu [11] and Bian et al. [12]. In the present study, the analysis was extended by comparing two estimation criteria, identifying groups of distributions with similar goodness of fit, and evaluating parameter variability. Comparison of maximum likelihood estimation with maximum goodness-of-fit estimation based on the Anderson–Darling statistic made it possible to determine whether distribution selection was stable with respect to the adopted fitting criterion. This was relevant because the Anderson–Darling statistic is sensitive to differences in distribution tails and, consequently, to extreme observations [39]. Bootstrap resampling additionally enabled the uncertainty of the estimated parameters to be evaluated, which was particularly important for small samples, datasets containing numerous zero values, and datasets affected by extreme observations [40, 41].

Differences among user groups were particularly evident when urban and suburban buses were compared. The passenger service process was more regular for urban buses, resulting in closer agreement between observed and simulated values. This greater regularity was associated with more consistent service organization and the typical use of vehicles with multiple doors. Suburban buses exhibited greater variability because their passenger service process involved more diverse vehicle types, more frequent single-door service, possible ticket purchases from the driver, and greater variation in vehicle stopping position relative to waiting passengers. This interpretation is consistent with the markedly greater dispersion of boarding and alighting time observed for suburban buses and with the higher verification error obtained for this group. The differences also reflect the more directional passenger demand observed at many stops for suburban buses, where boarding or alighting strongly predominated depending on the function of the stop and service direction. Combining these groups into a single dataset could therefore produce parameters that are acceptable for aggregated data but difficult to interpret in relation to the actual passenger service process.

The event-based representation of boarding and alighting time related passenger service time not only to passenger counts but also to passenger distribution along the platform, vehicle stopping position, passenger access to the doors, and individual service times. This extends approaches in which dwell time or passenger service time is represented primarily as a function of passenger counts, such as those proposed by Sun et al. [3] and Meng and Qu [11]. In practice, two vehicle stop events with similar passenger counts may result in different passenger service times when vehicle stopping position, passenger distribution, or boarding organization differs. The detailed microsimulation algorithm for this process was presented in [33], whereas the present study incorporated it into the group-specific parameterization of passenger service time components.

The separate treatment of post-service technical time was also justified because this component depended on the user group. For urban buses, post-service technical time exhibited a more regular pattern and was represented by the gamma distribution. For *other users*, the fitting results were less conclusive, and the logistic distribution was adopted as a practical approximation for simulation purposes. This represents an important interpretive limitation: adopting the logistic distribution does not imply full agreement with the empirical distribution but provides a means of reproducing the dominant variability of this component within the simulation model.

Several limitations should be considered when interpreting the results. The numerical parameter values were derived from shared-use bus stops in two Polish cities and therefore primarily represent the operating conditions observed at these locations. The suburban bus group was heterogeneous and included regional, intercity, and long-distance services with different vehicle and passenger service characteristics. Separate parameterization of these subgroups was not possible at all locations because of limited sample sizes. For taxis and other vehicles, distribution fitting was performed using pooled data because individual-stop samples were small. The logistic distribution adopted for the post-service technical time of this group should also be regarded as a practical approximation, since none of the examined theoretical distributions fully satisfied the goodness-of-fit criterion.

The proposed approach also accounts for situations in which the formally designated number of bus stop berths does not reflect the actual organization of passenger service. At shared-use bus stops, the effective number of service positions and the spacing between vehicles may vary depending on the user group, vehicle length,

stopping position, and order of arrival. Through group-specific parameterization, the model does not rely on a single average passenger service time. Instead, it generates boarding and alighting passenger counts, boarding and alighting time, and post-service technical time separately for the individual user groups.

From a practical perspective, the proposed parameterization can support the preparation of input data for microscopic bus stop simulation when detailed route- or operator-level passenger data are unavailable. In particular, the parameters of the negative binomial distribution can be approximated from the mean and standard deviation of boarding and alighting passenger counts. When incorporated into a broader bus stop simulation model, the group-specific passenger service parameters can be used to examine how changes in the composition of stop users, service organization, or use of available berths affect berth occupancy, queue formation, and stop capacity. This may support decisions concerning the organization of shared-use stops and the allocation or separation of different service groups.

The transferability of the proposed approach concerns primarily the modeling framework rather than the numerical parameter values obtained in this study. The same group-specific procedure can be applied to other passenger transport systems in which vehicle groups with different passenger service characteristics use the same stopping area or passenger-service infrastructure. The numerical parameters obtained in this study may be directly applicable under comparable operating conditions, whereas substantially different passenger demand, vehicle characteristics, or service organization should be considered when transferring the parameterization to other systems.

Future development should focus on extending the empirical database to additional cities and operating environments and on independent validation of the parameterization outside the two cities analyzed in this study. Larger datasets would also allow the suburban group to be divided into more homogeneous vehicle and service categories. Further development of passenger-arrival modeling would require integration with broader passenger-flow and service-assignment models, particularly for suburban services, for which detailed information on passenger allocation among routes and operators is often difficult to obtain. Other potentially relevant factors, including vehicle occupancy, uneven use of individual doors, fare collection procedures, and passenger characteristics, may also be incorporated as additional explanatory variables.

CONCLUSIONS

Based on the analyses performed, the following conclusions were drawn:

1. The study developed and verified a group-specific probabilistic parameterization of passenger service time at shared-use bus stops. Urban buses, suburban buses, and taxis and other vehicles differed in passenger demand structure, boarding and alighting time, and post-service technical time. These findings support representing passenger service through group-dependent components rather than using a single common dwell-time parameterization.
2. The negative binomial distribution provided a practical basis for modeling boarding and alighting passenger counts. For urban buses, it provided the best fit at most analyzed stops, whereas the results for suburban buses were less consistent because of greater demand variability and more frequent zero values. In 69 of 78 complete fits, parameters derived from the mean and standard deviation were within the uncertainty range of the maximum likelihood estimates determined using bootstrap resampling.
3. Comparison of maximum likelihood estimation and maximum goodness-of-fit estimation based on the Anderson–Darling statistic, supplemented by bootstrap resampling, allowed both distribution fit and parameter stability to be evaluated. This was particularly relevant for datasets with small samples, numerous zero values, and extreme observations.
4. Boarding and alighting time was determined not only by passenger counts but also by passenger distribution along the platform, vehicle stopping position, passenger access to the doors, and individual service times. The results therefore support an event-based representation in which passenger service time reflects the combined effects of demand, spatial conditions, and service organization.
5. Post-service technical time also differed among user groups. The gamma distribution adequately represented this component for urban buses, whereas the logistic distribution provided the most suitable practical approximation for

the remaining users. Treating this component separately from boarding and alighting time provides a more detailed representation of passenger service dwell time.

6. Simulation-based verification showed good agreement between observed and simulated passenger service times. The mean absolute error was 0.81 seconds for urban buses, 1.91 seconds for suburban buses, and 2.04 seconds for taxis and other vehicles. The resulting group-specific parameterization can be used as input to microscopic models of shared-use bus stops operating under comparable conditions.

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