

Theoretical substantiation of the intensification of the seed material separation process in a pseudo-fluidized bed

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ABSTRACT

The results of the scientific research presented in this paper are focused on solving an important scientific and technical problem – enhancing the efficiency of the grain material separation process using a vibro-pneumatic method with the application of mechanical intensifiers to obtain high-quality seed material. The relevance of the study is substantiated, and the key factors influencing the increase in productivity and the improvement of quality in seed material preparation processes are analyzed. A critical review of the existing literature related to the research topic is conducted, and promising approaches for addressing the identified challenges are determined. The paper presents mechanical and mathematical models of the separation process of seed materials based on seed density on working surfaces equipped with mechanical intensifiers. In developing multidimensional mathematical models of the separation process, it is assumed that under the combined action of a directed airflow and oscillations of the working surface, effective vertical stratification of the pseudo-fluidized seed material occurs. As a result, N layers of particles are formed, each characterized by their own density, aerogravitational, and hydrodynamic properties. The motion of each particle layer is considered within the framework of multiphase continuum mechanics, where each layer is modeled as a separate continuous medium, and the interactions between layers are described through boundary conditions at their interfaces. Thus, the dynamics of the vertically stratified pseudo-fluidized particle bed can be described as the motion of an $(N + 1)$ -phase system consisting of N layers of discrete particles and one continuous gas phase (air). When modeling particle density, it is assumed that limiting density values exist for the “light” fraction (including difficult-to-separate impurities, damaged and substandard seeds with reduced density) and the “heavy” fraction (biologically viable and active seeds with higher density). Between these limiting values lies the “medium” fraction, which contains particles with intermediate density characteristics.

Keywords: seed material, pneumatic sorting table, seed density, multiphase medium, separation, process intensification.

INTRODUCTION

Military hostilities taking place on the territory of Ukraine have had a significant impact on agricultural production and the export of grain

and leguminous crops, as prior to 2022 Ukraine was one of the leading global producers of agricultural commodities [1]. This impact is associated with substantial losses of cultivated areas, logistical disruptions, and the destruction of

agricultural and transport infrastructure [2]. In addition, countries of the European Union are also affected by political and economic factors, as well as pronounced climatic variability, including droughts and excessive precipitation. Since 2024, signs of recovery in agricultural production have begun to emerge, which can be attributed partly to improved climatic conditions and partly to the adaptation of agricultural producers to new operating environments [3].

According to USDA/WASDE reports [4], wheat, maize, and barley production declined significantly during the period from 2022 to 2023, as crops were severely affected by droughts, particularly in Southeastern Europe. A stabilization of production was expected for the 2023–2024 period; however, moisture deficits continued to negatively affect yields. USDA estimates indicate that grain production in European Union countries will increase by approximately 4–5%. According to Eurostat data [5], soybean production increased by about 14% between 2022 and 2024. At the same time, chickpea production in the 2025–2026 season is projected to increase by 1.4%, reaching 17.8 million tons.

The analysis shows that the consumption of grain and leguminous crops demonstrates a steady upward trend, which in turn necessitates an increase in production volumes. However, limited land resources, ongoing military actions in Ukraine, and political and economic instability encourage the search for approaches aimed at intensifying the production processes of grain and leguminous crops.

One effective approach to addressing this challenge is the use of high-quality seed material, which, according to [6], significantly increases crop yields. Despite the fact that yield improvements resulting from high-quality seed material depend on a complex set of factors (crop type, soil and climatic conditions, and agricultural technologies) [7], the use of certified seeds with high germination energy and vigor reduces the risk of viral diseases and ensures a consistently higher yield increase – ranging from 50% to 80% compared to uncertified seeds [8]. The production of high-quality seed material is a complex process that integrates genetic, agrotechnological, biological, and engineering stages.

According to studies reported in [9], the engineering stage involves the efficient separation of high-quality seed material from bulk grain using machine-based technologies. The separation

process enables the formation of seed batches with the required purity and uniformity. Methods such as optical separation [10], magnetic separation [11], and separation based on seed density are commonly employed to obtain high-quality seed material. Increasing the productivity of the separation process is primarily achieved through process intensification, as expanding the working surface area of separation machines is inherently limited [12]. To address this issue, mechanical “intensifiers” are installed on the working surface to enhance particle redistribution into layers corresponding to specific material densities. For the modernization of existing separation machines and the development of new designs, theoretical modeling of the separation process is required, considering the structural and operating parameters of the separation machine as well as the physical and mechanical properties of the processed material [13].

Data and problem statement

A considerable number of scientific studies have been devoted to addressing the problem of improving the productivity and quality of post-harvest grain processing. In [14], the authors compare magnetic separation methods with separation in liquids and demonstrate the efficiency and economic feasibility of using “dry” separation techniques. Study [15] examines the dependence of separation machine productivity on the efficiency of vertical pneumatic sorting channels. The authors investigate the effect of preliminary stratification on the intensity of redistribution of light impurity particles and develop theoretical models for determining particle motion trajectories, considering the preliminary stratification of the mixture and the corresponding operating parameters. However, this study considers only the motion of individual particles without analyzing interlayer interactions.

In [16, 33], the authors investigate the separation process of seed material on the working surface of a pneumatic sorting table and develop mathematical models describing the motion of particle layers with different densities. Nevertheless, the specific features of mechanical process intensification are not addressed. In [17, 32], a separation machine for removing seed husks is proposed and studied [34–36]. The machine operates based on frictional interaction and separation in an airflow. However, the investigated impurity

differs significantly from the main crop seeds in terms of density and aerodynamic properties, which limits the applicability of this approach for separating difficult-to-remove impurities. Study [18] discusses technical measures for intensifying the separation process of granular materials. The heat and mass transfer processes occurring between granular particles are analyzed; however, the research focus is placed on the separation of individual particles, without adequately considering the physical and mechanical properties of the processed material [19–22].

The conducted analysis of the literature reveals an insufficient level of investigation of the separation processes of seed (granular) materials [23,24]. This gap leads to complications and inaccuracies in the design and calculation of separation machines, as well as to improper machine settings, which significantly reduce the quality and economic efficiency of the separation process [23–27].

The development of mathematical models for technological processes requires the application of computational methods capable of predicting seed behavior and identifying optimal operating regimes of the equipment [28–31].

Thus, the results of this study have both fundamental and applied significance, as they contribute to the effective solution of granular material sorting problems and can be utilized to improve the efficiency of agricultural technologies.

RESULTS AND DISCUSSION

The scientific novelty of this study lies in the development of a multiphase mechanical–mathematical model for seed material separation in a pseudo-fluidized bed on a pneumatic sorting table. The proposed model represents the seed material as a system of interacting particle layers with different densities and a continuous air phase. Unlike existing approaches, which describe either the motion of individual particles or treat the seed material as a homogeneous continuum, the proposed framework accounts for the interaction between particle layers. This provides a more realistic description of the vertical stratification process occurring under the combined action of airflow and vibration.

The proposed modeling approach is based on the assumption that the directed airflow and oscillatory motion of the working surface produce an effective vertical stratification of the

pseudo-fluidized seed material. As a result, the material is represented by N interacting particle layers, each characterized by its own density, aerodynamic, and hydrodynamic properties. The motion of these layers is described within the framework of multiphase continuum mechanics, enabling the physical mechanisms governing particle redistribution during separation to be represented more accurately than in conventional single-phase models.

Another contribution of this work is the integration of the governing equations of continuum mechanics with a weak variational formulation and finite element approximation for solving the boundary-value problem of seed motion. This unified computational framework simultaneously considers airflow velocity, vibration frequency and amplitude, deck inclination, frictional interaction, and aerodynamic forces, thereby improving the predictive capability of the model under various operating conditions of pneumatic sorting tables.

In addition, a probabilistic approach for evaluating separation efficiency has been developed, in which the calculated trajectories of particle fractions are combined with probability density functions to estimate the purity of the target seed fraction. This approach makes it possible to predict separation quality and determine rational operating parameters without extensive experimental investigations, providing an effective tool for the analysis, optimization, and modernization of pneumatic seed-sorting equipment.

As a result, N layers of particles are formed, each characterized by their own density, aerogravitational, and hydrodynamic properties.

The motion of each particle layer is considered within the framework of multiphase continuum mechanics, where each layer is modeled as an individual continuous medium, and the interactions between layers are described through boundary conditions at their interfaces. Thus, the dynamics of the vertically stratified pseudo-fluidized particle bed can be represented as the motion of an $(N + 1)$ -phase system consisting of N layers of discrete particles and one continuous gas phase (air).

In modeling particle density, it is assumed that limiting density values exist for the “light” fraction (including difficult-to-separate impurities, damaged, and substandard seeds with reduced density) and the “heavy” fraction (biologically viable and active seeds with higher density). Between these extreme values lies the “medium”

fraction, which contains particles with intermediate density characteristics.

For the mathematical description of the process, it is assumed that the vertical distribution of particles by density follows a normal distribution, and the coordinates of layers with different densities correspond to the mathematical expectation of this distribution. Since particle density varies continuously and the exact distribution of particle quantity by density is unknown, this formulation complicates obtaining an exact analytical solution. To simplify the calculations, the distribution functions are constructed for averaged density values of the “light,” “medium,” and “heavy” seed fractions, while the variance of the coordinate for each layer is assumed to be equal to unity.

Pneumatic sorting tables are widely used for separating granular materials based on density, particle size, and aerodynamic properties. The efficiency of such separation largely depends on several parameters, among which the most significant are the inclination angles and vibration frequencies of the working surface, as well as the airflow velocity.

Optimization of these parameters makes it possible to improve the quality of seed material separation, reduce losses of biologically valuable seeds, and increase the overall efficiency of the separation process.

One of the key aspects of effective separation is the determination of optimal operating parameters of the pneumatic sorting table, such as the inclination angles and vibration frequency of the working surface, as well as the velocity and direction of the airflow. The selection of these parameters has a substantial influence on particle motion dynamics, the degree of stratification, and the quality of the final sorting. At the same time, the physical and mechanical properties of the seed material (in particular, density, shape, size, and surface texture) complicate the mathematical description of the process and require the development of appropriate models.

The development of adequate mathematical models describing seed motion under pneumatic sorting conditions makes it possible not only to improve separation quality but also to optimize equipment operation without conducting many experimental studies. This, in turn, contributes to cost reduction, increased productivity, and enhanced stability of the technological process. In this study, a multiphase approach is employed, which allows the seed material to be treated as a

pseudo-fluid system composed of individual layers, each possessing its own hydrodynamic properties. This approach enables the consideration of the physical features of separation and the description of particle transport with different densities along distinct trajectories, thereby improving the understanding of sorting mechanisms.

In the present study, seed material is modeled as a system consisting of a finite number of pseudo-fluid layers, each differing in hydrodynamic properties, particle density, and aerodynamic characteristics. Such an approach allows the layer-by-layer motion of seeds to be interpreted as a physically justified model of material separation into individual fractions through selective transport of particle layers with different densities along different trajectories.

Pneumatic sorting tables are widely applied for separating granular materials by density, size, and aerodynamic properties of particles. The effectiveness of such separation strongly depends on parameters such as the inclination angles and vibration frequency of the working surface, as well as the airflow velocity. Optimization of these parameters improves the quality of seed separation, reduces losses of biologically valuable seeds, and increases the overall efficiency of the separation process. In this study, a flat pneumatic sorting table used the general view of its working surface is shown in Figure 1.

The fundamental equations governing fluid motion that will be used in the following analysis are presented below. We begin by considering a single layer.

To describe fluid motion, the basic equations of continuum mechanics are employed. All continuum models involve the determination of the fields of stresses σ_{ij} , strains ε_{ij} , displacements u_i (or velocities V_i), as well as temperature T , pressure p , and density ρ as functions of time. These fields are obtained using the following relationships:

- the equations of motion under the action of external body forces X_i , formulated in terms of stress

$$\sigma_{ij,j} + X_i = \rho \frac{\partial^2 u_i}{\partial t^2}; \quad i, j = 1, 2, 3, \quad (1)$$

- the Cauchy strain–displacement relations, which relate small strains to the displacement field

$$\varepsilon_{ij} = \frac{1}{2} (u_{i,j} + u_{j,i}) \quad (2)$$

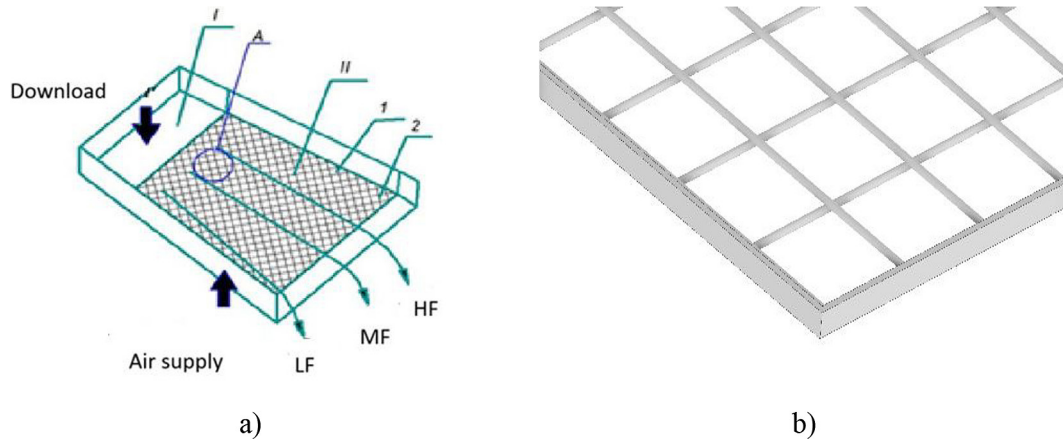


Figure 1. General view of the upgraded work surface with a mesh-type mechanical booster installed (a) and a cross-section of the surface (b)

In terms of velocities, the Cauchy relations take the form:

$$\varepsilon_{ij} = \frac{1}{2}(V_{i,j} + V_{j,i}) \quad (3)$$

- the strain compatibility conditions, which ensure the consistency of the deformation field; the prescribed boundary and initial conditions; and the constitutive equation of state-related pressure, density, and temperature.

$$\frac{\partial^2 \varepsilon_{ik}}{\partial x_j \partial x_l} + \frac{\partial^2 \varepsilon_{jl}}{\partial x_i \partial x_k} = \frac{\partial^2 \varepsilon_{il}}{\partial x_j \partial x_k} + \frac{\partial^2 \varepsilon_{jk}}{\partial x_i \partial x_l} \quad (4)$$

However, the above relationships are insufficient for the unique determination of σ_{ij} , ε_{ij} , and u_i ($i, j = 1, 2, 3$). Therefore, in fluid mechanics, constitutive relations between stresses and rates of deformation are introduced. It is assumed that the components of the stress tensor are linearly related to the components of the strain-rate tensor.

Different forms of these relationships correspond to different fluid models. The following assumptions are introduced for the pseudo-fluid medium: the fluid is considered compressible and viscous; pressure perturbations are assumed to be small, and thus the problem is treated within a linear formulation; the fundamental laws of fluid mechanics are applied, the law of mass conservation (the continuity equation) in the following form:

$$\frac{\partial \rho_f}{\partial t} = -\text{div}(\rho_f V_f) \quad (5)$$

Next, the equations of fluid motion expressed in terms of stress (because of the law of

conservation of momentum) are employed, considering the body forces $\mathbf{F}$, in the following form:

$$\rho_f \frac{dV_f}{dt} = \rho_f F + \text{div}(\sigma_f) \quad (6)$$

To determine the components of the stress tensor, the Stokes hypothesis is adopted:

$$\sigma_f = -\left(P + \frac{2}{3}\mu\right)I + \dot{S} \quad (7)$$

where: $\dot{S}$ – denotes the deviator of the strain-rate tensor, μ – the dynamic viscosity coefficient.

$$\dot{S} = \begin{pmatrix} \dot{\varepsilon}_{11} - \dot{\varepsilon}_0 & \dot{\varepsilon}_{12} & \dot{\varepsilon}_{13} \\ \dot{\varepsilon}_{12} & \dot{\varepsilon}_{22} - \dot{\varepsilon}_0 & \dot{\varepsilon}_{23} \\ \dot{\varepsilon}_{13} & \dot{\varepsilon}_{23} & \dot{\varepsilon}_{33} - \dot{\varepsilon}_0 \end{pmatrix} \quad (8)$$

$$\dot{\varepsilon}_0 = \frac{1}{3}(\dot{\varepsilon}_{11} + \dot{\varepsilon}_{22} + \dot{\varepsilon}_{33})$$

The equation of state is applied in the form $p = p(\rho)$ together with the following relation:

$$\frac{\partial P}{\partial \rho} = c^2 \quad (9)$$

where: c – the speed of sound in the medium.

Now consider the motion of multilayer fluid. Let each layer be characterized by the following parameters: density ρ_n , dynamic viscosity coefficient μ_n , and mean layer thickness h_n . The equations of motion for each layer, in invariant form, can be written as follows:

$$\rho_n \frac{dV^{(n)}}{dt} = -\nabla P^{(n)} + \mu_n \Delta V^{(n)} + \rho_n F^{(n)} \quad (10)$$

$$\text{div} \mathbf{V}^{(n)} = (\nabla \cdot \mathbf{V}^{(n)}) = 0 \quad (11)$$

In Equations 10–11 the velocity vector $\mathbf{V}^{(n)}$ has the components $\mathbf{V}^{(n)} = (V_1^{(n)}, V_2^{(n)}, V_3^{(n)})$, and the body-force vector $\mathbf{F}^{(n)}$ in component form is written as $\mathbf{F}^{(n)} = (F_1^{(n)}, F_2^{(n)}, F_3^{(n)})$. The material (substantial) derivative operator $\frac{d}{dt}$ is $\frac{d}{dt} = \frac{\partial}{\partial t} + (\mathbf{V}^{(n)} \cdot \nabla)$.

Each of the systems (10–11) constitutes a closed set of four equations for four unknowns: the components of the velocity vector $V_1^{(n)}, V_2^{(n)}, V_3^{(n)}$ and the pressure $P^{(n)}$. The first vector equation corresponds to the Navier–Stokes equation, while Equation 8 is the continuity equation for an incompressible fluid. If the fluid is compressible, Equation 11 is replaced by Equation 5 in the form:

$$\frac{\partial \rho_n}{\partial t} + (\nabla \cdot \rho_n \mathbf{V}^{(n)}) = 0 \quad (12)$$

This equation implies that the mass of each phase (fraction) remains conserved in the absence of sources or sinks. It is used to monitor the particle density within each control volume and allows the determination of particle distribution under the action of external forces. In the context of pneumatic sorting, this equation enables the description of the dynamic variation of the density distribution of fractions along the sorting table.

Next, the fundamental equations of motion for a multiphase system modeling the pseudo-fluidized layer of seed material (SM) are presented. It is assumed that the pseudo-fluidized SM consists of two phases: a discrete phase formed by solid particles (seeds) and a continuous phase represented by air.

According to [13], the discrete phase is considered as consisting of N discrete components, each formed by solid particles with density $\bar{\rho}_n$, $n = 1, 2, \dots, N$.

Based on this assumption, the density of the i -th particle layer (the n -th component of the discrete phase) is determined as follows:

$$\rho_n = \bar{\rho}_n \delta_n, \quad (13)$$

where: δ_n – the volume fraction of particles of the n -th component in the pseudo-fluidized seed material.

The density of the discrete phase is defined as:

$$\rho_P = \sum_{n=1}^N \rho_n \quad (14)$$

Since the medium is diluted, each discrete phase contains both air and solid particles. The number of solid particles in each phase is characterized by the parameter δ_n . Accordingly, the volume of the gaseous medium (air) in each discrete phase is reduced by the amount $\bar{\rho} \delta_n$ compared to a medium filled with air, where $\bar{\rho}$ – denotes the density of the gaseous medium. Thus, the density ρ of the continuous phase is calculated as follows:

$$\rho = \bar{\rho} \left(1 - \sum_{n=1}^N \delta_n \right) \quad (15)$$

The density of the pseudo-fluidized seed material is given by:

$$\rho_c = \rho + \rho_P \quad (16)$$

From this, the following equation for the velocity V_c of the pseudo-fluidized seed material is obtained:

$$\rho_c V_c = \sum_{n=1}^N \rho_n V^{(n)} + \rho V \quad (17)$$

where: V – the airflow velocity.

For each discrete layer, the continuity equation is used in the following form:

$$\frac{\partial \rho_n}{\partial t} + (\nabla \cdot \rho_n \mathbf{V}^{(n)}) = 0 \quad (18)$$

For the continuous phase, the following continuity equation is applied:

$$\frac{\partial \rho}{\partial t} + (\nabla \cdot \rho \mathbf{V}) = 0 \quad (19)$$

From Equations 17 and 18, it follows that the continuity equation (mass conservation) for the pseudo-fluidized seed material takes the form:

$$\frac{\partial \rho_c}{\partial t} + (\nabla \cdot \rho_c \mathbf{V}_c) = 0 \quad (20)$$

Next, we consider the momentum conservation Equation 5 and express it in Cartesian (component) form for each discrete phase:

$$\begin{aligned} & \rho_n \left(\frac{\partial V_i^{(n)}}{\partial t} + (\mathbf{V}^{(n)} \cdot \nabla) V_i^{(n)} \right) = \\ & = \frac{\partial}{\partial x_j} \left[-P^{(n)} \delta_{ij} + \mu_n \left(\frac{\partial V_i^{(n)}}{\partial x_j} + \frac{\partial V_j^{(n)}}{\partial x_i} \right) \right] + \\ & \quad \rho_n F_i^{(n)} + \rho_n \sum_{m=1}^N \alpha_{nm} (V_m^{(n)} - V_n^{(m)}), \\ & \quad i, j = 1, 2, 3, n, m = 1, 2, \dots, N. \end{aligned} \quad (21)$$

where: $V_i^{(n)}$ is the i -th component of the velocity of the n -th discrete-phase component; μ_n is the effective dynamic viscosity coefficient of the n -th component in the seed material; $F_i^{(n)}$ is the i -th component of the body force acting per unit mass of the n -th component of the mixture; $P^{(n)}$ is the static pressure of the n -th component in the seed material; δ_{ij} is the Kronecker delta; summation over repeated indices is implied.

The last term on the right-hand side of Equation 21 characterizes the interaction of the n -th component with another, m -th component of the seed material. In this case, the following equality holds:

$$\alpha_{nm}\rho_n = \alpha_{mn}\rho_m \quad (22)$$

The force $F^{(n)}$ acting in each layer includes components that account for the influence of the vibrating table, friction forces, normal reactions, and other interactions. In the case of a pneumatic sorting table, the principal forces are:

Gravity, which is resolved into components acting along the inclination of the table and perpendicular to it.

Normal reaction of the table N , acting perpendicular to the working surface.

Friction force $\mu_f N$, which opposes motion along the table surface.

The aerodynamic force is defined as $F_{aero} = 0.5 S C_d \rho_{air} v_{air}^2$ where it depends on the airflow velocity.

The force induced by the vibration of the table is characterized by a vibration amplitude A and oscillation frequency f and can be described as harmonic motion. Thus, the vibrating table is modeled as a harmonic oscillator with a specified amplitude and frequency.

In the present study, the airflow velocity is considered within the range of 1–2 m/s. At airflow velocities exceeding 2 m/s, the material enters a continuous “boiling” or highly turbulent state, leading to constant mixing and, consequently, to the cessation of the separation process. Conversely, at airflow velocities below 1 m/s, pseudo-fluidization does not occur, and effective separation is not achieved.

Seeds with different densities respond differently to the action of the airflow and oscillations of the working surface, and they also exhibit different responses to airflow-induced and inertial forces. Light seeds tend to be lifted more intensively by the airflow, whereas heavy seeds

predominantly roll downward along the inclined surface. Vibrations create an additional separation mechanism based on particle size and shape.

In this study, soybean and chickpea seeds are considered. For each type of seed material, three fractions are introduced: light, medium, and heavy. The indices of the three fractions for each seed material are denoted as $i = 1, 2, 3$, with the following relationship between their densities:

$$\rho^{(1)} < \rho^{(2)} < \rho^{(3)}.$$

where: $\rho^{(1)}$ corresponds to the light fraction (LF), $\rho^{(2)}$ represents the density of the medium fraction (MF), and $\rho^{(3)}$ refers to the heavy fraction (HF).

The differences between the fractions can be summarized as follows. The airflow has a minimal effect on the heavy fraction, which moves primarily along the table surface and is less affected by vibrations. Vibrations have a greater influence on the medium fraction, allowing its trajectory to change under certain conditions. The light fraction is subject to the strongest influence of airflow and vibration and may even remain suspended in the airflow.

The weighted residuals method is applied to transform the strong form of the governing equations into their weak form. As a result, the following expressions are obtained:

$$\begin{aligned} \iiint_{\Omega_{n-1}} \left[\frac{\partial (\rho V_1^{(n)})}{\partial t} + \frac{\partial (V_1^{(n)} V_1^{(n)})}{\partial x_1} + \frac{\partial (V_1^{(n)} V_2^{(n)})}{\partial x_2} + \frac{\partial (V_1^{(n)} V_3^{(n)})}{\partial x_3} \right] w_1^n d\Omega = \\ = \iiint_{\Omega_{n-1}} w_1^n \left(-\frac{\partial p^{(n)}}{\partial x_1} + \rho g_1 + F_1 \right) d\Omega + \\ + \iiint_{\Omega_{n-1}} w_1^n \left[\frac{\partial}{\partial x_1} \left(\mu \frac{\partial V_1^{(n)}}{\partial x_1} \right) + \frac{\partial}{\partial x_2} \left(\mu \frac{\partial V_1^{(n)}}{\partial x_2} \right) + \frac{\partial}{\partial x_3} \left(\mu \frac{\partial V_1^{(n)}}{\partial x_3} \right) \right] d\Omega \\ \iiint_{\Omega_{n-1}} \left[\frac{\partial (\rho V_2^{(n)})}{\partial t} + \frac{\partial (V_2^{(n)} V_1^{(n)})}{\partial x_1} + \frac{\partial (V_2^{(n)} V_2^{(n)})}{\partial x_2} + \frac{\partial (V_2^{(n)} V_3^{(n)})}{\partial x_3} \right] w_2^n d\Omega \end{aligned} \quad (23)$$

To determine particle trajectories, it is necessary to solve the equation of motion (23) for each fraction, subject to the prescribed initial conditions. Numerical computations of the fraction trajectories were performed for the seed materials considered (soybean and chickpea).

Figure 2 presents the “average” trajectories of motion of three soybean fractions – light (Lf), medium (Mf), and heavy (Hf) – on the pneumatic sorting table under different operating conditions. The governing parameters were the table inclination angle (the longitudinal and transverse angles were assumed equal), selected as $\theta = 0^\circ, 5^\circ, 8^\circ$, and the vibration frequency, $f = 13.3$ Hz, 15 Hz, and 16.7 Hz.

The x-axis runs along the slope of the table; the y-axis represents the height of the seed layer; the z-axis runs across the table.

The motion trajectories demonstrate how the fractions change their positions on the table under the combined action of gravitational, aerodynamic, and vibrational forces.

It is observed that at smaller inclination angles ($\theta < 5^\circ$), the fractions move more smoothly and remain longer within the initial placement region. With increasing inclination angles ($\theta < 5^\circ$ and $\theta = 8^\circ$), a more rapid transport of the fractions toward the discharge edge of the table is observed, particularly for the heavy fraction (Hf), which exhibits the least sensitivity to the airflow (Figure 3).

As the inclination angle increases, the trajectories of the fractions become more distinctly separated. It should be noted that at a lower vibration frequency ($f = 13.3$ Hz), the motion of the fractions is more uniform and predictable. With

increasing vibration frequency, a greater number of turbulent and chaotic deviations appear in the trajectories, particularly for the light fractions. Higher frequencies promote more effective separation of the light fractions; however, they may simultaneously cause undesirable mixing of the medium and heavy fractions.

Trajectory analysis makes it possible to assess fraction purity. When the trajectories are well separated spatially, the sorting process can be considered effective. Conversely, if intersections or chaotic deviations are observed, optimization of the operating parameters (inclination angle, vibration frequency, and airflow velocity) is required.

An increase in airflow velocity results in a stronger influence of aerodynamic forces, leading to a significant vertical displacement of the light fractions compared to the case with an airflow velocity of 2 m/s.

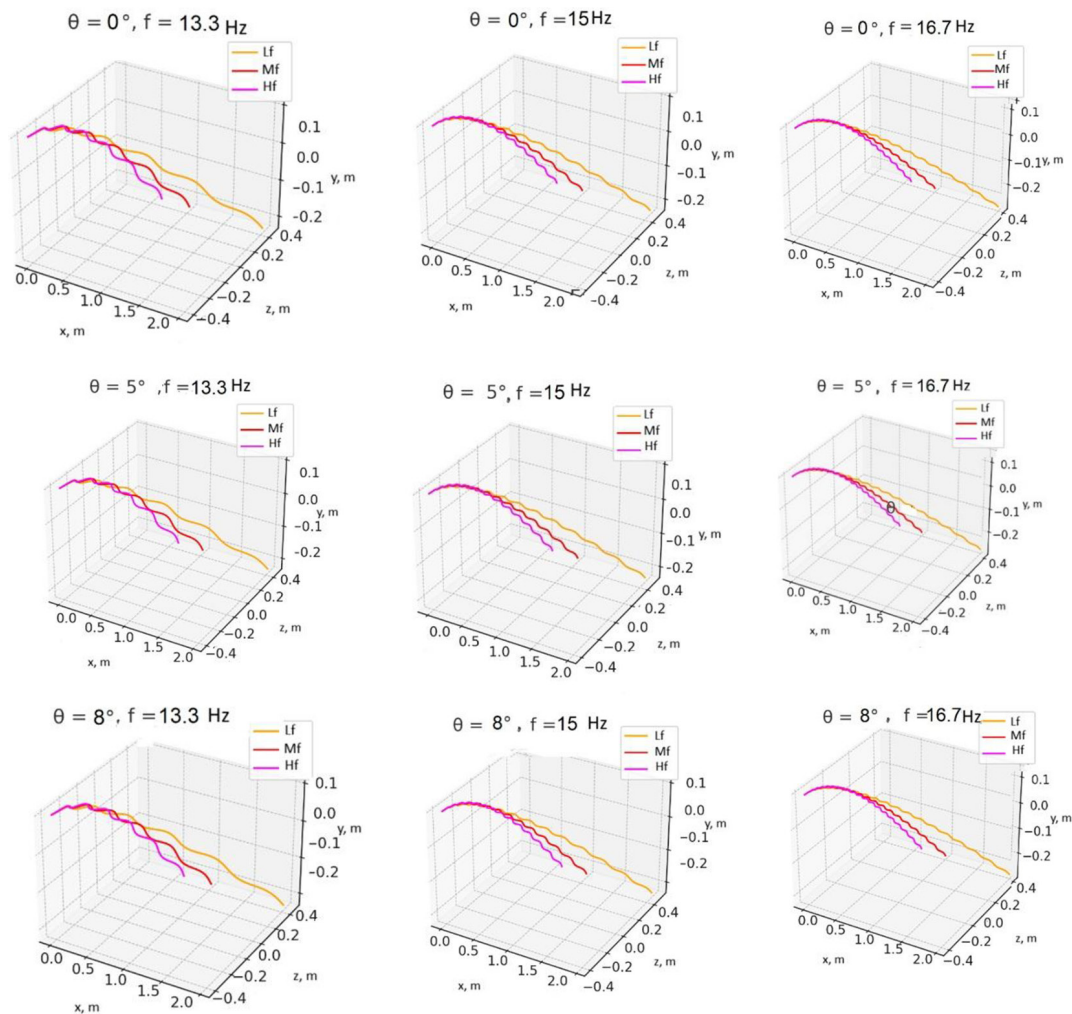


Figure 2. Trajectories of soybean fraction motion at an airflow velocity of 2 m/s and a vibration amplitude of 5 mm

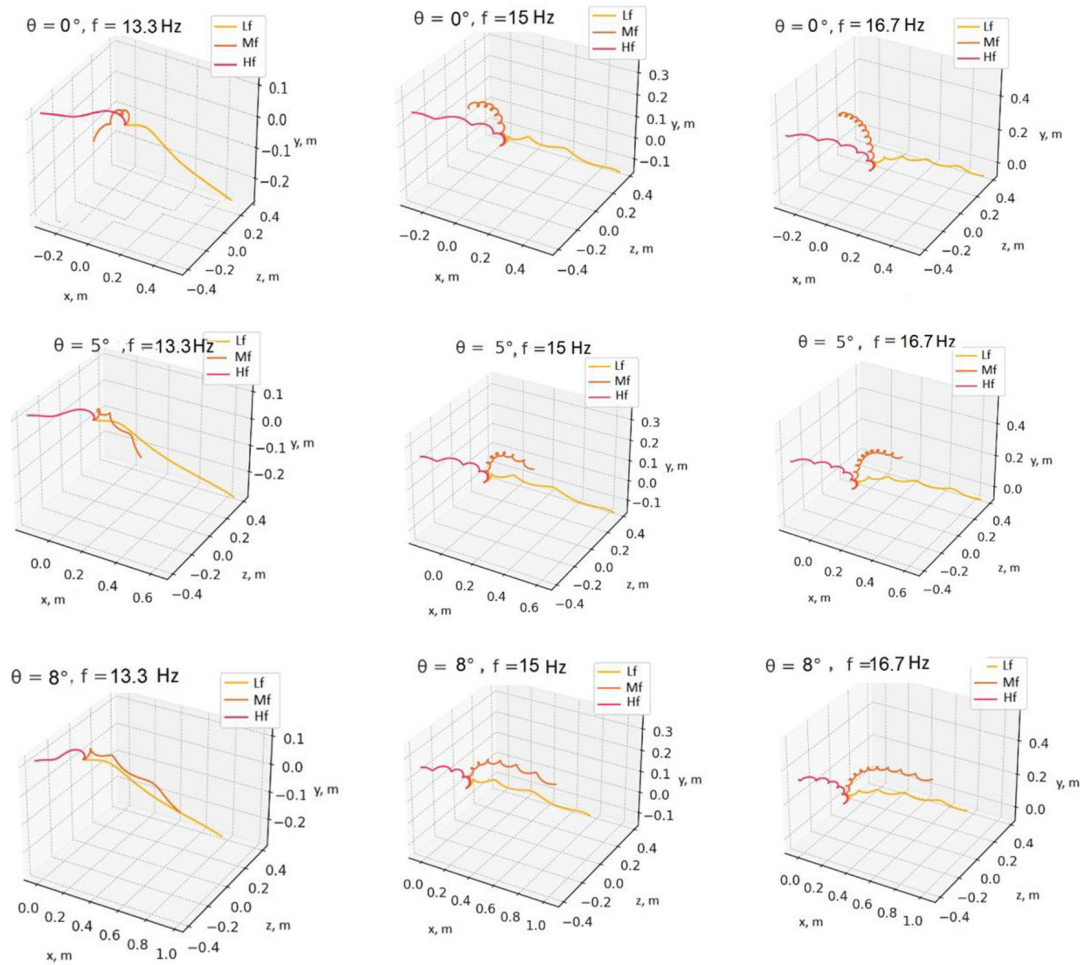


Figure 3. Presents the “average” motion trajectories of chickpea fractions at an airflow velocity of 2 m/s and a vibration amplitude of 5 mm

The proposed mathematical model has several limitations that should be considered when interpreting the obtained results. First, the motion of the seed material is described using averaged trajectories of particle fractions rather than trajectories of individual particles. Such an approach significantly reduces computational complexity and allows the general separation behavior to be predicted; however, it cannot reproduce local stochastic particle interactions occurring within the pseudo-fluidized layer.

Second, the interaction between particles is represented within the framework of a multi-phase continuum approach, in which each fraction is treated as a continuous medium characterized by average physical and mechanical properties. Consequently, microscopic phenomena such as particle collisions, rolling, rotation, and local contact forces are not modeled explicitly but are incorporated indirectly through effective material parameters. This approximation is

appropriate for engineering analysis of industrial pneumatic sorting processes but may require further refinement for detailed investigations of individual particle dynamics.

Finally, the proposed model is valid within the range of operating conditions adopted in this study, namely airflow velocities that ensure stable pseudo-fluidization and the investigated ranges of vibration frequency, amplitude, and deck inclination. Extension of the model to substantially different operating regimes or other seed materials may require recalibration of the model parameters and experimental validation. Nevertheless, within the considered operating ranges, the developed model provides a reliable tool for analyzing separation processes and selecting rational operating parameters for pneumatic sorting tables.

The exact distribution of particle density within the initial seed batch is generally unknown because seed density varies continuously owing

to natural biological variability. Therefore, for the development of a computationally efficient mathematical model, the seed material is represented by a finite number of average fractions corresponding to the light, medium, and heavy components. Under this assumption, the spatial distribution of particles belonging to each fraction is approximated by a normal probability distribution. This assumption is introduced as a simplifying modeling hypothesis rather than as an experimentally established law.

The selection of the normal distribution is justified by both its mathematical properties and the physical nature of the separation process. The final position of a particle on the pneumatic sorting table is governed by the combined influence of numerous independent or weakly correlated factors, including particle density, size and shape, aerodynamic drag, friction, local airflow fluctuations, and vibration-induced disturbances. According to the central limit theorem, the cumulative influence of these factors can be reasonably approximated by a Gaussian distribution. Consequently, the normal distribution provides a physically meaningful and computationally efficient representation of the average spatial distribution of particle fractions while avoiding unnecessary complications in the mathematical formulation.

It should be emphasized that the adopted probability distribution is not intended to reproduce the exact statistical distribution of individual seeds. Instead, it serves as an engineering approximation for describing the average behavior of particle fractions and for estimating the probability of their occurrence within different regions of the pneumatic sorting table. Such an approach makes it possible to evaluate the purity of the target fraction and compare different operating conditions without requiring detailed experimental information on the initial density distribution of the seed material.

The assessment of fraction separation quality on a pneumatic sorting table can be based on the following criteria:

Spatial separation of fractions by analyzing the obtained trajectories, it is possible to evaluate how well the light, medium, and heavy fractions are separated. If the trajectories of different fractions do not intersect and are clearly separated at the discharge end of the table, this indicates an effective sorting process. The presence of significant trajectory intersections may indicate poor

separation performance and the need for adjustment of process parameters.

Residence time of fractions on the table The time during which particles remain on the table is an important indicator of separation efficiency. Optimal sorting occurs when heavy particles remain on the table for a longer time, gradually moving downward along the surface, while light particles are rapidly removed by the airflow.

In this study, it is assumed that the probability of particles with a given density reaching a specific coordinate follows a normal distribution. In this case, the coordinates of fractions with different densities correspond to the mathematical expectation of the normal distribution:

$$f_i(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x - x_i)^2}{2\sigma^2}\right) \quad (24)$$

where: $f_i(x)$ is the probability density function of the coordinate x ; σ is the variance of the coordinate; x_i is the coordinate of the i -th fraction of the raw material with the corresponding density at the outlet of the pneumatic sorting table (the mathematical expectation).

The solution of the boundary-value problem yields the trajectories of phases with different densities and the coordinate at which a particle with a given density will be located at the outlet of the pneumatic sorting table deck (PST). Thus, the coordinates of the locations of material particles with different densities are known immediately before they enter the collection hopper. These coordinates x_i are obtained after constructing the trajectories of the fractions.

For the pneumatic sorting table, the index i varies from 1 to 3, where index 1 corresponds to the light fraction, index 2 to the medium fraction, and index 3 to the heavy fraction.

It should be noted that the distribution of the number of particles within a material layer by density is an important characteristic that largely depends on the quality indicators of the raw material. Various factors influencing this distribution, particularly the physical and chemical properties of the material, can significantly complicate accurate process modeling. Accounting for these factors increases the complexity of problem formulation and solution, as it requires a detailed analysis of many variable parameters, such as particle

shape, aerodynamic behavior, and interactions between different fractions.

Figure 4 illustrates the distribution functions for the average density values of the light, medium, and heavy fractions of the seed material. Averaging makes it possible to substantially reduce the number of variables that must be considered and to simplify the calculations. In addition, the variance of the coordinate, which characterizes the variability of particle density, was assumed to be equal to unity, which is a typical approximation for such calculations.

The coordinate x in Figure 4 is normalized with respect to the maximum width of the pneumatic sorting table deck, $x_{\max}$, from which the separated seed material is discharged into the collection bins. The coordinate $x = 0$ corresponds to the section of the deck from which the light fraction is discharged, whereas the coordinate $x_{\max}$ corresponds to the section of the deck from which the heavy fraction is discharged.

It is assumed that the portion of material entering the interval from 0 to $0.33 x_{\max}$ is collected in the bin for the light fraction. Material entering the interval from $0.33 x_{\max}$ to $0.66 x_{\max}$ is directed to the bin for the medium fraction, while material entering the interval from $0.66 x_{\max}$ to $x_{\max}$ is collected in the bin for the heavy fraction.

It is evident that there exists a probability that particles whose densities correspond to the light and medium fractions may enter the bin intended for the heavy fraction. Similarly, particles with densities corresponding to the heavy fraction may enter the bins intended for the light and medium fractions.

The number of particles of the light, medium, and heavy fractions of the seed material that enter the bins designated for the light ($i = 1$), medium ($i = 2$), and heavy ($i = 3$) fractions, respectively, can be determined as follows:

$$S_1 = N_1 \int_{-\infty}^{0.33x_{\max}} f_1(x) dx + N_2 \int_{-\infty}^{0.33x_{\max}} f_2(x) dx + N_3 \int_{-\infty}^{0.33x_{\max}} f_3(x) dx \quad (25)$$

$$S_2 = N_1 \int_{0.33x_{\max}}^{0.66x_{\max}} f_1(x) dx + N_2 \int_{0.33x_{\max}}^{0.66x_{\max}} f_2(x) dx + N_3 \int_{0.33x_{\max}}^{0.66x_{\max}} f_3(x) dx \quad (26)$$

$$S_3 = N_1 \int_{0.66x_{\max}}^{\infty} f_1(x) dx + N_2 \int_{0.66x_{\max}}^{\infty} f_2(x) dx + N_3 \int_{0.66x_{\max}}^{\infty} f_3(x) dx \quad (27)$$

where: S_1 , S_2 , and S_3 denote the numbers of particles in the bins designated for the light ($i = 1$), medium ($i = 2$), and heavy ($i = 3$) fractions, respectively; $f_1(x)$, $f_2(x)$, and $f_3(x)$ are the probability density functions for particles of the light ($i = 1$), medium ($i = 2$), and heavy ($i = 3$) fractions, respectively; and N_1 , N_2 , and N_3 represent the total numbers of particles in the corresponding fractions.

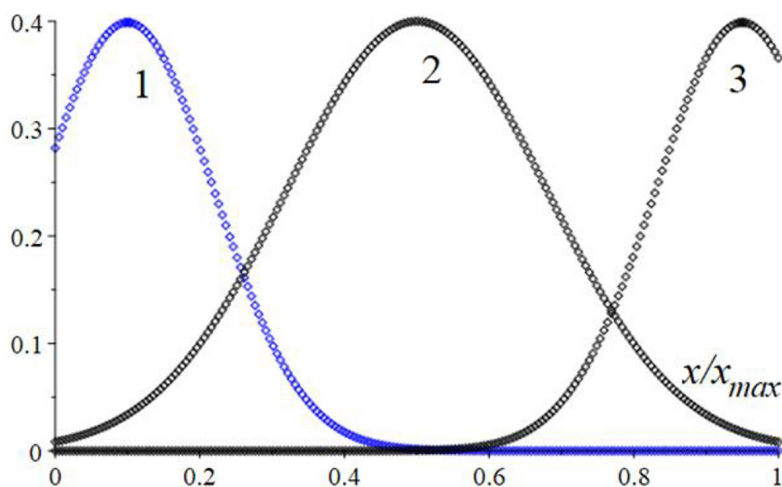


Figure 4. Probability density functions of the coordinates of the “light,” “medium,” and “heavy” fractions of seed material on the pneumatic sorting table: 1 – light fraction; 2 – medium fraction; 3 – heavy fraction

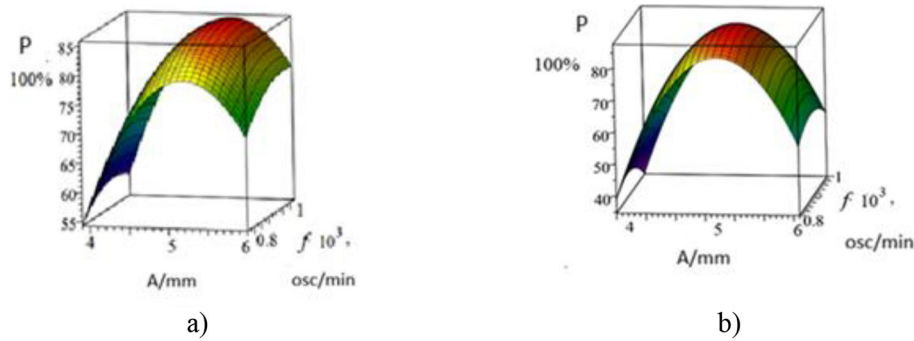


Figure 5. Variation of the purity of the main seed fraction as a function of vibration frequency (f) and amplitude (A) of the pneumatic sorting table:(a) soybean; (b) chickpea

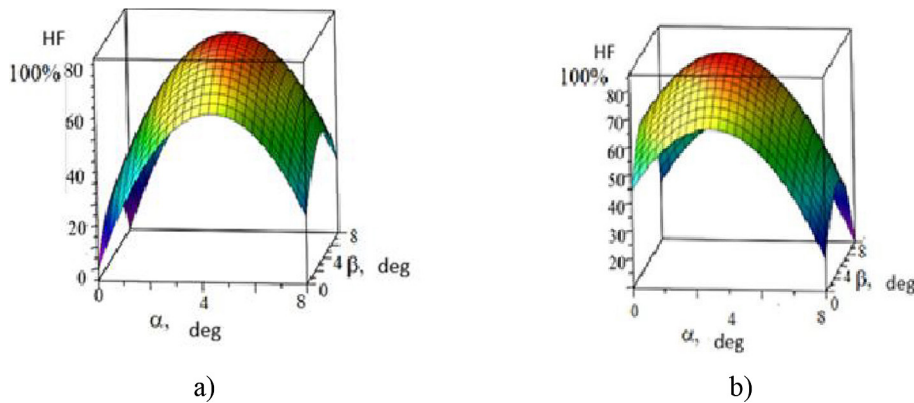


Figure 6. Variation of the purity of the main seed fraction as a function of the longitudinal (α) and transverse (β) inclination angles of the pneumatic sorting table deck: (a) soybean; (b) chickpea

Considering that there is no predefined distribution of particles by density in the initial seed material within the boundaries between the light, medium, and heavy fractions, it is assumed that $N_1 = N_2 = N_3$.

Since the goal of material separation on the pneumatic sorting table is to obtain high-quality, biologically active seed material, the purity of the heavy fraction is adopted as the primary criterion for evaluating the efficiency of the separation process. The purity of the heavy fraction is defined as the ratio of the number of heavy-fraction particles in the corresponding bin ($i = 3$) to the total number of particles collected in that bin:

$$U_B = N_3 \int_{0.66x_{max}}^{\infty} f_3(x) dx / S_3 \cdot 100\%. \quad (28)$$

Given that the density distribution of particles in the initial seed material within the transition ranges between the “light,” “medium,” and “heavy” fractions is not specified, it is assumed that $N_1 = N_2 = N_3$. With this assumption, Equation 28 can be rewritten as A and f .

$$U_B = \frac{\int_{0.66x_{max}}^{\infty} f_3(x) dx}{\int_{0.66x_{max}}^{\infty} f_1(x) dx + \int_{0.66x_{max}}^{\infty} f_2(x) dx + \int_{0.66x_{max}}^{\infty} f_3(x) dx} \cdot 100\% \quad (29)$$

Figure 5 shows the distribution of the heavy fraction purity $HF(A, f)$ for soybeans and chickpea.

The numerical simulations indicate that the optimal operating parameters depend on the physical and mechanical properties of the processed seed material. In particular, the highest purity of the target fraction was obtained at an airflow velocity of 1.2–1.4 m/s for soybean and 1.5–1.8 m/s for chickpea. These results demonstrate that the proposed mathematical model can be used to determine crop-specific operating conditions for pneumatic sorting tables while maintaining a high degree of separation efficiency (Figure 6).

CONCLUSIONS

The obtained numerical results can be used as a practical basis for selecting the operating

parameters of pneumatic sorting tables depending on the physical properties of the processed seed material. The calculated particle trajectories make it possible to predict the degree of spatial separation of light, medium, and heavy fractions before conducting experimental trials. Consequently, the proposed model enables the preliminary adjustment of airflow velocity, vibration frequency, vibration amplitude, and deck inclination in accordance with the density characteristics of a particular crop, thereby reducing the time and cost required for machine setup.

The developed model also provides an effective tool for identifying operating regions that ensure maximum purity of the target fraction while minimizing the loss of biologically valuable seeds. Analysis of the calculated trajectories and probability distributions demonstrates that excessive airflow velocity or vibration intensity may increase the mixing of neighboring fractions, whereas insufficient values of these parameters reduce the degree of stratification. Therefore, the model allows rational operating regimes to be selected by balancing separation efficiency and seed recovery, rather than relying solely on empirical adjustments.

From a practical perspective, the proposed approach can be applied not only during the design of new pneumatic sorting tables but also during the modernization and optimization of existing equipment. By replacing many trial-and-error experiments with numerical simulations, manufacturers and operators can determine the most appropriate operating conditions for different crops, including soybean and chickpea, while accounting for their physical and mechanical properties. This significantly improves process stability, increases the purity of the separated seed fraction, and reduces operating costs associated with repeated machine adjustment and experimental calibration.

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