

Parameter identification of a macroscopic hardening law for metals based on crystal plasticity theory assisted by fuzzy logic

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ABSTRACT

Modeling the mechanical behavior of metallic materials under loading, while accounting for both macroscopic deformation and microstructural evolution, is computationally demanding, particularly when crystal plasticity models requiring numerous hardening parameters are employed. Consequently, the reliable identification of these parameters represents a key challenge for the practical application of such models. This study investigated the application of a macroscopic phenomenological hardening function containing only four material parameters for the description of strain hardening in selected metals. An optimization-based procedure was employed to identify the hardening parameters from experimental stress–strain curves. A sensitivity analysis was additionally performed to quantify the influence of individual hardening parameters on the model response. Since the objective function may exhibit multiple local minima, a fuzzy logic-based algorithm is introduced to improve the robustness of parameter selection and identify the most representative solution under uncertainty. The proposed methodology was validated using experimental data for selected metals with different structures, demonstrating very good agreement between numerical predictions and experimental hardening curves. The results indicate that the combined optimization and fuzzy logic approach enables reliable parameter identification while significantly reducing the complexity and computational cost associated with conventional crystal plasticity hardening models. The proposed framework provides an efficient methodology for identifying the parameters of a macroscopic hardening law derived from crystal plasticity theory. Since the constitutive model was developed for subsequent application in crystal plasticity-based polycrystal modeling, the identified parameters constitute a reliable basis for their future implementation in CPFEM simulations.

Keywords: strain hardening, crystal plasticity, hardening function, modeling, fuzzy logic.

INTRODUCTION

Strain hardening represents a significant phenomenon in mechanical and materials engineering, as it strongly influences the mechanical performance of materials and determines their suitability for various engineering applications. This process leads primarily to an increase in yield strength and ultimate tensile strength, while simultaneously causing a decrease in ductility and impact resistance of the material [1]. From a microscopic point of view,

one of the mechanisms responsible for hardening in face-centered cubic (FCC) metals is the collinear interaction of dislocations. This interaction results from the mutual attraction of dislocations moving along the same slip plane [2]. As deformation progresses, the density of dislocations within the material increases, leading to a state of saturation that limits the formation and movement of additional dislocations [3].

At the macroscopic level, the strain hardening behavior of materials can be characterized by obtaining stress–strain relationships through

conventional uniaxial tensile or shear tests. The shape of the hardening curve varies depending on the material properties and several influencing parameters, including the applied loading conditions and the number of available slip systems. Consequently, distinct stages of hardening can be identified during plastic deformation (Figure 1). The evolution of the hardening response is also affected by the crystal structure and grain size of the material.

According to Roth et al. [4], the shear stress–shear strain relationship can be divided into three main stages of deformation hardening (Figure 2). Stage I, referred to as the easy-glide stage, appears directly after yielding and is typically observed in single crystals undergoing deformation through a single slip system. This stage is characterized by a very low hardening coefficient ($\frac{d\tau}{d\gamma} = \frac{G}{3000}$) [5]. The limited increase in stress during this region results from the relatively low dislocation density and the absence of significant obstacles to dislocation motion along the active slip plane. It should be noted that Stage I is generally not observed in the metals with a body-centered cubic (BCC) crystal structure [6]. Stage II corresponds to the linear hardening regime, where plastic deformation involves both primary and secondary slip systems [7]. During this stage, the material experiences an increase in dislocation density, the generation of point defects, and the activation of Frank–Read sources, which

contribute to continuous dislocation multiplication and plastic strain development [8]. The linear character of this region is associated with a high hardening rate ($\frac{d\tau}{d\gamma} = \frac{G}{3000}$), resulting mainly from the increasing interactions between dislocations, including their mutual obstruction and accumulation [9]. Stage III represents the parabolic hardening region, where the hardening rate gradually decreases with increasing deformation [10]. This behavior is related to processes such as dynamic recovery and dislocation annihilation, which reduce the efficiency of further strengthening mechanisms [11].

Some studies [12–14] additionally distinguish two further stages of deformation. Stage IV is associated with a nearly constant average grain size and a low, approximately constant hardening rate, particularly under low-temperature deformation conditions. Stage V occurs when the characteristics of Stage IV disappear, resulting in a gradual reduction of the hardening rate toward zero [15].

The strain hardening phenomenon is an essential aspect that must be incorporated into numerical modeling approaches. Nevertheless, choosing an appropriate constitutive model capable of reproducing experimental hardening behavior with sufficient accuracy remains a challenging issue and has been widely discussed in the literature [16–17]. Over the years, numerous constitutive relationships have been proposed to describe strain hardening behavior at both

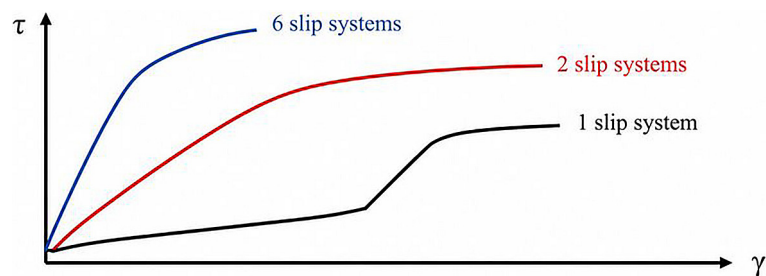


Figure 1. Shape of the shear stress-shear strain curve for a single and several slip systems in FCC metals

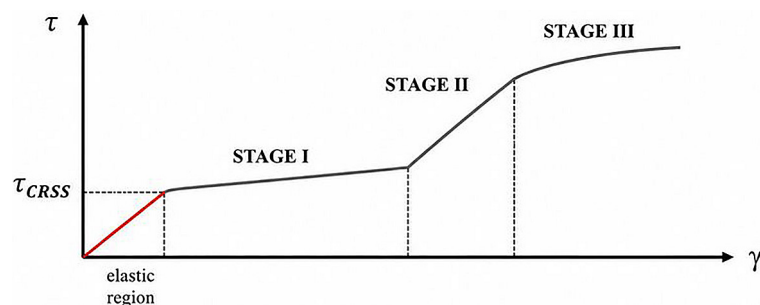


Figure 2. Stages of work hardening on the shear stress-shear strain curve for a single slip system in FCC metals

macroscopic and microscopic scales. Among the most frequently applied formulations are exponential and power-law functions, including the models introduced by Hollomon [18], Swift [19], Voce [20], and Ludwik [21]. Despite their widespread use, conventional macroscopic models have certain limitations. In particular, they often fail to account for the saturation stress observed in experimental hardening curves. Moreover, many of these empirical relationships are suitable only for representing specific regions of the strain hardening response rather than the entire deformation process. For instance, the Hollomon equation provides a satisfactory description mainly of the later stage of hardening, which restricts its applicability for comprehensive modeling of the complete stress–strain behavior.

Crystal plasticity (CP) approaches provide another possibility for describing strain hardening behavior in numerical simulations. These models are commonly formulated as power-law relationships, where individual parameters are assigned to specific slip systems. Such an approach enables effective prediction of hardening responses for various materials under a broad range of loading conditions [22]. An important advantage of crystal plasticity-based formulations is their ability to account for stress saturation effects, as well as their applicability to both small and large deformation ranges. Among the hardening formulations most frequently implemented within crystal plasticity frameworks are the models proposed by Ludwigson [23], Kocks–Mecking [24], and Estrin–Tóth [25]. Although these approaches generally require only a limited number of material parameters, their accurate identification remains a complex and computationally demanding procedure. The optimization-based calibration process often requires numerous iterations to achieve satisfactory agreement and convergence between experimental observations and numerical predictions.

The limitations associated with neglecting saturation stress in conventional macroscopic hardening laws, together with the difficulties involved in calibrating parameters of crystal plasticity models, motivated Sahoo et al. [26] to propose a new macroscopic hardening formulation derived from crystal plasticity theory. The developed exponential relationship contains only four material parameters and incorporates the stress saturation phenomenon observed during experimental investigations. In their study, Sahoo et al.

[26] demonstrated that the calibrated parameters can be directly used in polycrystal viscoplastic self-consistent (VPSC) modeling for crystal plasticity-based incremental strain-hardening simulations, while significantly reducing the computational effort compared with conventional crystal plasticity hardening models.

Although the constitutive model proposed by Sahoo et al. [26] was developed for subsequent implementation in crystal plasticity frameworks, the reliable identification of its material parameters remains a critical prerequisite for such applications. Therefore, the present study focuses exclusively on the calibration of the macroscopic hardening law from experimental stress–strain curves and on improving the robustness of the identification procedure through optimization and fuzzy logic. The implementation and validation of the calibrated parameters in CPFEM or polycrystal simulations are beyond the scope of the present work and constitute a subject of future research.

In the present study, the applicability of the hardening equation proposed by Sahoo et al. was investigated through simulations of simple loading conditions, including uniaxial tensile deformation. An optimization procedure for the identification of the corresponding hardening parameters was also employed. The obtained results demonstrated that the proposed model provides good agreement with experimental observations. Future work will focus on applying this formulation to simulations at the single-crystal level, where plastic deformation is described by dislocation slip mechanisms.

Although the macroscopic hardening formulation proposed by Sahoo et al. represents a promising alternative to traditional crystal plasticity hardening models due to its limited number of material parameters, its effective implementation depends on the availability of a reliable parameter calibration strategy. A significant challenge in this process is the possibility of obtaining multiple local optima during optimization, which may result in non-unique solutions and difficulties in selecting a physically meaningful parameter set. Therefore, an efficient identification approach is required to improve both the accuracy and reliability of the parameter estimation procedure. The objective of this study was to investigate the applicability of the four-parameter macroscopic hardening model for selected face-centered cubic (FCC) and body-centered cubic (BCC) metals and to propose a fuzzy

logic-supported framework for material parameter identification. The developed methodology is intended to reduce the uncertainties associated with the inverse determination of hardening parameters. The novelty of the presented approach lies in combining a physically based hardening formulation with fuzzy logic-assisted decision-making, providing a robust solution to the non-uniqueness problem commonly encountered in parameter identification. By integrating optimization techniques with fuzzy logic, the proposed framework enables the determination of representative hardening parameters that allow accurate reconstruction of experimental hardening curves while preserving computational efficiency. As a result, the proposed methodology extends the applicability of the macroscopic hardening formulation by providing a robust and reliable parameter identification procedure. The identified parameters constitute a suitable basis for future implementation of the constitutive model in crystal plasticity simulations.

THE PROBLEM FORMULATION

The following hardening function on a macroscopic scale is used here (Equation 1):

$$\dot{\sigma}_0(\bar{\epsilon}) = h \left(1 - \frac{\sigma_0(\bar{\epsilon})}{\sigma_{sat}} \right)^a \bar{\epsilon} \quad (1)$$

where: $\bar{\epsilon}$ is a macroscopic von-Mises equivalent strain rate, σ_0 is a reference strength of the material, σ_{sat} is a saturation stress, h and a are hardening parameters which determine the hardening rate and the shape of the hardening curve.

After the integration of Equation 1, the following equation can be obtained (Equation 2):

$$\sigma_0(\bar{\epsilon}) = \sigma_{sat} - \sigma_{sat} \left[\left(\frac{h(a-1)}{\sigma_{sat}} \right) \bar{\epsilon} + \left(1 - \frac{\sigma_{00}}{\sigma_{sat}} \right)^{1-a} \right]^{\frac{1}{1-a}} \quad \text{for } a \neq 1 \quad (2)$$

where: σ_{00} is the yield strength of the material at $\bar{\epsilon} = 0$.

Compared with the hardening formulations commonly used in crystal plasticity theory (Equation 3), the model considered in this study requires the identification of only four unknown parameters, which significantly simplifies the calibration

procedure. These parameters, appearing in the proposed hardening equation, are denoted as σ_{00} , σ_{sat} , h , and a . The initial yield stress of the material (σ_{00}) is directly obtained from experimental results acquired during the uniaxial tensile test at zero plastic strain ($\epsilon_p = 0$). The remaining parameters are determined through an optimization procedure based on the scalar objective function defined in Equation 4, using the experimental stress-strain data as the reference.

$$\tau_0^i = \sum_{j=1}^n q_c^{ij} h_c \left(1 - \frac{\tau_0^i}{\tau_{sat}^i} \right)^{a_c} |\dot{\gamma}^j| \rightarrow \min \quad (3)$$

where: τ_0^i is a resolved shear stress on the i slip system, $\dot{\gamma}^j$ is the resolved shear strain rate on the j slip system, n is the number of slip systems, q_c^{ij} is a square $n \times n$ matrix assuming the self and the latent hardening, h_c i a_c are hardening parameters and τ_{sa}^i is the stress related with the saturation of the material.

$$f = \sum_{i=1}^N |\sigma_i^{sim}(\bar{\epsilon}_i) - \sigma_i^{exp}(\bar{\epsilon}_i)| \quad (4)$$

where: N is the number of points used for the determination of hardening parameters for the analytical function obtained from the stress-strain curve.

Additionally, the proposed macroscopic formulation simplifies the description of self and latent hardening effects, which are not explicitly taken into account in the present model. This simplification significantly reduces the computational effort required for numerical simulations.

To identify the aforementioned parameters, the optimization algorithm proposed in [26] was employed. At the first stage of the identification procedure, the admissible ranges of the hardening parameters were established according to literature recommendations and preliminary numerical tests. The search space was discretized into 100 intervals for each parameter and all generated parameter combinations were evaluated using the objective function. The parameter set providing the lowest objective function value was subsequently used to define a reduced search domain. The optimization process was repeated iteratively until the prescribed convergence criterion was achieved. The identification process was continued until the minimum value of the objective function was obtained with the required accuracy of 0.01 MPa.

RESULTS AND DISCUSSION

The macroscopic hardening equation presented in the previous section (see Equation 2), together with the proposed parameter identification algorithm, was evaluated through simulations of strain hardening curves for various metallic materials subjected to uniaxial tensile loading at room temperature. It should be emphasized that the present study was limited to the calibration and validation of the macroscopic constitutive equation using experimental uniaxial stress–strain data. No crystal plasticity finite element (CPFEM) or polycrystal simulations were performed, as the objective of this work was to establish a reliable parameter identification methodology for subsequent applications of the model. The experimental stress–strain datasets used for calibration were obtained for four metallic materials exhibiting different hardening behaviors.

Three of the investigated materials, namely A1 aluminum, D63 brass, and B100 copper, are characterized by a face-centered cubic (FCC) crystal structure. These materials were selected to assess the capability of the proposed hardening formulation for metals in which plastic deformation is primarily governed by dislocation slip mechanisms. Additionally, DC04 low-carbon steel with a body-centered cubic (BCC) crystal structure was considered as a reference material to evaluate the robustness of the proposed parameter identification approach and its potential applicability beyond FCC metals. The main characteristics of the analyzed materials are presented in Table 1.

The hardening parameters presented in Table 2 were identified from the experimental stress–strain curves of the investigated materials using the proposed optimization-based identification procedure. A comparison between the experimental and numerically predicted hardening curves is presented in Figure 3, while the corresponding hardening rates are shown in Figure 4. The parameters used to evaluate the accuracy of the curve fitting are summarized in Table 3.

The obtained results indicate that the proposed hardening formulation is capable of accurately describing the experimental hardening response of all investigated materials. The calculated RMSE values were within the range from 1.13 MPa for DC04 steel to 9.22 MPa for D63 brass, whereas the MAE values remained below 8.4 MPa for each analyzed material. Although the highest absolute deviation was observed for D63 brass, the maximum relative error was limited to 3.5%. Additionally, the coefficient of determination exceeded 0.99 in all cases, confirming a very high level of correlation between the numerical predictions and experimental measurements. These findings verify that the proposed parameter identification methodology enables reliable determination of hardening parameters and provides accurate reconstruction of experimental hardening curves for the considered metals.

Following the identification of the hardening parameters, a sensitivity analysis was performed to quantify the influence of individual parameters on the predicted hardening response. The purpose of this analysis was to determine the relative importance of each parameter for the prediction of the material behavior. The investigated

Table 1. Characteristic of materials used in tests

No:	Material designation	Material	Crystal structure
1.	DC04	Low-carbon steel	BCC
2.	A1	Commercially pure aluminum	FCC
3.	D63	– brass	FCC
4.	B100	Commercially pure copper	FCC

Table 2. Hardening parameters of the function for different materials

No:	Material	Yield stress σ_{00} [MPa]	Saturation stress σ_{sat} [MPa]	Hardening coefficient h [MPa]	Hardening exponent a [-]
1.	DC04 steel	175	1690	14010	14.23
2.	A1 aluminum	27	477.8	6364	19.80
3.	D63 brass	103	1962	1888	8.40
4.	B100 copper	43	772.5	2906	1.62

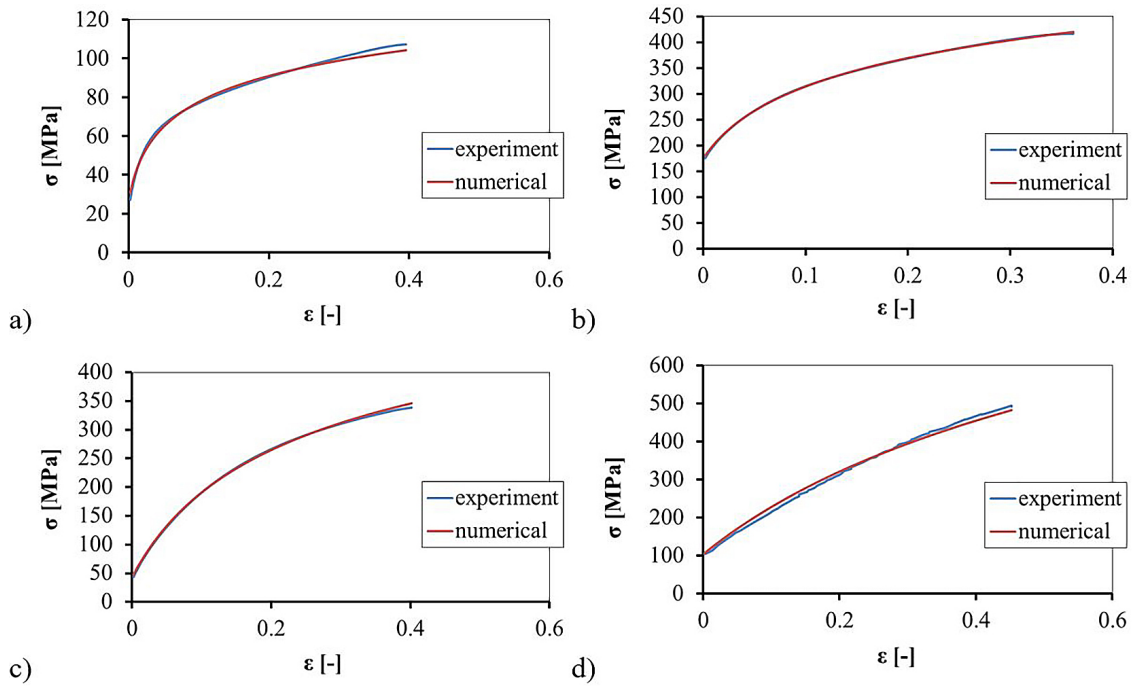


Figure 3. Comparison of experimental and numerical hardening curves for: a) aluminum, b) steel, c) copper and d) brass

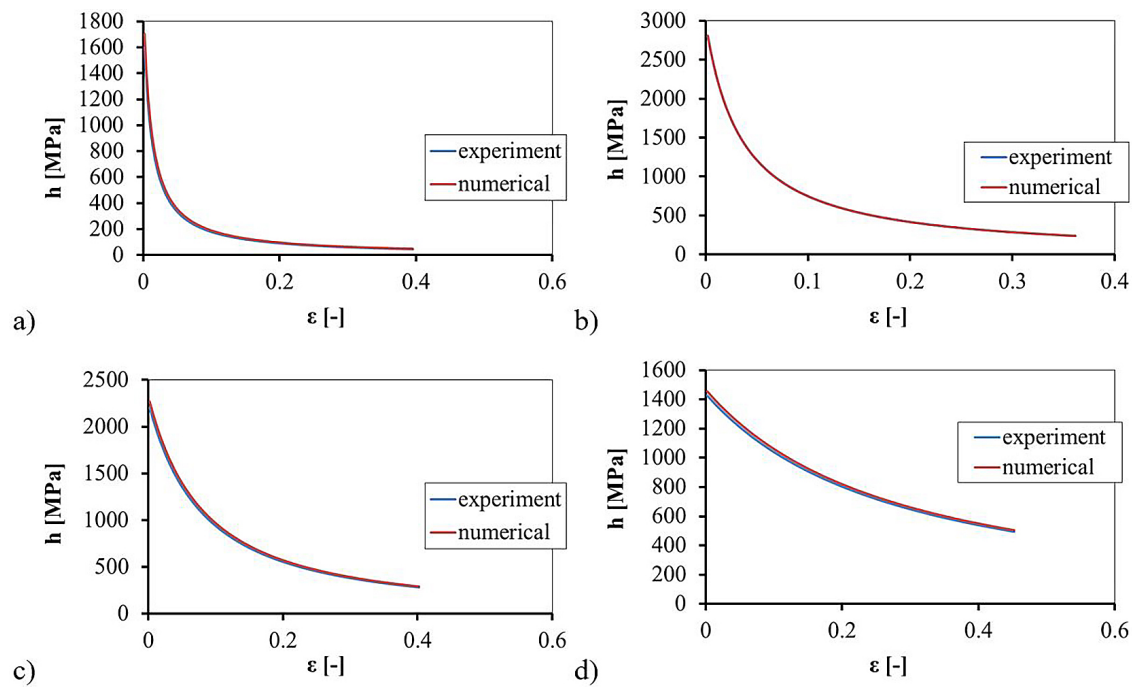


Figure 4. The strain hardening rate for a) aluminum, b) steel, c) copper and d) brass

Table 3. The quality of the model fitting

No:	Material	RMSE [MPa]	MAE [MPa]	Maximum error [%]	R^2 [-]
1.	DC04 steel	1.13	0.79	1.31	0.999
2.	A1 aluminum	1.34	1.07	3.50	0.995
3.	D63 brass	9.22	8.36	2.81	0.993
4.	B100 copper	2.03	1.62	2.26	0.999

Note: RMSE – root mean square error, MAE – mean absolute error, R^2 – coefficient of determination.

Table 4. Ranking of hardening parameter sensitivity based on the mean sensitivity coefficient

No:	Material	Mean sensitivity coefficient [-]			Sensitivity ranking
		$\bar{S}_h$	$\bar{S}_{\sigma_{sat}}$	$\bar{S}_a$	
1.	DC04 steel	0.52	1.39	1.27	$\sigma_{sat} > a > h$
2.	A1 aluminum	0.36	1.11	1.02	$\sigma_{sat} > a > h$
3.	D63 brass	0.75	0.44	0.42	$h > \sigma_{sat} > a$
4.	B100 copper	0.50	0.65	0.43	$\sigma_{sat} > h > a$

Note: $\bar{S}_h$ – mean sensitivity coefficient of h parameter, $\bar{S}_{\sigma_{sat}}$ – mean sensitivity coefficient of σ_{sat} parameter, $\bar{S}_a$ – mean sensitivity coefficient of a parameter.

hardening model was described by the parameter vector (Equation 5). The initial yield stress σ_0 was considered separately, since it defines the initial stress level and does not directly control the evolution of strain hardening.

$$p = [h, a, \sigma_{sat}] \quad (5)$$

The sensitivity analysis was performed by perturbing each identified parameter by $\pm 10\%$, while keeping the remaining parameters unchanged. The influence of each parameter on the predicted hardening behavior was quantified using the mean sensitivity coefficient $\bar{S}_i$, calculated as the average value of the absolute normalized sensitivity coefficient $S_i(\varepsilon_i)$ over the investigated strain range (Equation 6).

$$\bar{S}_i = \frac{1}{N} \sum_{i=1}^N |S_i(\varepsilon_i)| \quad (6)$$

where: N is the number of strain points from the stress-strain curve.

The obtained mean sensitivity coefficients were used to establish the parameter ranking for each investigated material. The results are

summarized in Table 4. The sensitivity ranking demonstrates that the influence of the hardening parameters is material-dependent and is related to the specific characteristics of the stress–strain response. For steel and aluminum, the saturation stress σ_{sat} was identified as the most influential parameter, followed by the shape parameter a and the hardening coefficient h . This indicates that the hardening behavior of these materials is mainly governed by the development of the saturated dislocation structure and the final strengthening level achieved during plastic deformation [27]. For copper, the saturation stress remained the dominant parameter; however, the hardening coefficient h showed a greater influence than the shape parameter a . This behavior is associated with intensive dislocation multiplication and rearrangement typical of highly ductile FCC metals [28]. In the case of brass, the highest sensitivity was obtained for the hardening coefficient h , followed by σ_{sat} and a . This can be attributed to the influence of solute atoms on dislocation mobility and the resulting dependence of the flow stress on the strain-hardening rate [29].

Table 5. Hardening parameters for different materials

Material	Saturation stress σ_{sat} [MPa]	Hardening coefficient h [MPa]	Hardening exponent a [-]	Error [MPa]
DC04 steel	1690	14010	14.23	5.46
	1567	13545	14.01	5.49
	1789	12097	13.97	5.44
A1 aluminum	477.8	6364	19.80	3.76
	481.0	6298	19.78	3.77
	475.5	6311	19.23	3.75
D63 brass	1962	1888	8.40	13.90
	1957	1867	8.32	13.88
	1998	1891	8.54	13.93
B100 copper	772.5	2906	1.62	7.65
	771.0	2909	1.67	7.69
	773.4	2915	1.73	7.64

Despite the satisfactory agreement obtained between the numerical predictions and experimental hardening curves, the optimization procedure revealed the presence of several local minima of the objective function. As a consequence, different combinations of hardening parameters resulted in comparable objective function values (Table 5), indicating the non-uniqueness of the inverse identification problem. Therefore, selecting the optimal parameter set exclusively according to the lowest objective function value may result in solutions with physically inconsistent or unrealistic parameter combinations.

To overcome this limitation, a novel fuzzy logic-based selection procedure was developed. The proposed fuzzy inference system assesses the identified parameter sets using predefined membership functions and inference rules, enabling the elimination of solutions that are less consistent with the expected material behavior. From the remaining acceptable parameter combinations, the final set of hardening parameters is chosen based on the minimum value of the objective function. Consequently, the proposed methodology is intended to improve the reliability of the parameter calibration stage, which constitutes a prerequisite for the subsequent application of the constitutive model in crystal plasticity-based simulations.

The application of fuzzy logic additionally enables the incorporation of expert knowledge and uncertainty related to parameter identification into the decision-making process. This approach allows the selection of the most physically representative solution among several parameter sets exhibiting similar fitting accuracy. Consequently, the proposed hybrid optimization–fuzzy methodology reduces the ambiguity associated with conventional inverse identification procedures and provides a reliable framework for determining physically meaningful hardening parameters while maintaining close agreement with experimental data.

Fuzzy logic represents a soft-computing approach that allows the description of systems characterized by uncertainty, imprecision, or incomplete information. This methodology is particularly suitable for problems in which the available knowledge about the analyzed phenomenon is limited, or the governing relationships are not fully defined [30]. Unlike classical deterministic approaches, fuzzy logic enables the representation of input quantities as fuzzy variables with a certain degree of variability. Through an appropriate mapping procedure, fuzzy input

variables are transformed into fuzzy output quantities, which can subsequently be converted into discrete values during the defuzzification stage.

The application of fuzzy-based methods is especially advantageous in the cases where experimental investigations are costly, time-consuming, or difficult to reproduce. A detailed description of fuzzy theory and examples of its implementation in various engineering applications can be found in [31–34]. The possibility of applying fuzzy logic for hardening parameter identification for another models has also been investigated by the authors in previous studies [35–37].

The α -level approach was applied in this research. The principle of this procedure is based on the transformation of fuzzy input parameters into the result space through the mapping procedure. In the proposed approach, the parameters of the Sahoo constitutive model, namely h , a , and σ_{sat} are treated as fuzzy input variables. The fuzzification step describes the possible variation of the model parameters and allows the influence of their uncertainty on the model response to be evaluated. Triangular membership functions associated with the Sahoo model parameters were applied. The uncertainty range of $\pm 10\%$ was assumed to describe the expected dispersion of the identified Sahoo model parameters. This range was selected based on preliminary calibration analyses and the observed sensitivity of the model response to parameter variations. It provides a compromise between sufficient exploration of the parameter uncertainty and maintaining physically meaningful values of the material parameters. The selected range allows the influence of parameter variability on the stress–strain response. The triangular membership functions for one of input parameter h , were defined using the following equation (Equation 7)

$$\mu(h) = \begin{cases} 0 & \text{for } h < h_L \\ \frac{h-h_L}{h_0-h_L} & \text{for } h_L \leq h \leq h_0 \\ \frac{h_U-h}{h_U-h_0} & \text{for } h_0 \leq h \leq h_U \\ 0 & \text{for } h > h_U \end{cases} \quad (7)$$

where: h_0 represents the nominal value of the fuzzy parameter, while h_L and h_U denote the lower and upper limits of the assumed uncertainty range, respectively. The same triangular membership function was applied independently to another Sahoo model parameters – a and σ_{sat} .

The membership functions of the fuzzy input variables were divided into a sufficient number of α -levels. For each selected α -level, the boundary values of the fuzzy parameters were determined. For the defined intervals of fuzzy parameters, the stress–strain response was calculated using the Sahoo constitutive equation. The minimum and maximum values of the approximation error were searched for each α -level. These values define the shape of the fuzzy output membership function associated with the approximation error. The searching procedure was repeated for all considered α -levels. The fuzzy output variable (z), representing the approximation error between experimental and calculated stress–strain curves, was obtained. The developed MATLAB procedure performs the searching process by generating randomly distorted model parameters within the defined fuzzy ranges and evaluating the corresponding model response. The transformation of fuzzy results into the output crisp value was

performed using the mass center method (Figure 5). The most reliable discrete value of the approximation error z_0 was determined from the output membership function $\mu(z)$ using the equation (Equation 8).

$$z_0 = \frac{\int z \cdot \mu(z) dz}{\int \mu(z) dz} \quad (8)$$

The hardening parameters identified using the fuzzy logic-based procedure are summarized in Table 6. The comparison between the experimental and numerically predicted hardening curves is presented in Figure 6. Although the approximation error remained nearly unchanged, the fuzzy-based approach eliminated physically inconsistent parameter combinations and reduced the ambiguity of the inverse identification problem. Therefore, the proposed methodology provides a more realistic representation of the variability observed in experimental measurements, as well as uncertainties related to material processing conditions

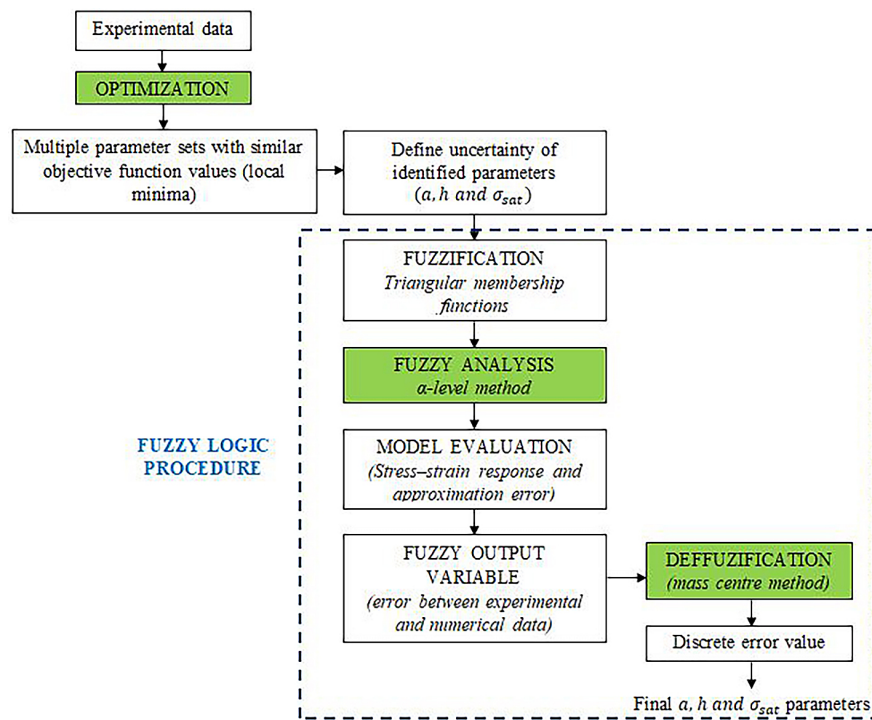


Figure 5. The idea of the fuzzy logic algorithm used in this paper

Table 6. Hardening parameters obtained by fuzzy logic algorithm

No:	Material	Yield stress σ_{00} [MPa]	Saturation stress σ_{sat} [MPa]	Hardening coefficient h [MPa]	Hardening exponent a [-]	Error [MPa]
1.	DC04 steel	175	1876	15117	16.6	5.52
2.	A1 aluminum	27	475	5522	18.5	3.71
3.	D63 brass	103	2303	2207	6.9	13.96
4.	B100 copper	43	1561	3251	9.5	7.72

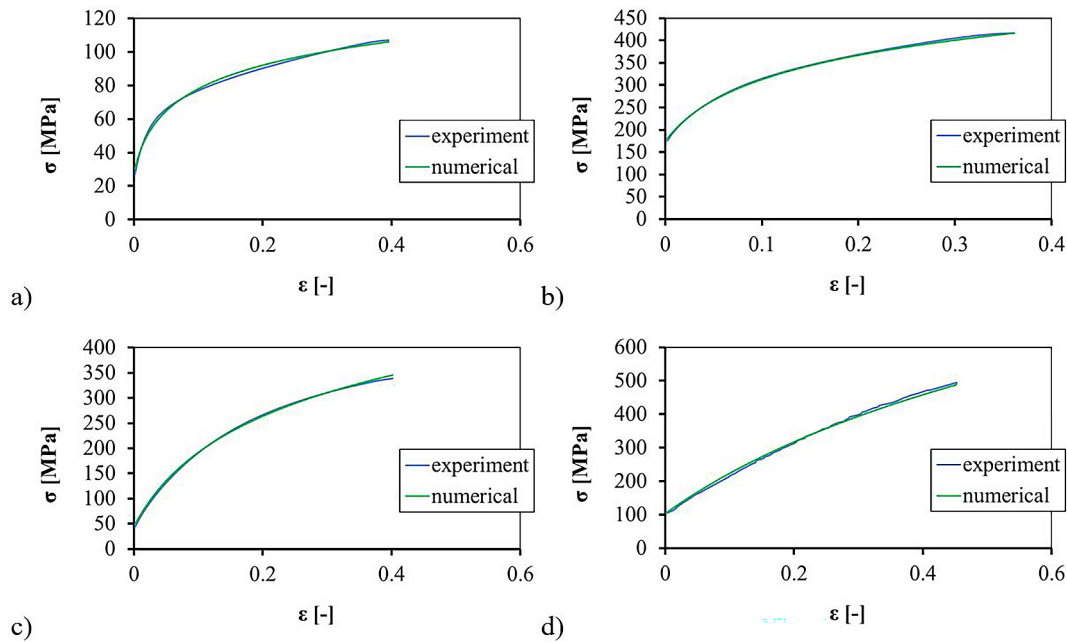


Figure 6. Comparison of experimental and numerical hardening curves for: a) aluminum, b) steel, c) copper and d) brass; after the application of fuzzy logic algorithm

and measurement inaccuracies. In contrast to conventional deterministic calibration methods, the fuzzy-based framework does not restrict the material parameters to single fixed values. Instead, it represents them as fuzzy ranges characterized by corresponding membership degrees. Such a formulation improves the robustness of the identification process and enables the selection of parameter sets that are more consistent with the physical behavior of the investigated materials.

The obtained results confirm that the proposed optimization–fuzzy logic framework improves the robustness of parameter identification while maintaining excellent agreement with the experimental stress–strain curves. Since the investigated constitutive equation was originally developed by Sahoo et al. [26] for subsequent application in crystal plasticity-based polycrystalline modeling, improving the reliability of its calibration is an important prerequisite for such simulations. Nevertheless, the implementation and verification of the calibrated parameters within CPFEM or VPSC models remain beyond the scope of the present work and will be addressed in future studies.

It should be emphasized that the identified value of σ_{sat} in the Sahoo model represents a constitutive fitting parameter and should not be interpreted directly as the experimentally measured tensile strength or physical saturation stress of the investigated material. The parameters of the constitutive equation are strongly coupled, and

the identification procedure determines their optimal combination rather than independent physical quantities. Therefore, an increase in the value of one parameter may be compensated by variations of other parameters, while maintaining or improving the agreement between the calculated and experimental stress–strain curves. The higher values of σ_{sat} obtained using the fuzzy-based approach result from this parameter interaction and from the broader exploration of the admissible parameter space during the calibration procedure.

CONCLUSIONS

The four-parameter macroscopic hardening formulation proposed by Sahoo et al. was employed in this study to describe the mechanical response of materials subjected to external loading. The model, derived from the crystal plasticity framework, incorporates both self-hardening and latent hardening effects. The initial yield stress (σ_{00}), saturation stress (σ_{sat}), and hardening-related coefficients (a and h) constitute the unknown parameters of the formulation. These parameters can be obtained either directly from experimental stress–strain data or through an optimization-based identification procedure. The hardening curves predicted using the proposed macroscopic model were compared with experimental results to assess the accuracy of the formulation. Since

the objective function applied during parameter calibration may exhibit multiple local extrema, leading to ambiguity in the identified solution, a fuzzy logic-based procedure was introduced to improve the reliability of parameter selection.

The main conclusions drawn from this study are summarized as follows:

1. The four-parameter macroscopic hardening law provides an accurate representation of the strain-hardening behavior of the investigated metals, showing very good agreement with the experimental stress–strain responses.
2. The proposed parameter identification methodology demonstrates its effectiveness for FCC materials and, following appropriate validation, may also be extended to selected BCC metals.
3. The presence of multiple local minima in the objective function highlights the non-uniqueness of the inverse identification problem and demonstrates the limitations of conventional optimization methods based solely on objective function minimization.
4. The integration of fuzzy logic into the identification procedure enables the consideration of expert knowledge and parameter uncertainty, supporting the selection of physically meaningful and robust parameter sets while maintaining high fitting accuracy.
5. The proposed optimization–fuzzy inference framework provides a computationally efficient and reliable methodology for identifying the parameters of the macroscopic hardening law while explicitly accounting for the uncertainty associated with multiple optimization solutions. The resulting constitutive parameters are therefore more robust and representative than those obtained from a single optimum solution. Since the constitutive equation was originally developed for crystal plasticity-based polycrystal modeling, the identified parameters constitute a reliable basis for future implementation as well as validation in CPFEM and VPSC simulations.

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