

Effect of cylinder diameter and unequal loading on the motion asymmetry of two independent pneumatic actuators

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ABSTRACT

This study investigated the influence of cylinder diameter and unequal external loading on the motion asymmetry of two independent double-acting pneumatic actuators. The analysis was motivated by rehabilitation-device applications in which two actuators are expected to perform simultaneous or symmetrical motion despite differences in external resistance. A reduced-order mathematical model was developed in MATLAB and included piston dynamics, nonlinear friction, variable chamber and tube volumes, isothermal pressure evolution, and compressible airflow through effective inlet and outlet restrictions. Two actuator configurations, with piston diameters of 32 and 40 mm and a nominal stroke of 0.4 m, were analysed under two unequal-loading conditions derived from biomechanical calculations and rehabilitation-device operating conditions. The results demonstrated that load asymmetry produced measurable differences in piston-rod displacement, velocity, and stroke-completion time. Under the minimum unequal-load condition, both actuators completed the full stroke. The maximum displacement discrepancy reached 15.03 mm for the 32 mm cylinders and 12.88 mm for the 40 mm cylinders. Under the maximum unequal-load condition, the discrepancy increased to 91.73 mm and 71.02 mm, respectively, while the more heavily loaded actuator did not reach 99% of the nominal stroke within the 25 s simulation interval. Increasing the piston diameter from 32 to 40 mm reduced the maximum displacement discrepancy by approximately 14% under the minimum unequal-load condition and by approximately 23% under the maximum unequal-load condition. The results show that a larger piston diameter improves resistance to unequal loading but does not ensure synchronous motion. The developed model can support preliminary actuator selection and assessment of whether an additional pneumatic, mechanical, or electronic synchronisation system is required.

Keywords: pneumatic actuators; piston diameter; pneumatic-cylinder modelling; rehabilitation devices; motion asymmetry.

INTRODUCTION

Pneumatic actuation is widely used in industrial automation, mechatronic systems, assistive technologies, and rehabilitation equipment because of its low mass, favourable force-to-weight ratio, structural simplicity, cleanliness, and inherent compliance. These characteristics are especially valuable in devices that interact directly with patients. Pneumatic cylinders, artificial muscles, and soft actuators have consequently been applied in orthopaedic rehabilitation devices and hand or wrist exoskeletons [1,2], as well as in

parallel ankle-rehabilitation robots and soft rehabilitation gloves [3,4]. Recent developments also confirm the continuing importance of compliant actuation in rehabilitation systems [5].

The dynamic response of pneumatic drives is strongly nonlinear because it depends on air compressibility, pressure-dependent mass flow, variable chamber and dead volumes, tube capacity, friction, external loading, and the mechanical dynamics of the driven system. Established pneumatic-actuator models describe these coupled effects using force-balance and thermodynamic equations [6,7], while system-level formulations

provide practical methods for design and simulation [8]. Motion initiation and low-velocity operation are additionally governed by static friction, Coulomb friction, the Stribeck effect, and viscous resistance [9,10]. More detailed friction models can also account for presliding displacement and hysteresis [11].

Seal condition may further modify actuator behaviour. Polyurethane rod seals are exposed to reciprocating sliding, cyclic deformation, contaminants, and wear debris. Service-induced changes in surface morphology and material condition may affect friction and sealing performance, as demonstrated by the EPR and SEM investigations of the seals used in pneumatic drives [12].

When two pneumatic actuators are expected to move simultaneously, even nominally identical cylinders can develop different displacement and velocity histories because of unequal external loads, seal friction, manufacturing tolerances, valve characteristics, and local flow resistance. The observed increase in displacement discrepancy under unequal loading is consistent with earlier studies showing that simultaneous pneumatic-actuator motion may require a dedicated synchronising element [13] or cross-coupled and coordinated-control strategies [14,15]. Unequal loading is particularly relevant in rehabilitation devices, where resistance may differ between the two limbs because of variations in joint mobility, muscular stiffness, limb mass, patient participation, mechanism geometry, or joint friction. Pneumatic-muscle mechanisms likewise illustrate the importance of coordinated actuation in parallel rehabilitation structures [16]. More generally, structural response can be highly sensitive to load distribution [17].

The broader engineering literature confirms that local changes in component condition and boundary conditions can alter global system response. Relevant examples include the influence of degraded paint layers on aerodynamic performance [18], coating effects on structural vibration and sound absorption [19,20], variable stiffness in composite structures [21], rolling resistance associated with tread geometry [22], and damage or fatigue evolution in polymer-matrix composites [23,24]. Although these studies concern different systems, they support the general need to quantify how local asymmetry propagates into overall dynamic behaviour.

Reduced-order mathematical modelling provides an efficient means of separating and

quantifying the mechanisms governing pneumatic-cylinder motion. Such models combine piston force balance, ideal-gas relationships, mass conservation, variable chamber volumes, and compressible airflow, while requiring substantially less computational effort than full CFD analysis [6,8]. Compressible-flow models must distinguish between choked and subcritical conditions and use absolute pressures in pressure-ratio and mass-flow equations [25,26]. Standardised procedures for determining pneumatic-component flow characteristics are provided in ISO 6358-1 [27]. Gauge pressure may be used in mechanical force calculations, whereas absolute pressure is required in pressure-ratio and mass-flow equations.

Cylinder geometry, supply pressure, flow-control settings, external load, friction, and response time are among the principal parameters governing pneumatic positioning performance [28]. Advanced backstepping, sliding-mode, fuzzy, and adaptive controllers have been proposed to compensate for nonlinear airflow and varying loads [29,30]. Digital pneumatic bridge circuits and fuzzy algorithms provide another approach to large-stroke positioning under variable loading [31]. Friction-reduction concepts based on high-frequency vibration [32] and air-floating actuator construction [33] can further improve motion quality, but they do not remove the need to quantify the natural divergence of uncoupled cylinders.

Despite extensive research on rehabilitation robots, pneumatic modelling, friction compensation, and closed-loop synchronisation, the combined influence of piston diameter and unequal loading on the baseline motion asymmetry of two independent double-acting pneumatic cylinders remains insufficiently quantified. In particular, a comparative assessment should determine not only whether both actuators complete the intended stroke, but also the displacement discrepancy, velocity difference, and stroke-completion delay.

Accordingly, the aim of this study was to develop a reduced-order mathematical model of two independent double-acting pneumatic cylinders subjected to unequal external loads. Two configurations with piston diameters of 32 and 40 mm were analysed using loading-pressure combinations derived from a rehabilitation-device research programme. The model included piston dynamics, nonlinear friction, variable chamber and tube volumes, pressure

evolution, and compressible airflow through effective restrictions.

The novelty of the study lies in the quantitative comparison of the sensitivity of the two cylinder configurations to unequal loading before any mechanical, pneumatic, or electronic synchronisation system is introduced. The analysis establishes the baseline motion asymmetry that a future synchronisation device or control algorithm would need to compensate for using displacement histories, stroke-completion times, maximum, mean, root-mean-square, and relative displacement discrepancies, and maximum velocity differences.

NUMERICAL MODEL

A reduced-order mathematical model was developed to analyse the motion of two independent double-acting pneumatic cylinders subjected to unequal external loads. The cylinders operated without a mechanical, pneumatic, or electronic synchronising element. Therefore, the motion of each piston rod was determined independently by the pressure forces acting in the actuator chambers, the external load, friction forces, and compressed-air flow conditions.

The model included the mechanical dynamics of both piston rods, pressure variations in the actuator chambers, variable chamber volumes, compressible airflow through effective flow restrictions, and the additional volume of the pneumatic tubes connecting the valves and cylinders.

The purpose of the model was to quantify the differences in piston-rod displacement, velocity, and stroke-completion time under asymmetric loading conditions. The mathematical formulation was implemented in MATLAB as a system of coupled ordinary differential equations.

Mechanical dynamics of the pneumatic cylinders

For each pneumatic cylinder, the motion of the piston rod was described using the force balance equation. The forces acting on the moving assembly included the pressure forces generated in the cap-side and rod-side chambers, the external load, friction and motion resistance, and the reaction associated with the physical stroke limits.

Gauge pressures were used in the mechanical force balance, whereas absolute pressures were used in the compressible-flow equations. The gauge and absolute chamber pressures were related by:

$$p_{Ai,g} = p_{Ai,abs} - p_{atm} \quad (1)$$

$$p_{Bi,g} = p_{Bi,abs} - p_{atm} \quad (2)$$

where: $p_{Ai,g}$ and $p_{Bi,g}$ denote the gauge pressures used in the mechanical force balance, $p_{Ai,abs}$ and $p_{Bi,abs}$ denote the absolute chamber pressures used in the pressure-dynamics and compressible-flow equations, and p_{atm} is the atmospheric pressure.

The equation of motion for the i -th pneumatic actuator was expressed as:

$$m_i \ddot{x}_i = p_{Ai,g} A_A - p_{Bi,g} A_B - F_{Li} - F_{fi} + F_{Si} \quad (3)$$

where: m_i – equivalent mass of the moving components of the i -th actuator, x_i – piston-rod displacement, $\ddot{x}_i$ – piston-rod acceleration, $p_{Ai,g}$ – gauge pressure in the cap-side chamber, $p_{Bi,g}$ – gauge pressure in the rod-side chamber, A_A – effective piston area on the cap side, A_B – effective piston area on the rod side, F_{Li} – external load force acting on the i -th actuator, $i = 1, 2$, F_{fi} – friction and motion-resistance force, F_{Si} – reaction force associated with the physical stroke limits.

The effective piston area on the cap side was calculated as:

$$A_A = \frac{\pi D^2}{4} \quad (4)$$

The effective piston area on the rod side was calculated as:

$$A_B = \frac{\pi(D^2 - d^2)}{4} \quad (5)$$

where: D – piston diameter, d – piston-rod diameter.

The pressure force generated on the cap side was therefore:

$$F_{Ai} = p_{Ai,g} A_A \quad (6)$$

whereas the pressure force acting on the rod side was:

$$F_{Bi} = p_{Bi,g} A_B \quad (7)$$

The resultant pneumatic force acting on the piston was expressed as:

$$F_{pi} = p_{Ai,g} A_A - p_{Bi,g} A_B \quad (8)$$

Piston motion occurred when the resultant pneumatic force exceeded the combined external load and mechanical resistance. For extension, this condition can be written as:

$$p_{Ai,g}A_A - p_{Bi,g}A_B > F_{Li} + F_{fi} \quad (9)$$

The motion of each actuator was restricted to the physical stroke range:

$$0 \leq x_i \leq L \quad (10)$$

where: L denotes the nominal piston stroke. In the analysed system: $L = 0.4$ m.

The term F_{si} was introduced numerically to prevent the calculated piston displacement from exceeding the physical limits of the cylinder stroke.

Friction and motion-resistance model

The mechanical resistance acting on each piston rod was represented using a nonlinear friction model. The formulation included static friction, Coulomb friction, the Stribeck effect, and viscous resistance. The friction force acting on the i -th actuator was expressed as:

$$F_{fi} = \left[F_c + (F_s - F_c) \exp \left(- \left| \frac{v_i}{v_s} \right|^2 \right) \right] \tanh \left(\frac{v_i}{v_\epsilon} \right) + bv_i \quad (11)$$

where: F_s – static friction force, F_c – Coulomb friction force, v_i piston-rod velocity, v_s – characteristic Stribeck velocity, v_ϵ – smoothing velocity parameter, b – viscous damping coefficient.

The piston-rod velocity was defined as:

$$v_i = \frac{dx_i}{dt} \quad (12)$$

At very low piston velocities, the friction force approaches the static-friction level F_s . As the velocity increases, the friction force decreases toward the Coulomb-friction level F_c , which represents the Stribeck effect. The exponential component describing this transition was written as:

$$F_{Str,i} = (F_s - F_c) \exp \left(- \left| \frac{v_i}{v_s} \right|^2 \right) \quad (13)$$

The viscous component of the motion resistance was expressed as:

$$F_{vi} = bv_i \quad (14)$$

The hyperbolic tangent function:

$$\tanh \left(\frac{v_i}{v_\epsilon} \right) \quad (15)$$

was used to provide a continuous change in the friction-force direction near zero velocity. This numerical smoothing prevents discontinuities associated with the classical sign function and improves the stability of numerical integration.

The total mechanical resistance, therefore, depended on both the piston-rod velocity and the transition between static and sliding friction. The same friction-model structure was applied to both pneumatic cylinders. The adopted formulation was intended to reproduce the dominant influence of friction on piston initiation and motion without introducing a detailed seal-contact model. Therefore, the friction parameters were treated as effective model parameters.

External load model

The external loads acting on the pneumatic actuators were generated by separate loading cylinders. The loading pressure applied to each loading cylinder was assumed to remain constant throughout each simulation case.

The external force acting on the i -th actuator was calculated as:

$$F_{Li} = p_{Li}A_L \quad (16)$$

where: p_{Li} – gauge pressure applied to the corresponding loading cylinder, A_L – effective piston area of the loading cylinder.

The loading-cylinder area was determined from:

$$A_L = \frac{\pi D_L^2}{4} \quad (17)$$

where: D_L – denotes the loading-cylinder piston diameter.

In the present study: $D_L = 0.025$ m.

Therefore:

$$A_L = 4.91 \times 10^{-4} \text{ m}^2 \quad (18)$$

The difference between the forces applied to the two actuators was defined as:

$$\Delta F = |F_{L1} - F_{L2}| \quad (19)$$

Unequal external loading was introduced by assigning different pressures p_{L1} and p_{L2} to the two loading cylinders. The resulting forces were constant during each simulation and independent of piston-rod displacement.

This assumption enabled direct assessment of the influence of load asymmetry on actuator displacement, velocity, and stroke-completion time.

Chamber and tube volumes

The volumes of the pneumatic-cylinder chambers changed continuously with piston displacement. In addition, the internal volume of the tubes connecting the valves and the actuator chambers was included in the model. For the cap-side chamber of the i -th actuator, the total effective volume was expressed as:

$$V_{Ai} = A_A(l_{A0} + x_i) + V_t \quad (20)$$

For the rod-side chamber:

$$V_{Bi} = A_B(l_{B0} + L - x_i) + V_t \quad (21)$$

where: V_{Ai} – total effective volume of the cap-side chamber, V_{Bi} – total effective volume of the rod-side chamber, l_{A0} – initial dead length of the cap-side chamber, l_{B0} – initial dead length of the rod-side chamber, V_t – internal volume of the pneumatic tube.

The internal tube volume was calculated as:

$$V_t = \frac{\pi d_t^2}{4} l_t \quad (22)$$

where: d_t – internal tube diameter, l_t – tube length.

The pneumatic tubes used in the model had an outer diameter of 6 mm, an inner diameter of 4 mm, and a length of 0.30 m, $d_t = 0.004$ m, $l_t = 0.30$ m. The resulting internal tube volume was $V_t = 3.77 \times 10^{-6}$ m³.

The tube volume was added to the corresponding actuator-chamber volume because the compressed air contained in the tube also participated in the pressure dynamics of the system. The inclusion of tube volume was particularly important at the beginning and end of the piston stroke, where the chamber volume was relatively small, and the additional volume of the connecting line had a noticeable influence on the rate of pressure change.

Pressure dynamics in the actuator chambers

The pressure changes in the actuator chambers were determined from the mass balance of compressed air and the variation of chamber volume during piston motion. An isothermal process was assumed, and air was treated as an ideal gas.

The isothermal assumption $T = \text{const}$ applies to the pressure evolution inside the actuator chambers. The specific heat ratio $\kappa = 1.4$, in contrast, is used in the compressible mass-flow formulation to determine the critical pressure ratio and to

distinguish between choked and subcritical flow. Therefore, the use of $\kappa = 1.4$ is not inconsistent with the isothermal chamber model. A fully non-isothermal formulation would require additional energy-balance equations and heat-transfer parameters. For the cap-side chamber of the i -th actuator, the pressure dynamics were expressed as:

$$\frac{dp_{Ai,abs}}{dt} = \frac{RT}{V_{Ai}} \dot{m}_{Ai} - \frac{p_{Ai,abs} A_A v_i}{V_{Ai}} \quad (23)$$

For the rod-side chamber:

$$\frac{dp_{Bi,abs}}{dt} = \frac{RT}{V_{Bi}} \dot{m}_{Bi} + \frac{p_{Bi,abs} A_B v_i}{V_{Bi}} \quad (24)$$

where: R – specific gas constant for air, T – absolute air temperature, $\dot{m}_{Ai}$ – net mass-flow rate into the cap-side chamber, $\dot{m}_{Bi}$ – net mass-flow rate into the rod-side chamber,

The first term in each equation describes the influence of compressed-air mass flow on chamber pressure. The second term represents the effect of chamber-volume variation caused by piston motion.

During piston extension, the cap-side chamber volume increases, whereas the rod-side chamber volume decreases. Therefore:

$$\frac{dV_{Ai}}{dt} = A_A v_i \quad (25)$$

and

$$\frac{dV_{Bi}}{dt} = -A_B v_i \quad (26)$$

The pressure equations can also be written in the general form:

$$\frac{dp}{dt} = \frac{RT}{V} \dot{m} - \frac{p}{V} \frac{dV}{dt} \quad (27)$$

This relation shows that chamber pressure depends simultaneously on the mass-flow rate and the instantaneous rate of volume change. During extension, compressed air flowed into the cap-side chamber and was discharged from the rod-side chamber. Therefore:

$$\dot{m}_{Ai} = \dot{m}_{in,i} \quad (28)$$

$$\dot{m}_{Bi} = -\dot{m}_{out,i} \quad (29)$$

The corresponding pressure equations were:

$$\frac{dp_{Ai,abs}}{dt} = \frac{RT}{V_{Ai}} \dot{m}_{in,i} - \frac{p_{Ai,abs} A_A v_i}{V_{Ai}} \quad (30)$$

$$\frac{dp_{Bi,abs}}{dt} = -\frac{RT}{V_{Bi}} \dot{m}_{out,i} + \frac{p_{Bi,abs} A_B v_i}{V_{Bi}} \quad (31)$$

The chamber pressures were limited to physically meaningful values and were not allowed to fall below atmospheric pressure. The same

pressure-dynamics formulation was applied to both pneumatic actuators. The model assumes a uniform pressure distribution within each chamber and does not resolve local pressure gradients or wave propagation in the pneumatic lines.

The isothermal assumption was adopted as a reduced-order approximation suitable for the comparative character of the present analysis. During cylinder filling, air compression may produce a temporary temperature increase, whereas expansion and exhaust may cause cooling. These temperature variations can modify the transient pressure build-up and decay rates and may therefore affect piston acceleration and stroke-completion time. A non-isothermal formulation would require an additional energy-balance equation for each actuator chamber, including instantaneous gas temperature, heat transfer between the compressed air and the cylinder walls, wall temperature, and an appropriate heat-transfer coefficient. This would increase the number of model states and introduce additional parameters requiring experimental identification. The isothermal assumption is therefore considered adequate for comparing the relative influence of cylinder diameter and unequal loading, although it may limit the quantitative accuracy of the predicted pressure and motion transients.

Compressible airflow model

The mass-flow rate through the pneumatic restrictions was calculated using a compressible-flow formulation. The restrictions represented the combined influence of valves, fittings, and tubing on the airflow between the pressure source, actuator chambers, and atmosphere. The pressure ratio was defined as:

$$r = \frac{p_d}{p_u} \quad (32)$$

where: p_u – upstream absolute pressure, p_d – downstream absolute pressure.

The critical pressure ratio was calculated from:

$$r_{cr} = \left(\frac{2}{\kappa+1} \right)^{\frac{\kappa}{\kappa-1}} \quad (33)$$

where: κ – specific heat ratio of air.

For air, the following value was assumed $\kappa = 1.4$.

When:

$$\frac{p_d}{p_u} \leq r_{cr} \quad (34)$$

The flow was treated as choked. Under choked-flow conditions, the mass-flow rate was expressed as:

$$\dot{m} = C_d A_e p_u \sqrt{\frac{\kappa}{RT}} \left(\frac{2}{\kappa+1} \right)^{\frac{\kappa+1}{2(\kappa-1)}} \quad (35)$$

where: $\dot{m}$ – mass-flow rate, C_d – discharge coefficient, A_e – effective flow area,

When:

$$\frac{p_d}{p_u} > r_{cr} \quad (36)$$

The flow was treated as subcritical. The mass-flow rate was then calculated as:

$$\dot{m} = C_d A_e p_u \sqrt{\frac{2\kappa}{RT(\kappa-1)}} \left[\left(\frac{p_d}{p_u} \right)^{\frac{2}{\kappa}} - \left(\frac{p_d}{p_u} \right)^{\frac{\kappa+1}{\kappa}} \right] \quad (37)$$

The same general relation was used for both chamber filling and chamber exhaust. During filling, the upstream pressure was the supply pressure, and the downstream pressure was the instantaneous chamber pressure:

$$p_u = p_{s,abs} \quad (38)$$

where: $p_{s,abs}$ – denotes the absolute supply pressure.

The absolute supply pressure was calculated from the gauge supply pressure as:

$$p_{s,abs} = p_{s,g} + p_{atm} \quad (39)$$

$$p_d = p_{Ai,abs} \quad (39)$$

During exhaust, the upstream pressure was the chamber pressure, and the downstream pressure was atmospheric pressure:

$$p_u = p_{Bi,abs} \quad (40)$$

$$p_d = p_{atm} \quad (41)$$

The inlet and outlet restrictions were described by separate effective flow areas A_{in} and A_{out} . These parameters represented the combined influence of the directional valve, fittings, and pneumatic tubes. Because complete manufacturer flow characteristics were not available, the effective flow areas were treated as calibrated model parameters. The mass-flow rates for the cap-side chamber and the rod-side chamber were therefore written as:

$$\dot{m}_{in,i} = f(p_{s,abs}, p_{Ai,abs}, A_{in}) \quad (42)$$

$$\dot{m}_{out,i} = f(p_{Bi,abs}, p_{atm}, A_{out}) \quad (43)$$

where: the function f was evaluated using either the choked-flow or subcritical-flow equation, depending on the instantaneous pressure ratio.

The airflow model allowed the chamber pressures to respond dynamically to both the available pressure difference and the instantaneous piston position.

NUMERICAL ANALYSIS CONDITIONS

The numerical analysis was performed for two pneumatic-cylinder configurations differing in piston diameter. The first configuration included cylinders with a piston diameter of 32 mm and a piston-rod diameter of 10 mm. The second configuration included cylinders with a piston diameter of 40 mm and a piston-rod diameter of 16 mm. In both cases, the nominal piston stroke was 0.4 m.

These diameters were considered as two practical actuator configurations for the intended lower-limb rehabilitation device. Their selection accounted for the required actuator force, supply pressure, and actuator dimensions. The 40 mm cylinder provides a larger effective piston area and therefore enables the required force to be generated at a lower supply pressure, whereas the 32 mm cylinder represents a more compact configuration but requires a higher supply pressure. The comparison of these two variants makes it possible to assess how piston diameter influences motion asymmetry under unequal external loading.

Model parameters

The actuator geometry, stroke, supply pressures, loading-cylinder diameter, and loading-pressure values were derived from biomechanical

calculations and operating conditions previously used in a lower-limb rehabilitation-device research programme. The remaining quantities were treated as effective model parameters. All supply and loading pressures reported in this section are gauge pressures (Table 1).

The remaining friction, flow, initial-condition, and numerical-integration parameters were treated as effective model parameters and were kept constant for both actuators within each analysed configuration.

The effective friction parameters and flow areas were treated as calibrated model parameters. For the 32 mm cylinders, the effective inlet and outlet flow areas were $7.7373 \times 10^{-8} \text{ m}^2$ and $8.8979 \times 10^{-8} \text{ m}^2$. For the 40 mm cylinders, the corresponding values were $1.1963 \times 10^{-7} \text{ m}^2$ and $1.3757 \times 10^{-7} \text{ m}^2$.

The supply pressures were calculated for the maximum assumed lower-limb force of approximately 150 N, taking into account the effective annular piston area, an actuator efficiency coefficient of 0.75, and an additional 5% allowance for internal friction. This resulted in supply gauge pressures of 3.5 bar for the 32 mm cylinders and 2.4 bar for the 40 mm cylinders.

The loading conditions were selected on the basis of biomechanical and anthropometric calculations performed for a lower-limb rehabilitation device. The masses of the thigh, shank, and foot were estimated using anthropometric body-segment parameters commonly applied in biomechanical modelling [34,35]. For the considered body-mass range of 40–110 kg, the calculated total lower-limb mass ranged from approximately

Table 1. Main geometric and operating parameters used in the numerical model

Parameter	Symbol	Value	Unit
Piston diameter	D	32 / 40	mm
Piston-rod diameter	d	10 / 16	mm
Piston stroke	L	0.4	m
Supply gauge pressure	$p_{s,g}$	3.5 / 2.4	bar
Loading-cylinder diameter	D_L	0.025	m
Minimum loading gauge-pressure pair	p_{L1}, p_{L2}	1.01 / 1.32	bar
Maximum loading gauge-pressure pair	p_{L1}, p_{L2}	3.05 / 3.97	bar
Tube inside diameter	d_i	4	mm
Tube length	l_i	0.30	m
Air temperature	T	293.15	K
Specific gas constant for air	R	287.05	J/(kg·K)
Specific heat ratio		1.4	-
Simulation time	t	25	s

5.5 to 15.2 kg, corresponding to a gravitational force of approximately 54–149 N. Comparable loading ranges were also adopted in previous experimental investigations of two pneumatic cylinders operating with a synchronising element in a rehabilitation-device configuration [36].

External loading was represented using loading cylinders with a piston diameter of 25 mm. To reproduce the asymmetric resistance associated with the movement of two lower limbs, the pressure acting on the second loading cylinder was assumed to be 30% higher than that acting on the first cylinder. This value was treated as a design assumption adopted in the rehabilitation-system calculation procedure, rather than as a universal clinical asymmetry coefficient.

Two unequal-load cases were analysed for each cylinder diameter. The minimum loading-pressure pair was: $p_{L1} = 1.01$ bar, $p_{L2} = 1.32$ bar. The maximum loading-pressure pair was: $p_{L1} = 3.05$ bar, $p_{L2} = 3.97$ bar. All loading pressures were treated as gauge pressures.

Numerical results

The numerical simulations were performed in MATLAB R2025b. The coupled nonlinear differential equations were integrated using the variable-step stiff solver ode15s. The relative and absolute error tolerances were set to 10^{-7} and 10^{-9} , respectively, while the maximum integration step was limited to 0.02 s. The simulations covered the time interval from 0 to 25 s, and the numerical results were recorded at intervals of 0.1 s.

The numerical simulations were performed for four operating cases combining two cylinder diameters and two unequal-loading conditions. For each case, the piston-rod displacement histories of both actuators were analysed over a simulation time of 25 s. The comparison focused on the influence of piston diameter and load asymmetry on displacement divergence, stroke-completion time, and motion uniformity. The analysed cases were defined as follows:

- D=32 mm, minimum load difference,
- D=32 mm, maximum load difference,
- D=40 mm, minimum load difference,
- D=40 mm, maximum load difference.

Figures 1–4 present the calculated piston-displacement histories for the four analysed cases. The results were evaluated using the maximum, mean, root-mean-square, and relative displacement discrepancies, together with stroke-completion time and maximum velocity difference.

The numerical results demonstrate that piston-displacement asymmetry depended strongly on both the loading condition and cylinder diameter. Under the minimum unequal-load condition, both actuators completed the full stroke, whereas under the maximum unequal-load condition, the more heavily loaded actuator did not reach 99% of the nominal stroke within the 25 s simulation interval. For the cylinders with a piston diameter of 32 mm, the maximum displacement discrepancy under the minimum unequal-load condition was 15.03 mm. Cylinder 1 completed the stroke after approximately 16.8 s, whereas Cylinder 2 reached the end position after approximately 17.6 s.

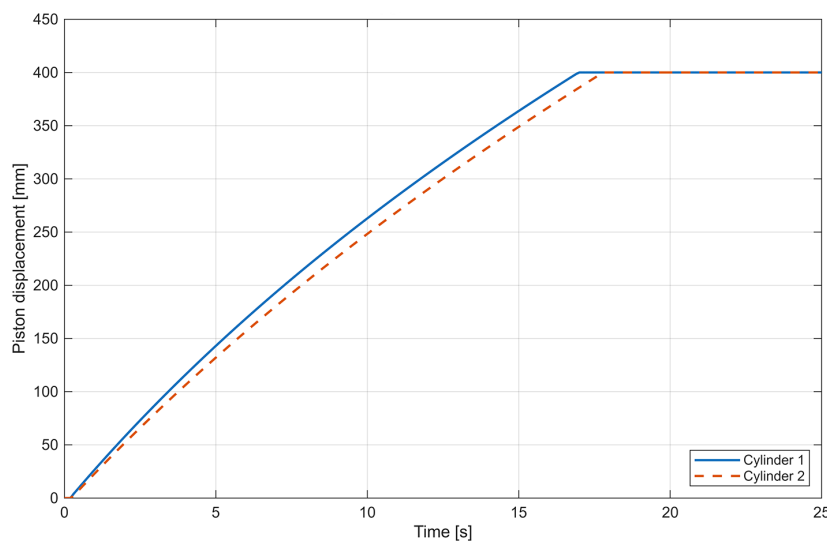


Figure 1. Numerical piston-displacement histories for two independent pneumatic cylinders with D=32 mm, under the minimum unequal-load condition: $p_{L1} = 1.01$ bar, and $p_{L2} = 1.32$ bar

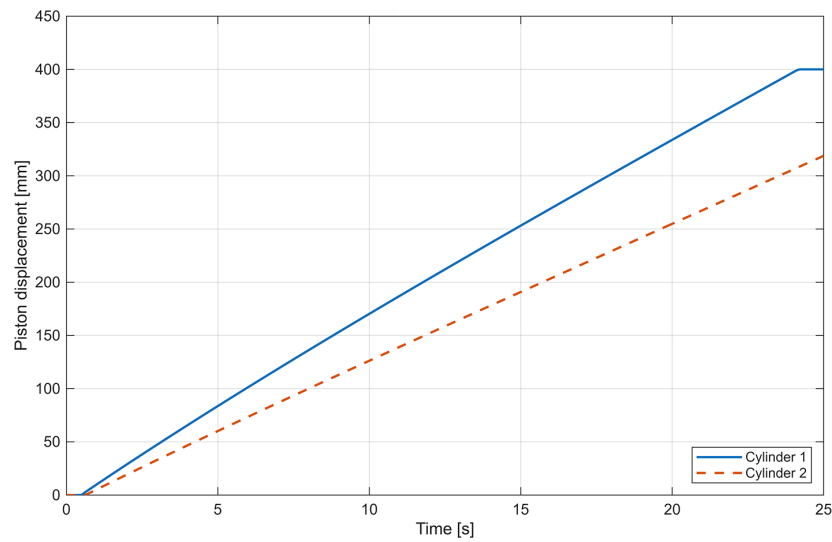


Figure 2. Numerical piston-displacement histories for two independent pneumatic cylinders subjected to unequal external loading $D=32$ mm, maximum load difference; $p_{L1} = 3.05$ bar, and $p_{L2} = 3.97$ bar

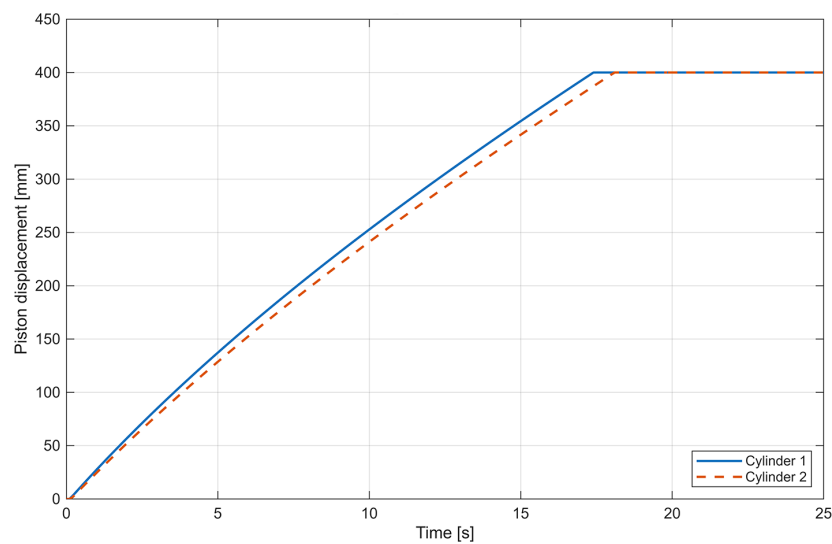


Figure 3. Numerical piston-displacement histories for two independent pneumatic cylinders subjected to unequal external loading: $D=40$ mm, minimum load difference; $p_{L1} = 1.01$ bar, and $p_{L2} = 1.32$ bar

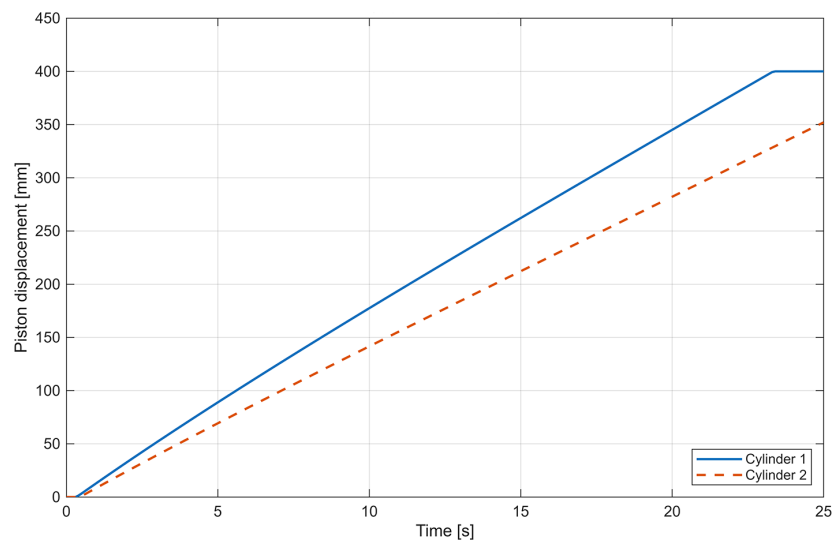


Figure 4. Numerical piston-displacement histories for two independent pneumatic cylinders subjected to unequal external loading: $D=40$ mm, maximum load difference, $p_{L1} = 3.05$ bar, and $p_{L2} = 3.97$ bar

For the cylinders with a piston diameter of 40 mm, the maximum displacement discrepancy under the same loading condition was lower and reached 12.88 mm. The corresponding stroke-completion times were approximately 17.2 s for Cylinder 1 and 17.9 s for Cylinder 2.

These results indicate that increasing the piston diameter reduced the displacement difference under moderate unequal loading. Under the maximum unequal-load condition, the motion asymmetry increased substantially. For the cylinders with a piston diameter of 32 mm, the maximum displacement discrepancy reached 91.73 mm. Cylinder 1 completed the nominal stroke after approximately 24.0 s, whereas Cylinder 2 did not reach 99% of the nominal stroke within the 25 s simulation interval.

For the cylinders with a piston diameter of 40 mm, the maximum displacement discrepancy was lower and reached 71.02 mm. Cylinder 1 completed the stroke after approximately 23.1 s, whereas Cylinder 2 again did not reach 99% of the nominal stroke within the simulation time.

The comparison shows that increasing the piston diameter from 32 to 40 mm reduced the maximum displacement discrepancy under both loading conditions. The reduction was particularly evident under the maximum unequal-load condition, where the discrepancy decreased from approximately 91.7 mm to 71.0 mm.

Comparison of maximum displacement discrepancy

The comparison of the analysed cases confirmed that the maximum displacement discrepancy increased markedly along with load asymmetry. For both cylinder diameters, the minimum unequal-load condition produced only moderate differences between the piston-rod trajectories. In contrast, the maximum unequal-load condition led to a pronounced loss of motion uniformity.

For the 32 mm cylinders, the maximum displacement discrepancy increased from 15.03 mm under the minimum unequal-load condition to 91.73 mm under the maximum unequal-load condition. This represents more than a sixfold increase. For the 40 mm cylinders, the corresponding discrepancy increased from 12.88 mm to 71.02 mm. In this case, the increase was approximately 5.5 times.

The larger cylinder diameter reduced the maximum displacement discrepancy in both

analysed loading cases. Under the minimum unequal-load condition, the discrepancy was reduced by approximately 14%. Under the maximum unequal-load condition, the reduction was approximately 23%.

The same trend was observed for the mean and root-mean-square displacement discrepancies. For the 32 mm cylinders, the mean displacement discrepancy increased from 8.32 mm to 51.05 mm, whereas the root-mean-square discrepancy increased from 10.46 mm to 57.67 mm. For the 40 mm cylinders, the mean displacement discrepancy increased from 6.96 mm to 40.38 mm, while the root-mean-square discrepancy increased from 8.69 mm to 45.15 mm.

The relative maximum displacement discrepancy reached approximately 3.76% of the nominal stroke for the 32 mm cylinders under the minimum unequal-load condition and 22.93% under the maximum unequal-load condition. For the 40 mm cylinders, the corresponding values were approximately 3.22% and 17.76%.

These results show that a larger piston diameter improved the resistance of the actuator pair to unequal loading. However, increasing the diameter did not eliminate the motion asymmetry. Under the highest analysed loading difference, the discrepancy remained large enough to prevent the more heavily loaded actuator from completing the full stroke within the simulation time.

Figure 5 compares the maximum piston-displacement discrepancies obtained for the two cylinder diameters under the minimum and maximum unequal-load conditions.

Final remarks

The numerical analysis demonstrated that unequal external loading had a direct and substantial influence on the motion of two independent pneumatic cylinders. Even though the actuators had the same nominal geometry and were supplied from the same pressure source, differences in the applied loads produced measurable discrepancies in piston-rod displacement and stroke-completion time.

Under the minimum unequal-load condition, both actuators completed the full stroke for both analysed cylinder diameters. However, the more heavily loaded actuator moved more slowly, which resulted in a time-dependent displacement difference. This confirms that load asymmetry

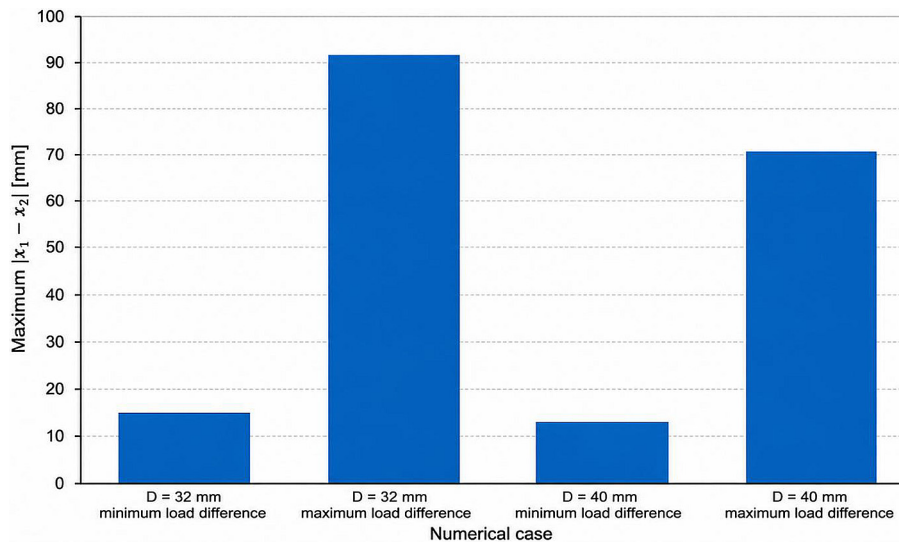


Figure 5. Comparison of the maximum piston-displacement discrepancy for the analysed cylinder diameters and unequal-loading conditions

affects actuator motion even when both cylinders remain fully operational.

The effect became much more pronounced under the maximum unequal-load condition. In both cylinder configurations, the more heavily loaded actuator failed to reach 99% of the nominal stroke within the 25 s simulation interval. This indicates that the available pneumatic force and airflow capacity were insufficient to maintain comparable motion under the higher load difference.

The comparison between the 32 mm and 40 mm cylinders showed that the larger piston diameter reduced the sensitivity of the actuator pair to unequal loading. This can be attributed to the larger effective piston area, which increased the pneumatic force available for overcoming the external load and friction resistance. As a result, the 40 mm cylinders exhibited lower maximum, mean, and root-mean-square displacement discrepancies.

However, the increase in piston diameter did not eliminate the asymmetry completely. Under the maximum unequal-load condition, the displacement difference remained substantial, and the more heavily loaded actuator did not complete the full stroke. This means that actuator oversizing alone cannot guarantee synchronous motion when the load difference is sufficiently high.

From an engineering perspective, the results indicate that unequal loading should be considered at the design stage of systems using parallel pneumatic actuators. This is particularly important in rehabilitation devices, lifting mechanisms,

positioning systems, and other applications in which both piston rods are expected to move simultaneously.

The analysis also shows that piston diameter should be selected not only on the basis of nominal force requirements, but also with regard to expected load asymmetry. A larger cylinder may reduce displacement differences, but a dedicated synchronisation mechanism or closed-loop control system may still be necessary when high motion uniformity is required.

The obtained results therefore justify the use of additional synchronisation solutions in the systems in which unequal external loads cannot be avoided. Such solutions may include pneumatic synchronising elements, flow-control valves, proportional valves, position sensors, or electronic control of individual actuators.

Limitations of the study

The developed model was intended for comparative analysis of piston-rod motion under unequal loading conditions and should not be interpreted as a high-fidelity representation of all physical phenomena occurring in the pneumatic system.

The model was not directly validated using experimental tests of two independent cylinders operating without a synchronising element. Therefore, the obtained results should be interpreted as a comparative numerical assessment, rather than as a fully validated prediction of a specific physical system.

The effective inlet and outlet flow areas were calibrated because complete valve flow characteristics, fitting losses, and detailed tubing data were not available. These values therefore represent equivalent parameters of the complete flow path rather than directly measured valve-orifice areas.

The friction model included static friction, Coulomb friction, the Stribeck effect, and viscous resistance. However, the friction parameters were treated as effective values and were not identified independently for each actuator. Real pneumatic cylinders may exhibit differences in seal friction, lubrication, wear, and manufacturing tolerances.

The thermodynamic process was assumed to be isothermal. In a real pneumatic actuator, compression during chamber filling may increase the air temperature, whereas expansion and exhaust may produce cooling. These effects can modify the transient chamber-pressure evolution and consequently influence piston acceleration and stroke-completion time. Their inclusion would require a separate energy-balance equation for each chamber, together with heat-transfer coefficients, wall-temperature assumptions, and temperature-dependent gas properties. The isothermal model was retained, because the present study focused on a comparative assessment of cylinder diameter and unequal loading rather than on high-fidelity prediction of thermal transients. The model also neglected external leakage, elastic deformation of the pneumatic tubes, local pressure losses, valve-switching dynamics, and pressure-wave propagation.

The external loads were assumed to remain constant during each simulation. In a real rehabilitation device or mechanical system, the load may vary with piston position, mechanism geometry, user interaction, or time. Only two cylinder diameters and two unequal-load conditions were analysed. Therefore, the conclusions should be interpreted within the range of the considered geometries and operating parameters.

Despite these limitations, the model captures the main mechanisms governing motion asymmetry and provides a useful basis for comparing the influence of cylinder diameter and unequal external loading. The observed increase in displacement discrepancy under unequal loading is consistent with earlier studies showing that simultaneous pneumatic-actuator motion requires either a dedicated synchronising element or cross-coupled control [13–15]. The present results extend these findings by quantifying the baseline asymmetry of uncoupled

cylinders and demonstrating that an increase in piston diameter reduces, but does not eliminate, the displacement divergence.

CONCLUSIONS

Unequal loading produced clear differences in piston-rod displacement and stroke-completion time. Under the minimum unequal-load condition, both actuators completed the full stroke, although the more heavily loaded actuator moved more slowly. Under the maximum unequal-load condition, the displacement discrepancy increased substantially. For both cylinder diameters, the more heavily loaded actuator did not reach 99% of the nominal stroke within the 25 s simulation interval. Increasing the piston diameter from 32 to 40 mm reduced the maximum displacement discrepancy under both analysed loading conditions. The reduction was particularly evident for the maximum unequal-load case.

The larger piston diameter improved the resistance of the actuator pair to load asymmetry but did not ensure synchronous motion. Therefore, increasing actuator size alone cannot replace a dedicated synchronisation solution when high motion uniformity is required. The results confirm that unequal loading should be considered during the design of pneumatic systems containing parallel actuators. The developed model can support preliminary actuator selection and help determine whether additional pneumatic, mechanical, or electronic synchronisation is required.

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