Advances in Science and Technology Research Journal
Volume 8, No. 22, June 2014, pp. 26–30
DOI: 10.12913/22998624.1105158

Research Article

PERIODIC SIGNAL DETECTION WITH USING DUFFING SYSTEM POINCARE MAP ANALYSIS

Valeriy Martynyuk[1], Mykola Fedula[2], Oleksii Balov[3]

[1] Depatrment of Radio Engineering and Communications, Faculty of Programming, Computer and Telecommunication Systems, Khmelnytskyi National University, Khmelnytskyi, Ukraine, e-mail: martynyuk.valeriy@gmail.com; mailfm2000@gmail.com; superbalov@gmail.com

Received: 2014.04.30
Accepted: 2014.05.20
Published: 2014.06.05

ABSTRACT
In this article the periodic signal detection method on the base of Duffing system chaotic oscillations analysis is presented. This work is a development of the chaos-based signal detection technique. Generally, chaos-based signal detection is the detection of chaotic-to-periodic state transition under input periodic component influence. If the input periodic component reaches certain threshold value, the system transforms from chaotic state to periodic state. The Duffing-type chaotic systems are often used for such a signal detection purpose because of their ability to work in chaotic state for a long time and relatively simple realization. The main advantage of chaos-based signal detection methods is the utilization of chaotic system sensitivity to weak signals. But such methods are not used in practice because of the chaotic system state control problems. The method presented does not require an exact system state control. The Duffing system works continuously in chaotic state and the periodic signal detection process is based on the analysis of Duffing system Poincare map fractal structure. This structure does not depend on noise, and therefore the minimum input signal-to-noise ratio required for periodic signal detection is not limited by chaotic system state control tolerance.

Keywords: periodic signal detection, chaotic system, Duffing oscillator, Poincare section, SNR.

INTRODUCTION

Weak periodic signal detection is one of the most important problems in many fields of modern technology. The main of these fields is the diagnostics of industrial machinery and equipment, biomedicine signal processing, radar information detection and communications. The new chaos-based signal detection method was proposed in 1992 [1]. This method is based on the Duffing oscillator state transition under weak periodic signal influence. The modern works related to the above focus on the chaos identification algorithms and methods [2]. Also there are different modifications of the Duffing oscillator which have a higher sensitivity [3]. These works are based on the results of chaotic system modeling and numerical simulation. But they are not connected closely with the general signal processing theory.

It should be noted that such methods are not popular in practical applications because of the oscillator critical state control errors. So we propose a new detection method with using Duffing oscillator without the state transition.

CHAOS-BASED SIGNAL DETECTION

The Duffing oscillator model is given by Eq. (1):

$$\ddot{x} + k\dot{x} - x + x^3 = s(t), \qquad (1)$$

where: $s(t)$ – the driving signal,
x – the output signal,
k – the damping constant [2, 4, 5].

In this work we consider the damping constant value $k = 0.5$. The driving signal is defined by Eq. (2):

$$s(t) = A_0 \sin(\omega_0 t + \varphi_0) + A_s \sin(\omega_s t + \varphi_s) + N(t) \quad (2)$$

where: $A_0\sin(\omega_0 t)$ – the exciting signal;
$A_s\sin(\omega_s t)$ – the input signal to-be-detected;
$N(t)$ – the input noise.

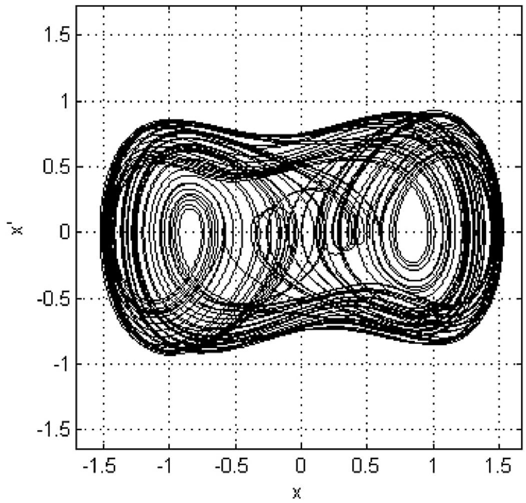

Fig. 1. Typical phase trajectories in chaotic state

The potential function [6, 7] of Duffing oscillator is defined by expression:

$$H(x) = \frac{1}{2} \cdot x^4 - x^2 \quad (4)$$

The state of Duffing oscillator depends on the average potential value. For the Duffing oscillator (1,2) the chaotic oscillations are possible in the interval:

$$0 < H(x) < 1 \quad (5)$$

This is shown in Figure 2.

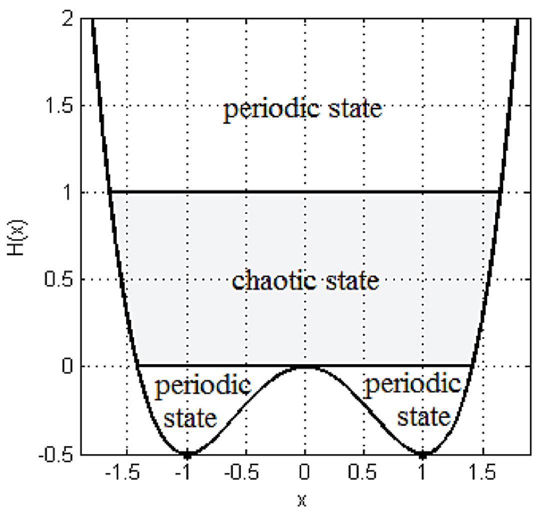

Fig. 2. Phase trajectories in the chaotic state

Generally, the dynamics of Duffing oscillator can be described by the phase trajectories which are projected on the Hamiltonian surface [8, 9, 10]. The Hamiltonian surface $H(x, x')$ of unforced Duffing oscillator is given by expression (3).

$$H(x, x') = (x')^2 + \frac{1}{2}x^4 - x^2 \quad (3)$$

The projection of Duffing oscillator phase trajectories on its Hamiltonian surface is shown in Figure 3.

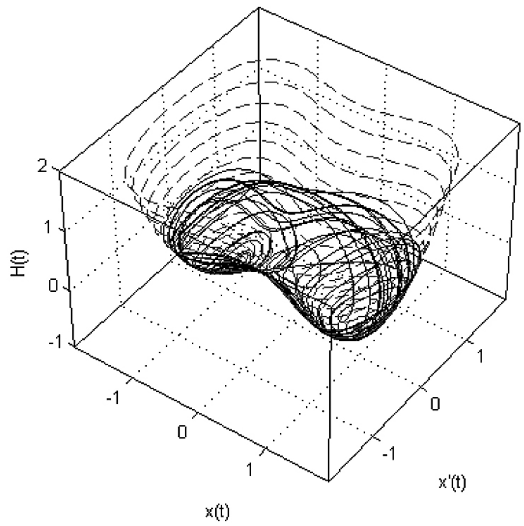

Fig. 3. Phase trajectories projection on the Hamiltonian surface in the case of chaotic state

For periodic signal detection the initial Duffing oscillator state is set to critical chaotic by the exciting periodic signal amplitude $A_0 = 0.8245$.

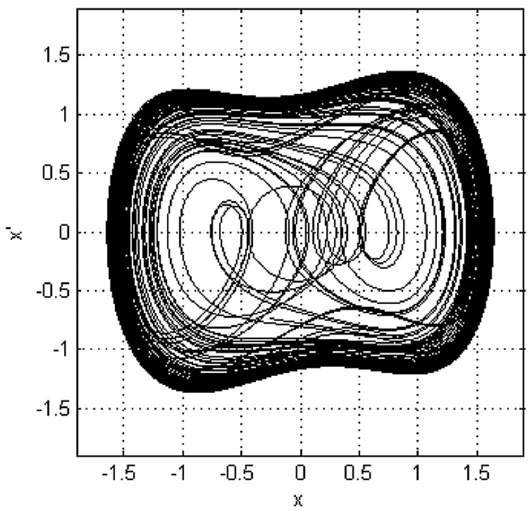

Fig. 4. Phase trajectories near the critical chaotic state

If $\omega_0 \approx \omega_s$ and the input signal periodic component A_s exceeds its threshold value, the system transforms from chaotic state to periodic state [2, 11, 12].

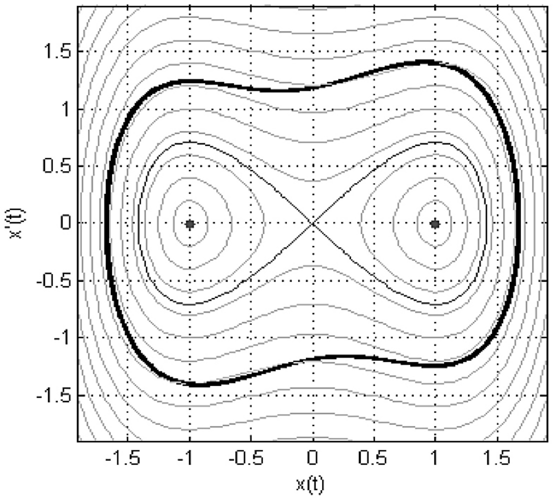

Fig. 5. Phase trajectories in the periodic state

The method described allows to detect weak periodic signals under strong background noise conditions. The main disadvantages of this method are related to the critical state control problem. In critical state the Duffing oscillator is sensitive to non-stationary input signals but in periodic state it is not sensitive to weak periodic signals. These factors may cause significant errors during the periodic signal detection process.

THE FRACTAL-BASED DETECTION METHOD

In this paper we propose the signal detection method which is based on Duffing oscillator Poincare map fractal structure analysis.

Let us consider the point $(x(kT,\varphi_0); x'(kT,\varphi_0))$ on the phase plane with respect to the Duffing oscillator Poincare map structure. This point moves along oscillator Poincare map structure with time.

If we increase the input signal periodic component, the point $(x(kT,\varphi_0); x'(kT,\varphi_0))$ shifts along the Poincare map structure, as shown in Figure 6.

The direction of this shift is noise-independent. But it varies with respect to the chaotic dynamics of the Duffing oscillator.

It is known that the Duffing oscillator Poincare map has a complex fractal structure. At any time the $(x(kT,\varphi_0); x'(kT,\varphi_0))$ point shift direction depends on its position in fractal structure of Duffing oscillator Poincare map.

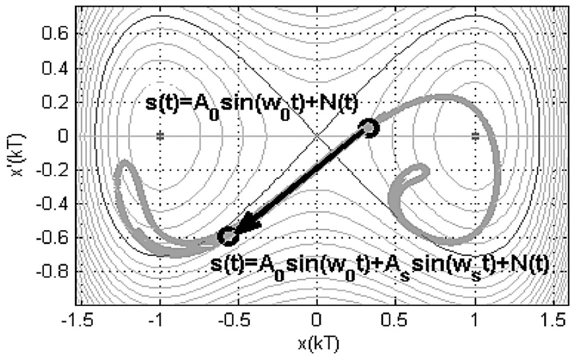

Fig. 6. The effect of periodic component increase

The main bifurcation point of Duffing oscillator has coordinates (0; 0). Near this point the phase trajectory divergence speed is much higher than in other places. When a segment of Poincare map structure passes this region, then it is stretched as shown in Figure 7.

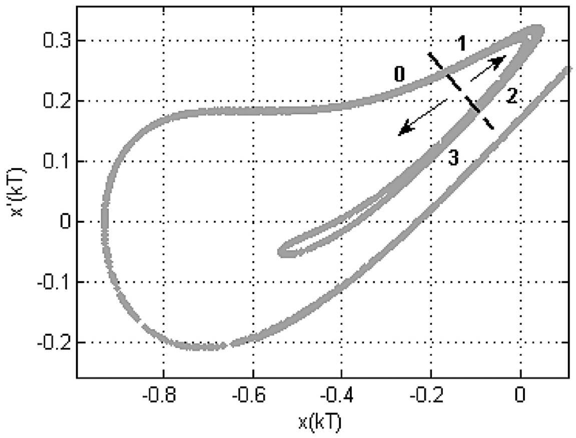

Fig. 7. The stretching of Poincare map fractal structure

If the fractal structure segment passes the stretch region, it can be divided into four parts with different dynamics. In Figure 7 the parts are numbered in accordance with the input periodic component amplitude increase direction. Parts 1 and 2 move from left center to right center. But the parts 0 and 3 stay at the orbit of the current center. After some time the parts 0, 1, 2, 3 stretch as well as the segment {0123}, and the fractal structure complexity increases.

Denote the length of the $(x(kT,\varphi_0); x'(kT,\varphi_0))$ point path along the fractal structure by $\lambda(t, A)$. If we consider the signal frequency $\omega_s = \omega_0$, $A = A_0 + A_s$ is the input periodic component amplitude.

Therefore, generally $\lambda(t, A)$ depends on the input periodic component amplitude monotonically. So the expressions $\lambda(t)\big|_{A=const}$ and $\lambda(A)\big|_{t=t_0}$

are monotonic increasing functions, where t_0 is any time value from the experiment time interval. Thus, it is advisable to estimate the path length $\lambda(t,A)$ with the tolerance up to one fractal structure element. Then the path length $\lambda(t,A)$ can be estimated as a normalized number $\Lambda(t,A)$ in the quaternary system value:

$$\Lambda(t,A) = L_M \cdot 4^0 + L_{M-1} \cdot 4^{-1} + ... + L_0 \cdot 4^{-M} \quad (6)$$

where: M – the number of digits,
L_0, ..., L_M – the weights of the corresponding digits.

The weight value can be taken from the array $\{0, 1, 2, 3\}$. The fractal structure corresponding to $\Lambda(t,A)$ is shown in Figure 8.

The fractal structure in Fig.8 shows that in the case of input noise absence we can define the input periodic signal amplitude $A = A_0 + A_s$ from Duffing oscillator output signal with very high accuracy. If we know the $\Lambda(t_k, A_0)$, then we can compare this value to $\Lambda(t_k, A_0 + A_s)$, and determine the presence or absence of periodic component with amplitude A_s at the input. The detection process is the decision between two hypotheses, H_0, signal absent, and H_1, signal present.

For convenience, we consider the discrete functions $V_{H0}(k)$ and $V_{H1}(k)$:

$$V_{H0}(k) = \Lambda(t_k, A_0), \quad (7)$$

$$V_{H1}(k) = \Lambda(t_k, A_0 + A_s). \quad (8)$$

The fractal-based periodic signal detection process was simulated in MATLAB/Simulink environment under the folowiwng conditions:

$$A_0 = 0.4V, \; A_s = 0.005V,$$
$$\omega_0 = \omega_s = 1 \, rad/\sec.$$

The noise is white and the signal-to-noise ratio is 3dB in the band $[0.1\omega_0; 10\omega_0]$. The graphs of the functions $V_{H0}(k)$ and $V_{H1}(k)$ are presented in Figure 9.

The Figure 9 shows that if the signal $A_s \cdot \sin(\omega_s t)$ is present, then $V(k)$ is much greater, than if it is absent. The case of input noise presence is shown in Figure 10.

The results in Figure 10 show that the input noise causes the increase of $V_{H0}(k)$ value, but $V_{H1}(k) > V_{H0}(k)$ as well.

The experiments show that if the input SNR is higher than 0dB by power, we can set a constant threshold value $V_{thr} \in (V_{H0\max}; V_{H1\min})$. Therefore, the detection process is robust and reliable.

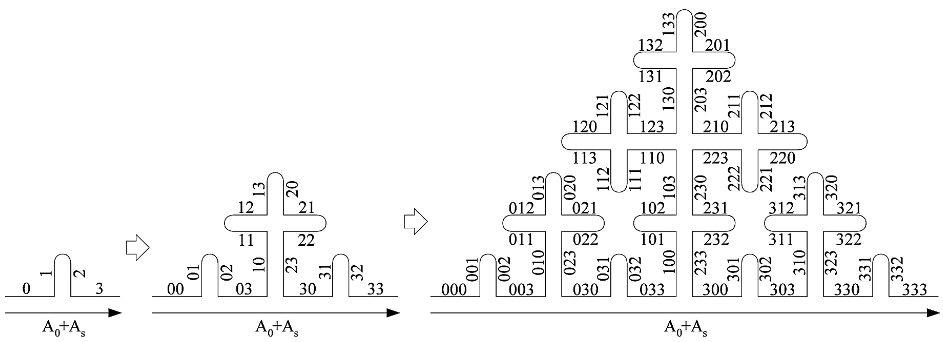

Fig. 8. The fractal structure corresponding to path length estimation $\Lambda(t,A)$

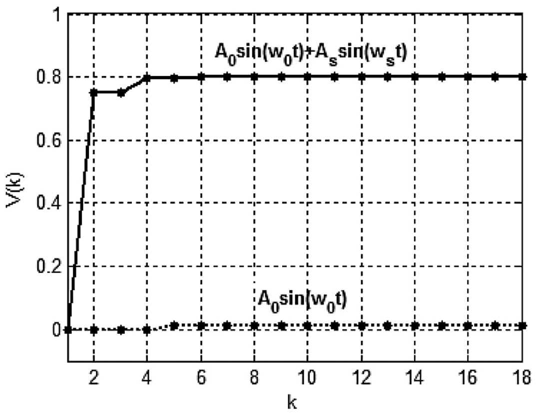

Fig. 9. Functions $V_{H0}(k)$, $V_{H1}(k)$ without input noise

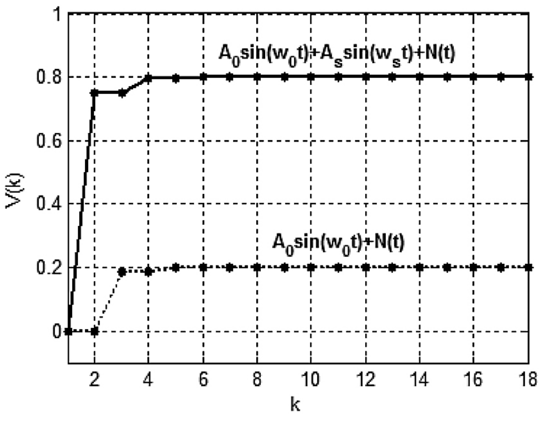

Fig. 10. Functions $V_{H0}(k)$, $V_{H1}(k)$ with input noise

If the input SNR is lower than 0 dB by power, then the first digits $L_M,...,L_{M-p}$ of $\Lambda(t_k, A_0)$ can be greater than the same digits of $\Lambda(t_k, A_0 + A_s)$. So we cannot detect the signal $A_s \cdot \sin(\omega_s t)$ directly with the constant threshold value. In this case we must analyze the next digits $L_{M-p-1},...,L_1$ of $\Lambda(t_k, A)$ without respect to the first $L_M,...,L_{M-p}$. This is the direction of our future work.

CONCLUSION

The main advantages of the method proposed are the following features:

- the critical state setting and control are not required;
- Duffing oscillator works continuously in chaotic state and the chaos identification is not required;
- the estimation of input periodic component amplitude is robust and reliable;
- the minimum input SNR required for periodic signal detection is not limited by chaotic system state control tolerance.

This method is more resistant to non-stationary and pulse noise in comparison with the known method [2]. The direction of the future work is connected with the analysis of Duffing oscillator Poincare map fractal structure under SNR less than 0 dB.

REFERENCES

1. Birx D. I. Chaotic oscillators and CMFFNS for signal detection in noise environments. IEEE International Joint Conference on Neural Networks, Vol. 22, 1992, 881–888.
2. Jalilvand A., Fotoohabadi H. The Application of Duffing Oscillator in Weak Signal Detection. ECTI Transactions on Electrical Eng., Electronics, and Communications, 9(1), 2011, 1–6.
3. Li Yue, Yang Baojun. Chaotic system for the detection of periodic signals under the background of strong noise. Chinese Science Bulletin, 48(5), 2003, 508–510.
4. Dai Y. The Weak Signal Detection Method and Instrument. The National Defence Industrial Publishing House, Beijing 1994, 265–278.
5. Liyun S., Qian Y., Yuli Z., Jiaojun L. Noise Immunity of Duffing Oscillator and its Applications in Weak UWB Signal Detection. Journal of Networks, 7(3), 2012, 540–546.
6. Thomson J.M.T., Stewart H.B. Nonlinear Dynamics and Chaos. John Wiley & Sons, Ltd, Baffins Lane, Chichester, West Sussex, England 2002.
7. Ott E. Chaos in Dynamical Systems. Cambridge University Press, Cambridge 1993.
8. Moon F.C. Chaotic Vibrations. An Introduction for Applied Scientists and Engineers. John Wiley & Sons, Inc., Hoboken, New Jersey 2004.
9. McLachlan N.W. Ordinary Non-linear Differential Equations in Engineering and Physical Sciences, 2nd Ed. Clarendon, Oxford 1958.
10. Armitage J.V. and Eberlein W.F. Elliptic Functions. LMS 67, Cambridge 2006.
11. Ferrer S., Lara M., Families of Canonical Transformations by Hamilton-Jacobi-Poincare Equation. Application to Rotational and Orbital Motion, Journal of Geometric Mechanics, Vol. 2, 2010, 223-241.
12. Wang Guanyu, Chen Dajun, Lin Jianya, Chen Xing. The application of chaotic oscillators to weak signal detection. IEEE Transactions on industrial electronics, 46(20), 1999, 440-443.
13. Yang Jianhong, Zhang Rencheng, Fang Huaiying, et al. Application of Duffing Oscillator signal detection to arcing faults protection. Automation of Electric Power System, 30(24), 2006, 69-72.