

An investigation on the Mullins effect of high damping rubber

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ABSTRACT

The Mullins effect refers to stress softening in rubber-like materials during loading after prior deformation. Although this phenomenon has been extensively studied, its strain-history dependence and recovery behavior in high-damping rubber (HDR) remain insufficiently quantified for engineering applications. This study experimentally investigates the Mullins effect of HDR under cyclic shear loading with variable and constant strain amplitudes. A cyclic shear test with variable amplitudes was used to evaluate the effects of maximum experienced strain and prior loading history, whereas repeated cyclic shear tests with a constant amplitude were performed after 1 day, 1 week, and 3 weeks to examine time-dependent recovery. The results show clear stress softening at all investigated strain levels, with the stress reduction decreasing from 37.62% at a strain level of 0.5 to 17.84% at a strain level of 2.5. The stress amplitude recovered rapidly, reaching 88.90%, 94.81%, and 97.43% after 1 day, 1 week, and 3 weeks, respectively. Based on the relationship between the stress ratio and accumulated absorbed energy, a phenomenological model is proposed to reproduce the Mullins effect in HDR.

Keywords: Mullins effect, high damping rubber, stress-softening, Mullins recovery.

INTRODUCTION

The Mullins effect was first mentioned by Bouasse and Carriere [1] and was later investigated in detail by Mullins [2,3]. It describes the stress-softening response observed when rubber-like materials are reloaded after a previous deformation. Under the first loading path, the stress-strain response is generally unique; after unloading and reloading, the material follows a softer path that cannot reproduce the initial response. This phenomenon is now widely referred to as the Mullins effect [4].

Considerable research has been devoted to explaining the mechanisms of the Mullins effect and to developing mechanical models capable of representing stress softening. Existing explanations include damage-like mechanisms in filled elastomers, changes in polymer-chain networks, filler-polymer interactions, internal sliding, and strain-induced microstructural evolution. On this basis, several constitutive approaches have been proposed, including conventional mechanical

models [5–7], thermodynamic theories [8, 9], and microstructural interpretations supported by neutron-scattering observations [10].

The recovery of the Mullins effect is also important because the softened response may partially or almost fully recover after a rest period or thermal treatment. This recovery has been related to the reformation of polymer-chain networks and the restoration of filler-polymer interactions. Previous studies [11–14] have shown that stress softening may recover in both virgin and previously used rubber specimens. Yan et al. [13] reported substantial recovery of a nanoparticle-filled elastomer within less than 17 h at room temperature, whereas Chu et al. [14] showed that recovery in pre-deformed filled rubbers depends strongly on annealing time and temperature.

High-damping rubber bearings are widely used for seismic isolation because of their favorable energy-dissipation capacity and strength [15–18]. These properties were achieved by adding fillers such as carbon black, oils, silica, and other particles during the vulcanization of

natural rubber [16,19]. However, the resulting HDR material response was highly nonlinear and history dependent. Important behaviors, including stress softening associated with the Mullins effect [5] and temperature-dependent cyclic response [20-22], remain difficult to describe accurately in engineering practice.

Compared with conventional rubber materials, fewer studies have focused specifically on the Mullins effect in HDR. Dall'Asta and Ragni [23] performed experiments and developed an analytical model for HDR dissipating devices that accounted for the Mullins effect at room temperature. Their results indicated that recovery can occur rapidly after 1 day, although complete recovery was not observed even after a long rest period. More recently, Tan et al. [24] developed a random polymer model that considered both low-temperature dependence and the Mullins effect in HDR bearings.

Despite these advances, the Mullins effect in HDR remains challenging because the response depends simultaneously on the maximum experienced strain, cyclic loading history, dissipated energy, and recovery time. In particular, limited experimental evidence is available on the combined influence of strain amplitude, prior strain history, and rest period on stress recovery, damping recovery, and stiffness recovery of HDR under cyclic shear loading.

To provide further insight into these issues, an experimental program was conducted on HDR specimens. A cyclic shear test with variable amplitudes was performed on a virgin specimen to investigate the dependence of the Mullins effect on strain amplitude and previous strain history. Additional cyclic shear tests with a constant strain amplitude were conducted to examine recovery after 1 day, 1 week, and 3 weeks. Finally, a phenomenological model was proposed using the experimental relationship between stress ratio and accumulated absorbed energy.

MATERIALS AND METHODS

High damping rubber specimens

The HDR specimens were designed of ISO 22762-1 [25] for dynamic property testing. As shown in Figure 1, each specimen consisted of two HDR blocks bonded to three steel plates. Each rubber block had a plan area of 25×25

mm² and a thickness of 4 mm. Figure 2 shows the testing machine (Shimadzu Autograph AG-100kNG), this is a computer-controlled testing machine, the input data is applied in the computer in Figure 2(a) and the HDR specimen is installed in the testing machine in Figure 2(b). All tests were conducted on HDR specimens at room temperature. The specimens and testing machine were the same as those used by Nguyen [26].

Cyclic shear test with variable amplitudes (CSVA test)

The Mullins effect is strongly dependent on the maximum strain level previously experienced by the virgin rubber material [3], the Mullins effect of HDR is also depends on past strain histories [23, 24]. Therefore, a cyclic experiment in Figure 3 is conducted to investigate the effect of the strain amplitude on the Mullins effect. The strain amplitude increases from 0.5 to 2.5. There are 10 cycles at each strain level. The dependence of the response on the maximum experienced strain level and the previous strain amplitude is determined when the strain history shown in Figure 3 is applied to a virgin specimen.

Cyclic shear test with a constant strain amplitude (CSCA test)

Mullins, 1969 [3] indicated that the Mullins effect of rubber vulcaniztes can be recovered. In addition, Dall'Asta and Ragni, 2006 [23] indicate that the recovery time of the Mullins effect of HDR is quite fast. In order to get information about the recovery time of the Mullins effect, the strain histories in Figure 4 would be applied on the specimens repeated by a day, 1 week and 3 weeks (three different specimens). First, the strain histories were placed on all the HDR specimens, then these specimens were reloaded after 1 day, 1 week, and 3 weeks with the same strain histories to examine the recovery of the Mullins effect in HDR.

RESULTS AND DISCUSSION

CSVA test results

Figures 5 and 6 show the CSVA test results, the stress response of the 1st cycle with the Mullins effect is maximum, then this response reach to the stable cycle without the Mullins effect

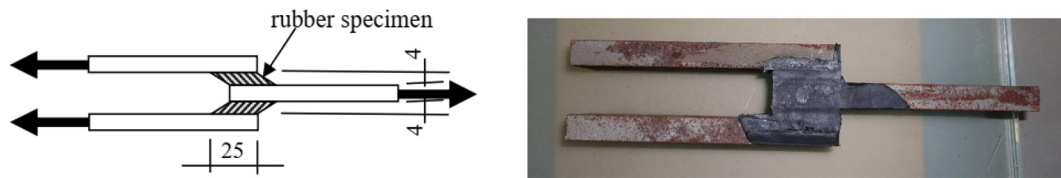


Figure 1. HDR specimen

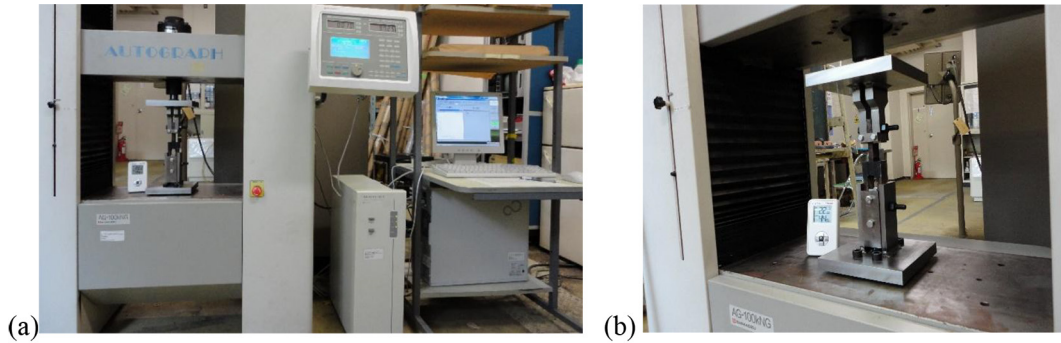


Figure 2. (a) Testing machine – Shimadzu, Autograph AG-100 kNG; (b) Specimen in the machine

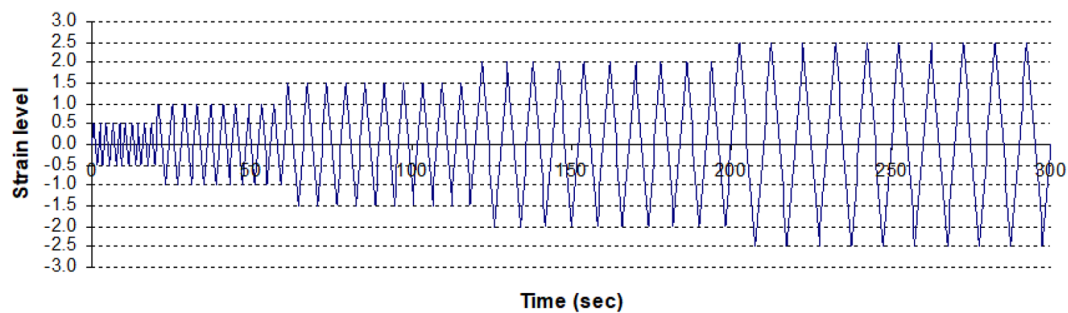


Figure 3. Strain histories applied in the cyclic shear test with variable amplitude
Test conditions: + Strain level: $\pm 0.5, \pm 1.0, \pm 1.5, \pm 2.0, \pm 2.5$; + Strain rate: 1/sec;
+ Number of cycles: 10 cycles x5

after some cycle loading. These stress softening describe the Mullins effect appearing at each strain level.

$$\frac{\tau_{\max}^i}{\tau_o^i} \quad (1)$$

where: $\tau_{\max}^i$ is the stress amplitude at the 1st cycle in each strain level including the Mullins effect; τ_o^i is the stress amplitude at the 10th cycle in each strain level excluding the Mullins effect.

Table 1 shows the amount of stress softening induced by the Mullins effect in CSA test. It can be observed that stress softening occurs at all strain levels, but the percentage of this stress reduction decreases gradually with increasing

load-strain levels. This result can be explained by the degree of virginity of the CSA test specimen at each load-strain level, the degree of virginity at the higher strain levels gradually decreases because the specimen has undergone 10 cycles of action to eliminate the Mullins effect at the each lower load-strain level.

CSCA test results

The stress responses of the two tests are compared in Figure 7, the stress of the repeated test is closely the stress of the 1st test and after 3 weeks they almost equal each other. It implies that the Mullins effect can be most recovered after 3 weeks.

The stress-softening phenomenon and the recovery of the Mullins effect are more clearly

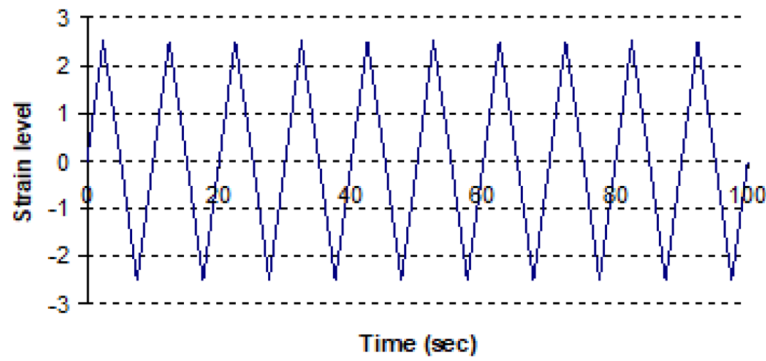


Figure 4. Strain histories applied in the cyclic shear test with constant amplitude
Test conditions: + strain level: ± 2.5 ; + strain rate: 1/sec; + number of cycles: 10 cycles;
+ time interval for repeating: 1 day, 1 week, and 3 weeks.

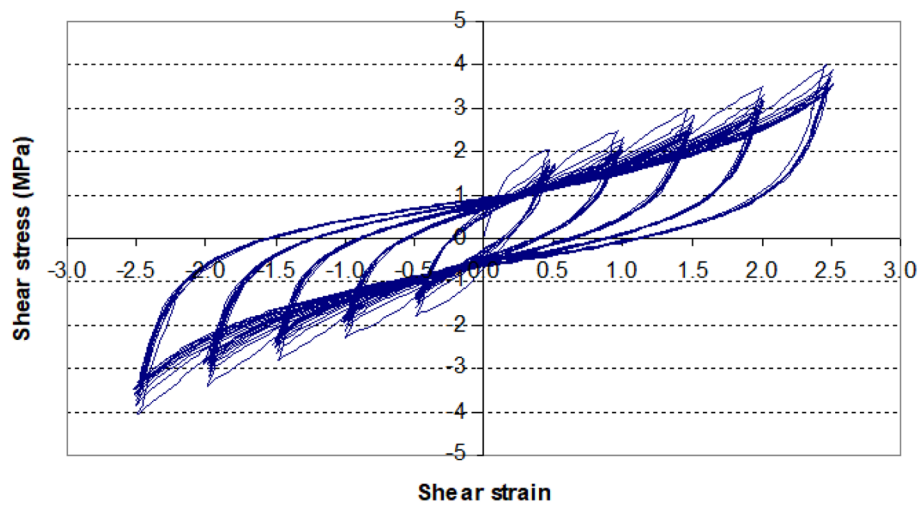


Figure 5. Stress-strain curves of CSVA test

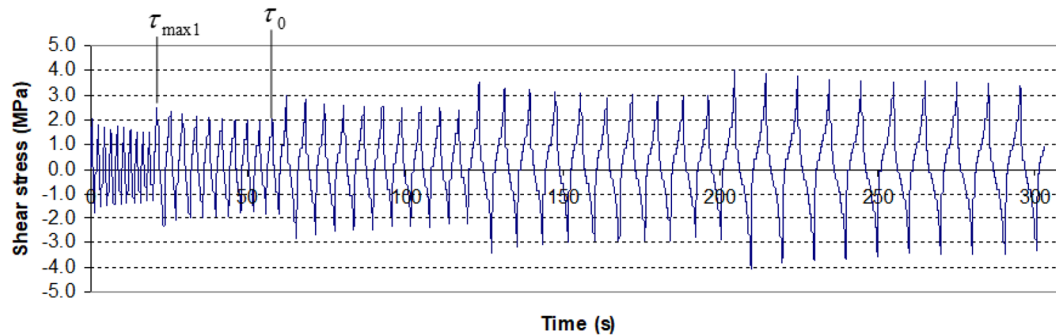


Figure 6. The stress responses of CSVA test

In order to express the Mullins effect to appear in the CSVA test, the max stress ratio is used as a function of the maximum experienced strain level in Equation 1.

observed in Figure 8. The stress-strain curve at the 1st cycle and the 10th cycle are selected to plot, because the 1st cycle includes Mullins effect and the 10th cycle excludes Mullins effect. The difference between 1st cycle and 10th cycle demonstrates the Mullins effect, the difference between 1st test

and the repeated test demonstrates the recovery of Mullins effect. It is clear that Mullins effect's recovery increases over time. The stress-strain curve at 10th cycle of both the tests are almost identical because the response of this cycle is not affected by Mullins effect.

Table 1. The stress softening induced by the Mullins effect under CSVA test

Level	τ_{max1}	τ_0	$\Delta\tau=\tau_{max}-\tau_0$	$\Delta\tau/\tau_0 \times 100$
	(MPa)	(MPa)	(MPa)	(%)
0.5	2.051	1.490	0.561	37.62
1.0	2.494	1.903	0.592	31.09
1.5	2.965	2.429	0.537	22.10
2.0	3.539	2.947	0.592	20.07
2.5	4.017	3.409	0.608	17.84

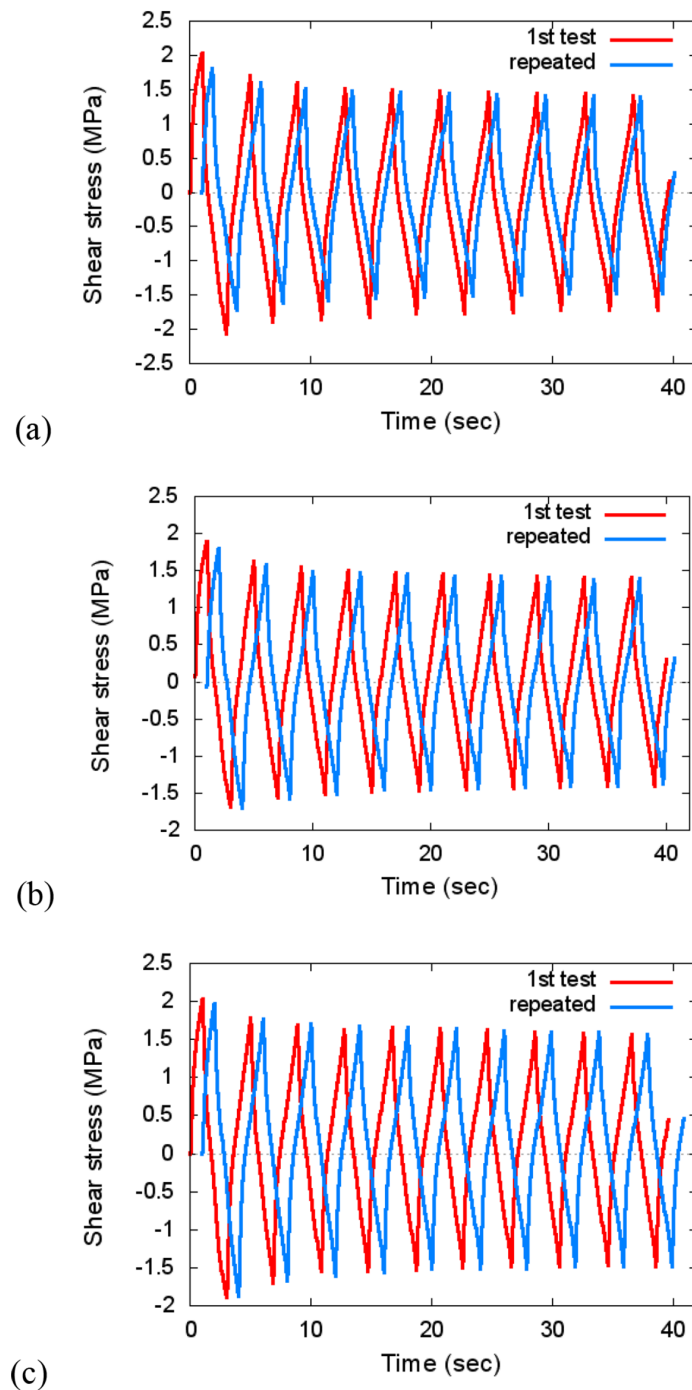


Figure 7. Stress responses of the 1st test and the repeated test after: (a) 1 day, (b) 1 week, (c) 3 weeks from the 1st test

Similar to the CSVA test, the stress ratio is used to quantify the stress softening induced by the Mullins effect as in Equation 2:

$$\frac{\tau_2}{\tau_1} \quad (2)$$

where: τ_1 is the stress amplitude of the 1st test; τ_2 is the stress amplitude of the repeated test.

Table 2 shows the stress softening induced by the Mullins effect under CSCA test. The stress amplitude- τ recovers 88.90%, 94.81%, and 97.43% after 1 day, 1 week, and 3 weeks, respectively.

Because the Mullins effect appears mainly at the 1st cycle, the graph of the recovery of the stress ratio- τ at the 1st cycle is plotted in Figure 9. The stress softening due to the Mullins effect can recover very quickly, after 1 day the stress amplitude recovers by 88.90% and it recover almost after 3 weeks by 97.43% under CSCA tests.

The equivalent damping constant h_B and the equivalent shear modulus G_B are important mechanical properties of rubber bearings. Changes in these two quantities affect the performance of structures using HDR bearings.

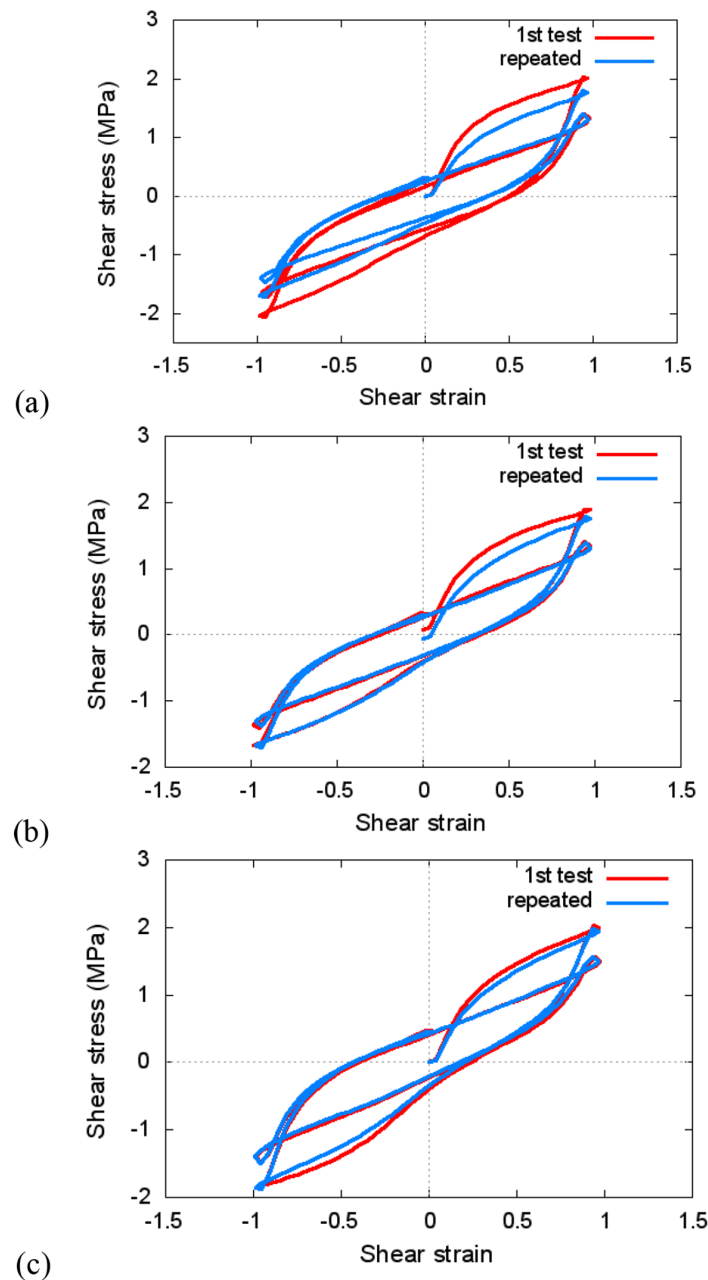


Figure 8. Stress-strain curves at the 1st cycle and the 10th cycle of the 1st test and the repeated test after: (a) 1 day, (b) 1 week, (c) 3 weeks from the 1st test

Table 2. The stress softening induced by the Mullins effect under CSCA test

Cycle	Test-1day			Test-1week			Test-3weeks		
	τ_1	τ_2	τ_2/τ_1	τ_1	τ_2	τ_2/τ_1	τ_1	τ_2	τ_2/τ_1
	(MPa)	(MPa)	(%)	(MPa)	(MPa)	(%)	(MPa)	(MPa)	(%)
1	2.030	1.805	88.90	1.894	1.796	94.81	2.030	1.978	97.43
2	1.708	1.593	93.27	1.626	1.579	97.11	1.778	1.761	99.02
3	1.602	1.517	94.68	1.546	1.495	96.71	1.694	1.707	100.76
4	1.515	1.474	97.27	1.500	1.475	98.35	1.631	1.671	102.45
5	1.498	1.456	97.16	1.472	1.457	98.99	1.659	1.657	99.89
6	1.480	1.436	97.04	1.452	1.432	98.62	1.641	1.644	100.20
7	1.454	1.425	98.03	1.434	1.419	98.94	1.631	1.606	98.46
8	1.447	1.419	98.03	1.419	1.410	99.40	1.592	1.601	100.53
9	1.445	1.407	97.36	1.412	1.382	97.86	1.580	1.594	100.86
10	1.404	1.393	99.19	1.403	1.387	98.90	1.570	1.566	99.70

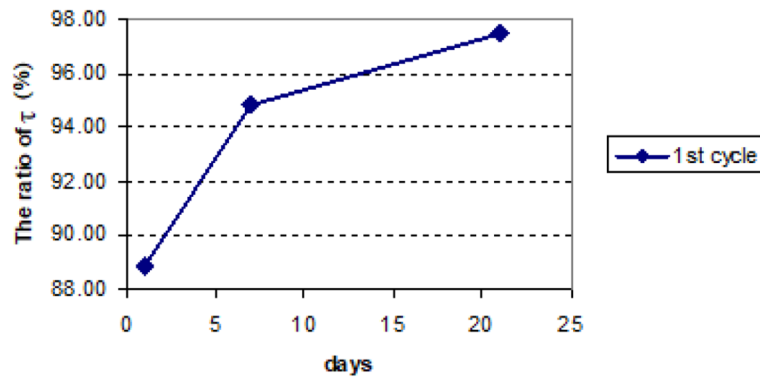


Figure 9. The recovery of the stress amplitude at the 1st cycle

The equivalent damping constant h_B is calculated by Equation 3:

$$h_B = \frac{\Delta W}{2\pi W} \quad (3)$$

where: ΔW is the dissipated energy, and equal to the area of the hysteretic curve of the shear stress- strain response in Figure 10; W is the elastic strain energy, and equal to the triangular area in Figure 10 and 11.

The h_B ratio describes the recovery of the equivalent damping constant in Equation 4:

$$\frac{h_B^2}{h_B^1} \quad (4)$$

where: h_B^1 is the equivalent damping constant of the 1st test; h_B^2 is the equivalent damping constant of the repeated test.

Table 3 describe the recovery of the equivalent damping constant h_B under CSCA test, h_B recovers 82.69%, 84.50%, and 90.34% after 1 day, 1 week, and 3 weeks, respectively. The influence of the recovery on the damping property of HDR is remarkable

The equivalent shear modulus – G_B is calculated by Equation 5:

$$G_{eq} = \frac{\tau_{\max}^+ - \tau_{\max}^-}{\gamma_{\max}^+ - \gamma_{\max}^-} \quad (5)$$

where: $\tau_{\max}^+$ and $\tau_{\max}^-$ are the maximum positive and negative shear stresses, respectively; $\gamma_{\max}^+$ and $\gamma_{\max}^-$ are the maximum positive and negative shear strains, respectively.

The G_B ratio describes the recovery of the equivalent shear modulus:

$$\frac{G_B^2}{G_B^1} \quad (6)$$

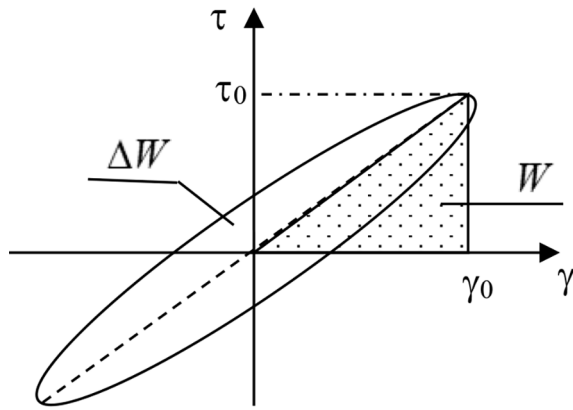


Figure 10. The stress-strain curve of a specimen under cyclic loading

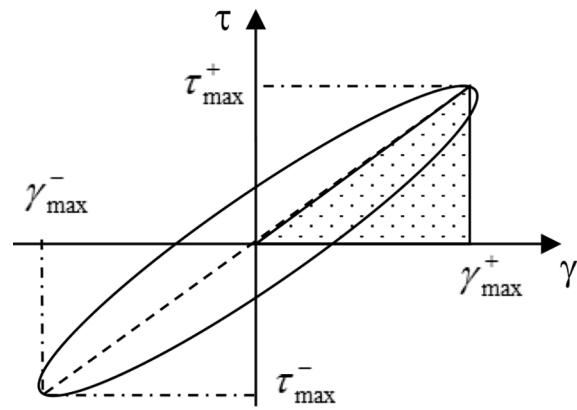


Figure 11. The stress-strain curve of a specimen under cyclic loading

where: G_B^1 is the equivalent shear modulus of the 1st test and G_B^2 is the equivalent shear modulus of the repeated test.

The recovery of the equivalent shear modulus under CSCA test is indicated in Table 4. The equivalent shear modulus-GB recovers 86.64%, 97.98%, and 98.64% after 1 day, 1 week, and 3 weeks, respectively. The recovery also strongly influences on the stiffness property of HDR.

The three introduced parameters in Table 2–4 are recovered very quickly after 1 day, and the recovery process continued over a long period. This recovery can be explained through the recognition of the Mullins effect as a rupture process of bonds between the rubber matrix and filler network [4] under the loading, and then there is the possibility of recross-link of the polymer chains, with associated strength recovery [11, 12]. This recross-linking is achieved by

bringing into proximities the broken links [11]. The microstructural discussion of the recovery of the Mullins effect is presented in [11, 12].

A PHENOMENOLOGICAL MODEL FOR THE MULLINS EFFECT OF HDR

The mechanical behavior of rubber materials is quite complicated such as large deformations, stress softening, nonlinearity. Therefore, most of its models are phenomenal models in Equation 7. Following this method, the Mullins effect of HDR is considered as a damaging process.

The general form of phenomenological models

$$W(F) = (1 - d)W_0(F) \quad (7)$$

Table 3. The recovery of the equivalent damping constant h_b under CSCA test

Cycle	Test-1day			Test-1week			Test-3weeks		
	h_B^1	h_B^2	h_B^2/h_B^1	h_B^1	h_B^2	h_B^2/h_B^1	h_B^1	h_B^2	h_B^2/h_B^1
	(%)	(%)	(%)	(%)	(%)	(%)	(%)	(%)	(%)
1	30.85	25.51	82.69	29.70	25.09	84.50	27.19	24.56	90.34
2	26.22	23.05	87.88	23.42	21.95	93.74	23.19	20.95	90.35
3	25.57	22.41	87.65	22.65	21.51	94.97	21.26	20.12	94.61
4	25.65	22.36	87.20	22.32	20.70	92.74	21.22	19.80	93.33
5	24.80	21.94	88.46	22.03	20.47	92.91	20.19	19.31	95.63
6	24.51	21.73	88.69	21.79	20.45	93.87	19.68	19.88	101.01
7	24.42	21.56	88.26	21.75	20.28	93.26	19.49	19.42	99.63
8	24.66	21.28	86.29	21.64	20.05	92.66	19.84	19.26	97.05
9	24.55	21.00	85.54	21.28	20.24	95.14	19.66	19.25	97.94
10	24.18	21.14	87.41	21.10	19.82	93.94	19.70	19.55	99.23

where: $W_0(F)$ is the classic elastic strain energy densities; F is the deformation gradient; d is the damage parameter.

According to this approach, a classical strain energy density function, $W_0(F)$, is defined for the undamaged material. The Mullins effect then reduces this energy according to the loading history. The damage parameter- d , describes the Mullins effect in terms of its evolution law and is characterized by the mechanical variables and the parameter d defines is a prior unknown and may cover any physical phenomenon like chain and multichain damage, microstructural damage, microvoid formation etc [5]. Based on Equation 6, various models have been defined [27–33]. They vary from each other by the mechanical variables or/and by the evolution law and some typical models are shown in Table 5. Following this way, a phenomenal model is also proposed in the next step.

An evolution equation for the Mullins effect

According to the discussion, the stress softening referred to the Mullins effect of HDR, then the stress ratio in Equation 8 can be used to describe the Mullins effect.

$$r = \tau_i / \tau_0 \quad (8)$$

where: τ_i is the maximum stress of the i^{th} cycle with the Mullins effect, τ_0 is the maximum stress of the stable cycle without the Mullins effect.

According to Equation 6, strain energy is an important quantity, so in this paper, the absorbed energy determined from experiments would be used in modeling. Base on the CSVA experiment, the absorbed anergy of the i^{th} cycle D_i is determined in Figure 12a then the accumulated absorbed energy $D = \sum D_i$. Figure 12b shows the gradient of r - D relationship, based on this result, an equation is proposed in Equations 9–10.

$$\frac{dr}{dD} \gamma_{\max}^m = c(1-r) \quad (9)$$

if $\gamma_{\max} = \text{constant}$

$$r = r_{\text{initial}} \quad (10)$$

if $\gamma_{\max}$ increases

where: m and c are constants and determined by fitting Equation 8 with the experimental data as in Figure 12(b).

$$r_{\text{initial}} = \alpha + \beta \exp(-\psi D) \quad (11)$$

where: α , β , ψ are positive constants and determined by fitting Equation 10 with the experimental data as in Figure 13. $\psi_{\max}$ are the maximum shear strains.

The constants of the proposed equation are determined from CSVA test data presented in Table 6.

Figure 14 compares the r - D relationships obtained from the CSVA test and the proposed model. The predicted curves followed the experimental trends well, indicating that the proposed

Table 4. The recovery of the equivalent shear modulus G_B under CSCA test

Cycle	Test-1day			Test-1week			Test-3weeks		
	G_B^1	G_B^2	G_B^2/G_B^1	G_B^1	G_B^2	G_B^2/G_B^1	G_B^1	G_B^2	G_B^2/G_B^1
	(MPa)	(MPa)	(%)	(MPa)	(MPa)	(%)	(MPa)	(MPa)	(%)
1	2.085	1.806	86.64	1.832	1.795	97.98	2.002	1.975	98.64
2	1.851	1.640	88.59	1.630	1.607	98.59	1.777	1.763	99.19
3	1.764	1.580	89.57	1.563	1.541	98.57	1.696	1.697	100.08
4	1.714	1.548	90.31	1.525	1.506	98.77	1.658	1.658	100.00
5	1.683	1.525	90.64	1.500	1.486	99.03	1.635	1.629	99.67
6	1.662	1.507	90.68	1.483	1.465	98.82	1.607	1.639	101.98
7	1.643	1.496	91.04	1.467	1.452	98.98	1.591	1.596	100.31
8	1.653	1.484	89.79	1.454	1.440	99.02	1.576	1.579	100.23
9	1.647	1.477	89.66	1.445	1.428	98.87	1.562	1.571	100.59
10	1.604	1.465	91.33	1.437	1.419	98.75	1.552	1.564	100.78

Table 5. Some the mechanical variables for phenomenological models defined by Equation 6

Variables	Evolution laws	Reference
$\gamma = \frac{\delta}{h}$ and $\dot{\gamma}$	$\dot{q}_e = \begin{cases} \zeta_e \dot{\gamma} (0.5 \gamma - q_e) & \text{if } q_e < 0.5 \gamma \\ 0 & \text{if } 0.5 \gamma \leq q_e \leq 1 \end{cases}$ $\dot{q}_v = \zeta_v \dot{\gamma} (1 - q_v)$	[23]
$\alpha(t) = \text{Max}_{s \in [0,t]} f(s)$ $\beta(t) = \int_0^t \dot{f}(s) ds$	$d = \hat{d}_\alpha(\alpha) + \hat{d}_\beta(\beta)$ $\begin{cases} \hat{d}_\alpha(\alpha) = d_\alpha^\infty \left[1 - \exp\left(-\frac{\alpha}{\eta_\alpha}\right) \right] \\ \text{and} \\ \hat{d}_\beta(\beta) = d_\beta^\infty \left[1 - \exp\left(-\frac{\beta}{\eta_\beta}\right) \right] \end{cases}$	[27]
$\lambda_{\max}^I$	$D = D_\infty \left[1 - \exp\left(-\frac{\lambda_{\max}^I - 1}{\eta}\right) \right]$	[28]
$W_m - \tilde{W}_0(\lambda)$	$\eta = 1 - \frac{1}{r} \tanh\left[\frac{W_m - \tilde{W}_0(\lambda)}{\mu m}\right]$	[29]
$\bar{\alpha} = \sqrt{I_1/3} - 1$	$\tilde{d}(\bar{\alpha}) = d_\infty \left[1 - \exp\left(-\frac{\bar{\alpha}}{\eta}\right) \right]$	[30]
$W_{\max} - W$	$\eta = \exp\left(-b\sqrt{(W_{\max} - W)}\right)$	[31]
$W_{0\max}^m(I_1, I_2)$	$d(\bar{I}_1, \bar{I}_2) = d_\infty \left[1 - \exp\left(-\gamma W_{0\max}^m\right) \right]$	[32]

relationship can reproduce the main features of the Mullins effect observed in HDR under the investigated loading conditions.

As shown in Figure 14, an increase in accumulated absorbed energy D reduced the stress

ratio r. In addition, r decreased as the maximum experienced strain $\gamma_{\max}$ increased. These observations are consistent with previous findings that the Mullins effect depends on both the maximum strain [3] and the previous strain history [24].

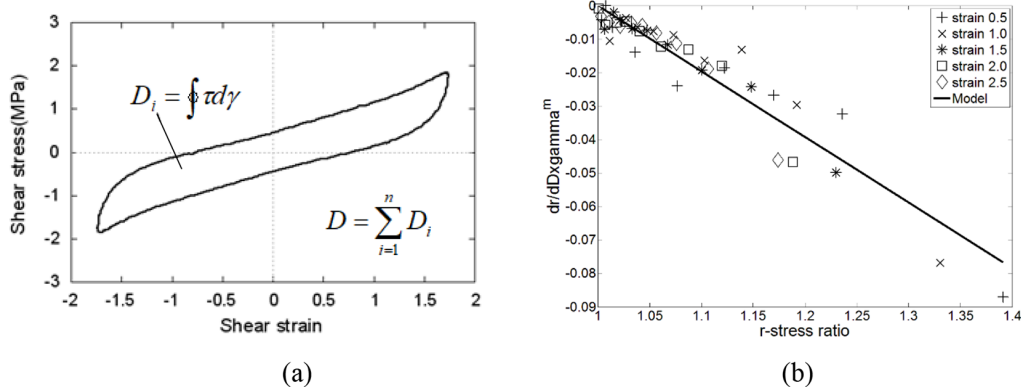


Figure 12. CSVA test results: (a) absorbed energy under cyclic loading and (b) relationship between r and the gradient of r-D

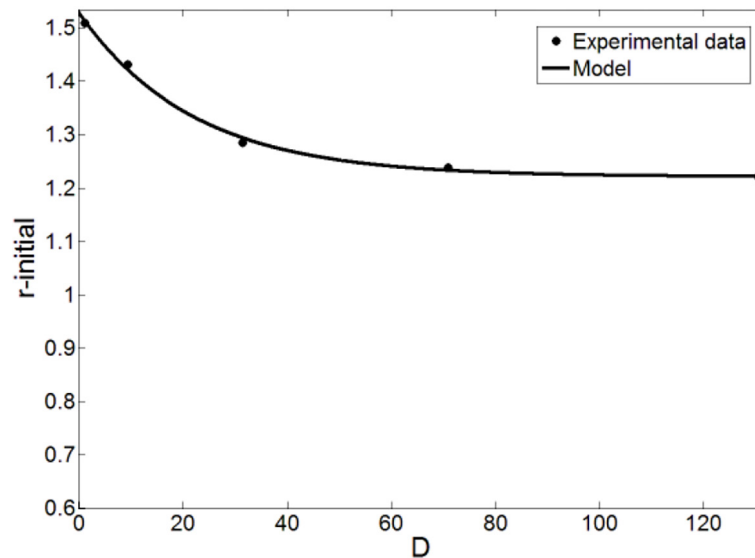


Figure 13. Fitting results between Equation 9 and CSVA test data

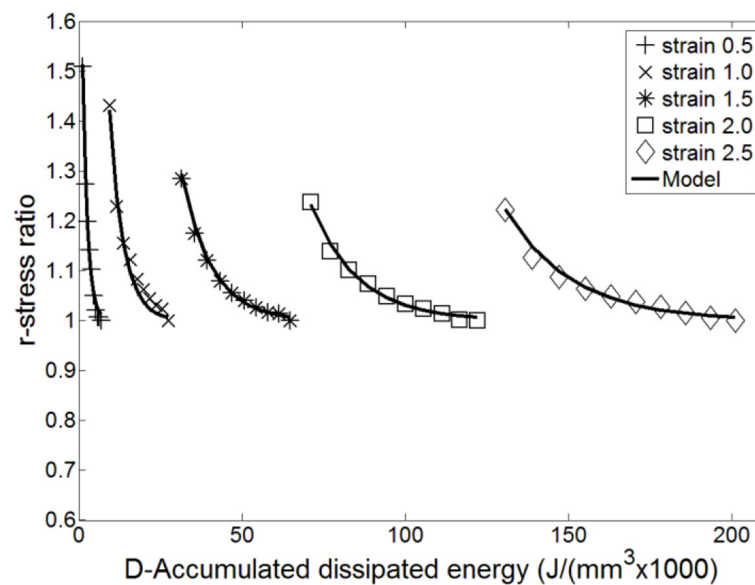


Figure 14. The r-D relationships obtained from the CSVA test and the proposed model

Table 6. Constants of the proposed equation

m	c	α	β	ψ
1.546	0.196	1.222	0.307	0.046

CONCLUSIONS

Cyclic shear test with variable amplitudes was carried out to investigate the Mullins effect of high damping rubber. Although the stress softening induced the Mullins effect at all strain levels, the percentage of this stress reduction decreases gradually with increasing load-strain levels from 37.62% at strain level of 0.5 to 17.84% at strain

level of 2.5. It implies that the dependence of Mullins effect on the degree of rubber virgin is stronger than the dependence on the past strain histories.

Based on the relationship between stress ratio and accumulated absorbed energy obtained from CSVA test result, an phenomenal model is proposed to describe the Mullins effect of HDR. The comparison demonstrated the agreement of the proposed model with the experimental data.

CSCA experiments were conducted to check the recovery of the Mullins effect. The all introduced parameters are recovered by more 80% after 1 day. The stress softening due to the Mullins effect can recover very quickly after 1 day by 88.90% and almost after 3 weeks by 97.43%.

Acknowledgements

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REFERENCES

1. Bouasse H, Carrière Z. Courbes de traction du caoutchouc vulcanisé. In: *Annales de la faculté des sciences de Toulouse 2e série* 1903; 5(3): 257–283. <https://doi.org/10.5254/1.3546914>
2. Mullins L. Effect of stretching on the properties of rubber. *Rubber Chem. Technol.* 1948; 21(2): 281–300. <https://doi.org/10.5254/1.3546914>
3. Mullins L. Softening of rubber by deformation. *Rubber Chem. Technol.* 1969; 42(1): 339–362. <https://doi.org/10.5254/1.3539210>
4. Florian K, Michael J, and Alexander L. Mullins effect: Overview of recent advances. *proceedings in technology transfer. Collaborative Research Advancing Engineering Solutions for Real-World Challenges* 2026; 3: 126–146. https://doi.org/10.1007/978-3-032-09384-4_9
5. Diani J, Fayolle B, Gilormini P. A review on the Mullins effect. *European Polymer Journal.* 2009; 45(3): 601–612. <https://doi.org/10.1016/j.eurpolymj.2008.11.017>
6. Plagge J, Klüppel M. Mullins effect revisited: Relaxation, recovery and high-strain damage. *Materials Today Communications*, 2019; 20: 100588. <https://doi.org/10.1016/j.mtcomm.2019.100588>
7. Man-Gong Z, Wenshuai X, Tao W, Xiang-Dong Z, Hao Z, Zhao L, Chao Z, Heng J, Meng C. Investigation on Mullins effect of rubber materials by spherical indentation method. *Forces in Mechanics*, Volume 4, October 2021; 100037. <https://doi.org/10.1016/j.finmec.2021.100037>
8. Cantounet S, Desmorat R, Besson J. Mullins effect and cyclic stress softening of filled elastomers by internal sliding and friction thermodynamics model. *International Journal of Solids and Structures*, 1 June 2009 46(11–12): 2255–2264. <https://doi.org/10.1016/j.ijsolstr.2008.12.025>
9. Alexander R, Nils HK, Peter W. Comparison of discontinuous damage models of Mullins-type. *Archive of Applied Mechanics*, August 2021; 91: 4097–4119. <https://doi.org/10.1007/s00419-021-02026-9>
10. Jing-Jing H, Ting-Ting Y, Chao-Wen Y, Li-Zhao H, Kun S, Guang-Ai S, Cheng-Sha W, Jia-Zhuang X, Zhong-Ming L, Dong L. The maximum-strain and strain-interval dependences of microstructural evolution underneath the Mullins effect. *Composites Part A: Applied Science and Manufacturing*, September 2023; 172: 107586. <https://doi.org/10.1016/j.compositesa.2023.107586>
11. D' Ambrosio P, De Tommasi D, Ferri D, Puglisi G. A phenomenological model for healing and hysteresis in rubber-like materials. *International Journal of Engineering Science*, April 2008; 46(4): 293–305. <https://doi.org/10.1016/j.ijengsci.2007.12.002>
12. De Tommasi D, Marzano S, Puglisi G, Zurlo G. Damage and healing effects in rubber-like balloons. *International Journal of Solids and Structures*, 2009; 46: 3999–4005. <https://doi.org/10.1016/j.ijsolstr.2009.07.020>
13. Yan L, Dillard DA, West Robert L., Lower LD, Gordon GV. Mullins effect recovery of a nanoparticle-filled polymer, *Journal of Polymer Science – Part B*, 2010, <https://doi.org/10.1002/polb.22102>
14. Hao C, Ji L, Dong L, Jin Q, Rui X. A network evolution model for recovery of the Mullins effect in filled rubbers. *International Journal of Applied Mechanics*, 2020; 12(9): 2050108 <https://doi.org/10.1142/S1758825120501082>
15. Anh DN, A rheology model of high damping rubber bearings for seismic analysis at room and low temperatures. *Doctoral dissertation, Graduate School of Science and Engineering, Saitama University, Japan*, 2017.
16. Kelly JM, *Earthquake resistant design with rubber*. 2nd edition, Springer-Verlag Berlin Heidelberg, New York, 1997.
17. Mateusz Ż, Dominik R, Robert Z. Physics-informed neural network for identifying the hysteresis model of the vacuum-packed particles torsional damper. *Advances in Science and Technology Research Journal (ASTRJ)*, 2026; 20(1): 301–317. <https://doi.org/10.12913/22998624/210455>
18. Gigih P, Fachruddin H, Fitriani I, Ubaidillah, Purbo S, Muhammad INuS. Development of a vibration isolator on the basis of a magneto-rheological elastomer. *Advances in Science and Technology Research Journal*, 2024; 18(8): 272–280. <https://doi.org/10.12913/22998624/194441>
19. Yoshida J, Abe M, Fujino Y. Constitutive model of high-damping rubber materials. *Journal of engineering mechanics*, 2004; 130(2), 129–141. [https://doi.org/10.1061/\(ASCE\)0733-9399\(2004\)130:2\(129\)](https://doi.org/10.1061/(ASCE)0733-9399(2004)130:2(129))

20. Nguyen DA, Dang J, Okui Y, Amin AFMS, Okada S, Imai T, An improved rheology model for the description of the rate-dependent cyclic behavior of high damping rubber bearings. *Soil Dynamics and Earthquake Engineering*, 2015; 77: 416–431. <https://doi.org/10.1016/j.soildyn.2015.06.001>
21. Dung NA, Phong LT. Low Temperature Effect on Dynamic Mechanical Behavior of High Damping Rubber Bearings. In *Modern Mechanics and Applications: Select Proceedings of ICOMMA 2020*; Springer: Singapore, 2022; 203–218. https://doi.org/10.1007/978-981-16-3239-6_16
22. Zahraa KR, Raed SS, Bashar DHA, Alaa SO. Thermal, tribological, and mechanical performance of nano-Al₂O₃ reinforced silicone rubber for high-temperature sealing applications. *Advances in Science and Technology Research Journal (ASTRJ)*, 2026; 20(7).
23. Dall'Asta A, Ragni L. Experimental tests and analytical model of high damping rubber dissipating devices. *Engineering Structures*, 2006; 28: 1974–1884. <https://doi.org/10.1016/j.engstruct.2006.03.025>
24. Yuqing T, Yanhui L, Ji D, Akira I, Takehiko H, Yuki H. Stress-softening behavior of high-damping rubber bearings at low temperatures. *ASCE Journal of Structural Engineering*, 2024; 150(6): 04024050-1-13. <https://doi.org/10.1061/JSENDH.STENG-13253>
25. International Organization for Standardization ISO 22762-1, 2010. Elastomeric seismic-protection isolators, Part 1: Test methods.
26. Nguyen AD, Quang HN. A new experiment manner to determinate design parameters of high damping rubber bearings. *International Journal of Innovative Technology and Exploring Engineering (IJITEE)*, 2019; 8(9): 1859–1566.
27. Miehe C. Discontinuous and continuous damage evolution in Ogden-type large-strain elastic materials. *Eur. J. Mech., A/Solids*, 1995; 14(5): 697–720.
28. Laiarinandrasana L, Piques R, Robisson A. Visco-hyperelastic model with internal state variable coupled with discontinuous damage concept under total Lagrangian formulation. *International Journal of Plasticity*, 2003; 19: 977–1000. [https://doi.org/10.1016/S0749-6419\(02\)00089-X](https://doi.org/10.1016/S0749-6419(02)00089-X)
29. Dorfmann A, Ogden RW. A constitutive model for the Mullins effect with permanent set in particle-reinforced rubber. *International journal of Solids and Structures*, 2004; 41: 1855–1878. <https://doi.org/10.1016/j.ijsolstr.2003.11.014>
30. Chagnon G, Verron E, Gornet L, Marckmann G, Charrier P. On the relevance of continuum damage mechanics as applied to the Mullins effect in elastomers. *Journal of the Mechanics and Physics of Solids*, 2004; 52: 1627–1650. <https://doi.org/10.1016/j.jmps.2003.12.006>
31. Elias-Zuniga A. A phenomenological energy-based model to characterize stress-softening effect in elastomers. *Polymer*, 2005; 46: 3496–3506. <https://doi.org/10.1016/j.polymer.2005.02.093>
32. Li J, Mayau D, Lagarrigue V. A constitutive model dealing with damage due to cavity growth and the Mullins effect in rubber-like materials under triaxial loading. *Journal of the Mechanics and Physics of Solids*, 2008; 56: 953–973. <https://doi.org/10.1016/j.jmps.2007.06.009>
33. Luo RK. Investigation on the full Mullins effect using time-dependent hyperelastic model with energy dissipation for rubber antivibration applications. *Mechanics of Time-Dependent Materials*, 2021; 25: 581–600. <https://doi.org/10.1007/s11043-020-09461-1>