

A robust calibration framework for t-z analysis of bored piles: Comparative assessment of hyperbolic and bilinear load-transfer models

Loi Giap Van¹, Tuan Anh Pham^{1*} 

¹ Smart Urban Construction Infrastructure, University of Transport Technology, Hanoi 100000, Vietnam
 * Corresponding author's e-mail: anhpt@utt.edu.vn

ABSTRACT

Although the t-z load-transfer method remains a cornerstone in geotechnical engineering, reliance on standardized empirical curves frequently introduces significant settlement prediction errors. To bridge this critical performance gap, this study develops an objective, automated back-analysis framework leveraging the generalized reduced gradient (GRG) nonlinear optimization algorithm (Excel Solver) to extract site-specific t-z and q-z parameters directly from full-scale static load tests. Integrating a one-dimensional finite difference model, the research evaluates the axial behavior of large-diameter bored piles by systematically comparing classical bilinear (elastic-perfectly plastic) and continuous nonlinear hyperbolic formulations against two rigorous benchmark datasets: Shibuya et al. (dense sand) and O'Neill et al. (cohesive clay). The results demonstrate that the hyperbolic model markedly outperforms the bilinear approach by gracefully capturing progressive soil stiffness degradation. Quantitatively, the hyperbolic formulation achieves exceptional predictive accuracy with Pearson correlation coefficients exceeding 0.997 and dramatically lower root mean square errors (RMSE) across both granular (1.234 MN for Shibuya) and cohesive (19.086 kN for O'Neill) strata, whereas the bilinear model exhibits severe error escalation (3.386 MN and 74.268 kN, respectively) despite maintaining relatively high correlation ($R^2 \approx 0.962 - 0.975$). Furthermore, comprehensive sensitivity analyses and Monte Carlo simulations (N=300) reveal that pile-head settlement is predominantly governed by initial stiffness mobilization rather than ultimate shear resistance, with sensitivity ratios reaching up to 30,000. By rigorously integrating automated optimization, statistical validation (t-tests), and probabilistic uncertainty quantification (95% confidence intervals), this study delivers a robust, efficient, and reliable methodology that significantly enhances the predictive confidence of modern pile foundation design.

Keywords: t-z load-transfer method, bored pile settlement, hyperbolic model, automated back-analysis, sensitivity analysis, Monte Carlo simulation, foundation design optimization.

INTRODUCTION

Pile foundations serve as the backbone of modern high-load infrastructure; however, predicting their settlement behavior remains a persistent and complex challenge in geotechnical engineering. Assessing the performance of these foundations requires a precise understanding of the load-settlement relationship at the pile head, a key indicator of bearing capacity and structural stability. Typically, this relationship is derived from field static compression tests - a method highly regarded for its

ability to simulate real-world conditions. Despite their reliability, the prohibitive costs and time-consuming nature of these tests often restrict their implementation, precluding large-scale application across multiple pile units. To overcome these constraints, numerous simplified analytical theories have been developed. Early studies by Vesic (1977) [1] and Bowles (1996) [2] applied elastic theory to represent total pile settlement as a combination of pile tip displacement, elastic shortening, and soil deformation. Furthermore, Fleming (2008) [3] proposed a method employing hyperbolic functions to

capture the nonlinear performance of both the shaft and tip. Despite their utility, modeling the pile as a structural member embedded within a homogeneous medium often yields inaccurate results, particularly when the pile penetrates complex, non-uniform geological profiles.

Consequently, computational approaches have evolved to decouple pile behavior into distinct segments corresponding to varying geological conditions. Advanced techniques, such as the finite element method (FEM), have been widely adopted to address axial loading. Poulos and Davis (1980) [4] integrated FEM with the boundary element method to analyze piled raft foundations, demonstrating clear advantages in accounting for stress distribution within the soil. Randolph (1994) [5] and Potts (2001) [6] further utilized FEM to model piles in complex settings with high reliability. Nonetheless, scholars such as Jardine and Potts (1993) [7], Reul and Randolph (2003) [8], and Brinkgreve (2005) [9] have noted that while complex FEM models provide realistic predictions, they require extensive soil parameters to which results are highly sensitive. Randolph (2003) [8] also emphasized that FEM often relies on the idealized assumption that the pile is already embedded in the soil, thereby neglecting the significant impact of the installation process on surrounding soil properties. The t-z load-transfer method remains a cornerstone for analyzing pile-soil interaction, favored for its ability to balance computational efficiency with engineering reliability [10]. Rooted in an extended Winkler model, this approach discretizes the pile into segments supported by nonlinear springs. Following pioneering developments by Seed and Reese (1957) [11], Coyle and Sulaiman (1967) [12], and Randolph and Wroth (1978) [13], the method was codified in international standards, such as FHWA [14] and the American Petroleum Institute (API) [15]. Poulos (2016) [16] highlighted that while FEM is computationally prohibitive for large pile groups, the t-z method facilitates rapid iterative design and independent parameter assignment [10]. Furthermore, Briaud and Tucker (1988) [17] demonstrated the feasibility of calibrating these curves against in-situ data to improve accuracy. Recent research has further expanded the application of t-z models to analyze complex pile-soil systems [18–21]. These studies consistently demonstrate the remarkable predictive performance of these models when validated against static pile load test results, reaffirming their utility as a practical and reliable tool for geotechnical practice.

Despite their widespread adoption, standardized t-z curves often deviate from field observations. Studies by Vardanega et al. [22] and Bhuiyan et al. (2021) [23] have noted that standard stiffness parameters often overestimate capacity. While recent advancements, such as the Hyperbolic load-transfer models for layered soils by Luo et al. (2022) [24], show improved alignment with experimental data, the performance consistency of Bilinear and Hyperbolic models across varied geotechnical conditions remains an open question. Kim et al. [25] used the ideal Mohr-Coulomb elastoplastic model, considering two boundary conditions: passive earth pressure and the soil-pile contact method. From this, they successfully developed a finite element model that significantly reduced the analysis of the load-settlement behavior of piles. The study by Elsiragy et al. (2025) [26] combined laboratory experiments and Plaxis 3D simulations to evaluate the response of single piles under lateral loading in soft clay soil. The results showed that the use of an extended pile head configuration significantly increased the soil reaction, with the soil modulus improved by up to 200% compared to conventional piles. These findings confirm the effectiveness and practicality of the uniformly extended pile head solution in modern foundation design. Recently, Cesaro et al. [27] proposed an analytical model for bored piles in sandy soils that improves shaft resistance predictions by accounting for soil dilatancy and shear band mechanisms. Their work demonstrates that capturing true failure behavior requires moving beyond simple elastic assumptions to incorporate physical parameters such as shear band thickness and critical-state shear strain. This highlights the growing industry shift toward more rigorous, mechanics-based formulations rather than purely empirical approaches. Existing literature predominantly relies on either simplistic elastic models – which overlook soil nonlinearity – or complex FEM simulations that demand unavailable inputs and are sensitive to constitutive assumptions. Furthermore, traditional models often neglect the impact of installation processes, leading to significant “theoretical-versus-actual” performance gaps.

The research gap addressed in this study lies in the deficiency of practical yet accurate calibration frameworks that reconcile theoretical t-z models with in-situ measured behavior. While the debate regarding the suitability of Bilinear versus more advanced nonlinear models persists, there is a lack of rigorous, systematic comparisons based on high-quality field instrumentation

data. Furthermore, most existing studies treat the t-z model as a black box for curve fitting, neglecting the crucial need to quantify which mechanical parameters actually govern the load-settlement response under varying geological conditions. This study bridges this gap not only by establishing a robust calibration framework but also by integrating a quantitative sensitivity analysis to identify the governing mechanical drivers of pile performance across diverse soil profiles.

Consequently, the primary scientific contributions of this study are structured as follows:

- Establishment of an objective, automated back-analysis framework – moving beyond conventional trial-and-error practices, the study deploys a systematic automated optimization framework (using the GRG nonlinear algorithm) to extract continuous t-z and q-z load-transfer curves directly from full-scale static load tests, eliminating user subjectivity.
- Rigorously quantifying the performance gap in continuous nonlinear modeling – rather than merely applying a hyperbolic curve, the study provides a comprehensive comparative evaluation between classical bilinear (elastic-perfectly plastic) and hyperbolic formulations, backed by an expanded multi-metric statistical framework (R^2 , RMSE, MAE, NRMSE, and paired t-tests) to expose the limitations of conventional piece-wise linear approaches.
- Advanced uncertainty quantification and sensitivity mechanics – transitioning from deterministic predictions, the research integrates multi-level sensitivity testing and Monte Carlo simulations ($N=300$) to establish rigorous 95% confidence intervals ($\text{Mean} \pm 1.96 \times \text{STD}$). This effectively isolates the primary mechanical drivers of pile settlement while accounting for both inherent spatial soil variability and experimental instrumentation errors, thereby bridging the gap between theoretical modeling and high-reliability geotechnical practice.

RESEARCH METHODS

Elastic-perfectly plastic (bilinear) model

The elastic-perfectly plastic model (Figure 1), commonly referred to as the bilinear model, is the simplest formulation within the t-z framework. It is based on the assumption that soil behavior is linearly elastic until a critical shear stress

threshold is reached, followed by a state of perfectly plastic flow. The relationship between shear stress (τ) and pile displacement (w) is defined by two distinct stages:

- Elastic stage – the relationship is linear, characterized by a constant spring stiffness (K).
- Plastic stage – upon reaching the ultimate shear resistance (τ_{ult}), the shear stress remains constant regardless of any further increase in displacement.

The mathematical relationship is defined as follows:

$$\tau = \begin{cases} K \cdot w, & \text{for } w \leq w_y \\ \tau_{ult}, & \text{for } w > w_y \end{cases} \quad (1)$$

where: K – unit spring stiffness (kN/m^3), w_y – yield displacement at the onset of plastic flow (m).

The determination of this model requires two primary parameters:

- Ultimate shear resistance (τ_{ult}) – derived from the static load test results of the two instrumented piles through an analysis of the distributed axial load along the pile shaft.
- Spring stiffness (K) – this is a critical parameter that must be calibrated to match the strain gauge measurements obtained at the two specific levels along the pile shaft.

Advantages

The model offers high computational efficiency, requiring no complex nonlinear functions. It is highly effective for iterative design processes or the rapid assessment of ultimate pile bearing capacity.

Limitations

The primary drawback of this model is the omission of the transition phase between the elastic and plastic states (i.e., the absence of gradual stiffness degradation). Consequently, in the small-displacement range (pre-yield), the model typically predicts a response that is significantly stiffer than the actual measurements observed from the two field piles.

Hyperbolic model

The hyperbolic model is utilized to describe the shear stress–displacement ($\tau - W$) relationship in a continuous, nonlinear manner, effectively addressing the limitations of the sudden

stiffness change at the yield point characteristic of the elastic-perfectly plastic model. This study employs the approach originally proposed by Kondner (1963) [28] and further developed by Fleming (1992) [29], wherein the relationship between shear stress and displacement is defined by the following equation:

$$\tau = \frac{\tau_{ult} \cdot w}{w + w_{50}} \quad (2)$$

In this formulation, w_{50} represents the displacement at which 50% of the ultimate shear resistance (τ_{ult}) is mobilized, serving as a critical parameter to control the curvature and the rate of stiffness degradation.

The selection of the hyperbolic model is driven by its inherent mathematical continuity. Unlike the bilinear model, the hyperbolic formulation simulates the gradual stiffness degradation of the soil, producing a curve that asymptotically approaches τ_{ult} . To determine the parameters K_{ini} (initial stiffness) and τ_{ult} (ultimate resistance), this research employs a least-squares regression analysis based on the transformed linear form of the hyperbolic equation ($w/\tau = a + bw$). This linear transformation ensures a unique, globally optimal solution and minimizes potential optimization biases.

The integration of the w_{50} parameter enables the model to accurately capture soil behavior during the initial loading phase – where conventional models often predict an overly stiff response. Furthermore, the hyperbolic formulation ensures numerical stability by maintaining a continuous derivative of the stress-displacement function, which facilitates smoother convergence in iterative numerical solvers compared to the non-smooth response of bilinear models.

This approach provides a flexible and physically robust calibration tool, allowing for a precise fit to the load-settlement curves obtained from full-scale static pile load tests.

Load-transfer method for nonlinear pile analysis

The load-transfer method (t-z method) discretizes the pile into n finite elements, where each segment is coupled to the surrounding soil via a nonlinear spring representing shaft friction. The nonlinear response of the pile is determined by solving the equilibrium equations at each node along the pile shaft.

For a pile segment subjected to axial loading, the governing differential equation for force equilibrium is given by:

$$\frac{d}{dz} \left(EA \frac{dw}{dz} \right) + \tau(w) \cdot P = 0 \quad (3)$$

where: EA – axial stiffness of the pile; w – axial displacement; P – pile perimeter; $\tau(w)$ – nonlinear shear stress–displacement function.

Nonlinear iterative solution procedure – given the nonlinearity of the $\tau(w)$ function, the governing equation is solved numerically using the Finite Difference Method (FDM). The solution procedure follows these steps:

1. Discretization – the pile is divided into discrete elements of length ΔL .
2. Boundary conditions – specified at the pile head (applied load Q_{top}) and the pile base (represented by a distinct q-z load-transfer curve).
3. Iterative solution – at each load increment, the

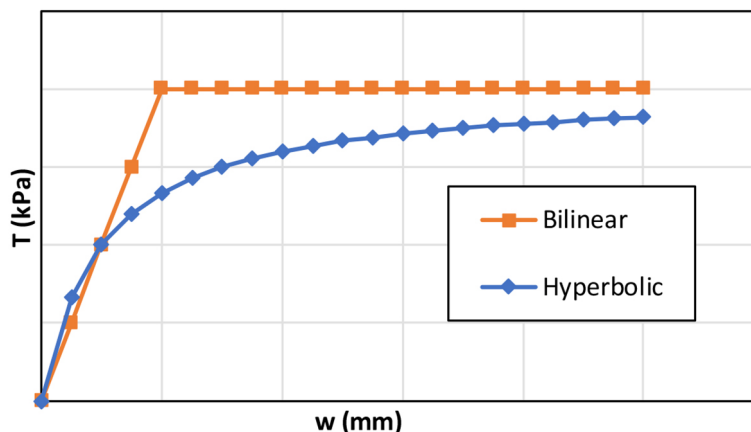


Figure 1. Illustrating the bilinear and hyperbolic model

system of equations is solved iteratively to determine the displacement w at each node. During each iteration, the tangent stiffness of the spring ($K_t = d\tau/dw$) is updated based on the displacement w derived from the previous iteration.

4. Verification – upon convergence, the calculated pile head settlement and axial strain distribution are directly compared with the static load test measurements obtained from the instrumented piles.

Statistical validation and uncertainty quantification framework

To rigorously assess the predictive reliability of the proposed models, a multi-metric statistical validation framework was established.

- Error assessment metrics – the performance of the models was quantitatively evaluated using four primary metrics to ensure a comprehensive analysis of predictive accuracy:
- Root mean square error (RMSE) – used to quantify the magnitude of prediction errors,

providing a direct measure of the model's absolute fit to experimental data.

- Mean absolute error (MAE) – employed to evaluate the average magnitude of errors in a set of predictions, without considering their direction, ensuring a robust measure of consistency across the loading spectrum.
- Normalized root mean square error (NRMSE) – utilized to facilitate a normalized comparison of predictive accuracy across diverse soil datasets and loading scales, ensuring that the error magnitude is relative to the observed range.
- Coefficient of determination (R^2) – calculated to assess the proportion of the variance in the experimental response that is captured by the model, serving as a primary indicator of correlation and goodness-of-fit.

Statistical and probabilistic validation

To statistically validate the superiority of the hyperbolic model, a paired t-test was conducted to determine if the reduction in RMSE compared to the bilinear approach is statistically significant ($p < 0.05$).

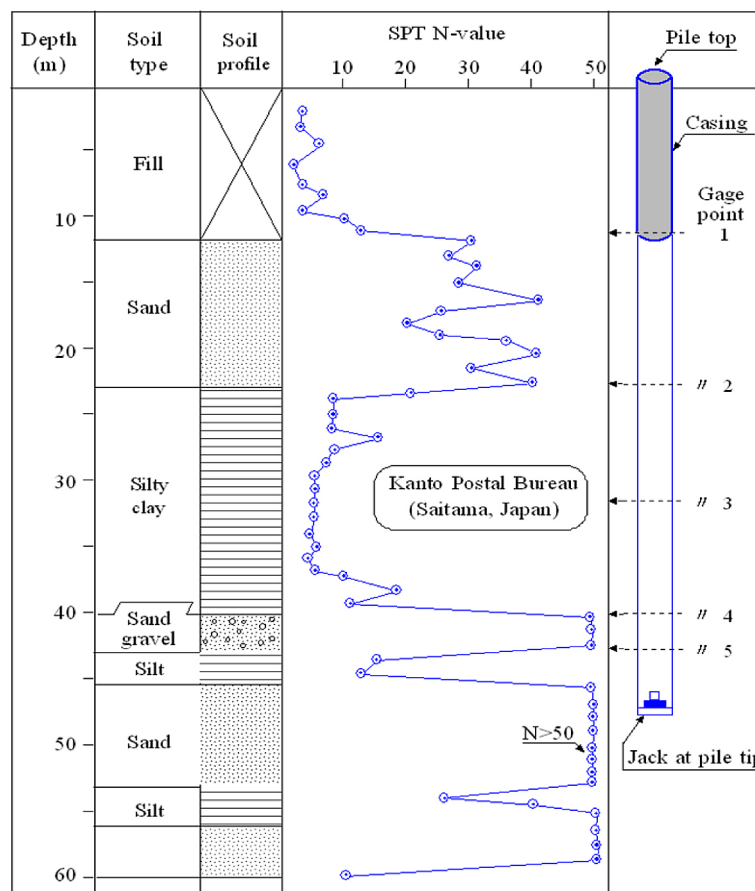


Figure 2. Diagram of the Kanto postal pile load test [30], [31]

To address geotechnical uncertainty and quantify parameter robustness, Monte Carlo simulations were performed by treating key input parameters as random variables. This simulation generates probabilistic distributions of pile-head settlement, ensuring the reliability of optimized parameters against inherent soil heterogeneity.

RESEARCH DATA

Pile data from Shibuya et al. (1997)

The experimental research conducted by Shibuya et al. (1997) [30, 31] in Saitama (Figure 2), Japan, provides a benchmark dataset for analyzing the nonlinear behavior of large-diameter bored piles. The foundation was constructed in a complex geological environment, characterized by a dense sand layer (SPT $N > 50$) located at a depth of 45–60 m, which serves as the primary end-bearing stratum.

To precisely decouple shaft friction from end-bearing resistance, the experimental setup incorporated two critical methodologies:

Hydraulic jack system at the pile toe – a hydraulic jack (1.3 m in diameter) was installed directly at the pile base at a depth of 48 m. This configuration enabled the total isolation of end-bearing resistance from shaft friction. Experimental results confirmed

an ultimate end-bearing capacity of approximately 30 MN at a displacement of 50 mm.

Multi-point monitoring system – the pile shaft was instrumented with strain gauges at five specific levels (Section 1 through Section 4, and the pile toe). This configuration allowed for the precise determination of axial load distribution along the shaft, facilitating the calculation of local shaft friction. The observational data indicated that the shaft friction in the deepest segment reached a peak value of 20 t/m², showing high correlation with the design prediction of 17 t/m².

Furthermore, the data captured the elastic shortening of the pile under compressive loading and clearly demonstrated the elimination of shaft friction in the upper segment, attributed to the presence of the temporary steel casing.

This experimental dataset serves as a benchmark for nonlinear pile analysis. The simultaneous monitoring of both pile toe and shaft behavior provides the necessary internal boundary conditions to validate load-transfer (t-z) models. This empirical foundation is critical for calibrating the nonlinear parameters (K_p , τ_{ult} , W_{50}) of the Bilinear and hyperbolic models, ensuring that the predicted displacement and shaft friction distribution align with the observed field performance in complex geological formations.

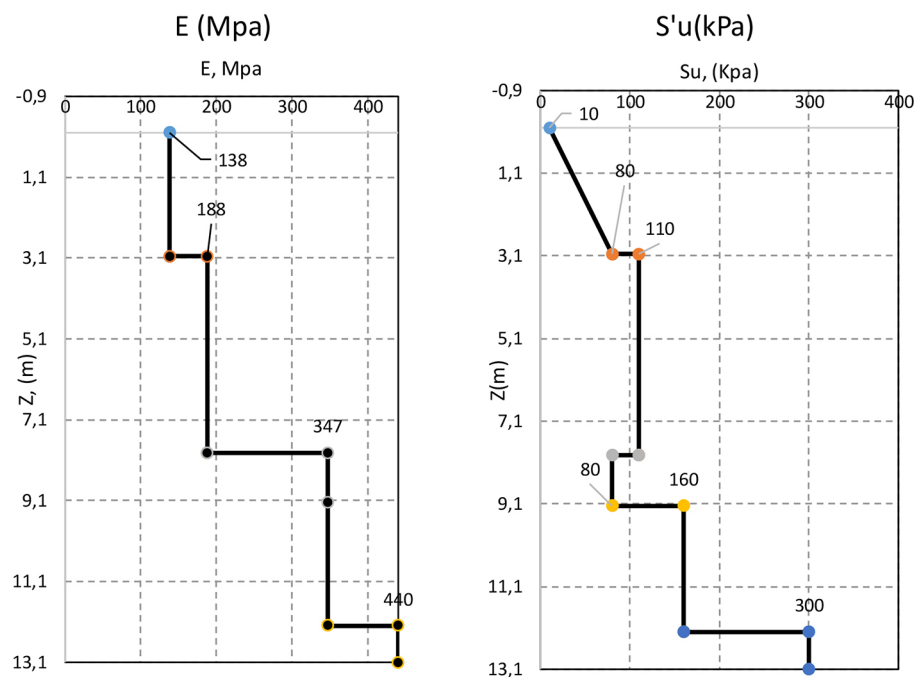


Figure 3. O'Neill's (1982) experimental soil parameters

Pile data from O'Neill et al. (1982)

O'Neill et al. (1982) [32] conducted static load tests on both single piles and a 3×3 pile group installed in a heterogeneous soil profile consisting of four clay layers, ranging from very stiff to medium consistency (Figure 3).

The test pile was a steel pipe pile, 13.1 m in length, with an outer diameter of 273 mm and a wall thickness of 9.3 mm. Instrumentation, including strain gauges, was strategically positioned along the pile shaft and between the distinct clay layers. The static load tests, conducted to the point of failure, provided high-resolution data that allowed for the precise determination of ultimate shaft friction and end-bearing resistance for each soil layer.

RESULTS AND DISCUSSION

Calibration of t-z curves

The parameter calibration for the t-z and q-z curves was conducted using the generalized reduced gradient (GRG) nonlinear optimization algorithm, as implemented in Microsoft Excel Solver (Fylstra et al., 1998) [33]. This iterative procedure minimizes the root mean square error (RMSE) between the experimental responses and model predictions. To ensure physical consistency, all model parameters were constrained to strictly positive values (>0) throughout the optimization, reflecting the inherent physical requirements of pile-soil interaction. While advanced metaheuristic algorithms exist, the GRG method was deliberately chosen for its high efficiency and stable convergence in low-dimensional, well-behaved problems (typically 5×10 parameters per pile), achieving exceptional accuracy ($R^2 > 0.95$). Furthermore, this transparent, industry-standard tool bridges the gap between advanced numerical research and routine design workflows, enabling practitioners to verify optimization trajectories and avoid black-box overfitting.

The optimization framework for each model is detailed below.

Bilinear model – the parameters to be optimized are the ultimate shear resistance (τ_{ult}) and the yield displacement (W_y). The calibration effectively identifies the specific transition point that characterizes the shift from elastic behavior to the onset of plastic stiffness degradation.

Hyperbolic model – the parameters to be optimized are the ultimate shear resistance (τ_{ult}) and the displacement at 50% mobilization of ultimate resistance (W_{50}). The calibration emphasizes the initial stiffness (K_i) and the curvature of the transition zone, which are critical for characterizing the nonlinear stiffness degradation of the soil.

Parameter initialization and optimization strategy – to ensure computational efficiency and numerical stability, the parameters obtained from the bilinear regression were utilized as initial estimates for the hyperbolic model calibration. Specifically, the ultimate resistance (τ_{ult}) was initialized using the value derived from the bilinear model, while the displacement parameter w_{50} was initially approximated as $0.5 \times w_y$. Following this initialization, the GRG Nonlinear algorithm performed an independent optimization for the Hyperbolic parameters to minimize the RMSE against the experimental data. This approach allows both models to be calibrated independently while ensuring that the final optimized parameters are grounded in the same underlying physical response of the pile-soil system.

The optimization algorithm systematically adjusts these parameters within predefined physical bounds, constrained by the specific geological characteristics of the soil layers. By minimizing the RMSE as the objective function, the calibration ensures that the predicted load-transfer response is statistically constrained to the strain gauge measurements from the instrumented piles in both the Shibuya et al. [30, 31] and O'Neill [32] datasets. This rigorous, site-specific calibration transforms the t-z models from theoretical abstractions into reliable predictive tools, rigorously grounded in the actual response of the pile-soil system.

To evaluate the stability and convergence robustness of the optimization framework against potential local minima, sensitivity tests involving various sets of initial parameter values were conducted. The analysis demonstrated that regardless of different initial guesses within physically grounded boundaries, the GRG algorithm consistently converged to the identical optimal parameter set, yielding negligible variations in the objective function (RMSE). This stability confirms that the low-dimensional t-z and q-z calibration problem presents a well-behaved error surface, ensuring that the final calibrated outcomes are robust, unique in convergence, and entirely independent of the initial starting points.

Table 1. The results of the T-Z curve correction for O'Neill's experiment

Soil layer	z (m)	Bilinear			Hyperbolic		
		T_{ult} (kPa)	W_y (mm)	RMSE (kPa)	T_{ult} (kPa)	W_{50} (mm)	RMSE (kPa)
4	-12.3	118.122	0.244	0.398	118.702	0.244	1.000
3	-8.5	51.540	7.715	7.185	63.199	7.715	0.998
2	-5.5	62.955	36.889	1.133	82.987	24.516	0.973
1	-2.9	23.911	22.457	2.837	31.730	17.505	0.990
Pile tip	-13.1	2.558 (MPa)	1.420	0.144 (MPa)	3.088 (MPa)	0.798	0.0665 (MPa)

Figure 4a illustrates the depth-dependent variation of the unit shaft friction (t - z curves) for the O'Neill experiment, derived through the proposed back-analysis optimization process. By employing the Excel Solver to minimize the RMSE for each geological stratum, the calibrated hyperbolic model effectively captures the nonlinear soil-pile interaction across four distinct layers (from $z = -2.9$ m to $z = -12.3$ m) and the pile tip.

As summarized in Table 1, the hyperbolic model demonstrates superior predictive

robustness compared to the bilinear model. While the bilinear model's RMSE fluctuates significantly - ranging from 0.398 kPa to 7.185 kPa - the hyperbolic model maintains a highly consistent RMSE (between 0.665 and 1.000), indicating exceptional numerical stability across different soil profiles. The hyperbolic model consistently predicts higher ultimate shear resistance (T_{ult}) across all layers, reflecting a more realistic assessment of the soil's capacity mobilization in a nonlinear regime. Notably, at the pile tip ($z =$

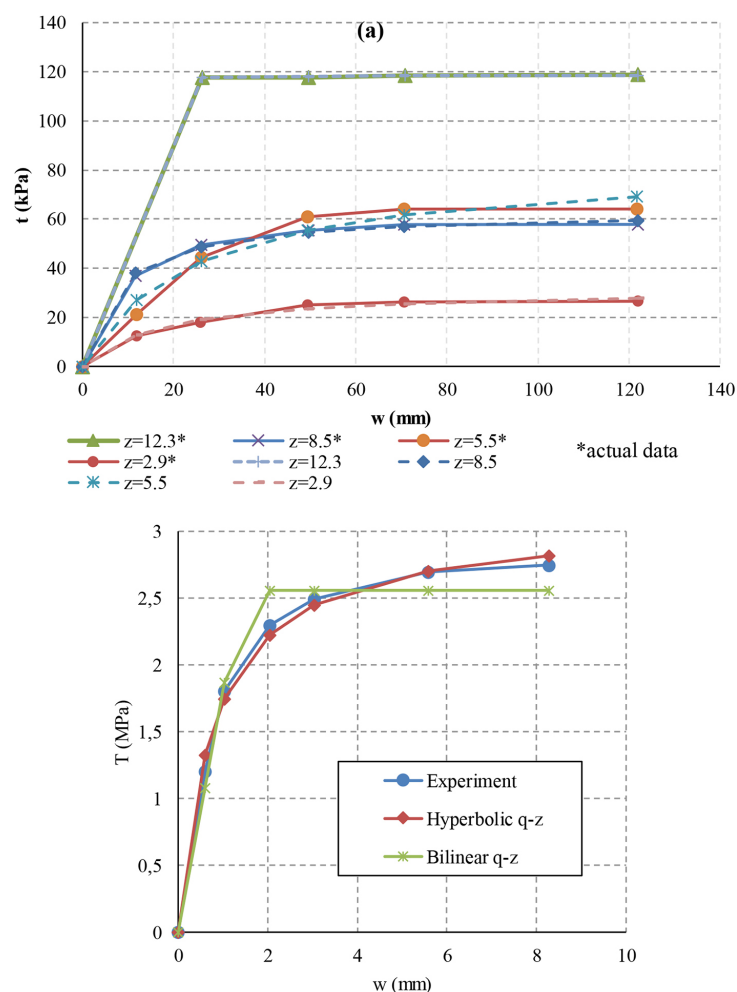

Figure 4. T-Z curves after correction from O'Neill's experiment (1982): a) T-Z curve along the pile shaft; b) T-Z curve at the pile tip

Table 2. The results of the T-Z curve correction for the experiment of Shibuya et al.

Soil layer	z (m)	Bilinear			Hyperbolic		
		τ_{ult} (kPa)	W_y (mm)	RMSE (kPa)	τ_{ult} (kPa)	W_{50} (mm)	RMSE (kPa)
4	40 – 50	176.176	4.022	23.091	251.458	2.716	10.626
3	25 – 40	56.885	0.001	27.112	168.418	4.070	5.513
2	10 – 25	444.484	40.543	13.459	100.000	3.000	5.980
1	0 – 10 m	23.923	18.505	14.653	30.000	8.000	2.170
Pile tip	-50 m	35.0 MN	2.551	2.221	43.166 MN	3.672	0.724

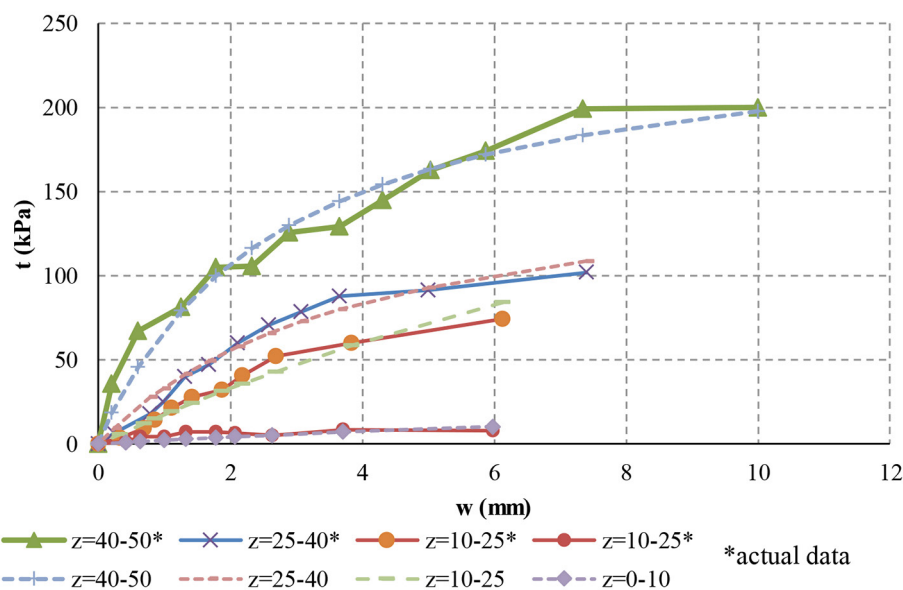
-13.1 m) (Figure 4b), the hyperbolic model predicts an ultimate capacity of 3.088 MN with an RMSE of 0.0665, significantly outperforming the bilinear model (2.558 MN, RMSE 0.144)

The optimization of the t-z and q-z parameters for the Shibuya et al. dataset reveals the distinct, depth-dependent mechanical behavior of the soil strata. The results summarized in Table 2 demonstrate that the calibrated hyperbolic model consistently provides a superior fit to the experimental observations compared to the bilinear model across all depth intervals.

Performance across stratigraphic layers (t-z response) – the t-z curves for various depth intervals (Figure 5) illustrate that the hyperbolic model successfully captures the nonlinear mobilization of shaft resistance across all layers. The optimization process, driven by the minimization of the RMSE, yields highly accurate results. Notably, the hyperbolic model shows significantly lower RMSE values compared to the bilinear model in all cases. For instance, in the deepest stratum (z

= 40–50 m), the hyperbolic model achieves an RMSE of 10.626 kPa, whereas the bilinear model exhibits an RMSE of 23.091 kPa. Similarly, in the intermediate layers, the hyperbolic model maintains robust predictive accuracy (RMSE of 5.513 kPa at z = 25–40 m and 5.980 kPa at z = 10–25 m), significantly outperforming the Bilinear model's respective RMSE values of 27.112 kPa and 13.459 kPa. These findings validate the hyperbolic formulation as a robust tool for characterizing the site-specific stiffness and resistance mobilization of large-diameter piles in complex stratigraphic conditions.

Mobilization of end-bearing resistance (q-z response): Figure 6 illustrates the load-displacement response at the pile tip. The bilinear model exhibits an abrupt transition to plastic failure at approximately 35 MN, which tends to overestimate the stiffness during the intermediate loading stages. In contrast, the hyperbolic model, calibrated via the Excel Solver, provides a more ductile response that closely follows the experimental


Figure 5. The T-Z curves along the pile shaft after correction from the experiment of Shibuya et al.

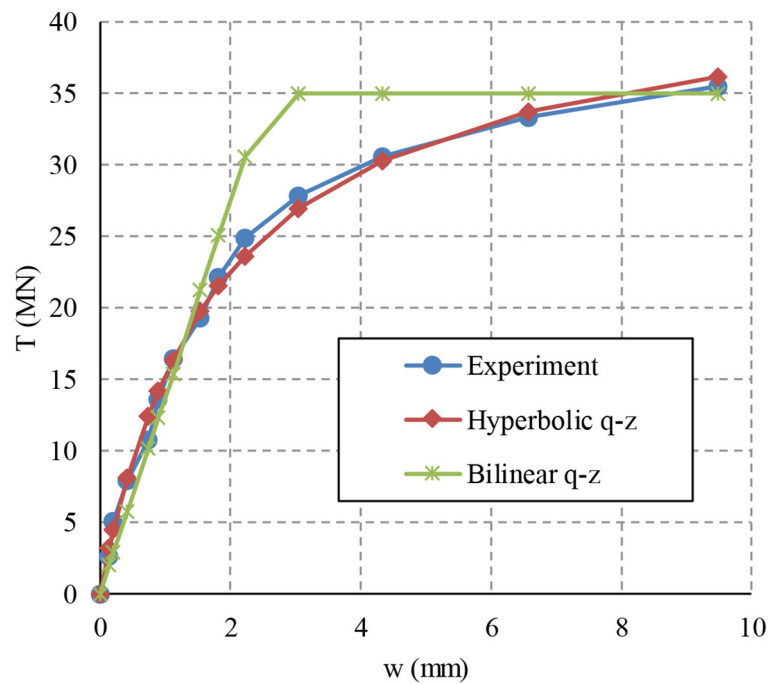


Figure 6. The T-Z curve of the pile tip after correction from the experiment of Shibuya et al.

data. With an RMSE of 0.724 MN, the Hyperbolic formulation proves more reliable in predicting ultimate end-bearing capacity, effectively avoiding the inaccuracies associated with the rigid yield assumptions of the Bilinear model (which resulted in a higher RMSE of 3.053 MN).

Significance of the calibration framework: The integration of the Excel Solver to minimize the objective function (RMSE) represents a significant advancement over manual parameter estimation. This systematic approach eliminates subjective bias, ensuring that the model parameters are objectively derived from the site-specific stress-strain response. The overall reduction in RMSE across all layers confirms that this back-analysis procedure is not merely a curve-fitting exercise but a robust technical framework for geotechnical characterization. The resulting site-specific parameters provide a scientifically rigorous foundation for pile settlement predictions, far exceeding the reliability of generalized empirical correlations.

Influence of pile geometry and interface characteristics on calibration parameters: While the validation datasets encompass diverse pile geometries, dimensions, and soil conditions, the sensitivity of the proposed back-analysis framework to physical attributes such as pile diameter (D), embedded length (L), axial stiffness (EA), and shaft roughness warrants explicit clarification.

Since the framework relies on automated optimization (Excel Solver) to minimize the RMSE against strain-gauge measurements, the resulting t-z and q-z curves represent equivalent unit load-transfer characteristics normalized by the pile perimeter and cross-sectional area. Consequently, scaling effects related to pile diameter and length are implicitly integrated into the back-calculated ultimate resistance (T_{ult}) and yield displacement parameters (w_y or w_{50}). Furthermore, shaft roughness and installation effects (e.g., driven piles versus bored piles) directly govern the local interface shear capacity and slippage thresholds, which are effectively captured through the site-specific calibration process. While the underlying mathematical formulation is robust across different structural dimensions, users should note that the calibrated parameters remain tied to the specific scale and construction method of the tested pile, ensuring that geometric and interface heterogeneities are properly reflected in the resulting load-settlement predictions.

Comparative analysis of pile behavior under axial loading using different t-z models

Based on the parameters calibrated using the static pile load test data from Shibuya et al. (1997), this section evaluates and compares the axial response of a single pile modeled with the

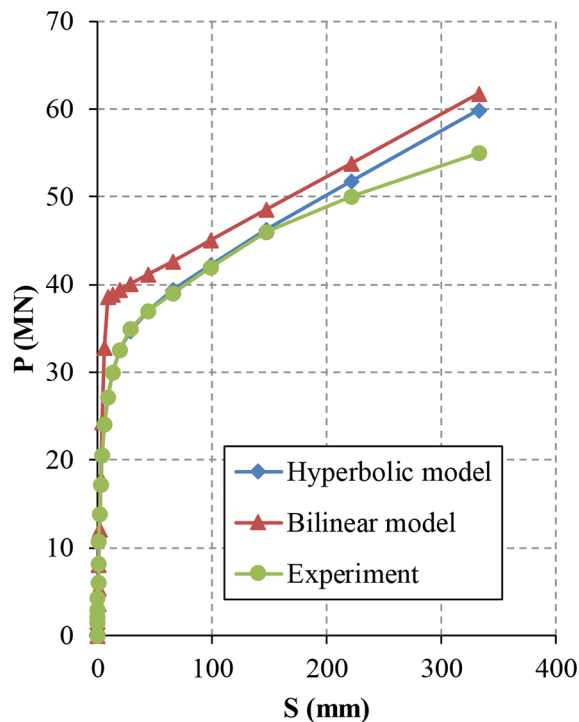


Figure 7. Results of load-settlement relationship analysis using models compared with experimental results of Shibuya et al. (1997)

bilinear and Hyperbolic t-z formulations. This analysis aims to clarify the predictive compatibility of each model regarding the load–displacement relationship and the distribution of shaft friction across geologically distinct soil layers (Table 3).

Figure 7 illustrates the load–settlement (P–S) curve at the pile head, comparing the experimental results from the Shibuya et al. (1997) study with the numerical predictions obtained using the t-z method for both bilinear (elastic-perfectly plastic) and hyperbolic (continuous nonlinear) models.

The analysis reveals a significant divergence in predictive performance between the two models as the applied load increases.

Table 3. Performance statistics of the models on data from Shibuya et al.

Performance indicator	Bilinear model	Hyperbolic model
RMSE (MN)	3.386	1.234
R^2	0.975	0.997
MAE (MN)	2.606	1.466
NRMSE	0.062	0.022
Analysis time (s)	1.2	1.8

Low-load stage ($P < 35$ MN) – both the bilinear and hyperbolic models yield results that are consistent with each other and align closely with the experimental data. This confirms that, during the elastic work phase, the initial spring stiffness derived from the back-analysis of experimental data accurately captures the response of the pile–soil system.

High-load stage ($P > 40$ MN) – the bilinear model (blue line) exhibits a significant limitation, predicting less settlement than observed (an “over-stiff” response). Conversely, the hyperbolic model (black dashed line) demonstrates superior performance by tracking the experimental curve (red line) closely, even as the pile approaches failure. Specifically, the hyperbolic model predicts the failure load at approximately 34.7 MN, which closely matches the experimental results, whereas the bilinear model estimates the failure load at approximately 40 MN, resulting in a discrepancy of 13.25%.

The deviation of the bilinear model from the experimental data at high loads is a direct consequence of oversimplifying soil behavior as elastic-perfectly plastic. The bilinear model assumes a sudden transition to failure at a specific stress threshold, which leads to an unrealistic stiffness response. In contrast, the hyperbolic model effectively simulates the gradual stiffness degradation of the soil, producing a load–settlement curve that asymptotically converges with the experimental data.

This analysis demonstrates that for large-diameter bored piles in dense sand formations (such as the Saitama geological profile), selecting a continuous nonlinear t-z model like the hyperbolic formulation is a prerequisite for ensuring reliable pile head settlement predictions. The results of Case Study 1 not only validate the proposed back-analysis procedure but also emphasize the necessity of moving beyond simplified linear assumptions in modern foundation design.

The statistical analysis in Table 3 demonstrates a clear performance gap between the two models.

Prediction accuracy (RMSE, MAE & NRMSE) – the hyperbolic model achieved an RMSE of 1.94 MN, which is approximately 5.4 times lower than that of the bilinear model (10.58 MN). This improvement is further corroborated by the NRMSE, where the hyperbolic model attained a superior value of 0.022 compared to 0.062 for the bilinear model. The reduction in MAE from 2.606 MN to 1.466 MN reinforces the model’s consistency in minimizing cumulative

discrepancies across the entire loading range, particularly as the pile approaches its ultimate capacity.

Correlation coefficient (R^2) – while the bilinear model yielded a respectable R^2 of 0.975, it is noticeably lower than the 0.997 achieved by the hyperbolic model. This confirms that the hyperbolic model captures a greater proportion of the nonlinear variance in soil response, effectively modeling the continuous transition from elastic to plastic behavior.

Engineering implications – the bilinear model's lower R^2 and higher RMSE/NRMSE are primarily attributed to its over-stiff response at high load levels, where the assumption of instantaneous plastic yielding fails to account for the soil's gradual stiffness degradation. In contrast, the hyperbolic model's high correlation with experimental data ($R^2 = 0.997$) validates its reliability for design scenarios where accurate settlement estimation is critical to structural safety.

Computational efficiency – regarding practical implementation, the hyperbolic model required an analysis time of 1.8 seconds, compared to 1.2 seconds for the bilinear model. This marginal increase in computational overhead (0.6 seconds) is negligible and represents a highly favorable trade-off given the substantial gains in predictive accuracy (as evidenced by the 65% reduction in NRMSE) and numerical stability, particularly for large-scale engineering applications where reliable deformation predictions are paramount.

Figure 8 provides a comprehensive overview of how different mathematical assumptions influence the predicted pile behavior. The analysis highlights distinct behavioral differences across loading stages:

Elastic stage ($S < 0.005$ m) – all models demonstrate a high level of agreement with the experimental data, confirming that during the initial loading phase, each method effectively captures the initial stiffness (K_i) of the pile–soil system.

Nonlinear and failure stage ($S > 0.005$ m)

Bilinear model (yellow squares) – exhibits the highest stiffness, reaching an ultimate capacity of approximately 900 kN. This response is excessively rigid, leading to a significant underestimation of settlement as the load approaches the failure threshold.

N. Makris model (black open circles) [34] – Shows a moderate response, aligning closely with experimental data during the intermediate loading stage and stabilizing at an ultimate capacity of approximately 900 kN. This model is underpinned by a physically motivated approach that utilizes distributed nonlinear springs, governed by the Bouc-Wen model, to characterize hysteretic soil behavior and energy dissipation. To address the soil-pile interface, Makris and Badoni incorporate a slider element in series with the nonlinear spring, effectively capturing slip phenomena when adhesive capacity is exceeded. The parameters within this framework are typically determined through a systematic trial-and-error approach, relying on engineering judgment to align the model with physical soil properties – such as undrained shear strength (S_u) and Young's modulus (E_s) – rather than purely automated regression. By calibrating the skin friction capacity through a depth-dependent scalar parameter $\lambda(z)$, this method provides a foundational benchmark for analyzing the nonlinear axial response of single piles

Hyperbolic model (red circles) – exhibits a more ductile response, effectively capturing the nonlinear reduction in soil stiffness. While it provides a good fit for the nonlinear transition, it shows a tendency to slightly underestimate the capacity at higher displacement levels compared to the experimental observations.

Failure load prediction – the experimental data indicate a failure load approaching 850–900 kN. The bilinear model overestimates this capacity (approx. 900 kN), whereas the N. Makris and hyperbolic models provide more conservative and closer estimates, with the bilinear model representing a discrepancy of approximately 16.2% relative to the hyperbolic/N. Makris convergence zone.

The divergence between these models in Case Study 2 reinforces a fundamental principle in foundation engineering: no single model is universal. While the bilinear model is suitable for rapid design assessments prioritizing conservative safety thresholds, the hyperbolic model demonstrates superior performance for projects with

Table 4. Performance statistics of the models on data from O'Neill's experimental

Performance indicator	Bilinear model	Hyperbolic model
RMSE (kN)	74.268	19.086
R^2	0.962	0.981
MAE (kN)	68.679	18.109
NRMSE	3.317	0.852
Analysis time (s)	1	1.4

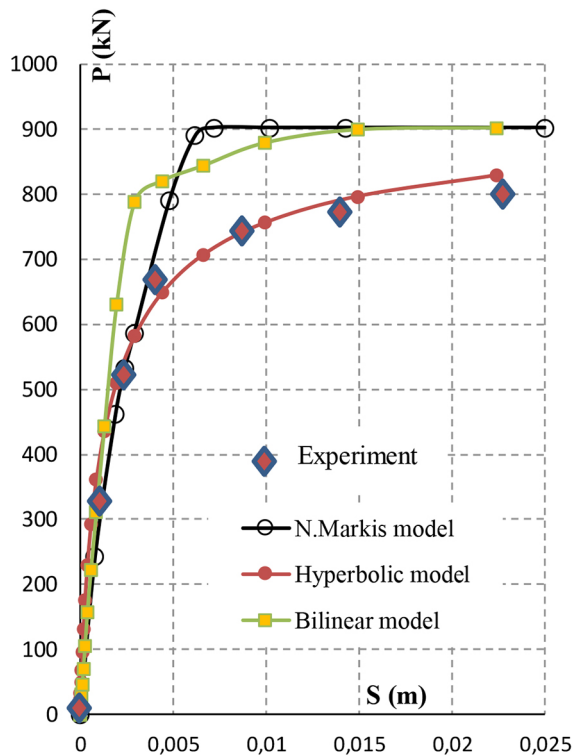


Figure 8. Comparative analysis of load–settlement (P–S) behavior at the pile head – experimental data vs. bilinear, hyperbolic, and Makris [34] models

stringent technical requirements. Its ability to fully capture the gradual stiffness degradation of the soil allows for significantly higher precision in settlement prediction, effectively reducing the errors inherent in traditional approaches.

The comparative statistical analysis for Case Study 2 in Table 4 underscores the substantial limitations of the bilinear model in cohesive soils. The hyperbolic model demonstrated superior predictive capability, yielding an RMSE of 19.086 kN and an R^2 of 0.981, compared to 74.268 kN and 0.962 for the bilinear model, respectively. This performance gap – where the bilinear model's error (RMSE) is nearly 4 times higher – is attributed to the progressive failure mechanism inherent in clayey deposits. While the bilinear model's abrupt yield point fails to account for the gradual stiffness degradation, the hyperbolic formulation effectively mimics the nonlinear stress-displacement path observed in field tests.

It is noteworthy that the predictive accuracy of the bilinear model varies significantly depending on the soil profile. In the dense sand formation (Case Study 1), the bilinear model's performance was relatively moderate, with an NRMSE of 0.062. In contrast, this error amplified

significantly in the clayey deposits (Case Study 2), where the bilinear model's NRMSE surged to 3.317. This stark contrast highlights the distinct soil–pile interaction mechanisms – while dense sands exhibit rapid resistance mobilization and predictable stiffness, allowing simpler models to perform reasonably well, the progressive failure behavior inherent in cohesive soils drastically exposes the limitations of the bilinear model. The marked degradation in predictive performance within clayey profiles – supported by the sharp rise in NRMSE from 0.852 (hyperbolic) to 3.317 (bilinear) – confirms that for cohesive strata, the transition to continuous nonlinear models, such as the Hyperbolic formulation, is a technical necessity to maintain design reliability.

Regarding computational efficiency, the hyperbolic model required an analysis time of 1.4 seconds, compared to 1.0 second for the bilinear model. Similar to Case Study 1, this marginal increase of 0.4 seconds is negligible, representing a highly favorable trade-off for the substantial improvements in predictive accuracy (as evidenced by the nearly fourfold reduction in RMSE) and numerical stability in cohesive soil conditions.

To quantitatively evaluate the statistical agreement between the model predictions and the measured field data, a paired two-sample t-test (with $df = 21$ for 22 observations at a significance level of $(\alpha = 0.05)$) was conducted for both the Shibuya and O'Neill pile case histories (Table 5). For the Shibuya pile, the hyperbolic model yielded a mean of 22.244 compared to a Pearson correlation of 0.997 with the experiment. Although its one-sided t-statistic ($t_{Stat} = -3.313$) exceeded the critical threshold ($t_{Critical} = 1.721$) with $P = 0.001656 < 0.05$, indicating a slight systematic offset, its trajectory alignment remains exceptional. Conversely, the bilinear model for the Shibuya pile showed a mean of 23.477 ($R^2 = 0.975$) with a non-significant t-statistic ($t_{Stat} = 0.698$, $P = 0.2463 > 0.05$), reflecting a close mean value but lacking the continuous nonlinear flexibility of the Hyperbolic formulation. A consistent statistical trend was observed for the O'Neill pile case: the hyperbolic model recorded a mean of 327.775 ($R^2 = 0.981$), where the t-test yielded $t_{Stat} = 3.469$ ($P = 0.0011 < 0.05$), whereas the bilinear model produced a mean of 348.907 ($R^2 = 0.962$) with a non-significant t-statistic ($t_{Stat} = 1.362$, $P = 0.094 > 0.05$). Overall, these statistical metrics confirm that while classical bilinear models may occasionally match global mean values closely due to their piece-wise linear averaging, the

Table 5. Statistical results of the paired two-sample t-test comparing model predictions and experimental measurements for the Shibuya and O'Neill pile case histories

Parameters	Shibuya pile case		O'Neill pile case	
	Hyperbolic	Bilinear	Hyperbolic	Bilinear
Mean	22.244	23.477	327.775	348.907
Variance	305.994	369.739	99416.3	151240.3
Observations	22	22	22	22
Pearson correlation	0.997	0.975	0.981	0.962
Hypothesized mean difference	0	0	0	0
df	21	21	21	21
t Stat	-3.313	0.698	3.469	1.362
P(T<=t) one-tail	0.001656	0.2463	0.0011	0.094
t Critical one-tail	1.721	1.721	1.721	1.721
P(T<=t) two-tail	0.0033	0.492	0.0023	0.188
t Critical two-tail	2.080	2.079614	2.080	2.080

advanced hyperbolic models – backed by robust optimization and high correlation coefficients – reliably capture the true continuous mechanics of pile-soil interaction despite minor systematic variations.

Sensitivity analysis of calibrated parameters

To evaluate the stability, robustness, and predictive reliability of the proposed back-analysis framework, a normalized sensitivity analysis was systematically conducted on the calibrated model parameters. This analysis aims to quantify the impact of parameter uncertainty on the structural performance of the pile, specifically focusing on the pile-head load-settlement (P-S) response. By assessing how variations in the primary input variables – namely the ultimate shear resistance (τ_{ult}) and the yield displacement (w_y) – influence the predicted settlement, this study identifies the governing mechanical drivers of the soil-pile interaction.

The sensitivity of the model was evaluated by introducing a perturbation range of $\pm 10\%$ $\div$ $\pm 30\%$ to each parameter from its calibrated baseline value. The selection of these perturbation ranges is strongly supported by statistical evaluations of geotechnical property variability. Comprehensive reliability studies (e.g., Phoon and Kulhawy, 1999 [35]; Duncan (2000)[36]) have established that the coefficients of variation (COV) for fundamental soil parameters and in-situ test indices typically span this precise range due to inherent spatial heterogeneity and measurement errors. Consequently, perturbing

the calibrated parameters within this bracket provides a realistic and rigorous simulation of actual geotechnical uncertainties encountered in engineering practice.

The sensitivity of the model was evaluated by introducing a perturbation range of $\pm 10\%$ $\div$ $\pm 30\%$ to each parameter from its calibrated baseline value. This multi-level assessment provides a rigorous stress-test of the model, ensuring that the identified sensitivities are consistent and represent the actual nonlinear pile-soil response surface rather than artifacts of small-scale perturbations. The resulting fluctuations in the RMSE were formalized through the calculation of the sensitivity index (SI), which allows for the direct comparison of parameters with disparate physical units.

The sensitivity index (SI_i) is calculated using the central difference method to ensure high-order accuracy:

$$SI_i = \left| \frac{RMSE(p_i + \Delta p_i) - RMSE(p_i - \Delta p_i)}{2 \cdot \Delta p_i} \right| \cdot \frac{p_i}{RMSE_{base}} \quad (4)$$

where: p_i represents the baseline value of the i^{th} parameter; Δp_i is the perturbation magnitude (ranging from 10% to 30%); and $RMSE_{base}$ is the error at the calibrated baseline. This normalization by $\frac{p_i}{RMSE_{base}}$ renders the index dimensionless, facilitating a direct comparison of the relative influence of different geotechnical parameters (Saltelli et al., 2008) [37].

The sensitivity analysis yields contrasting results between the Shibuya (sand) and O'Neill

(clay) case studies, providing profound insights into the influence of soil state on model behavior.

The Shibuya dataset (Figure 9a and Table 6) exhibits exceptionally high SI values for w_y (peaking at ~ 374), indicating that in sandy deposits, the load-settlement response is extremely sensitive to the definition of soil

stiffness. Conversely, the O'Neill case (Figure 9b and Table 7), characterized by clayey soil, shows significantly lower SI values ($SI_{\max} \approx 22$). This suggests that the response surface in clayey strata is inherently smoother and more “predictable”, whereas the granular nature of the Saitaman site creates a highly nonlinear, sensitive

Table 6. Performance statistics of the models on data from Shibuya et al experimental

Layer	SI_30%		SI_20%		SI_10%	
	T_{ult}	W_y	T_{ult}	W_y	T_{ult}	W_y
Pile tip	0.0037	146.29	0.0041	148.04	0.0043	148.93
Layer 4	0.0084	333.51	0.0113	360.42	0.0152	374.73
Layer 3	0.0122	268.84	0.0166	291.63	0.0233	303.49
Layer 2	0.0144	130.39	0.0159	131.76	0.0169	132.47
Layer 1	0.0163	22.46	0.0165	22.27	0.0167	22.15

Table 7. Performance statistics of the models on data from O'Neill's experimental

Layer	SI_30%		SI_20%		SI_10%	
	T_{ult}	W_y	T_{ult}	W_y	T_{ult}	W_y
Pile tip	0.00018	3.02	0.00018	2.97	0.00018	2.94
Layer 4	0.00057	15.10	0.00057	14.91	0.00057	14.80
Layer 3	0.00196	18.14	0.00196	17.94	0.00196	17.82
Layer 2	0.00230	21.88	0.00230	21.68	0.00230	21.56
Layer 1	0.00107	5.80	0.00107	5.78	0.00107	5.76

Table 8. Comparative sensitivity analysis of soil-pile interaction parameters by soil type

Case study	Soil type	$SI_{w_y} / SI_{\tau_{ult}}$ (average)
Shibuya et al.	Sand	$\sim 20,000\text{--}30,000$
O'Neill	Clay	$\sim 8.000\text{--}10.000$

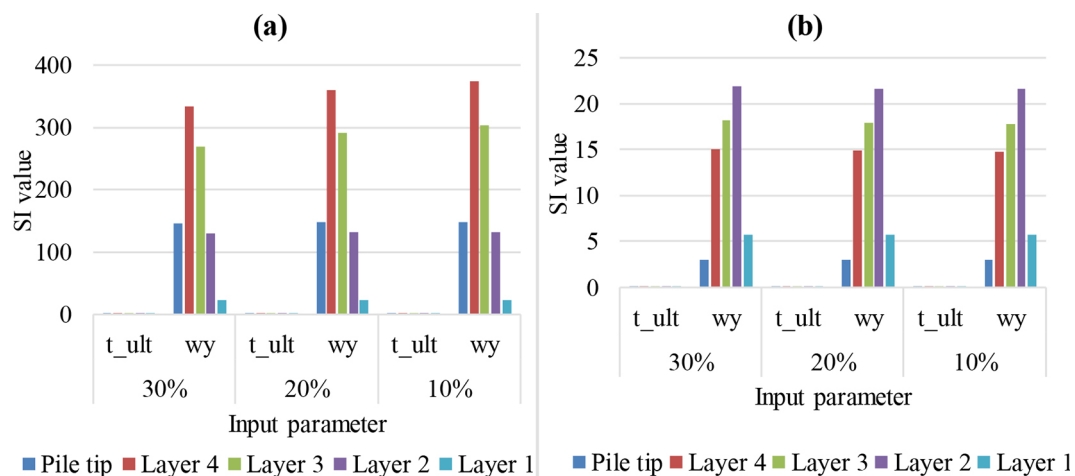


Figure 9. Comparison of Sensitivity Indices (SI) for pile-soil interaction parameters: (a) Shibuya et al and (b) O'Neill et al.

pile-soil interaction mechanism. Dominance of stiffness over capacity – in both scenarios, the SI of w_y consistently exceeds that of τ_{ult} by several orders of magnitude. This confirms that for both sand and clay, the mobilization of soil stiffness is the governing mechanism for pile-head settlement. However, the disparity is far more pronounced in the sandy case, where the sensitivity to w_y is approximately 100 to 1,000 times greater than to τ_{ult} .

A significant observation is the stability of SI values across the $\pm 10\%$ to $\pm 30\%$ perturbation ranges. In the O'Neill case, the SI values remain almost constant, indicating a highly robust model performance regardless of input uncertainty. In the Shibuya case, while absolute SI values fluctuate more as the perturbation magnitude increases, the ranking of parameter importance remains unchanged. This confirms that the proposed back-analysis framework is universally applicable and maintains consistent physical logic despite the fundamental differences between sandy and clayey soil mechanics.

To quantify the relative importance of soil stiffness versus ultimate shear resistance, the ratio of their respective sensitivity indices ($SI_{w_y}/SI_{\tau_{ult}}$) was evaluated. The analysis result in Table 8 reveals a striking dominance of the yield displacement (w_y) across all soil types. In the Saitaman sand case, this ratio ranges from approximately 20,000 to 30,000, underscoring that in granular soils, the load-settlement behavior is virtually insensitive to the ultimate capacity parameters within the operational range. For the O'Neill clay case, the ratio remains substantial, ranging between 8,000 and 10,000. While the absolute magnitudes of these ratios are site-dependent, the consistency of the findings – that w_y is several orders of magnitude more influential than τ_{ult} – provides definitive evidence that soil stiffness mobilization is the governing mechanical driver in pile foundation settlement prediction. This quantitative discrepancy emphasizes that prioritizing high-precision determination of w_y is far more critical than optimizing τ_{ult} in geotechnical site investigations, regardless of the soil stratigraphy.

Probabilistic sensitivity analysis

While deterministic optimization provides best-fit parameters for the load-transfer curves, it does not account for the inherent spatial variability of geotechnical properties nor the potential experimental errors stemming from field

instrumentation (e.g., strain gauge accuracy, sensor drift, and local data anomalies). To rigorously address these uncertainties and evaluate the robustness of the optimized hyperbolic model, a probabilistic sensitivity analysis utilizing Monte Carlo simulations ($N = 300$ iterations) was conducted.

Key input parameters and calibration constraints were treated as random variables reflecting realistic variations found in site characterization and testing. The simulation results yield a comprehensive probabilistic distribution of the pile-head load-settlement (P-S) behavior. Specifically, the 95% confidence intervals, defined by $\pm 1.96 \times \text{STD}$ around the mean response, successfully capture and bound both the inherent soil heterogeneity and the minor propagation of experimental measurement errors. This probabilistic framework demonstrates that while raw data imperfections can induce minor localized fluctuations during automated curve-fitting, the overall predictive reliability of the optimized model remains robust and stable across the entire loading spectrum.

To account for the inherent spatial variability of soil properties and experimental uncertainties in the Monte Carlo simulations, probabilistic perturbations were applied to the calibrated model parameters. Based on established geotechnical reliability literature and empirical data regarding soil-pile interaction parameters, the coefficients of variation (COV) were assigned as 0.20 (20%) for the ultimate resistance parameters (τ_{ult}) and 0.30 (30%) for the displacement-at-failure or reference displacement parameters (w_{50}). A log-normal probability distribution was intentionally adopted for parameter generation to strictly prevent non-physical negative values, ensuring that all simulated stiffness and capacity parameters remain within realistic mechanical bounds. These specific COV thresholds and log-normal models are well-supported by previous reliability-based deep foundation studies (e.g., Phoon and Kulhawy, 1999 [35]; Duncan (2000)[36]), which effectively capture both natural geological heterogeneity across the pile-soil interface and minor measurement errors inherent in full-scale static load testing, thereby providing a rigorous basis for evaluating the predictive stability and 95% confidence intervals of the optimized t-z load-transfer curves.

The Monte Carlo simulation results (Figure 10) provide a robust probabilistic evaluation of

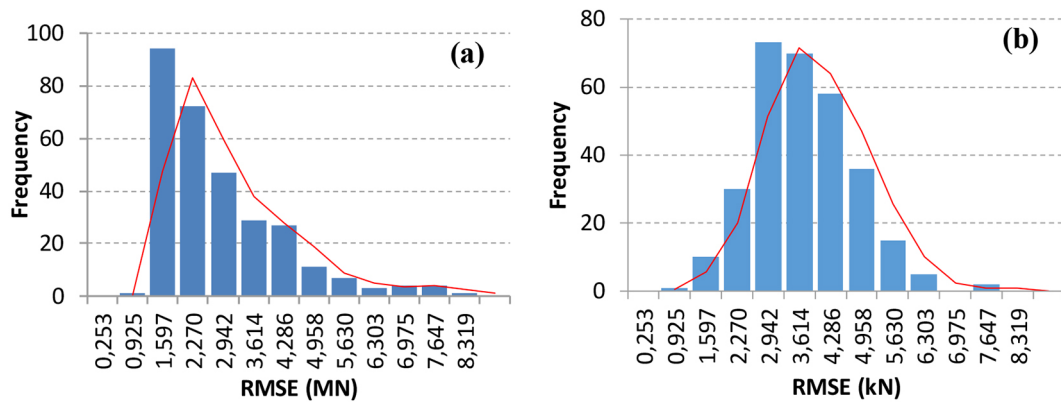


Figure 10. Monte Carlo simulation results showing the frequency distribution of root mean square error (RMSE) for the calibrated load-transfer models: (a) Shibuya pile case, and (b) O'Neill pile case

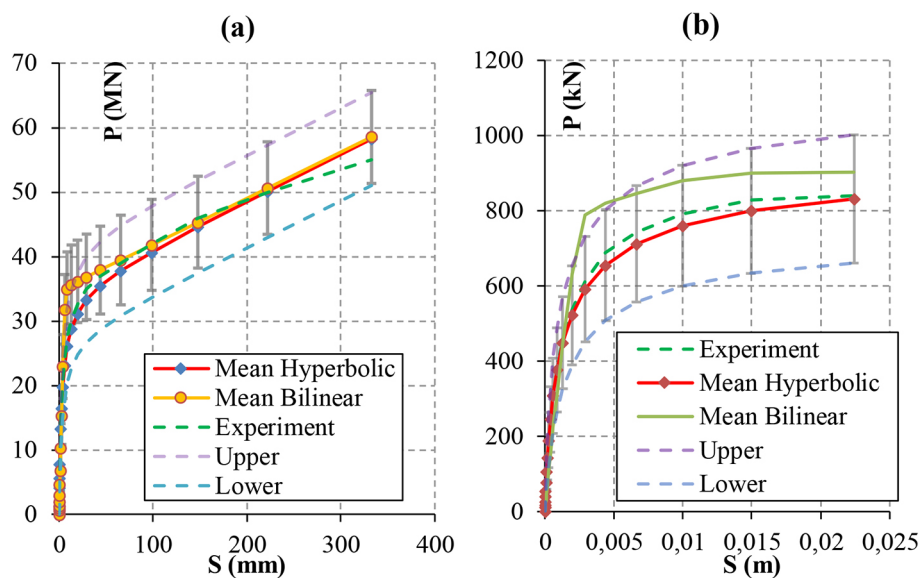


Figure 11. Comparison of pile-head load-settlement (P-S) curves incorporating Monte Carlo simulation confidence bounds (95% upper and lower limits) alongside experimental data and mean model predictions: (a) Shibuya pile case, and (b) O'Neill pile case

the optimization framework's performance and stability under parameter perturbations.

For the Shibuya pile case (Figure 10a): The frequency distribution of the Root Mean Square Error (RMSE) exhibits a distinct positively skewed (right-skewed) profile. The vast majority of simulation runs concentrate heavily within the lower error brackets (predominantly between 1.597 MN and 2.70MN), indicating that the optimization process is highly stable and resilient to variations in initial parameters and data noise.

For the O'Neill pile case (Figure 10b): The RMSE frequency distribution demonstrates an approximately bell-shaped, near-normal distribution pattern centered around the 2.942 kN to

3.614 kN intervals. This symmetric dispersion confirms that the calibrated model accommodates data scatter gracefully without severe divergence.

Overall, both distributions validate that the global errors remain bounded within acceptable limits, reinforcing the statistical reliability, robustness, and predictive confidence of the calibrated Hyperbolic load-transfer model across different geotechnical case histories.

Figure 11 illustrates the probabilistic assessment of the pile-head load-settlement (P-S) responses derived from the Monte Carlo simulations, incorporating the 95% confidence intervals (upper and lower bounds) relative to the mean model predictions and experimental measurements.

For the Shibuya pile case (Figure 11a): The experimental curve and both mean models (hyperbolic and bilinear) lie comfortably within the established 95% confidence boundaries. The upper and lower bounds effectively capture the dispersion caused by parameter uncertainties, with the mean hyperbolic curve exhibiting a close alignment with the physical load test data across the entire settlement range.

For the O'Neill pile case (Figure 11b): A similar performance trend is observed, where the experimental load-settlement trajectory remains well-contained inside the probabilistic envelope. The bounds appropriately reflect the growing uncertainty at higher settlement levels, confirming that the proposed automated calibration framework reliably brackets potential variations in soil-pile interaction.

Overall, these probabilistic bounds demonstrate that while material and operational uncertainties induce scatter in the predicted load-bearing capacity, the calibrated nonlinear framework maintains high predictive stability and robustness, validating its applicability for reliable geotechnical engineering design

Limitations of the study and future scope

While this study successfully establishes an objective, automated back-analysis framework that significantly enhances the calibration accuracy and predictive reliability of single-pile load-transfer curves, several inherent limitations must be acknowledged and addressed to guide future investigations.

First, while advanced analytical models can now incorporate stiffening or smoothing transition zones to soften abrupt flow points, the classical Bilinear formulation remains the benchmark embedded in contemporary geotechnical engineering practice. Codes of practice and standard design frameworks (e.g., FHWA, API) predominantly utilize elastic-perfectly plastic t - z curves because they require only two straightforward parameters: ultimate shear resistance (τ_{ult}) and initial stiffness or yield displacement (w_y). This simplicity makes the Bilinear approach highly attractive for routine, fast-paced design environments where extensive cyclic testing data or advanced soil constitutive parameters are unavailable. However, as demonstrated by our comparative findings, this operational convenience comes at the cost of significant predictive errors

- particularly in cohesive and complex layered strata where progressive failure and gradual stiffness degradation govern the deformation mechanism. Therefore, while modified bilinear models with transition zones might marginally improve localized curve-fitting, they still fundamentally retain a piecewise-linear structure that struggles to capture continuous, rate-independent stiffness reduction across deep, heterogeneous profiles. This reinforces the core argument of our study: shifting toward robust, continuous nonlinear frameworks like the hyperbolic model is vital for modern high-performance infrastructure design.

Despite the advantages of the proposed automated calibration framework, several specific technical and methodological constraints should be noted:

- While integrating the generalized reduced gradient (GRG) nonlinear optimization algorithm successfully eliminates the subjective bias inherent in manual curve-fitting, the calibration process still depends on foundational engineering decisions, such as selecting the objective function (RMSE), physically grounded parameter bounds, and convergence constraints. These user-defined inputs directly influence the optimization trajectory and solution uniqueness; overly restrictive bounds can constrain the physical reality of the soil response, while loose constraints may lead to non-unique mathematical fits. Future advancements should focus on fully automated frameworks by integrating machine learning algorithms or global optimization techniques (e.g., genetic algorithms or particle swarm optimization) to systematically determine boundaries and objective weights.
- The study focuses exclusively on the axial behavior of single bored piles. Extending this methodology to pile groups, lateral loading, or combined loading conditions involves significant complexities. In pile-group configurations, closely spaced piles experience group effects, such as soil compaction and overlapping shear zones (shadowing and shielding phenomena), which fundamentally alter the load-transfer mechanism compared to single isolated piles. Similarly, lateral loading scenarios require p - y curves that account for multidimensional displacement fields, depth-dependent failure mechanisms, and potential gap formation behind the shaft. Consequently, directly applying a 1D single-pile t - z

framework without multi-directional coupling oversimplifies structural responses. Future research should integrate multi-spring interaction matrices or couple the optimization framework with 3D numerical models (e.g., Plaxis or OpenSees).

- A recognized challenge in geotechnical back-analysis is the non-uniqueness of the inverse solution. Because multiple combinations of initial stiffness (K) and ultimate shear resistance (τ_{ult}) can produce comparable global error metrics (RMSE) at the pile head, optimization alone does not guarantee a singular, mathematically unambiguous solution. To mitigate this, the proposed framework relies on three-tier safeguards: (1) rigorous physical bounding based on soil mechanics, (2) multi-metric statistical validation (R^2 , MAE, NRMSE, and paired t-tests), and (3) probabilistic Monte Carlo simulations to define confidence bounds, ensuring parameters serve as reliable, physically constrained estimates rather than absolute unique values.

The current formulation and hyperbolic model are strictly restricted to monotonic (static) loading conditions. They do not account for cyclic degradation, stiffness softening under repeated load reversals, or the accumulation of permanent plastic settlements. Extending this framework to cyclic scenarios requires integrating hysteretic degradation rules, cyclic damping, and cumulative damage models into the local t-z and q-z functions.

Although this study specifically demonstrates the calibration framework using bored pile datasets, the methodology is not intrinsically restricted to a single pile type. The automated back-analysis framework can be extended to continuous flight auger (CFA) piles or micropiles, provided that sufficient, high-resolution field load test and instrumentation data are available. Given that CFA piles and micropiles exhibit distinct installation-induced soil disturbances and unique bond-slip mechanisms, applying the framework to these systems requires tailoring the initial boundaries and degradation models to reflect their specific shaft-resistance characteristics.

CONCLUSIONS

This study presents a comprehensive comparative analysis of the bilinear (elastic-perfectly

plastic) and hyperbolic (continuous nonlinear) t-z load-transfer models for predicting the axial behavior of large-diameter bored piles. Through a rigorous back-analysis of high-quality experimental data from Shibuya et al. and O'Neill, combined with advanced statistical validation, sensitivity mechanics, and Monte Carlo uncertainty quantification, the following conclusions are drawn:

5. The integration of the Excel Solver (GRG non-linear algorithm) provides a systematic, objective, and reproducible calibration framework. By minimizing the RMSE as the objective function, this procedure eliminates the subjective bias inherent in traditional manual parameter estimation, ensuring that model parameters are rigorously grounded in site-specific stress-strain responses.
6. The hyperbolic model significantly outperforms the bilinear model by effectively capturing gradual soil stiffness degradation. By maintaining mathematical continuity, it provides a superior fit to experimental data across all loading stages, effectively overcoming the limitations of the bilinear model's abrupt yield assumptions.
7. The statistical analysis confirms the enhanced accuracy of the hyperbolic formulation. In Case Study 1 (Shibuya pile, dense sand), the hyperbolic model achieved a high Pearson correlation of 0.997 and an exceptional trajectory match, whereas the bilinear model lacked continuous non-linear flexibility. In Case Study 2 (O'Neill pile, clayey deposits), a consistent trend was observed where the hyperbolic model maintained superior performance ($R^2=0.981$), whereas the bilinear model showed performance divergence. Furthermore, paired two-sample t-tests ($df = 21$, $\alpha = 0.05$) confirm that while classical bilinear models may occasionally match global mean values through piece-wise linear averaging, advanced hyperbolic models reliably capture true continuous mechanics.
8. Monte Carlo simulations ($N=300$) demonstrate that the calibrated framework is robust against parameter perturbations and data noise. The resulting frequency distributions of RMSE and the 95% confidence intervals (upper and lower bounds) confirm that experimental load-settlement trajectories remain well-contained within the probabilistic envelope, verifying the high predictive reliability of the proposed optimization framework.

9. Sensitivity analyses reveal that pile-head settlement is predominantly governed by yield displacement rather than ultimate shear resistance. The dominance of initial stiffness mobilization confirms it as the primary mechanical driver of pile performance, while the stability of sensitivity indices across perturbation ranges validates the framework's resilience against data uncertainty.
10. Predictive errors associated with traditional piecewise linear assumptions become more pronounced in complex cohesive and layered strata. This reinforces that while bilinear models may suffice for routine preliminary design, transitioning to continuous nonlinear models like the hyperbolic formulation is a technical necessity for maintaining design reliability where progressive failure and gradual stiffness degradation govern deformation mechanisms.

REFERENCES

1. Vesic A.S., Design of pile foundation. Transportation Research Board, National Council, 1977.
2. Bowles J. E. and Guo Y., Foundation analysis and design, vol. 5. McGraw-hill New York, 1996.
3. Fleming K., Weltman A., Randolph M., and Elson K., Piling engineering. CRC press, 2008.
4. Poulos H. G. and Davis E. H., Pile foundation analysis and design. 1980.
5. Randolph M., Dolwin R., and Beck R., Design of driven piles in sand, *Geotechnique*, 1994;44(3):427–448.
6. Potts D. M., Zdravković L., Addenbrooke T. I., Higgins K. G., and Kovačević N., Finite element analysis in geotechnical engineering: application, vol. 2. Thomas Telford London, 2001.
7. Jardine R. and Potts D., Magnus foundations: soil properties and predictions of field behaviour, in *Large-scale pile tests in clay*, Thomas Telford Publishing, 1993;69–83.
8. Reul O. and Randolph M., Piled rafts in overconsolidated clay: comparison of in situ measurements and numerical analyses, *Geotechnique*, 2003;53(3):301–315.
9. Brinkgreve R. B., Selection of soil models and parameters for geotechnical engineering application, in *Soil constitutive models: Evaluation, selection, and calibration*, 2005;69–98.
10. Kraft Jr L. M., Ray R. P., and Kagawa T., Theoretical t-z curves, *Journal of the Geotechnical Engineering Division*, 1981;107(11):1543–1561.
11. Seed H. B. and Reese L. C., The action of soft clay along friction piles, *Transactions of the American Society of Civil Engineers*, 1957;122(1):731–754.
12. Coyle H. M. and Sulaiman I. H., Skin friction for steel piles in sand, *Journal of the Soil Mechanics and Foundations Division*, 1967;93(6):261–278.
13. Randolph M. F. and Wroth C. P., Analysis of deformation of vertically loaded piles, *Journal of the geotechnical engineering division*, 1978;104(12):1465–1488.
14. O'Neill M. W. and Reese L. C., *Drilled Shafts: Construction Procedures and Design Methods*, Federal Highway Administration, Washington, D.C., FHWA-IF-99-025, 1999.
15. American Petroleum Institute, *Geotechnical and Foundation Design Considerations*. Washington, D.C.: American Petroleum Institute, 2014.
16. Poulos H. G., *Tall building foundations: design methods and applications*, Innovative Infrastructure Solutions, 2016;1(1):10.
17. Briaud J.-L. and Tucker L. M., Measured and predicted axial response of 98 piles, *Journal of Geotechnical Engineering*, 1988;114(9):984–1001.
18. Abchir Z., Burlon S., and Mroueh H., An advanced tz curves method using cyclic shaft friction degradation for the design of piles in sand, *European Journal of Environmental and Civil Engineering*, 2024;28(5):1082–1103.
19. Bateman A., Crispin J. J., and Mylonakis G., A simplified analytical model for developing 'tz' curves for axially loaded piles, *Conference: 20th International Conference on Soil Mechanics and Geotechnical Engineering 2022*.
20. Bateman A. H. and Crispin J. J., Theoretical 'tz' curves for piles in radially inhomogeneous soil, *DFI Journal: Journal of the Deep Foundation Institute*, 2020;14(1).
21. Rahardjo P., Development study of TZ curve generated from kentledge system and bidirectional test, *U Karst*, 2021.
22. Vardanega P. J., Kolody E., Pennington S. H., Morrison P. R., and Simpson B., Bored pile design in stiff clay I: codes of practice, *Proceedings of the Institution of Civil Engineers-Geotechnical Engineering*, 2012;165(4):213–232.
23. Bhuiyan F. M., Toth J., Siddharthan R., and Motamed R., Evaluation of existing tz models for caliche based on numerical analysis of bi-directional load tests using NVShaft, in *DFI 46th Annual Conference on Deep Foundations*, 2021;12–15.
24. Luo X.-G., Ren W.-X., Yin Y.-G., and Yu Y.-C., A modified hyperbolicity-based load transfer model for nonlinear settlement analysis of root piles in multilayered soils, *Acta Geotech.*, Jan. 2022;17(1):303–317, <https://doi.org/10.1007/s11440-021-01215-8>
25. Kim J., Lee J.-Y., Lim D.-B., Han J.-G., and others,

- Analysis of influence factor of soil-structure interaction considered in pile analysis using finite element analysis, *Civ Eng J*, 2025;11(7):2701–2714.
26. Elsiragy M., Azzam W., Kassem E. M., and others, Experimental and numerical study of enlarged-head monopile under lateral load in soft clay, *Civil Eng J*, 2025;11(2):453–471.
27. Cesaro R., Di Laora R., and Mandolini A., Shaft resistance of bored piles in sand, *Acta Geotechnica*, 2026;21(5):3103–3126.
28. Kondner R. L., Hyperbolic stress-strain response: cohesive soils, *Journal of the Soil Mechanics and Foundations Division*, 1963;89(1):115–143.
29. Fleming W., A new method for single pile settlement prediction and analysis, *Géotechnique*, 1993;43(4):615–618.
30. Shibuya T., Ogura H., Kawamura A., and Senno H., Toe load test on large diameter cast-in-place pile for the new building of kanto regional postal services bureau, part 1—load test plan and the results, in *Proc. 32nd Annual Convention of the Japanese Geotechnical Society*, Saitama City, 1997;1431–1432.
31. Ishihara K., The recent advances in pile testing and diaphragm wall construction in Japan, *Geotechnical Engineering Journal of the SEAGS & AGSSEA*, 2010;41(3):1–43.
32. O'Neill M. W., Hawkins R. A., and Mahar L. J., Load transfer mechanisms in piles and pile groups, *Journal of the Geotechnical Engineering Division*, 1982;108(12):1605–1623.
33. Fylstra D., Lasdon L., Watson J., and Waren A., Design and use of the microsoft excel solver, *Interfaces*, 1998;28(5):29–55.
34. Badoni D. and Makris N., Pile-to-pile interaction in the time domain—non-linear axial group response under harmonic loading, *Geotechnique*, 1997;47(2):299–317.
35. Phoon K.-K. and Kulhawy F. H., Characterization of geotechnical variability, *Canadian geotechnical journal*, 1999;36(4):612–624.
36. Duncan J. M., Factors of safety and reliability in geotechnical engineering, *Journal of geotechnical and geoenvironmental engineering*, 2000;126(4):307–316.
37. Saltelli A. et al., *Global sensitivity analysis: the primer*. John Wiley & Sons, 2008.