

Three-axle buses and lorries: Axle load dynamics and the design of brake systems

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ABSTRACT

The subject of this study is the operational processes within the braking system of a three-axle bus, examined in conjunction with the transient dynamic redistribution of vertical reactions on the axles during emergency braking. The scientific novelty of the work lies in establishing the patterns of redistribution of axial loads in multi-axle buses with a monocoque body and in justifying the design of the pneumatic brake actuator, which ensures compliance with UN Regulation No. 13 even in the event of a complete failure of one of the circuits, which is of critical importance for the development of modern braking systems. Three-axle buses and lorries differ significantly in terms of the dynamics of load redistribution across the axles during braking and acceleration. This is primarily due to two reasons: different loads on the second and third axles in buses with a monocoque body, as well as, accordingly, the independent suspension of each axle. The kinematics of the guide elements forming part of the suspension system also play a significant role in the distribution of loads across the axles, particularly in buses with air suspension. To minimise body roll during braking, it is necessary to bring the body's centre of difference as close as possible to the centre of mass and to determine the geometry of the longitudinal and transverse guide elements accordingly. Schematic of a three-axle, two-link bus, typically a city bus with a rear-engine layout, a driven third axle and a hinged swivel joint between the front and rear sections of the body, differs significantly from three-axle buses with a monocoque body in terms of both the configuration of the braking system and the anti-sway kinematics of the air suspension. Research into the braking characteristics of three-axle vehicles requires optimisation of the brake drive system. Consequently, it is necessary to take into account the varying reactions in the air suspension components of the supporting axle during dynamic load redistribution. A key aspect is the adjustment of the braking torque depending on the kinematics of the reaction rods, which minimises vertical oscillations and prevents wheel lock-up. This approach ensures directional stability and counteracts destabilising moments during emergency braking.

Keywords: three-axle buses, lorries, braking, brake drive systems, axle loads, longitudinal roll.

INTRODUCTION

Three-axle lorries have become widespread since the Second World War. The spring-balanced suspension of the 2nd and 3rd axles, which dominates in these designs, results in the equivalent dynamic configuration of such a vehicle during braking being reduced to a two-axle

configuration (with the equivalent centre of the rear axles located at the central axis of the balance suspension's attachment to the vehicle frame). The emergence of both lorries and, more recently, three-axle buses with monocoque bodies, where separate suspension systems are used for each axle, and ultimately combinations of axles with single and twin tyres (and consequently

different loads on the 2nd and 3rd axles) necessitates a detailed examination of this issue in terms of ensuring the effective operation of the wheel brakes. In the current context of the development of intelligent transport systems, the design of such assemblies is based on the integration of mechanical parameters with the algorithms of electronic braking systems (EBS/ABS), which requires accurate mathematical models of load distribution to ensure the correct operation of the pressure modulation valves.

As an analysis of recent research and publications has shown, unlike three-axle N3 category lorries, which have long been present on the market, the dynamics of braking and load redistribution on the axles have been examined in well-known studies [1–5]. The recent emergence of three-axle buses with a monocoque body and significant differences in suspension design has necessitated a separate study in this field. Naturally, these issues were not addressed in classical works on vehicle theory either [6–8]; publications on three-axle buses with monocoque bodies, which first appeared in Germany, are also limited to general information regarding layout solutions and assessments of manoeuvrability, stability and handling, however, in the context of market competition, they do not cover in detail design issues, in particular the dynamics of load redistribution on the axles, minimisation of longitudinal roll (body pitch) in unstable driving conditions, and the like [9–12].

Naturally, during the design process at JSC ‘Ukravtobusprom’ of the first domestic three-axle city bus with a monoblock body, the ‘Bogdan’ A231 [15], questions arose regarding the formation of an optimal drive scheme for the working brake system circuits [16,17] that would ensure compliance with regulatory legislative requirements regarding design certification and approval for operation [18]. A key consideration here is ensuring the necessary effectiveness of the backup (emergency) braking system (in the event of a failure in one of the drive circuits). In addition, the significant longitudinal body overhang during braking results in corresponding requirements regarding suspension kinematics [19]. Existing modern studies on the dynamics of three-axle vehicles are limited, however, to more common lorries with equal loads on the second and third axles [2–7]; the same applies to three-axle trailers [20].

It should be noted that although the basic design solutions for the Bogdan A231 bus platform were established earlier [15–18], and the kinematic aspects of the differential were investigated by the authors in [19], the relevance of revisiting this topic in 2026 stems from the need for a scientific synthesis of accumulated experience to develop a unified methodology for designing braking systems for modern three-axle low-floor buses. Unlike the counterparts [13,14], where emergency braking is often provided exclusively by EBS electronic components, this work addresses the fundamental engineering challenge of ensuring mechanical and pneumatic redundancy of the circuits, which guarantees the system’s operability even in the event of a complete failure of the on-board electrical system.

The aim of this study is to develop a mathematical model of the dynamics of load redistribution on the axles during braking of three-axle vehicles, taking into account layout configurations and suspension kinematics. Accordingly, the objective is to formulate an optimal wheel brake drive scheme that meets regulatory requirements [18].

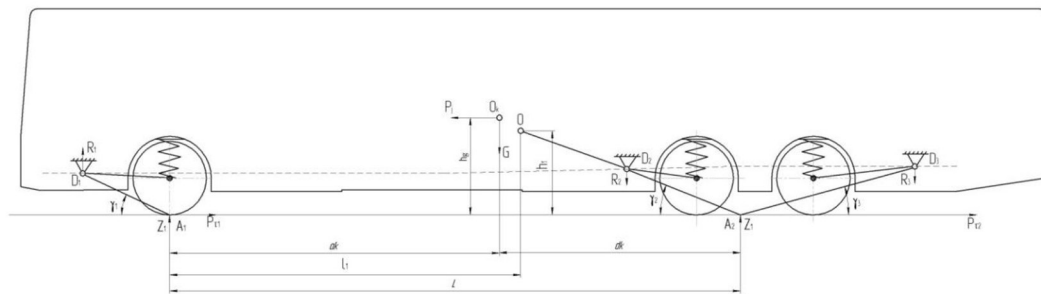
METHODOLOGY AND SUBJECT OF THE STUDY

The dynamic equivalent diagrams of the most common three-axle lorries with a rear balance suspension and extra-large buses (14–15 m overall length) and lorries with independent axle suspensions differ significantly (Figure 1). Conventional three-axle lorries with rear balance-beam suspension can be reduced to an equivalent two-axle diagram with the placement of a notional second axle, equivalent in terms of load and traction and braking forces to the actual second and third axles, centred on the point where the balance spring suspension is attached to the vehicle frame. The dynamics of load redistribution on the axles during braking have been examined in considerable detail in both domestic [1–4, 7] and international [5–6, 8–12] studies, however, with regard to three-axle buses with independent suspension of the 2nd and 3rd axles and unequal loads on these axles (Table 1), the issue remained largely unresolved at the time of the ‘Bogdan’ A231 project.

An assessment of the influence of suspension kinematics on the displacement of the nominal



a)



b)

Figure 1. ‘Bogdan A 231’ bus (a) and the forces acting on a braking bus with an anti-sway variant of the suspension kinematic diagram (b) [16]

centre of mass and the dynamic redistribution of axle loads, Figure 1b, for both the gross and kerb weights of the bus, which is also dictated by regulatory requirements for both weight variants [18], is considered in [19]. The corresponding suspension kinematics ensures that the coordinates of the centre of differential (h_1, l_1 , Figure 1b) are as close as possible to the reduced centre of mass of the body (h, a).

Calculation of dynamic vertical reactions on the axles

The equivalent diagram of a three-axle bus with air suspension and a monoblock body is distinguished primarily by the different loads on the second drive axle with twin tyres and the third axle with single tyres, and is shown in Figure 2. In this case, based on the conditions of coincidence of the equivalent centres of mass (vertical body oscillations) and longitudinal body roll [11].

Neglecting, as is customary, air resistance and tyre rolling resistance, which are insignificant in relation to the forces of inertia and braking, the dynamics of load redistribution on the axles Z_1, Z_2

and Z_3 can be determined from a system of three equations reduced to each of the axles.

$$\gamma G_a h - G_i \cdot a + z_2 \cdot (a + b) + z_3 \cdot (a + b + c) = 0 \quad (1)$$

$$\gamma G_a h - G_i \cdot b - z_1 \cdot (a + b) + z_3 \cdot c = 0 \quad (2)$$

$$\gamma G_a h + G_i \cdot (b + c) - z_1 \cdot (a + b + c) - z_2 \cdot c = 0 \quad (3)$$

where: γ – the specific deceleration, h – the height of the centre of mass, a – the distance from the front axle to the centre of mass of the bus/car, b – the distance from the centre axle to the centre of mass of the bus/car, c – the distance from the rear axle to the centre axle of the bus/car, G_i – the weight borne by a specific axle of the bus/car, G_a – the weight of the bus/car, Z_1, Z_2, Z_3 – vertical reactions on the 1st, 2nd and 3rd axles, F_1, F_2, F_3 – braking forces on the 1st, 2nd and 3rd axles.

Given the known values of γ , the weight of the bus G_a , the height of the centre of mass h and the distances between the axles a, b and c , from the

system of three Equations 1–3 we obtain expressions for the quantitative assessment of the change in static loads on the bus axles Z_1 , Z_2 and Z_3 .

$$z_1 = \frac{F_j \times h \left(1 - \frac{1}{c}\right) + G_a \times a}{a+b} \times \left(1 - \frac{1}{c}\right)^{-1} \quad (4)$$

$$z_2 = \frac{G \times a + F \times h + \frac{G \times a - F \times h}{a+b+c} + G_a \times a}{a+b} \times \left(1 + \frac{1}{a+b+c}\right)^{-1} \quad (5)$$

$$z_3 = \frac{G \times a - F \times h + \frac{G \times a - F \times h}{a+b}}{a+b+c} \times \left(1 + \frac{1}{a+b}\right)^{-1} \quad (6)$$

It is evident that this model of load redistribution dynamics on the axles is simplified and does not take into account the kinematics and characteristics of the suspensions, which influence the actual longitudinal roll of the body (deviation of the roll centre from the centre of mass) and the effect of unsprung masses.

Analysis of model assumptions and sensitivity analysis

The proposed model is based on the neglect of aerodynamic drag and tyre rolling resistance. Calculations show that at the maximum braking initiation speed ($v = 60$ km/h), the aerodynamic drag force of the body with an area of $S = 7.5$ m² accounts for less than 1.2% of the total braking force during intense deceleration. The effect of unsprung masses, accounted for in Equations 11–14, introduces a correction to the vertical reactions of the axles within the range of 2.4–3.1%, depending on the suspension travel. Thus, the simplifications adopted are physically justified, generally accepted and do not significantly affect the adequacy of the results of the drive circuit calculations.

For the most common lorries with rear balance suspension, the equivalent dynamic diagram is shown in Figure 3

The equations for the moments acting on the suspension and axles, relative to the centre of attachment of the suspension balance arm to the frame, will take the form:

$$z_3 c_2 - z_2 c_1 + G_{i23} c_3 - \gamma G_{i23} (h_3 - h_2) + (F_2 + F_3) h_3 = 0 \quad (7)$$

Table 1. Main design parameters of the ‘Bogdan A 231’ bus

Parameters	Value
Total mass of the bus, kg	22550
Wheelbase from front axle to 3rd axle	9.04
Wheelbase from front axle to 2nd axle	7.4
Centre of gravity coordinates at full load:	
- distance to front axle, a , m	5.696
- distance to the second axle, a , m	1.704
- distance to the rear axle, a , m	3.344
- distance to rear axle, b , m	1.704
- centre of gravity height, h , m	1.147
Kerb weight of the bus, kg	12420
Coordinates of the centre of mass at kerb weight:	
- Distance to the front axle, a , m	6.337
- distance to the 2nd axle, a , m	1.063
- distance to the third axle, a , m	2.703
- distance to rear axle, b , m	1.063
- centre of gravity height, h , m	0.9
Dynamic wheel radius, r_d , m	0.447

Accordingly, the vertical loads on the axles will be determined as:

$$z_1 = G_a \left(1 - \frac{a}{a+b+c_1} + \gamma \frac{h}{a+b+c_1}\right) - G_{i23} \left(\frac{h_2}{a+b+c_1} - \gamma \frac{h_3-h_2}{a+b+c_1}\right) - (\gamma G_a - F_1) \frac{h_3}{a+b+c_1} \quad (8)$$

$$z_2 = G_a \left(\frac{a}{a+b+c_1} - \gamma \frac{h}{a+b+c_1}\right) \frac{c_3}{c} + \frac{a+b+c}{c} \left[G_{i23} \left(\frac{c_3}{a+b+c_1} - \gamma \frac{h_3-h_2}{a+b+c_1}\right) + (\gamma G_a - F_1) \frac{h_3}{a+b+c_1} \right] \quad (9)$$

$$z_3 = G_a \left(\frac{a}{a+b+c_1} - \gamma \frac{h}{a+b+c_1}\right) \frac{c_1}{c} - \frac{a+b}{c} \left[G_{i23} \left(\frac{c_3}{a+b+c_1} - \gamma \frac{h_3-h_2}{a+b+c_1}\right) + (\gamma G_a - F_1) \frac{h_3}{a+b+c_1} \right] \quad (10)$$

where: G_{i23} – unsprung masses of the rear axle assembly (2nd and 3rd axles); h_2 – the centre height of unsprung masses; h_3 – the height from the balance suspension mounting point to the chassis; c_1 , c_2 , – distances from the wheel axles to the balance suspension mounting point; c_3 – the distance from the centre of the unsprung masses to the

balance suspension axis; G_{i23} – unsprung masses of the rear axle assembly.

Typically, for vehicles with independent leaf spring suspensions on the 2nd and 3rd axles, both the equivalent diagram and the expressions for the vertical reactions on the axles change accordingly.

A more complex system involves air suspension, which is primarily found on buses but is also available as an option on certain truck models with a gross vehicle weight of 23–24 tonnes (a second drive axle with twin tyres and a static load capacity of up to 11–12 tonnes, the third axle is passive with single tyres and the ability to lift when laden, with a load of up to 6 tonnes), which is not typical for lorries with a gross vehicle weight of up to 28–30 tonnes (both rear axles with twin tyres). A fundamental difference lies in the front-engine layout in lorries and the rear-engine layout in buses, which significantly affects the distribution of loads on the axles in laden condition and, consequently, the assessment of brake compliance with regulatory requirements [18].

Another difference in the bus layout is the placement of four air suspension cylinders on the second axle on a separate subframe, secured to the drive axle beam. The reaction arms of the beams on the 2nd and 3rd axles also, as a rule, differ in mounting heights both relative to the body frame base and to the axles themselves, which have different configurations, as well as in the distribution of the sprung masses of the 2nd and 3rd axles, which differ by more than a factor of two.

System of equations describing the loads on the wheels of the rear axles of the “Bogdan” A231 extra-large bus:

$$z_3 c_3 - R_{32} c_3 - G_3 c_3 - \gamma G_3 (h_3 - h_{03}) + F_3 h_{03} = 0 \quad (11)$$

$$z_2 c_{23} - R_{22} (c_{23} - c_{21}) - R_{23} c_2 - G_2 c_{23} - \gamma G_2 (h_2 - h_{02}) + F_2 h_{02} = 0 \quad (12)$$

$$z_2 - R_{21} - R_{22} - R_{23} - G_2 = 0 \quad (13)$$

$$z_3 - R_{31} - R_{32} - G_3 = 0 \quad (14)$$

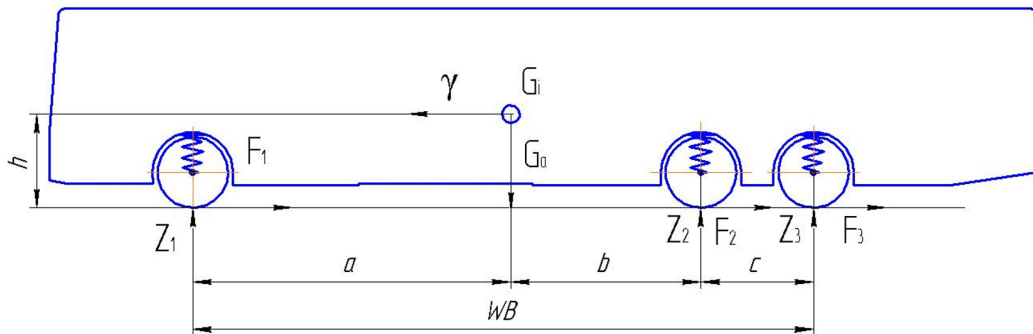


Figure 2. Dynamic equivalent diagram of a three-axe bus during braking

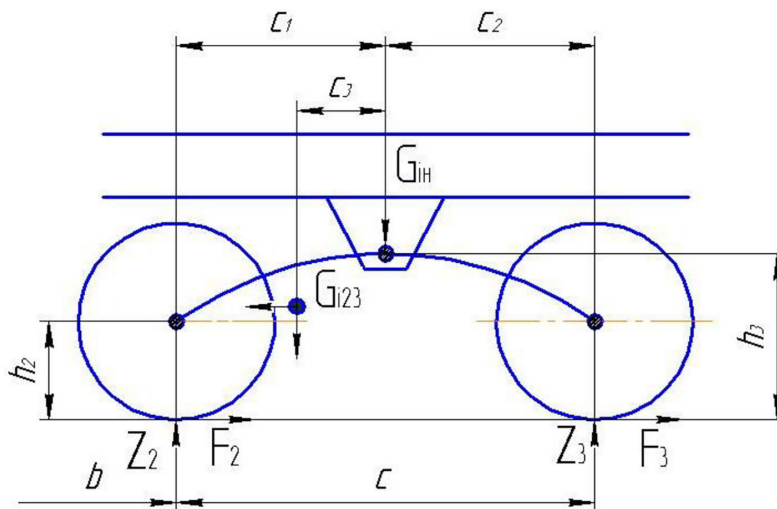


Figure 3. Dynamic equivalent diagram of a rear balance beam suspension

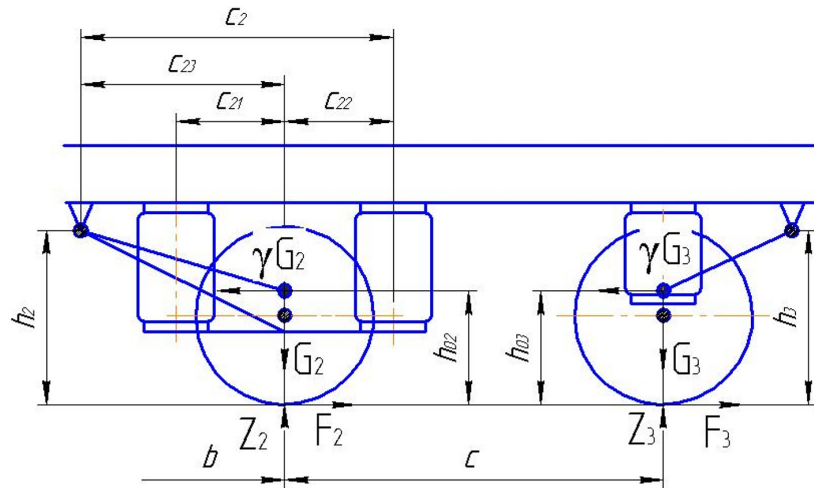


Figure 4. Equivalent diagram of the air suspension of the rear axles (2 and 3) of the 'Bogdan' A231 extra-large bus

where: z_2, z_3 are the loads on the 2nd and 3rd axles, respectively; G_2, G_3 – unsprung masses of the 2nd and 3rd axles; h_2, h_3 – the height at which the reaction rods and guide rods of the air suspension systems of the 2nd and 3rd axles are fixed to the body; h_{02}, h_{03} – the height of the centres of the unsprung masses of the 2nd and 3rd axles; c_3, c_{23} – distances from the centres of the 3rd and 2nd axles to the centres of the suspension guide rod mountings on the body; c_2 – the distance between the air springs of the 2nd axle.

Different load values on the 2nd (z_2, F_3) and 3rd axles (z_3, F_2), where practically 2/3 of the total load on both axles falls on the 2nd drive axle with twin tyres, result in a corresponding shift of the equivalent centre of unsprung masses G_2 i G_3 along the c -axis. If we neglect the inequality between the braking forces F_2 and F_3 , then with a negligible error, the load dynamics on the axles of an A231-type bus can be described as

$$z_1 = G \left(1 - \frac{a}{a+b+\frac{c}{3}} + \gamma \frac{h}{a+b+\frac{c}{3}} \right) + \frac{c}{3(a+b+\frac{c}{3})} (F_2 - F_3) \frac{h_2}{c_{23}} \quad (15)$$

$$z_2 = \frac{2}{3} G \left(\frac{a}{a+b+\frac{c}{3}} - \gamma \frac{h}{a+b+\frac{c}{3}} \right) + \frac{a+b}{2(a+b+\frac{c}{3})} (F_2 - F_3) \frac{h_3}{c_3} \quad (16)$$

$$z_3 = \frac{1}{3} G \left(\frac{a}{a+b+\frac{2c}{3}} - \gamma \frac{h}{a+b+\frac{2c}{3}} \right) - \frac{a+b}{2(a+b+\frac{2c}{3})} (F_2 - F_3) \frac{h_2}{c_{23}} \quad (17)$$

As is customary, the same applies to the front axle, the equivalent diagram of which is practically identical to that of the third axle (Figure 4), it is possible to formulate mathematical models in a similar manner to (11), taking into account the change in the direction of the inertial force γG_a , as in (1); however, in general, this refinement, like the aforementioned simplifications regarding the rear axles, has no significant impact on the determination and assessment of the dynamics of the load change on the axle during braking.

Analysis of possible configurations for the separation of brake system circuits

When designing braking systems for modern three-axle buses, it is necessary to take into account common engineering practices used in models such as the Mercedes-Benz Citaro L [14] or the Neoplan Centroliner N4420 [13]. Modern designs rely predominantly on electronic braking systems (EBS) (Figure 5) for dynamic modelling of pressure on the third axle and to compensate for load redistribution during braking. Although such systems are highly efficient, they are heavily dependent on the on-board electrical system. In contrast, the methodological approach of this study focuses on ensuring

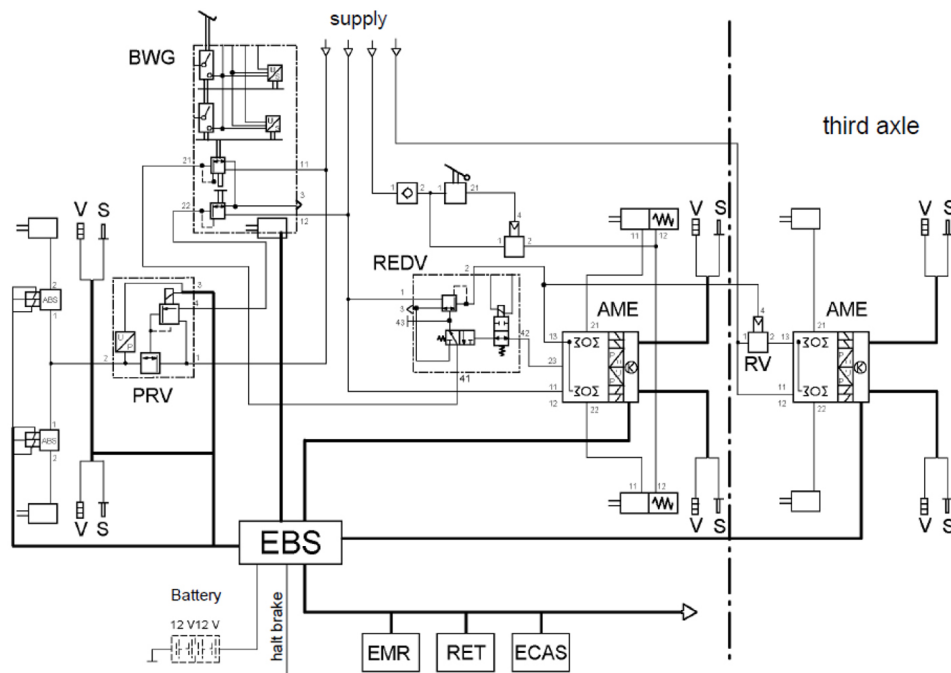


Figure 5. Schematic diagram of the pneumatic braking system of a Mercedes-Benz Citaro three-axe bus

fail-safe braking solely through the mechanical and pneumatic separation of circuits. This methodology aims to ensure that, even in the event of a complete electrical failure, the vehicle maintains optimal directional stability and meets the deceleration requirements for emergency braking in accordance with UN Regulation No. 13. Accordingly, to establish a reliable mechanical basis, it is necessary to evaluate several possible configurations of pneumatic braking circuits.

For a 3-axe chassis layout, three basic wheel brake drive circuit configurations are theoretically possible; we have also considered a fourth configuration utilising an accelerator valve with a function

to provide two brake drive inputs for the third axle from both the front and centre axle circuits. The design verification and assurance of the technical reproducibility of this system are based on the actual parameters of the braking system components, where the brake chambers of the front and third (support) axles are of type 24, and those of the second (driving axle with spring energy accumulators) are of type 24/24, and the operating pressure in the drive circuits is set at 0.80 MPa.

1) the front drive circuit controls the front axle brake mechanisms, and the rear circuit controls the brake mechanisms of the middle and rear axles (Figure 6).

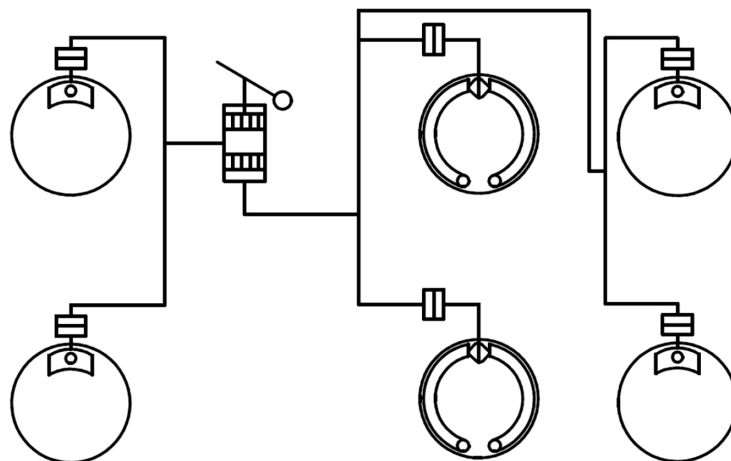


Figure 6. Schematic diagram of the wheel brake drive circuits for a 3-axe bus with a monoblock body, variant 1

- 2) in the front drive circuit, the brake mechanisms of the front and rear axles, and in the rear circuit, the brake mechanisms of the middle axle (Figure 7);
- 3) in the front drive circuit, the brake mechanisms of the front and centre axles, and in the rear – the brake mechanisms of the rear axle (Figure 8).
- 4) in the front drive circuit, the brake mechanisms of the front and rear axles, and in the rear circuit, the brake mechanisms of the middle and rear axles (Figure 9)

The selection of disc and drum brake mechanisms was carried out on the condition of sufficient efficiency to bring the wheels to the locking limit on a dry road with a coefficient of friction $\varphi = 0.7$. Changes in the centre of mass coordinates L_c and L_f for the kerb and gross weights of the bus (12420 kg and 22550 kg) remained constant for all four drive configurations, for each of which the values of the braking forces on the axles used

as the emergency braking system for the specific drive configuration varied accordingly.

The values of the bus deceleration when using only one of the drive circuits were determined on the basis of known relationships [3–6] and the simultaneous solution of four equations that determine the deceleration J_f and the specific γ_r , the normal reaction Z and the limiting braking force P_r from the condition of maximum tyre-road adhesion.

ASSESSMENT OF THE EFFECTIVENESS OF EMERGENCY BRAKING FOR THE GIVEN VARIANTS OF BRAKE SYSTEM CIRCUIT SEPARATION

The calculation results (Table 2) for the deceleration of each axle of the ‘Bogdan’ A231

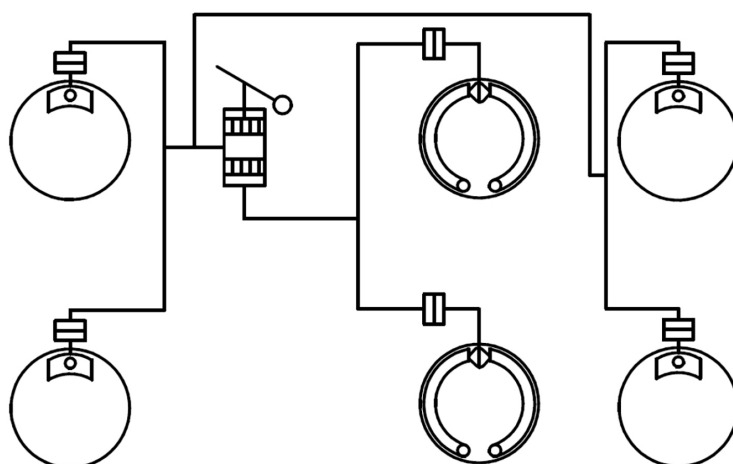


Figure 7. Schematic diagram of the wheel brake drive circuits for a 3-axe bus with a monoblock body, variant 2

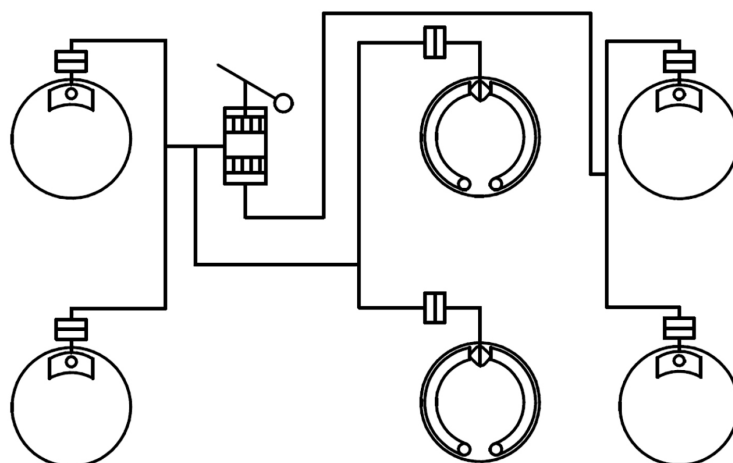


Figure 8. Schematic diagram of the wheel brake drive circuits for a 3-axe bus with a monoblock body, variant 3

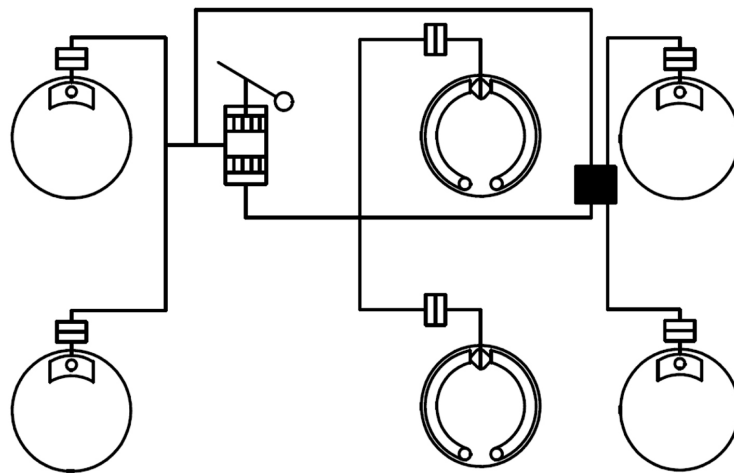


Figure 9. Schematic diagram of the wheel brake drive circuits for a 3-axle bus with a monoblock body, variant 4

three-axle bus indicate a significant unevenness in their braking potential depending on the load condition and the dynamic redistribution of vertical reactions. It has been established that at full mass (22550 kg), the deceleration provided by the front axle alone is 1.69 m/s^2 , by the middle axle – 2.40 m/s^2 , and by the rear axle – approximately 2.19 m/s^2 . In calculations at kerb weight (12420 kg), these values change to 1.03 m/s^2 for the first axle, 2.70 m/s^2 for the second and 2.51 m/s^2 for the third, respectively. Analysis of the data obtained in the context of the requirements [13] shows that, under full load, none of the bus's axles, in the event of isolated braking, meets the regulatory minimum for the reserve system of 2.5 m/s^2 . The second (driving) axle remains the most effective and stable, bearing approximately half of the total load, which justifies the need to form combined drive circuits with redundancy of braking forces on the third axle to achieve the required efficiency of the braking system.

The obtained values of individual axle deceleration indicate a significant deficit in braking efficiency during their autonomous operation.

In particular, the established fact that the standard requirement of 2.5 m/s^2 cannot be met by the efforts of a single (front or rear) axle alone determines the critical need to move from the analysis of individual force factors to the synthesis of a rational structure for the braking system as a whole. Such a transition is justified by the concept of ensuring the reliability of buses and the reliability of the braking system, which is a key requirement [18]. Since the performance of the backup system is assessed by the vehicle's ability to stop in the event of a failure in any of the working system's circuits, a scientific and practical task arises: to form such combinations of axles within individual circuits that would guarantee a total deceleration exceeding the established safety threshold. The need for further calculation of four circuit distribution options is dictated by the specific three-axle design of the A231 bus. The long wheelbase and high centre of gravity create conditions under which a simple redundant scheme may not ensure adequate stability. Therefore, a comparative assessment of the drive split options will not only verify compliance with quantitative

Table 2. Maximum possible deceleration of a super-large three-axle bus during emergency braking on one of the three axles

Braking axle	Bus mass	Deceleration, m/s^2	Compliance with Regulation No. 13 ($>2.5 \text{ m/s}^2$)
1	Total	1.69	Non-compliant
	Equipped	1.03	Does not apply
2	Full	2.4	Almost meets
	Fully equipped	2.7	Meets
3	Full	2.19	Does not meet
	Equipped	2.51	Complies

deceleration standards but also provide a qualitative justification for selecting a system architecture that minimises the risk of skidding and ensures the stability of the body's dynamic differential during emergency braking.

The values of the bus deceleration when using only one of the drive circuits were determined on the basis of known relationships [1–4, 21] and the simultaneous solution of four equations that determine the deceleration J_r and the specific γ_r , the normal reaction Z and the limiting braking force P_r subject to the condition of maximum tyre-road adhesion (Table 3).

An analysis of the four proposed options for separating the brake circuits of the A231 three-axle bus suggests that, for vehicles of this configuration, traditional approaches to circuit separation do not ensure full compliance with international safety standards. Calculations have confirmed a critical non-compliance of the basic schemes, as with classical configurations, the failure of one of the circuits leads to a drop in deceleration efficiency below the regulatory standard. The most vulnerable solutions were those where emergency braking relies exclusively on the front or trailing axle, whose braking capacity is significantly limited by the dynamic redistribution of vertical forces and the position of the centre of mass.

The only technical solution that guarantees compliance with the requirements for a backup braking system at both kerb and gross vehicle weights is the fourth circuit configuration. The introduction of an accelerator valve [22] with a circuit separation function allows the third axle to be engaged in both circuits simultaneously, ensuring stable deceleration in the range of 2.52–4.17 m/s² in the event of a failure in either circuit, exceeding the minimum permissible standard.

To verify the adequacy of the mathematical model and validate the effectiveness of the fourth variant of the brake drive circuit, a series of full-scale road tests of a prototype 'Bogdan A231' bus was conducted at a test track in accordance

with the procedures of UN Regulation No. 13. In accordance with Type 0 test procedures under UN Regulation No. 13 for M3 category vehicles, the braking tests were carried out on a dry asphalt surface with a high coefficient of friction ($\phi \approx 0.75$). The initial braking speed was 60 km/h. To ensure the transparency and accuracy of the experimental data, deceleration parameters were continuously recorded using the specialised Corrsys-Datron dynamic measurement system. The full-mass condition (22,550 kg) was accurately simulated by evenly distributing ballast within the passenger compartment to match the calculated centre of mass coordinates. For each simulated circuit failure, several test runs were carried out to ensure the statistical reliability of the recorded deceleration values presented in Table 3. Based on the results of these tests and a number of other tests, the 'Bogdan A231' bus with the described brake system configuration received a certificate of conformity and was approved for use on public roads. A comparative statistical analysis of the calculated and experimentally obtained deceleration values in the event of circuit failure (for Variant 4) is presented in the table (Table 3).

A comparative analysis of the results of classical mathematical modelling ($j_{\max}$) and road certification tests (d_m) revealed a systematic deviation of the experimental values from the theoretical limit of adhesion. All indicators recorded on the road are 13.0–17.8% lower than the calculated potential, which has a sound physical justification linked to the integral nature of the experimental parameter and the gas dynamics of the pneumatic drive.

The theoretical mathematical model determines the peak, instantaneous potential for tyre adhesion to the road surface. It assumes idealised conditions under which the pressure in the brake chambers rises instantaneously and ensures the full realisation of the axle contact force. In contrast, the road test result is determined based on

Table 3. Comparison of theoretical and experimental deceleration values (Variant 4)

Bus weight	Drive separation diagram	Deceleration upon failure of circuit j, m/s ²		
		Deceleration upon failure of circuit j, m/s ²	Theoretical	Experimental Relative difference, %
Gross	Failure of circuit I (axes II + III)	4.47	3.89	13.0
	Failure of circuit II (axes I + III)	3.03	2.52	16.8
Kerb	Failure of circuit I (axes II + III)	5.07	4.17	17.8
	Failure of circuit II (axes I + III)	3.40	2.82	17.1

the total braking distance. It directly incorporates the response time of the pneumatic system and the time taken for air pressure to build up in the line, and thus the inertia involved in generating the corresponding braking forces on the wheels within approximately 2 seconds [17]. During this phase of pneumatic lag, the vehicle travels a significant distance without maximum deceleration, which inevitably leads to a mathematical reduction in the average integral value relative to the theoretical limit.

The application of such a circuit not only allows for the achievement of regulatory efficiency but also optimises the bus's directional stability. By maintaining substantial braking force on the rear axles in the event of a failure of the front circuit, the risk of a dangerous longitudinal body roll is minimised and the stability of the vehicle's trajectory is ensured. Thus, the research results provide sound grounds for recommending the fourth circuit separation scheme as the most rational for the design of braking systems for modern three-axle buses with a monocoque body, which ultimately formed the basis for the bus's production design and was confirmed as effective and compliant with regulatory requirements.

CONCLUSIONS

As a result of the study of the dynamics of load redistribution on the axles of a three-axle bus with a separated, non-balanced suspension of the rear 2nd and 3rd axles, and particularly the uneven distribution of loads on the axles, as in the A231 bus under emergency braking conditions, the need to review traditional approaches to the design of brake circuit separation schemes has been scientifically substantiated. Mathematical modelling has confirmed that the specific layout of extra-large buses with a monocoque body, a long wheelbase and a high centre of mass results in significant unevenness in the dynamic vertical reactions on the axles. It has been established that, given the vehicle's gross weight, the individual deceleration values of the separate axles do not ensure compliance with the regulatory safety threshold of 2.5 m/s^2 , as set by UN Regulation No. 13 for the secondary braking system. This highlights the critical vulnerability of conventional drive separation schemes, where the failure of one circuit leads to a drop in braking efficiency below permissible limits.

A comparative analysis of four alternative brake drive circuit configurations demonstrated the superiority of the developed circuit utilising an acceleration valve for circuit separation, which ensures the safe engagement of the third (auxiliary) axle in the operation of each of the two brake system circuits. This configuration allows deceleration values to be stabilised within the range of $3.88\text{--}4.59 \text{ m/s}^2$ in the event of a failure in one of the circuits of the service braking system, which significantly exceeds the established requirements and ensures a high level of reliability for the braking system. The results obtained are of practical significance for improving active safety and directional stability of multi-axle passenger vehicles, allowing the amplitude of longitudinal body roll to be minimised during emergency braking. It is evident that a two-section bus with the power unit located in the rear overhang and a drive to the third axle with twin tyres (push configuration) will require a separate analysis and the development of an optimal drive configuration using a similar methodology, as will 4-axle lorries and 3-axle vehicles with non-standard layouts.

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