

Approximative stochastic model for production risk assessment and batch completion time in manufacturing lines

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ABSTRACT

This study developed an approximate analytical model for assessing production risks in manufacturing systems with sequential technological operations under uncertainty. While stochastic models allow detailed representation of production processes, their practical application is often limited by high computational complexity. To address this issue, an analytical approximation framework was proposed, enabling efficient evaluation of processing time under the influence of production risks. The model is based on a probabilistic interpretation of risk using a normalized risk priority index (WJRPN), which allows integrating the impact of 4M factors (Man, Machine, Material, Method) into the stochastic description of operation durations. Analytical expressions were derived for the expected value and variance of inter-operational delays, forming the basis for the proposed approximate model of batch processing time. A key advantage of this approach is a significant reduction in computational complexity compared to trajectory-based stochastic models while maintaining acceptable accuracy. Numerical results show that the relative error of the model does not exceed 5%, making it suitable for engineering applications. The developed model provides a practical tool for rapid analysis of production lines, enabling risk-aware planning, identification of critical operations, and improvement of manufacturing system reliability.

Keywords: stochastic flow shop, stochastic scheduling, production risk, risk assessment, FMEA, 4M factors, Monte Carlo simulation, makespan variability.

INTRODUCTION

Modern production lines operate under conditions of high uncertainty caused by technical equipment failures, variability in material quality, human factors and organizational failures [1]. Taken together, these factors create production risks that directly affect the fulfillment of planned production volumes and compliance with product delivery deadlines [2]. A classic tool for analyzing industrial risks is the failure mode and effects analysis (FMEA) method [3], widely used in mechanical engineering, mining and processing industries. However, the traditional risk priority index (RPN) has a number of limitations: lack of

standardization, impossibility of direct use in reliability calculations, and ignoring the probabilistic nature of the production process [4, 5]. In previously published works, a modification of FMEA using the normalized WJRPN index proposed [6], which allows risk to be interpreted as the probability of failure to fulfill the production plan.

Despite significant progress in deterministic and partially stochastic modeling of flow shop scheduling [7–9], the challenge of creating a unified stochastic model of a linear production line remains open [10, 11]. This model would:

a) observe full probability distributions of operation execution times (including multimodal or hyperexponential forms arising from a

mixture of different processing modes arising from the stochastic nature of risks across the 4M categories: Man – human factor, Machine – equipment failures and wear, Material – material quality variability, Method – organizational and technological deviations) [12, 13]. Such distributions arise from competing or superimposed modes of operation of the process equipment: the main (normal) processing mode with relatively low variance, as well as several “risky” modes like personnel absence, breakdown, additional setup, material defect, each characterized by its own time distribution law. The mixture of these components leads to a multimodal probability density function or a hyperexponential distribution of the full operation time, which is well described in classical works on the stochastic scheduling of flow systems [12]. In particular, multimodal forms clearly appear when taking into account risks according to 4M categories, as shown in recent studies on real production routes [6], where the distribution density graphs exhibit several pronounced modes corresponding to different equipment states ($z = 0$ – normal mode; $z > 0$ – risky modes). Ignoring the full form of the distribution (for example, reducing it to an exponential or normal law) leads to underestimation of the distribution tails and, as a consequence, to significant errors in predicting the probability of exceeding the planned batch time [13].

b) explicitly model inter-operational delays (technological downtime, buffer accumulation, flow synchronization) and their stochastic dynamics arising from random variations in processing times, equipment states, as well as the realization of risks in sequential operations that lead to cascading effects, such as blocking, starvation, and bottleneck migration depending on batch size and buffer constraints [14–17]. Inter-operational delays arise as a stochastic process due to random deviations in processing times in previous operations, equipment transitions to risky states (4M), and limited buffer capacity. This leads to blocking (the accumulation of parts in front of a busy operation), starvation (an idle operation due to a lack of parts), and downtime, as well as the need for flow synchronization to prevent bottleneck migration. The stochastic dynamics of these delays manifest as a nonlinear increase in the variance of the total batch time with increasing batch size N , especially with limited buffer

capacity, complicating analytical modeling and requiring explicit consideration of probabilistic state transitions [14, 15]. Studies on stochastic flow shops with blocking/no-wait and limited buffers have shown that ignoring these effects leads to an underestimation of the variance makespan and the risk of plan failure [16, 18]. Recent simulations on real production routes confirm that explicit inclusion of buffer accumulations and stochastic dynamics of inter-operation delays significantly improves the accuracy of batch time forecasting and the level of production risks [18].

c) allow for quantitative assessment of production risks as probabilistic measures of failure to fulfill the plan – specifically, the probability of exceeding the planned batch completion time (risk of delay or makespan threshold) and the risk of bottleneck migration (shifting of the limiting operation due to stochastic realization of risks across operations and batch sizes) [18, 20–22]. Quantitative assessment of production risks is achieved through the introduction of the risk function, defined as the probability of exceeding the planned production time for a batch of parts. This allows for direct measurement of the failure to fulfill the plan probability under stochastic conditions, including tail risks with large variances from 4M factors. Furthermore, the model reveals the bottleneck migration (or shifting bottleneck) effect, where a constraining operation moves along the route due to random risk occurrences (e.g., a failure at one operation increases the load on subsequent operations, causing cascading delays and a bottleneck shift).

d) take into account the dependence of the specified characteristics (statistical moments of batch processing time, risk functions, bottleneck positions) on the batch size N , type of production (small-, medium-, large-scale) and structural features of the line (sequence of operations, buffer capacity), where increasing the batch size N stabilizes variance and reduces relative risk due to averaging effects in stochastic environments, while buffer capacity, as well as operation sequence determine cascading delays, blocking/starvation phenomena, and bottleneck migration [23, 24].

Such a model is necessary for the transition from expert or static methods (such as classical FMEA) to probabilistic-oriented

reliability forecasting and real-time risk management of modern production systems with high uncertainty [25–28].

The aim of this paper was to develop a stochastic approximate analytical model of a production line with sequential arrangement of technological operations, which enables:

- explicit incorporation of production risks associated with the 4M categories, including their impact on operation time distributions and inter-operational dynamics;
- computation of key statistical characteristics of the batch manufacturing time (mean value, variance, higher moments, and probability distributions of completion time), as well as inter-operational delays and overall batch lead time under stochastic conditions [29];
- substantial reduction in computational complexity compared to the trajectory model of subject-technological description of the production process [30], while preserving acceptable accuracy for practical applications in real-time planning.

The proposed analytical model is based on the trajectory-based manufacturing model [30] and the probabilistic risk representation [19, 34] developed in authors' previous studies, which form the methodological basis of this paper. The scientific contribution of this paper lies in the analytical extension of the existing model, rather than its reformulation. The main innovation lies in replacing the computationally complex stochastic trajectory-based calculations with an analytical approximation that preserves the key probabilistic characteristics of the manufacturing process. This approximation enables analytical evaluation of the probability distribution, expected production completion time, and variance without explicitly modeling the trajectories of individual parts. Consequently, the proposed approach significantly improves computational efficiency (reducing the computational complexity for a batch of N parts and a process flow of M operations by up to $N \times M$ times), while maintaining the accuracy required for engineering applications, thereby extending the applicability of the previously developed methodology to real-time production planning and decision support problems.

This model is intended to support the probabilistic forecasting of production reliability, enable prioritization of risk mitigation measures on critical operations, and facilitate integration into

modern manufacturing execution systems (MES) for robust scheduling under high uncertainty.

MATERIALS AND METHODS

Formalization of the technological process

To formalize the manufacturing process for a basic part, parameters t_{ψ} and η , are introduced, defining the state of the part at time t , where $t_{\psi}(t)$ is the average processing time of the part at time t , characterizing its current position in the process flow (the average accumulated processing time for completed operations); $\eta(t)$ is the processing time of the part that is being processed at time t in the current process operation. The state of the part after completion of the m -th technological operation is characterized by the value of the technological processing time $t_{\psi}(t_m) = t_{\psi m}$, which is determined as follows:

$$t_{\psi k} = \sum_{m=1}^{k \leq M} \eta_{mean \ m}, \quad 1 \leq k \leq M, \quad (1)$$

$$\eta_{mean \ m} = E[\eta_m], \quad t_{d\psi} = \sum_{m=1}^M \eta_{mean}$$

where: M is the number of technological operations in the technological route; $\eta_{mean \ m}$ is average time of technological processing of a part $\eta_m = \eta(t_m)$, at the m -th operation in normal mode (without taking into account the risks); t_m is the time elapsed from the moment the part begins to be processed until the moment its processing is completed at the m -th technological operation; this time directly includes both the time the part is being processed and the time during which the part is awaiting processing; $t_{\psi k}$ is the average total technological time during which the part was processed from the first to the k -th technological operation inclusive; $\eta_m = \eta(t_m)$ time spent processing a part at the m -th operation; $t_{d\psi}$ is total average technological time for manufacturing a basic part along the entire route.

For a process operation to operate reliably, a continuous flow of resources is necessary, ensuring uninterrupted execution without downtime or blockages. To ensure the model's versatility and eliminate the influence of dimensional

characteristics (scale effects specific to a specific production process), the subsequent description of the production process is presented in dimensionless units:

$$\tau = \frac{t}{t_{d\psi}}, \tau_{\psi} = \frac{t_{\psi}}{t_{d\psi}}, \tau_{\psi} \in [0,1], \vartheta = \frac{\eta}{t_{d\psi}}, \quad (2)$$

where: τ is the dimensionless time coordinate corresponding to the normalized process time; τ_{ψ} is the average dimensionless technological processing time of a part, characterizing its position along the technological route; ϑ is a dimensionless parameter characterizing the processing time of a part during a technological operation at the current moment in time.

This parameterization allows describing the evolution of the state of a part in a multidimensional technological space, taking into account the stochastic nature of the execution time of operations and transitions between equipment states. The introduced dimensionless parameters enable representing the technological process in a standardized form, independent of the absolute values of processing time and cycle duration. This is achieved by dividing all time characteristics by the standard (average) technological time for manufacturing the base part (the total route time in normal mode, without taking risks into account).

The transition to a dimensionless representation allows for comparative analysis of production systems with different processing times of part, number of workdays per month, and planned throughput without losing the generality of the conclusions. This approach makes the resulting relationships universal and applicable to a wide range of production lines, regardless of industry, enterprise size, or absolute values of the time characteristics. Dimensionless quantities ensure the invariance of the model with respect to specific dimensional parameters, which is especially important in the case of stochastic processes and significant variability of input data.

The state of the part during the execution of the m -th technological operation at a fixed point in time τ can be completely described by a point in a multidimensional state space $(\tau, \tau_{\psi}, \vartheta)$. A sequence of such points, constructed for continuously successive moments in time, forms a trajectory in space $(\tau, \tau_{\psi}, \vartheta)$ [30], which reflects the evolution of the product parameters throughout the entire technological route – from the moment raw materials

or semi-finished products arrive at the first technological operation until the finished product is obtained. In the stochastic model, the trajectory becomes a random process subject to disturbances from the risks of the 4M categories, which leads to deviations from the nominal trajectory and the occurrence of inter-operational delays. This full trajectory model allows for accurate reproduction of the dynamics of the part but has a high computational complexity.

Probabilistic modeling of production risks

As a result, the effective execution time of an operation increases by the time required to detect, localize, and eliminate the risk, as well as to restore the equipment or workplace to normal operating conditions. This may lead to accumulated process downtime, greater variability in batch completion times, and a higher probability of exceeding the planned production time [33, 34]. In this study, production risks were classified according to the 4M categories: Man, Machine, Material, and Method. This classification is consistent with the Ishikawa diagram, which is widely used in management sciences and quality engineering to identify as well as structure the causes of process disturbances. Accordingly, production risks can be defined as stochastic events belonging to the 4M categories that cause a temporary halt or a significant slowdown in processing at a given operation [32]. A classification of typical risk factors occurring in production systems is presented in Table 1 [34]. An additional category corresponds to the standard manufacturing process performed in the absence of production risks, i.e., the nominal state, denoted as $z = 0$.

The failure mode and effects analysis (FMEA) method as a classic tool for classifying and analyzing technological processes and production risks is widely used in various industries. In mechanical engineering, FMEA is used to identify potential failures and assess their consequences during the design as well as production stages [37]. In the mining industry, the method is used to analyze risks associated with equipment, safety and technological processes, including reliability assessment under conditions of high uncertainty [38, 39]. In the process industry, FMEA is used to prioritize risks in continuous and discrete processes, with an emphasis on product quality and equipment reliability [40]. Modern extensions of FMEA include the integration

of stochastic models, probabilistic distributions of operation times, and consideration of 4M categories to quantify risks in flow manufacturing systems [18]. When calculating the probability of normal (standard) functioning of a technological operation r_{0m} ($z=0$) and the probabilities of the realization of production risks r_{zm} , ($z=1\dots6$) at the m -th operation when assessing the current state of the production line, the Failure Mode and Effects Analysis (FMEA) method was applied using the normalized risk priority index – risk priority number (RPN) [34].

The proposed approach made it possible to interpret the risk of a production system as the probability of failure to achieve the planned volume of output, expressed through three main parameters of a modified FMEA:

- Severity (S) – an assessment of the severity of the consequences of a defect or risk factor for achieving the system’s primary objective (meeting the planned production volume of finished goods). This parameter reflects the magnitude of the negative impact on output volume, product quality, and delivery times;
- Occurrence (O) – assessment of the frequency of occurrence of a given risk factor (equipment breakdowns, employee absenteeism, late or poor-quality deliveries of materials, etc.) during a representative analysis period;
- Detection (D) – an estimate of the probability that a potential cause of failure or the defect itself will not be detected in a timely manner by existing inspection methods before the semi-finished product is transferred to the next process step or before the finished product is shipped to the customer. High values indicate low inspection effectiveness.

To overcome the known shortcomings of the classical risk priority index (RPN), namely the lack of normalization, the impossibility of direct use in reliability calculations, and the ignoring of the interdependencies between the three parameters, a normalized unified risk priority index $WJRPN = (S \times O \times D) / 1000$, taking values in the range $[0; 1]$ was used. This modification enabled: a) the representation of the probability of the state of a technological operation taking into account the risk of each individual factor as a probability in the interval $[0; 1]$; b) correct aggregation of risks within the 4M groups for each technological operation; c) direct application of resulting values in reliability formulas for sequential and sequential-parallel structures by equating risk to unreliability of the production line; d) quantification of the contribution of each production element and each 4M category to the overall probability of failure to meet the monthly plan, and consequently to the magnitude of production losses, expressed in physical units. Thus, a modification of the FMEA method was used to calculate the production risk values by introducing the WJRPN coefficient, allowing for the ranking of production risks when constructing a comprehensive probabilistic model of the production line [36].

It should be noted that the method for calculating production risk values proposed in the article [34] is essentially a prototype of an expert approach in which the estimates of the parameters S, O and D are assigned in accordance with a discrete scale from 1 to 10. Despite the high practical value of this approach for small and medium-sized enterprises, its main limitation remains the significant influence of subjective factors and expert judgments on the final RPN

Table 1. Statistical characteristics of factors characterizing a technological operation [34]

Category	Type factor	Probability	Statistical characteristics of event duration (Time)		
			Distribution function	Average	Standard deviation
No risk	Characteristics workstation	r_{0m}	$f_{0m}(g)$	$g_{mean\ 0m}$	$g_{std\ 0m}$
Machine	Additional adjustments and settings	r_{1m}	$f_{1m}(g)$	$g_{mean\ 1m}$	$g_{std\ 1m}$
	Breakdowns	r_{2m}	$f_{2m}(g)$	$g_{mean\ 2m}$	$g_{std\ 2m}$
Material	Delays or shortages in the materials delivery	r_{3m}	$f_{3m}(g)$	$g_{mean\ 3m}$	$g_{std\ 3m}$
	Poor materials quality	r_{4m}	$f_{4m}(g)$	$g_{mean\ 4m}$	$g_{std\ 4m}$
Method	Technological and real time differences	r_{5m}	$f_{5m}(g)$	$g_{mean\ 5m}$	$g_{std\ 5m}$
Man	Absence	r_{6m}	$f_{6m}(g)$	$g_{mean\ 6m}$	$g_{std\ 6m}$

values, which significantly reduces the reproducibility, objectivity and comparability of results both between different enterprises and during repeated assessments at the same facility [41, 42, 43]. To improve the accuracy and reliability of forecasting production risks, further development of the method is the replacement (or supplementation) of expert assessments of the parameters S, O, D with statistically valid indicators obtained based on real experimental data on downtime of equipment, workplaces and technological operations [44–46]. This transition from purely qualitative expert scales to quantitative probabilistic characteristics (mathematical expectation, variance, downtime distribution density) allows for a significant increase in the accuracy of risk assessment as the probability of plan failure, a reduction in subjectivity, and the reproducibility of results in the context of the stochastic nature of production processes.

In particular, the stochastic nature of the execution time of each technological operation makes it necessary to model the production process as a random process of transferring technological resources during the sequential passage of parts through the route operations [47–49]. In this context, the technological trajectory of a part is the implementation of a stochastic process, where the processing time at each operation is a random variable formed under the influence of the entire complex of production factors (Table 1). These factors include both the standard (routine) processing of the part and the impact of 4M category production risks, which lead to additional delays, downtime and deviations from the nominal time. The process operation model based on accounting for production risks (Table 1) assumes that the processing of the part on manufacturing equipment exists in one discrete state. The probability of a part being in the z -th state is denoted by r_{zm} . On the probability space (Ω, F, P) , the random variable $\Psi: \Omega \rightarrow \{0, 1, \dots, 6\}$ is introduced to describe the operational state of the part during the m -th technological operation. The parameter Ω denotes the set of all possible outcomes for realizations of operation states; F is the σ -algebra of measurable events, $P: F \rightarrow [0, 1]$. The probability that the part is in state z is given by:

$$r_{zm} = P_m(\Psi = z) \geq 0, \quad \sum_{z=0}^6 r_{zm} = 1, \quad z = 0, \dots, 6. \quad (3)$$

The presented formula represents a basic approach to calculating the overall production risk for a linear production line with sequential technological operations [34]. The value of the total risk of the line was determined by an expression of the following form:

$$R_c = 1 - \prod_{m=1}^M \prod_{z=1}^Z (1 - r_{zm}), \quad Z = 6. \quad (4)$$

This method provides an aggregated risk assessment for the entire process route, but retains the limitations of classical FMEA: the subjectivity of the expert scales S, O, D and the lack of consideration of the stochastic dynamics of operation times, inter-operational delays and dependence on batch size.

The production risk assessment model for a linear production line was improved in [6] by additionally introducing the distribution density $f_m(\vartheta)$ of the processing time ϑ of a part at the m -th technological operation. The duration of processing of a part ϑ when the technological process at a given operation is in state $z = 0, \dots, 6$ is determined by the distribution function $f_{zn}(\vartheta)$. For states $z > 0$, this time includes both the time required to eliminate the resulting production risk and the direct time of processing the part. According to the law of total probability, the overall distribution density of the processing time for the m -th technological operation can be expressed as follows:

$$f_m(\vartheta) = r_{0m}f_{0m}(\vartheta) + \sum_{z=1}^6 r_{zm}f_{zm}(\vartheta - \vartheta_{mean\ 0m}) \quad (5)$$

where: the distribution density $f_{zn}(\vartheta)$ for a state $z > 0$ is characterized by two main statistical parameters: the mathematical expectation $\vartheta_{mean\ zn}$ and the standard deviation $\vartheta_{std\ zn}$ of the time, required to eliminate the consequences of a production risk as well as resume the normal functioning of the technological process (Table 1) and can be constructed on the basis of archival or experimental data on equipment breakdowns, absence of workers, delays and defective deliveries, accumulated over a sufficiently long observation period [50, 44, 45].

The main reason that in formula (7) the average standard technological time is used in the distribution density $f_{zn}(\vartheta - \vartheta_{mean\ 0n})$ in the state $z > 0$ and not the convolution of distribution functions

of the form $f_{0m} * f_{zn}$ is the relative simplicity of constructing the distribution density $f_{zn}(\vartheta)$ for the time that technological equipment is in the state of eliminating the consequences of production risk. This approximation significantly reduces the complexity of data collection and processing, while maintaining acceptable accuracy for the practical tasks of assessing production risks and predicting batch times under stochastic uncertainty. Unlike convolution, which requires the construction and verification of two independent distributions (the base processing time and the time to eliminate each risk), the proposed form (5) allows for function $f_{zn}(\vartheta)$ the direct use of empirical or parametric estimates obtained from the actual data on the duration of production risk elimination. In this case, the distribution $f_{0m}(\vartheta)$ is formed exclusively based on the observations risk-free operation periods and reflects the natural variability of the technological process as a stochastic process. It should be noted, however, that two fundamental limitations require further development of the model: a) it is assumed that the risk factor is realized before the start or after the completion of the technological processing of the current part, but does not take into account the possibility of its occurrence at an arbitrary moment within the processing cycle. Under real-production conditions, equipment failure, the discovery of a hidden material defect, or the sudden absence of an operator can interrupt the processing of a part at any stage; b) the model does not consider the cases where a part becomes irreparably defective (hidden wood defects, irreversible damage due to tool breakage). When such risk conditions are realized, the part is excluded from the further technological route, which requires the introduction of an additional absorbing state and modification of the distribution function of the time the system spends in this operation. Thus, the distribution density $f_m(\vartheta)$ of the processing time ϑ of a part at the m -th technological operation determines the output flow of parts from the technological operation, taking into account production risks. In particular, the stochastic nature of the execution time of each technological operation makes it necessary to model the production process as a random process of transferring technological resources during the sequential passage of parts through the route operations. The paper [18] explored a deterministic and stochastic model of a production line based on a subject-technological description of the production

process, which is used to assess production risks. This model implements a sequential calculation of the trajectories of parts as they move along the process route and requires a significant amount of computation, especially with a large number of risk realizations and operations.

Stochastic modeling of processing time distribution

In this paper, the approximated stochastic model of a production line with sequential execution of technological operations was developed, designed for the quantitative assessment of production risks during the manufacture of a batch of N parts. The model is based on a system of equations in general form [18], describing the trajectories of individual parts:

$$\begin{cases} \tau_{0,m} = \tau_{0,m-1} + \vartheta_{0,m} \\ \tau_{n,m} = \max(\tau_{n,m-1} + \vartheta_{n,m}, \tau_{n-1,m+1}), n > 0, m > 0 \\ \tau_{n,M} = \tau_{n,M-1} + \vartheta_{n,M}, \end{cases} \quad (6)$$

where: $m = 1, 2, \dots, M$ is the technological operation index; $n = 1, 2, \dots, N$ is the part within the batch index; $\tau_{n,m}$ is the start time of processing the n -th part at the m -th technological operation; $\vartheta_{n,m}$ is the stochastic processing time of a single n -th part at m -th operation according to the probability distributions (Table 1); $\tau_{n,m-1}$ is the completion time of this part at the previous operation.

This system of equations is used to calculate the technological trajectories of parts in a batch under given initial conditions for the start time of processing the n -th part in the first technological operation:

$$\begin{cases} \tau_{0,0} = 0, \\ \tau_{n,0} = \tau_{n-1,1}. \end{cases} \quad (7)$$

The processing time of the parts batch is defined as the time interval:

$$\tau_{batch} = \tau_{N,M} - \tau_{0,0}. \quad (8)$$

It is assumed that the movement of the first part of the batch is not limited by the previous batch, that is, the condition $\tau_{0,m} = 0$ is satisfied. The total time to produce a batch of parts can be represented as the sum of two components. The first component corresponds to the production time of the first part of the batch:

$$\tau_{1,M} = \tau_{1,0} + \sum_{m=1}^M g_{1,m} = \sum_{m=1}^M g_{1,m}. \quad (9)$$

The second term is determined by the sequence of output of the parts of the batch from the last technological operation at intervals of the time $\Delta\tau_{n,M}$:

$$\tau_{sum,M} = \sum_{n=2}^N \Delta\tau_{n,M}, \quad (10)$$

$$\Delta\tau_{n,M} = \tau_{n,M} - \tau_{n-1,M}.$$

In the particular case of deterministic case of processing a batch of N parts, the interval $\Delta\tau_{n,M}$ can be expressed through the maximum average time of processing parts in technological operations, $\vartheta_{mean\ max} \{ \vartheta_{mean\ m} : 1 \leq m \leq M \}$ with a deterministic time for performing an operation $\vartheta_{1m} = \vartheta_{1mean\ m}$, which allows writing an expression for a deterministic model of the process of manufacturing a batch of parts on a production line with a sequential arrangement of technological operations:

$$\tau_{batch} = \sum_{m=1}^M g_{1,mean\ m} + (N-1)g_{mean\ max}, \quad (11)$$

$$\tau_d = \frac{\tau_{batch}}{N} = \frac{1}{N} \sum_{m=1}^{M-1} g_{1,mean\ m} + g_{mean\ max},$$

Using the subject-technological description based on model (7), the processing of parts in two adjacent technological operations with numbers $(m-1)$ and numbers m is considered. Two scenarios are possible here.

First scenario: the part n has completed processing at $(m-1)$ -th technological operation and is waiting for the $(n-1)$ -th part to be processed at the m -th technological operation. In this case, the interval between the completion of the processing of the $(n-1)$ -th part and the n -th part in the m -th technological operation will be equal to the processing time $\vartheta_{n,m}$ for the n -th part at m -th technological operation: $\vartheta_{n,m} = (\tau_{n,m} - \tau_{n-1,m})$. For this scenario, there is no delay in equipment operation, $\Omega_{n,m} = (\tau_{n,m-1} - \tau_{n-1,m}) = 0$.

The second scenario: the n -th part continues to be processed in the $(m-1)$ -th operation, while the processing of the $(n-1)$ -th part in the m -th operation is completed. In this case, the m -th technological operation results in a

downtime of the technological equipment for a period of the time $\Omega_{n,m} = (\tau_{n,m-1} - \tau_{n-1,m})$. The interval between the completion of the processing of the $(n-1)$ -th part and the n -th part in m -th technological operation will be equal to the processing time $\vartheta_{n,m}$ for the n -th part in the m -th technological operation plus equipment downtime due to the absence of the part: $\vartheta_{n,m} + \Omega_{n,m} = (\tau_{n,m} - \tau_{n-1,m})$. In the second scenario, there is a delay in the operation of the equipment, $\Omega_{n,m} = (\tau_{n,m-1} - \tau_{n-1,m}) > 0$.

When constructing the approximated model, it is assumed that the parts are stored in a container in front of the production line and arrive at the start of the first technological operation without any downtime of the technological equipment, $\Omega_{n,1} = 0$. Then, the equipment downtime for the part during the first technological operation can be determined as follows:

$$\Delta\vartheta_{n,1} = \vartheta_{n,1} - \vartheta_{n-1,2} + \Omega_{n,1}, \Omega_{n,1} = 0, m = 1, \quad (12)$$

For subsequent technological operations, the downtime of the technological equipment is calculated recursively based on the value $\Delta\vartheta_{n,m-1}$:

$$\begin{aligned} \Delta\vartheta_{n,2} &= \vartheta_{n,2} - \vartheta_{n-1,3} + \Omega_{n,2}, \\ \Omega_{n,2} &= \max(\Delta\vartheta_{n,1}; 0), m = 2, \end{aligned} \quad (13)$$

$$\begin{aligned} \Delta\vartheta_{n,m} &= \vartheta_{n,m} - \vartheta_{n-1,m+1} + \Omega_{n,m}, \\ \Omega_{n,m} &= \max(\Delta\vartheta_{n,m-1}; 0), \end{aligned} \quad (14)$$

$$\begin{aligned} \Delta\vartheta_{n,M-1} &= \vartheta_{n,M-1} - \vartheta_{n-1,M} + \Omega_{n,M-1}, \Omega_{n,M-1} = \\ &= \max(\Delta\vartheta_{n,M-2}; 0), m = M-1, \end{aligned} \quad (15)$$

$$\Omega_{n,M} = \max(\Delta\vartheta_{n,M-1}; 0), m = M, \quad (16)$$

and accordingly, the formulas for calculating technological trajectories will take the form:

$$\begin{aligned} \tau_{1,M} &= \sum_{m=1}^M g_{1,m}, \quad \tau_{n,M} = \tau_{1,M} + \\ &+ \sum_{k=2}^n (g_{k,M} + \Omega_{k,M}), \quad k \leq n, \end{aligned} \quad (17)$$

$$\begin{aligned} \tau_{batch} &= \tau_{N,M} - \tau_{0,0} = \sum_{m=1}^M g_{1,m} + \sum_{n=2}^N (g_{n,M} + \Omega_{n,M}) = \\ &= \sum_{m=1}^{M-1} g_{1,m} + \sum_{n=1}^N (g_{n,M} + \Omega_{n,M}), \end{aligned} \quad (18)$$

In the absence of equipment downtime on the production line waiting for the part, the expression for calculating the production time of the batch of parts takes the form for a stochastic production process:

$$\tau_{batch} = \sum_{m=1}^M g_{1,m} + \sum_{n=2}^N g_{n,M} = \sum_{m=1}^{M-1} g_{1,m} + \sum_{n=1}^N g_{n,M}, \quad (19)$$

Analytical approximation of inter-operational delays

Let us define the statistical characteristics of a random variable $\Omega_{n,M}$. The statistical characteristics of random variables $\Omega_{n,1}, \Omega_{n,2}, \dots, \Omega_{n,M-1}$ can be defined recursively:

$$E[\Delta g_{n,m}] = E[g_{n,m} - g_{n-1,m+1} + \Omega_{n,m}] = E[g_{n,m}] - E[g_{n-1,m+1}] + E[\Omega_{n,m}] \quad (20)$$

$$E[(\Delta g_{n,m})^2] = E[(g_{n,m} - g_{n-1,m+1} + \Omega_{n,m})^2] = E[(g_{n,m})^2] + E[(g_{n-1,m+1})^2] + E[(\Omega_{n,m})^2] \quad (21)$$

$$\Omega_{n,m+1} = \max(\Delta g_{n,m}; \alpha) \quad \alpha = 0, \quad (22)$$

$$E[\Omega_{n,m+1}] = \alpha \int_{-\infty}^{\alpha} f(\Delta g_{n,m}) d\Delta g_{n,m} + \int_{\alpha}^{\infty} \Delta g_{n,m} f(\Delta g_{n,m}) d\Delta g_{n,m} = \int_0^{\infty} \Delta g_{n,m} f(\Delta g_{n,m}) d\Delta g_{n,m} \quad (23)$$

$$E[(\Omega_{n,m+1})^2] = \alpha^2 \int_{-\infty}^{\alpha} f(\Delta g_{n,m}) d\Delta g_{n,m} + \int_{\alpha}^{\infty} (\Delta g_{n,m})^2 f(\Delta g_{n,m}) d\Delta g_{n,m} = \int_0^{\infty} (\Delta g_{n,m})^2 f(\Delta g_{n,m}) d\Delta g_{n,m} \quad (24)$$

When a batch of parts moves along a technological route, the part is at the technological operation in one of the z -th states with a probability r_{zn} (4). At the same time, the previous part is in the next technological operation in one of the k -th states with the probability $r_{k(m+1)}$. The time interval, during which the part is in z -th state, is determined by the distribution density, Table 1. This distribution density is in practice approximated quite well by the normal distribution law [36]. To determine the static characteristics of a random variable $\Omega_{n,M}$, it is necessary to calculate integrals of the form (23), (24), and therefore, it is necessary to know the distribution law of the random variable $\Delta g_{n,m}$ for each technological operation. As a first step in constructing the approximated model of the production processing a batch of parts on a production line with a sequential arrangement of technological operations, it is assumed that the distribution law of a random variable $\Delta g_{n,m}$ is close to the normal distribution law.

Then, assuming a normal distribution law for the random variable $X = \Delta g_{n,m}$, let us define the statistical characteristics of a random variable Y :

$$Y = \max(X, 0), \quad X = N(m_x, \sigma_x^2), \quad f_x(x) = \frac{1}{\sigma_x \sqrt{2\pi}} \exp\left(-\frac{1}{2} \left(\frac{x - m_x}{\sigma_x}\right)^2\right), \quad (25)$$

$$X = \Delta g_{n,m-1} = (g_{n,m-1} - g_{n-1,m} + \Omega_{n,m-1}),$$

which can be obtained as a result of calculating integrals of the form (23), (24). Using the substitution:

$$z = \frac{x - m_x}{\sigma_x}, \quad x = \sigma_x z + m_x, \quad dx = \sigma_x dz, \quad a = \frac{m_x}{\sigma_x}, \quad (26)$$

and the properties of the distribution density $\varphi(z)$ and distribution function $\Phi(a)$:

$$\varphi(z) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{z^2}{2}\right), \quad \Phi(a) = \int_{-\infty}^a \varphi(z) dz, \quad \int_{-a}^{\infty} \varphi(z) dz = 1 - \Phi(-a) = \Phi(a), \quad (27)$$

$$\int_{-a}^{\infty} z \varphi(z) dz = \left| \frac{d\varphi(z)}{dz} = -z \varphi(z) \right| = - \int_{-a}^{\infty} \frac{d\varphi(z)}{dz} dz = -\varphi(z) \Big|_{-a}^{\infty} = \varphi(a), \quad (28)$$

$$\int_{-a}^{\infty} z^2 \varphi(z) dz = - \int_{-a}^{\infty} z d\varphi(z) = -z\varphi(z) \Big|_{-a}^{\infty} + \int_{-a}^{\infty} \varphi(z) dz = -a\varphi(a) + \Phi(a), \quad (29)$$

expressions are obtained for calculating the mathematical expectation m_y and standard deviation σ_y of a random variable Y , characterizing the delay in manufacturing a part for a technological operation:

$$m_y = E[Y] = \int_0^{\infty} x f_x(x) dx = \int_{-a}^{\infty} (\sigma_x z + m_x) \varphi(z) dz = \sigma_x \int_{-a}^{\infty} z \varphi(z) dz + m_x \int_{-a}^{\infty} \varphi(z) dz = \sigma_x \varphi(a) + m_x \Phi(a),$$

$$f_m(a) = \frac{E[Y]}{\sigma_x} = \varphi(a) + a\Phi(a), \quad (30)$$

$$E[Y^2] = \int_0^{\infty} (x - m_x)^2 f_x(x) dx = \int_{-a}^{\infty} (\sigma_x z + m_x)^2 \varphi(z) dz = \sigma_x^2 \int_{-a}^{\infty} z^2 \varphi(z) dz + 2m_x \sigma_x \int_{-a}^{\infty} z \varphi(z) dz + m_x^2 \int_{-a}^{\infty} \varphi(z) dz =$$

$$= \sigma_x^2 (-a\varphi(a) + \Phi(a)) + 2m_x \sigma_x \varphi(a) + m_x^2 \Phi(a) = (\sigma_x^2 + m_x^2) \Phi(a) + (2m_x \sigma_x - a\sigma_x^2) \varphi(a) = (\sigma_x^2 + m_x^2) \Phi(a) + m_x \sigma_x \varphi(a), \quad (31)$$

$$\sigma_y = E[(Y - E[Y])^2] = (\sigma_x^2 + m_x^2) \Phi(a) + m_x \sigma_x \varphi(a) - (E[Y])^2,$$

$$f_{\sigma}(a) = \frac{\sqrt{E[(Y - E[Y])^2]}}{\sigma_x} = \sqrt{(1 + a^2) \Phi(a) + a\varphi(a) - (\varphi(a) + a\Phi(a))^2}. \quad (32)$$

The distribution density of a random variable is represented as follows:

$$f_y(y) = \delta(y) \Phi\left(\frac{y - m_y}{\sigma_y}\right) + \frac{H(y)}{\sigma_y} \varphi\left(\frac{y - m_y}{\sigma_y}\right), \quad \int_0^y \delta(x) dx = H(y), \quad H(y) = \begin{cases} 1, & \text{if } y \geq 0, \\ 0, & \text{if } y < 0, \end{cases} \quad (33)$$

$$F(y) = \int_{-\infty}^y f_y(x) dx = H(y) \Phi(-b) + H(y) \Phi\left(\frac{y - m_y}{\sigma_y}\right), \quad F(y < 0) = 0, \quad F(0) = \Phi(-b), \quad b = \frac{m_y}{\sigma_y}, \quad F(\infty) = 1, \quad (34)$$

where: $\delta(y)$, $H(y)$ are the Dirac and Heaviside functions, respectively, and for generality of notation, the ratio of the mathematical expectation to the standard deviation is represented by the parameter $b = m_y / \sigma_y$.

The formulas for calculating the statistical characteristics of the inter-operational delay are written under the assumption that the random variable X (25) has a normal distribution law. Indeed, a production operation can be in one of the z -th states determined by the distribution density $f_{zn}(\vartheta)$ (5), which, as indicated above, is approximated quite well in practice by the normal distribution law [36]. However, the random variable characterizing the delay time of the technological operation $\Omega_{n,m-1}$ has a distribution law determined by the distribution density (33) with statistical characteristics (30) and (32). This distribution law differs from the normal distribution law. The error of the deviation between the theoretical distribution density and the calculated distribution density (33) depends on the value of the parameter $a = m_x / \sigma_x$, which is used to calculate the statistical characteristics of the random variable Y . For the purposes of applying the analysis, the values of the function $\varphi(a)$ and $\Phi(a)$, as well as the normalized values of the mathematical expectation $f_m(a)$ (30) and the standard deviation $f_{\sigma}(a)$ (32) are presented in Table 2 and in Figure 1.

As an approximation, the distribution density of a random variable Y is approximated by the normal distribution law, defining the tail for the region of negative values of the random variable Y using the function $\varphi\left(\frac{x - m_y}{\sigma_y}\right)$. This approach allows using the formulas (30–32) to calculate the statistical characteristics of a random variable Y . However, this transition, due to the addition of a tail to the original distribution density, will lead to a change in the value of the mathematical expectation and the standard deviation of the random variable Y .

$$\Delta m_y(a) = \int_{-\infty}^0 \left(\frac{1}{\sigma_y} \varphi \left(\frac{y-m_y}{\sigma_y} \right) - \delta(y) \Phi \left(\frac{y-m_y}{\sigma_y} \right) \right) y dy = \int_{-\infty}^0 \frac{1}{\sigma_y} \varphi \left(\frac{y-m_y}{\sigma_y} \right) y dy - \Phi \left(\frac{-m_y}{\sigma_y} \right) \cdot 0 = \sigma_y (a \Phi(-a) - \varphi(a)), \quad (35)$$

$$\begin{aligned} (\Delta \sigma_y)^2 &= \int_{-\infty}^0 \left(\frac{1}{\sigma_y} \varphi \left(\frac{y-m_y}{\sigma_y} \right) - \delta(y) \Phi \left(\frac{y-m_y}{\sigma_y} \right) \right) (y-m_y)^2 dy = \\ &= \int_{-\infty}^0 \frac{1}{\sigma_y} \varphi \left(\frac{y-m_y}{\sigma_y} \right) (y-m_y)^2 dy - \Phi \left(\frac{-m_y}{\sigma_y} \right) m_y^2 = \sigma_y^2 ((1-a^2) \Phi(-a) + a_y \varphi(a)), \end{aligned} \quad (36)$$

$$k_\sigma(a) = \sqrt{\frac{\sigma_y^2 + (\Delta \sigma_y)^2}{\sigma_y^2}} = \sqrt{1 + \sigma_y^2 ((1-a^2) \Phi(-a) + a_y \varphi(a))}. \quad (37)$$

In connection with the replacement of the original distribution law, given by expressions (33), (34), by a normal distribution, it becomes necessary to take into account the distortion of the main statistical characteristics of a random variable Y . In particular, such an approximation leads to a systematic shift in the mathematical expectation and to change the standard deviation. In this regard, the corrections are

Table 2. Functions of the normal distribution and normalized statistical characteristics

Name of a function	Parameter value, a								
	-4.0	-2.0	-1.0	-0.5	0.0	0.5	1.0	2.0	4.0
$\varphi(a)$	0.000	0.054	0.242	0.352	0.399	0.352	0.242	0.054	0.000
$\Phi(a)$	0.000	0.023	0.159	0.309	0.500	0.691	0.841	0.977	1.000
$f_m(a)$	0.000	0.008	0.083	0.197	0.399	0.698	1.083	2.008	4.000
$f_\sigma(a)$	0.002	0.075	0.262	0.413	0.584	0.744	0.867	0.980	1.000
$f_{\Delta m}(a)$	-	-	-	-	-0.399	-0.198	-0.083	-0.008	-0.0
$k_\sigma(a)$	-	-	-	-	1.225	1.186	1.114	1.020	1.000

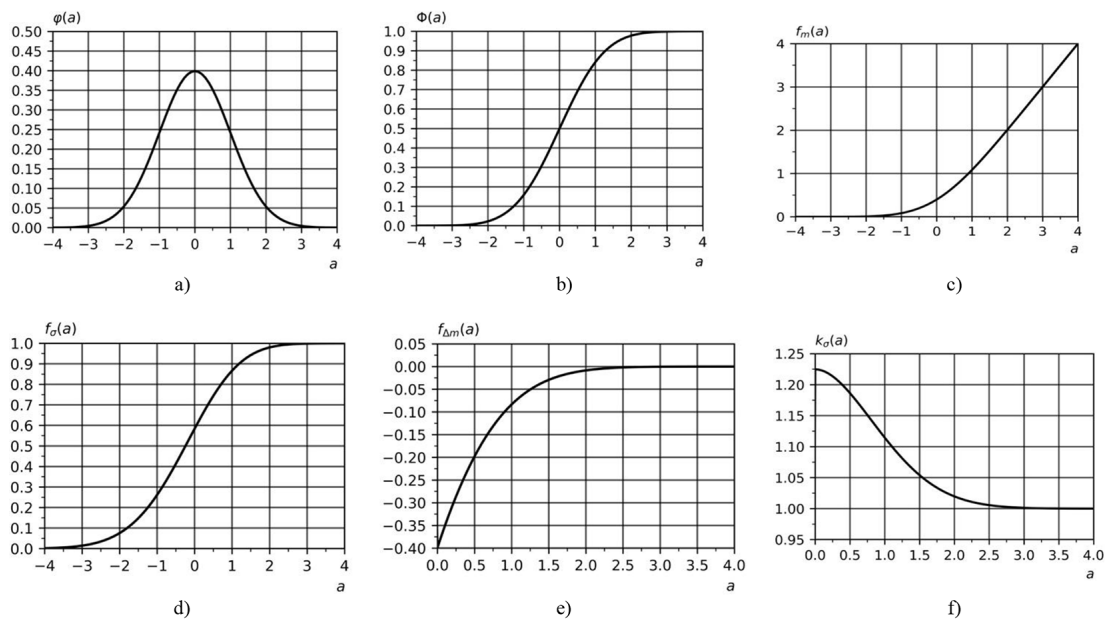


Figure 1. Functions of the normal distribution and normalized statistical characteristics
a) normal probability density function, b) normal distribution function, c) normalized mean,
d) normalized standard deviation, e) relative bias function of the mean, f) correction coefficient
for the standard deviation

introduced for the mathematical expectation and the standard deviation of the random variable Y , which transform the values for the mathematical expectation and the standard deviation:

$$m_{yN} = m_y + \Delta m_y(a), \quad \sigma_{yN} = k_\sigma(a)\sigma_y, \quad (38)$$

where: the coefficients $\Delta m_y(a)$, $k_\sigma(a)$ take into account the systematic bias and change in the scale of the distribution caused by replacing the original distribution of the random variable with a normal distribution.

The obtained expressions were used in this article to calculate the delay in the subsequent technological operation.

When considering the processing of a batch of parts under conditions of production risks at the m -th technological operation, when the distribution density of the time a part spends at the technological operation is represented by a mixture of distributions (7) with a probability r_{zm} in one of the states z , the mathematical expectation and standard deviation of the operation execution time are determined by the following expressions:

$$E[g] = r_{0m}g_{mean\ 0m} + \sum_1^6 r_{zm}(g_{mean\ zm} + g_{mean\ 0m}) = g_{mean\ 0m} + \sum_1^6 r_{zm}g_{mean\ zm}, \quad (39)$$

$$E[(g - E[g])^2] = r_{0m}(g_{mean\ 0m}^2 + g_{std\ 0m}^2) + \sum_1^6 r_{zm}(g_{std\ zm}^2 + (g_{mean\ 0m} + g_{mean\ zm})^2) - E[g]^2 \quad (40)$$

A batch of N parts is sequentially processed in M technological operations. To calculate the random variable of the delay Ω_{m,z_1z_2} , when the processing of parts in adjacent technological operations is in the state z_1 and z_2 , the random variable is introduced $\alpha_{m,z}$:

$$\alpha_{m,z} = \frac{1}{N} \sum_{n=1}^N g_{n,m,z}, \quad \Omega_{m,z_1z_2} = \max(\alpha_{m-1,z_1} - \alpha_{m,z_2} + \Omega_{m-1,z_1z_2}; 0), \quad (41)$$

$$E[\Omega_m] = \sum_{z_1=0}^6 \sum_{z_2=0}^6 r_{z_1m-1}r_{z_2m}E[\Omega_{m,z_1z_2}], \quad E[(\Omega_m - E[\Omega_m])^2] = \sum_{z_1=0}^6 \sum_{z_2=0}^6 r_{z_1m-1}r_{z_2m}E[\Omega_{m,z_1z_2}^2] - E[\Omega_m]^2, \quad (42)$$

$$E[\alpha_{m,z}] = \begin{cases} g_{mean\ 0m}, & z = 0, \\ g_{mean\ 0m} + g_{mean\ zm}, & z > 0, \end{cases} \quad E[(\alpha_{m,z} - E[\alpha_{m,z}])^2] = \frac{g_{std\ zm}^2}{N}. \quad (43)$$

When processing a batch of N parts, the delay value Ω_{m,z_1z_2} (41) for m -th technological operation, related to a single part, is recursively expressed through the delay Ω_{m-1,z_1z_2} at the $(m-1)$ -th technological operation for each pair of states (z_1, z_2) taking into account the statistical characteristics $g_{m-1,z}$ and $g_{m,z}$ with the statistical characteristics of the processing time (43), related to a single part of the batch. For each technological operation, if necessary, the statistical characteristics of the processing of a single part of a batch of N parts are determined by formulas (42).

For the technological operations of manufacturing enterprises, the relationship is $(g_{mean\ zm}/g_{std\ zm}) \sim 10$. This allows using approximate expressions for the normalized functions of the mathematical expectation $f_m(a)$ (30), the standard deviation $f_\sigma(a)$ (32) and the systematic bias of the mathematical expectation $f_{\Delta m}(a)$ (38) to calculate the delay value Ω_{m,z_1z_2} (41), Table 3. Indeed, for production lines, the processing of parts in adjacent operations in states (z_1, z_2) , are fair relationships:

$$\begin{cases} \frac{g_{mean\ 0m-1}}{g_{mean\ zm}} \approx (0.01 \div 0.1), & (0, z), z > 0, a \ll -1; \\ \frac{g_{mean\ zm-1}}{g_{mean\ 0m}} \approx (10 \div 100), & (z, 0), z > 0, a \gg 1; \\ \frac{g_{mean\ 0m-1}}{g_{mean\ 0m}} \approx (-0.1 \div 0.1), & (0, 0), a \approx 0; \end{cases} \quad (44)$$

Table 3. Analytical approximations of the normalized distribution characteristics: mean, standard deviation, and systematic bias

Analytical function	Parameter value, $a = m_x / \sigma_x$		
	$a \ll -1$	$a \sim 0.0$	$a \gg 1$
$f_m(a) = \varphi(a) + a\Phi(a)$	$\sim \frac{\varphi(a)}{a^2}$	$\sim \frac{1}{\sqrt{2\pi}} + \frac{a}{2} + \frac{a^2}{\sqrt{8\pi}} + \dots$	$\sim a + \frac{\varphi(a)}{a^2}$
$f_\sigma(a) = \sqrt{(1+a^2)\varphi(a) + a\varphi(a) - (\varphi(a) + a\Phi(a))^2}$	~ 0	$\sim \sqrt{\frac{\pi-1}{2\pi} + \frac{a}{\sqrt{2\pi}} + \frac{\pi-3}{2\pi}a^2 + \dots}$	~ 1
$f_{\Delta m}(a) = a\Phi(-a) + \varphi(a), a \geq 0$	–	$\sim -\sqrt{\frac{1}{2\pi}} + \frac{a}{2} - \frac{3}{\sqrt{8\pi}}a^2 + \dots$	$\sim -\frac{\varphi(a)}{a^2}$

for which the delay is either absent (case $a \ll -1$), or significantly exceeds the processing time of the part in the technological operation (case $a \gg 1$) or is comparable in order of magnitude to the standard deviation of the execution time of the operation.

The state in which the processing of parts occurs at adjacent operations, which are in states ($z_1 > 0, z_2 > 0$), is unlikely due to the smallness of the values of $r_{z1m} \ll 1$ and r_{z2m} . This allows using approximate expressions instead of exact expressions (42) to calculate the statistical characteristics of the delay Ω_m in the m -th technological operation:

$$E[\Omega_m] = r_{0m-1} \sum_{z=0}^6 r_{zm} E[\Omega_{m,0z}] \quad (45)$$

$$E[(\Omega_m - E[\Omega_m])^2] = r_{0m-1} \sum_{z=0}^6 r_{zm} E[\Omega_{m,0z}^2] - E[\Omega_m]^2,$$

which simplifies the recursive calculation of the delay in the last technological operation. Thus,

the processing time τ_d of a part from a batch of N parts on a production line with a sequential arrangement of technological operations:

$$\tau_d = \frac{\tau_{batch}}{N} = \frac{1}{N} \left(\sum_{m=1}^{M-1} \vartheta_{1,m} + \sum_{n=1}^N \vartheta_{n,M} \right) + \Omega_m. \quad (46)$$

The approximate model (46) is the basis for calculating the processing time τ_d of a part and, accordingly, can be used in a model for assessing production risks, replacing the analytical trajectory model (6) for processing a batch of parts on a linear production line and significantly reducing the computational complexity of the algorithm used in the model.

RESULTS

To analyze the accuracy of the approximation model (47) and the influence of the batch size on the accuracy of calculating the specific manufacturing time τ_{sp} of the final product, the technological

Table 4. List of technological operations and workstations in the production of single-leaf windows: mean processing time (min)

m	Name of the technological operation	Mean processing time, $\eta_{mean,z,m}$						
		z=0	z=1	z=2	z=3	z=4	z=5	z=6
E1	Cutting (WP: saw)	70	250	210	420	630	70	420
E2	Straightening (WP: PM)	60	120	120	360	540	60	360
E3	Planning (WP: PM – thicknesser)	70	120	140	420	630	70	420
E4	Profiling + cutting out (WP: BSMM)	163	340	640	700	1000	160	700
E5	Grinding (WP: wide belt grinder)	70	200	100	720	1000	70	720
E6	Assembling, drilling (WP: worktable)	140	360	700	680	950	140	680
E7	Impregnation and painting (WP: IPW)	120	130	500	600	900	120	600
Total		693						

Note: WP – work place; PM – planning machine; BSMM – bottom spindle milling machine; IPW – industrial painting workshop.

route containing seven consecutive technological operations was examined, see Table 4. During technological processing at the m -th technological operation, the part is in one of the z -th states during a random time interval. The time spent by the process equipment in the z -th state is characterized by the mathematical expectation $\eta_{mean\ z,m}$ and the standard deviation $\eta_{std\ z,m} = 0.1\eta_{mean\ z,m}$ for $z>0$ and $\eta_{std\ 0,m} = 0.2\eta_{mean\ 0,m}$ for $z=0$, the values of which are determined experimentally and correspond to the industry standards for the production of single-leaf windows. The probability that the equipment is in the z -th state is calculated based on the processing of experimental statistical information presented in Table 5.

Statistical data collection and parameter estimation

The parameter values used in this study are obtained based on statistical observations of the single-sash wooden window production process on an industrial production line over a period of 74 weeks. The experimental dataset consisted of the records describing the execution time of process operations, as well as the occurrence of production risk events related to 4M categories. The event recording was based on production logs, equipment downtime records, maintenance reports, and quality control documentation regularly collected during the production process. The probabilities of individual risk states were estimated based on the observed frequencies of occurrence, while the statistical characteristics of the operation durations were determined based on the corresponding experimental samples using standard estimates of the sample mean and standard deviation. The numerical values presented in Tables 4 and 5 represent the statistically obtained estimates used to demonstrate the application of

the proposed analytical model to a real production system. For practical applications, the model parameters should be determined based on the statistical production data collected for the specific production system under study. To test the proposed approach, a production batch of 60 wooden windows was analyzed. A comparative analysis of the experimental implementation of the production of a batch of 60 wooden windows was performed with the results obtained within the proposed model. Additionally, in this paper, the results obtained using the proposed analytical model were compared with those obtained using Monte Carlo simulation for batch of different sizes of wood windows.

Based on the introduced dimensionless parameters of the production line (3), the experimental data on the processing time during the manufacturing of a batch of wooden windows (Table 4) are presented in a dimensionless form in Table 6.

Figure 2 shows the distribution densities of part processing times for selected process operations. The graphs for operations E1 (cutting), E4 (profiling + cutting), and E6 (assembly and drilling) clearly demonstrate the impact of production risks (Table 5) on the stochastic execution time of each operation.

The distribution density $f_m(\vartheta_m)$ of the processing time ϑ_m of a part at the technological operation is a multimodal function. Each mode of this function corresponds to the probability of the technological equipment being in state z -th (Table 3).

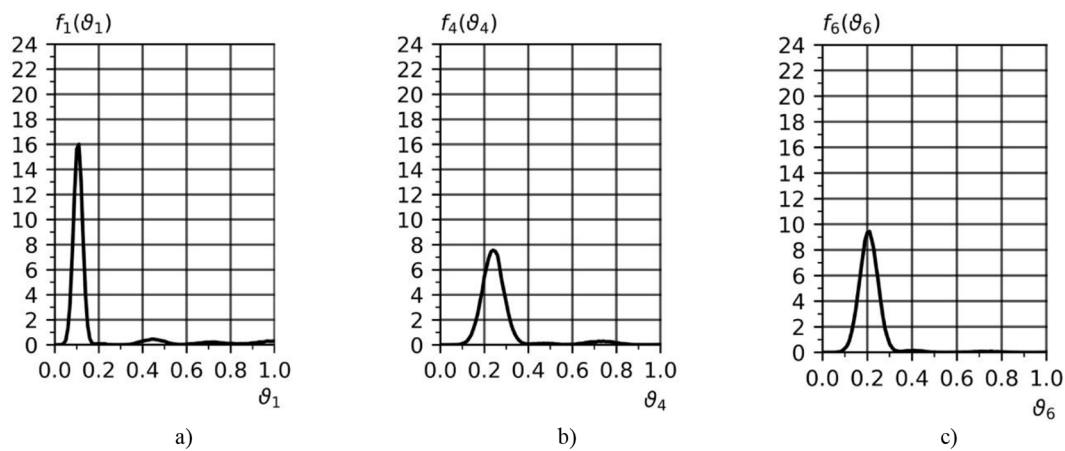
The graphs clearly illustrate that the main mode of operation of the production line for each technological operation is the standard (normative) state of the workplace ($z=0$) as indicated in Tables 1 and 5. The remaining modes of the distribution density $f_m(\vartheta_m)$ correspond

Table 5. The state probability, r_z

m	Name of the technological operation	state probability, r_z						
		z=0	z=1	z=2	z=3	z=4	z=5	z=6
E1	Cutting	0.834	0.036	0.020	0.024	0.072	0.006	0.008
E2	Straightening	0.963	0.002	0.002	-	-	0.009	0.024
E3	Planning	0.964	0.002	0.004	-	-	0.006	0.024
E4	Profiling + cutting out	0.896	0.048	0.025	-	-	0.015	0.016
E5	Grinding	0.903	0.025	0.042	-	-	0.006	0.024
E6	Assembling, drilling, removing finishing grinding	0.956	0.012	0.010	-	-	0.006	0.016
E7	Impregnation and painting	0.880	0.003	0.009	0.012	0.036	0.012	0.048

Table 6. Average dimensionless processing time

m	Name of the technological operation	average processing time, $\vartheta_{mean\ z,n}$						
		z=0	z=1	z=2	z=3	z=4	z=5	z=6
E1	Cutting	0.101	0.361	0.303	0.606	0.909	0.101	0.606
E2	Straightening	0.087	0.173	0.173	0.519	0.779	0.086	0.519
E3	Planning	0.101	0.173	0.202	0.606	0.909	0.101	0.606
E4	Profiling + cutting out	0.235	0.490	0.924	1.010	1.443	0.231	1.010
E5	Grinding	0.101	0.288	0.144	1.038	1.443	0.101	1.038
E6	Assembling, drilling, removing finishing grinding	0.202	0.519	1.010	0.981	1.371	0.202	0.981
E7	Impregnation and painting	0.173	0.188	0.721	0.865	1.298	0.173	0.866
Total:		1.000						


Figure 2. Density distribution $f_m(\vartheta_m)$ of the processing time ϑ_m . a) 1-th operation; b) 4-th operation; c) 6-th operation

to the production risks that lead to the failure to fulfill the production order on time. Using the approximate model (47), the statistical characteristics of the processing time of parts at the m -th technological operation were calculated. Figure 3 shows the statistical characteristics of the processing time of parts in the absence of production risks (when only the standard state $z=0$ is possible) for a batch of N parts on a line with a sequential arrangement of technological operations: black line - $N=1$; brown line - $N=2$; orange line - $N=4$; green line - $N=8$; blue line - $N=16$; purple line - $N=32$; red line - $N=64$. It is worth paying attention to a number of characteristic features of the curves shown in the graphs. The first feature is the absence of inter-operational delay in the technological operation $m=4$ (Figure 3a, d) with the maximum processing time of the parts $\vartheta_{mean\ 4} = \vartheta_{mean\ max}$ (Figure 3b, e). This behavior is expected when processing a batch of products and serves as confirmation

of the correctness of the developed approximate model. Indeed, this operation represents a bottleneck for the production line under study in the absence of production risks.

The second characteristic feature is that the total processing time $\vartheta_{mean\ sum\ m}$ of parts in subsequent technological operations after the bottleneck operation ($\vartheta_{mean\ 4} = \vartheta_{mean\ max}$) asymptotically approaches the maximum technological processing time $\vartheta_{mean\ max}$ for all operations:

$$\vartheta_{mean\ sum\ m} = \vartheta_{mean\ m} + \vartheta_{mean\ m} \rightarrow \vartheta_{mean\ max}. \quad (47)$$

As the batch size N increases, the value of the total processing time $\vartheta_{mean\ sum\ m}$ asymptotically tends to the maximum process time, which is clearly demonstrated in Figure 3c. The strict equality (47) is typical for a deterministic model described by the system of Equation 11, in which each part leaves the last operation of the production line after a constant time $\vartheta_{mean\ max}$ and serves as a second confirmation of the correctness of the

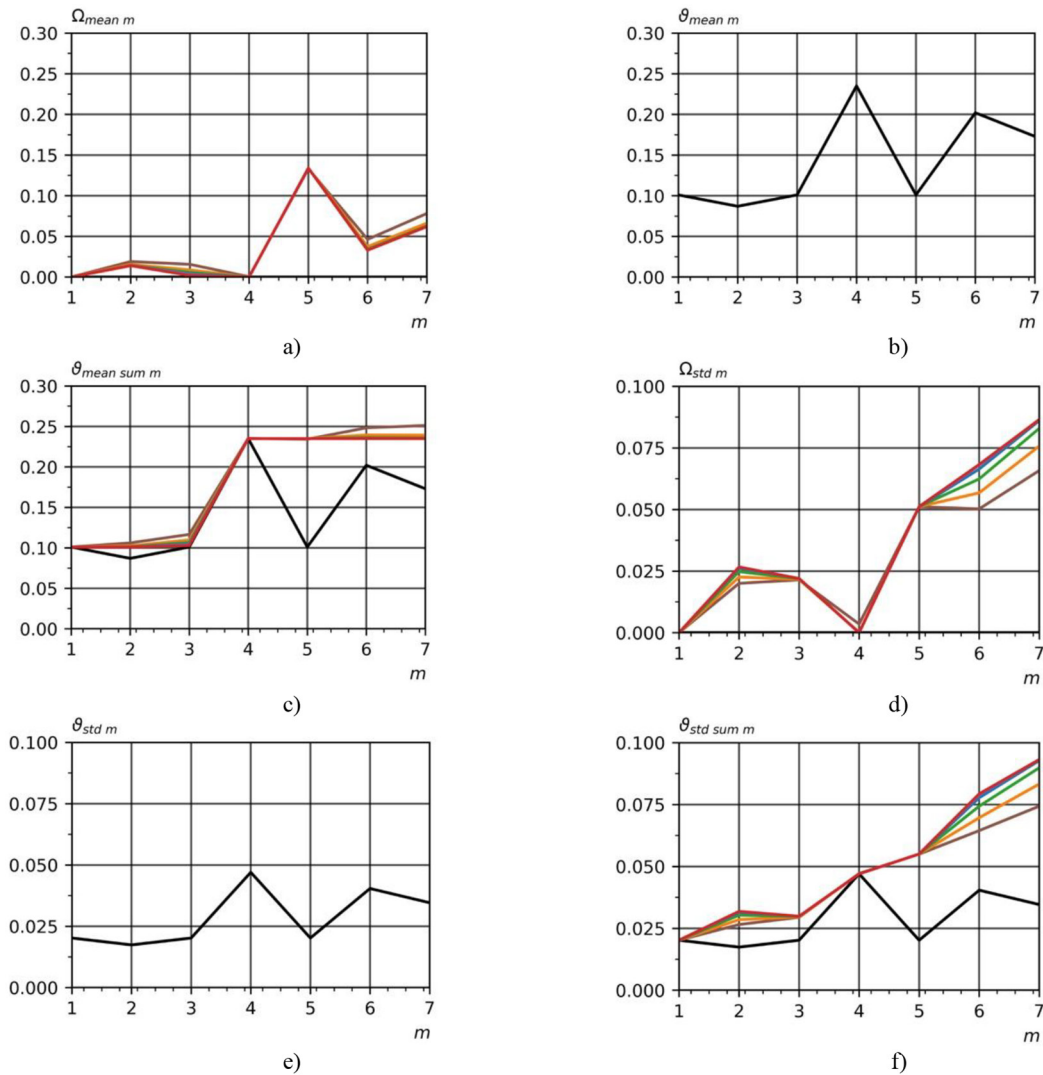


Figure 3. Statistical characteristics of the processing time of parts at the m -th technological operation in the absence of production risks depending on the size of the batch of parts (black: $N=1$; brown: $N=2$; orange: $N=4$; green: $N=8$; blue: $N=16$; purple: $N=32$; red: $N=64$). a) average delay per operation, $\Omega_{mean\ m}$; b) average operating time, $\vartheta_{mean\ m}$; c) average total processing time of a part per operation, $\vartheta_{mean\ sum\ m}$; d) standard deviation of delay per operation, $\Omega_{std\ m}$; e) standard deviation of operating time, $\vartheta_{std\ m}$; f) standard deviation the total processing time of a part per operation, $\vartheta_{std\ sum\ m}$

developed approximate model. Next, the statistical characteristics of the random variable were determined for operations $m > 4$, where a technological delay (downtime of technological equipment) was observed - a phenomenon that is not typical for ordinary non-synchronized production lines. The absence of inter-operational delay after the completion of the process operation $m=4$ (Figures 5a and 5d) with the maximum part processing time $\vartheta_{mean\ 4} = \vartheta_{mean\ max}$ (Figures 3b and 3d) is the expected behavior during part processing and thus confirms the validity of the developed approximate model. Indeed, the process operation becomes a bottleneck for the production

line in the absence of production risks. Figure 4 shows the statistical characteristics of the processing time of a batch of parts for an experimental production line for the manufacture of wooden windows with a sequential arrangement of technological operations (black line - $N=1$; brown line - $N=2$; orange line - $N=4$; green line - $N=8$; blue line - $N=16$; purple line - $N=32$; red line - $N=64$). An industrial monitoring system for the impact of industrial risks, as determined by the operational indicators given in Tables 4 and 5.

The last technological operation corresponds to the maximum value of the total processing time

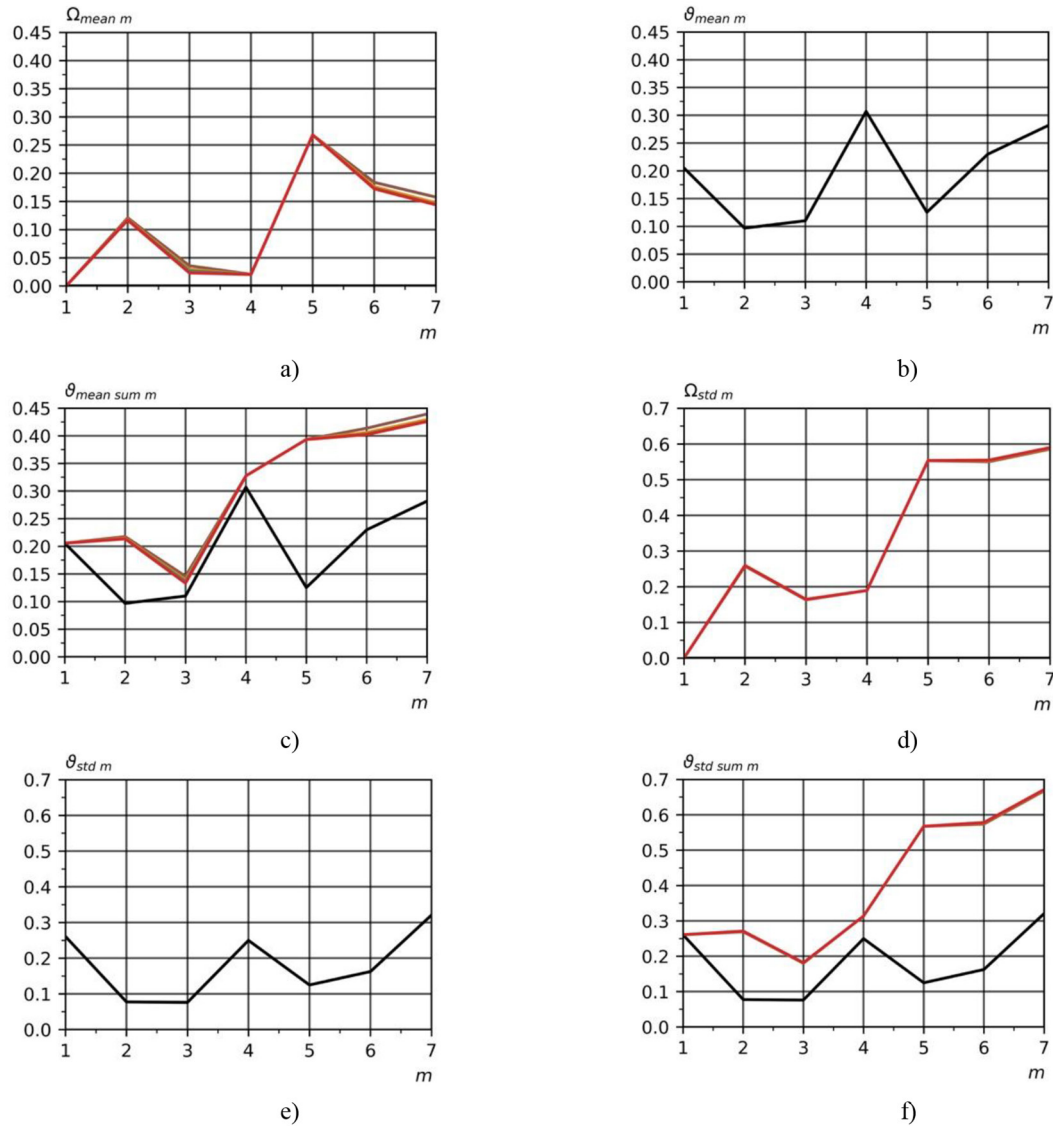


Figure 4. Statistical characteristics of the processing time of parts at the m -th technological operation in the presence of production risks depending on the size of the batch of parts (black: $N=1$; brown: $N=2$; orange: $N=4$; green: $N=8$; blue: $N=16$; purple: $N=32$; red: $N=64$). a) average delay per operation, $\Omega_{mean\ m}$; b) average operating time, $\theta_{mean\ m}$; c) average total processing time of a part per operation, $\theta_{mean\ sum\ m}$; d) standard deviation of delay per operation, $\Omega_{std\ m}$; e) standard deviation of operating time, $\theta_{std\ m}$; f) standard deviation the total processing time of a part per operation, $\theta_{std\ sum\ m}$

of the part $\theta_{mean\ sum\ m}$. At technological operation $m=4$, a technological delay is observed (Figure 4a) when processing batches of different sizes ($N = 1 \dots 64$), due to the presence of production risks. The standard deviation of the delay $\Omega_{std\ m}$ in the m -th technological operation is practically independent of the batch size of parts.

Figure 5 shows the dependencies of the statistical characteristics of the processing time τ_d of a part from a batch of N parts on a production line with a sequential arrangement of technological operations depending on the batch size in the absence of production risks: the black line is the

results of the Monte Carlo method; the red line is the results of the approximate model. Figures 5a and 5b show comparative curves for estimating the mathematical expectation and standard deviation of the processing time τ_d of a part from a batch of N parts. Figure 5c shows the relative error in calculating the mathematical expectation of the processing time $\tau_{d\ mean}$ of the parts using models based on the Monte Carlo method and using the proposed approximate model. For small-scale production, the relative error is less than 2% ($\Delta \tau_{d\ mean} / \tau_{d\ mean} < 0.02$), and for large-scale production, it is approximately 0.5%.

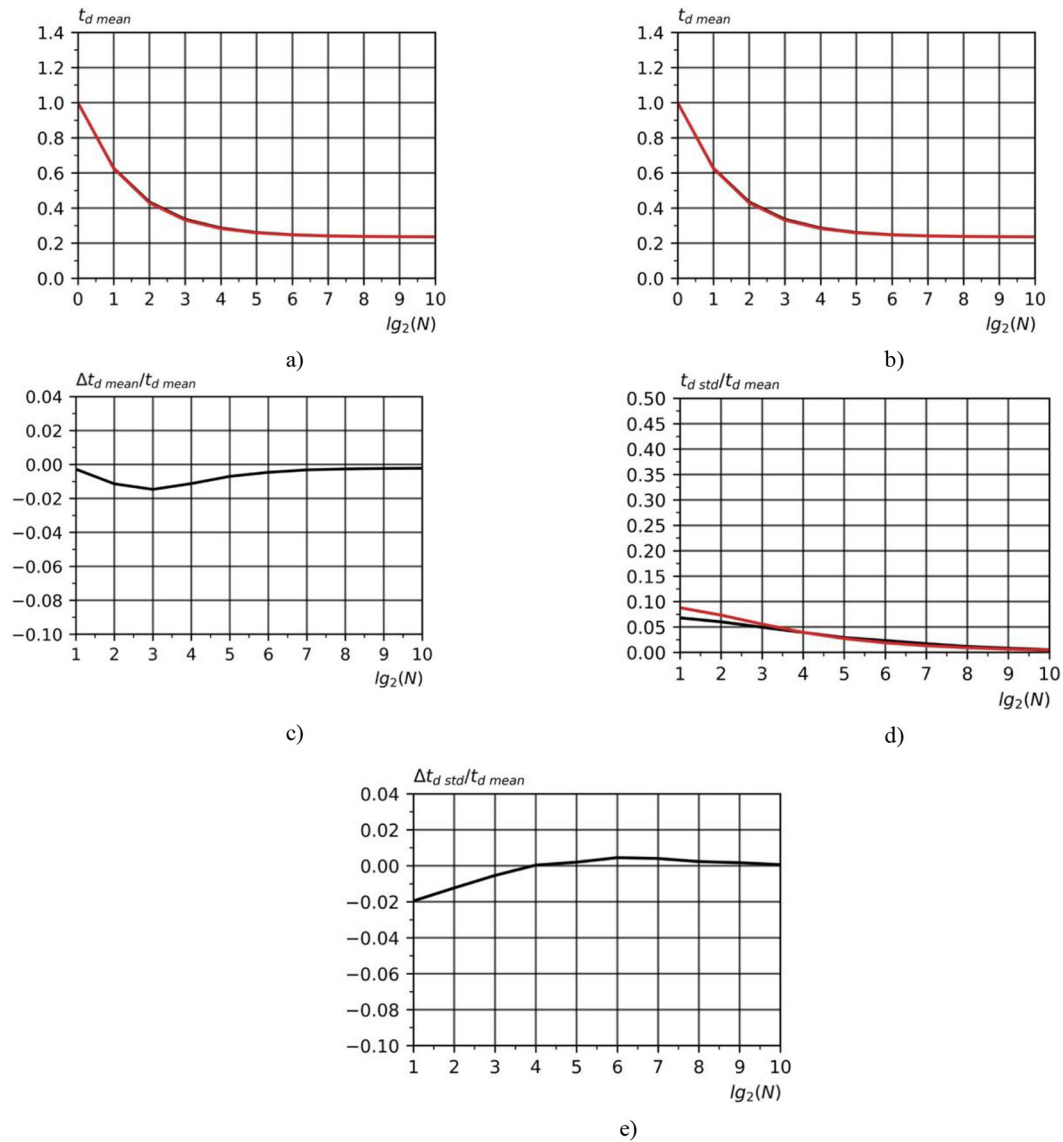


Figure 5. Statistical characteristics of the processing time of parts in the absence of production risks: black line – Monte Carlo method; red line – approximation model. a) mathematical expectation, $\tau_{d \text{ mean}}$; b) standard deviation, $\tau_{d \text{ std}}$; c) relative error, $\tau_{d \text{ mean}} / \tau_{d \text{ mean}}$; d) relative standard deviation, $\tau_{d \text{ std}} / \tau_{d \text{ mean}}$; e) relative error, $\tau_{d \text{ std}} / \tau_{d \text{ mean}}$

Figure 5d shows the ratio of the standard deviation $\tau_{d \text{ std}}$ to the mathematical expectation $\tau_{d \text{ mean}}$ of the processing time τ_d of a part from a batch of N parts, which corresponds to the coefficient of variation of this indicator. This graph allows estimating the relative error in calculating statistical characteristics using two approaches: the Monte Carlo method (reference model) and the approximate analytical model proposed in this paper. The absolute deviation $\Delta \tau_{d \text{ std}}$ between the calculated values of the standard deviation for the two models does not exceed 2% in the case of small-scale production ($\Delta \tau_{d \text{ mean}} / \tau_{d \text{ mean}} < 0.02$) and tends to zero when moving to large-scale batches of products ($\Delta \tau_{d \text{ mean}} / \tau_{d \text{ mean}} \rightarrow 0$), which confirms the

high accuracy of the approximation model with increasing batch size. The scale of the graphs in Figure 5d was chosen specifically for the convenience of direct comparative analysis with the results obtained in Figure 6 using the same two models, but in the presence of production risks.

Figures 5a and 6a show the curves of the mathematical expectation $\tau_{d \text{ mean}}$ of the processing time of a part from a batch of N parts on an enlarged scale, which allows for a direct comparison of the results in the absence and presence of production risks. The differences due to the influence of risks are most clearly seen in Figure 6f, which shows the additional bias in average processing time caused by risk conditions alone. As

the batch size increases, the effect of production risks on the mathematical expectation of processing time gradually weakens: the value $\tau_{d \text{ only risk mean}}$

decreases, and for large-scale and mass production it asymptotically approaches a constant level. This behavior significantly simplifies predicting

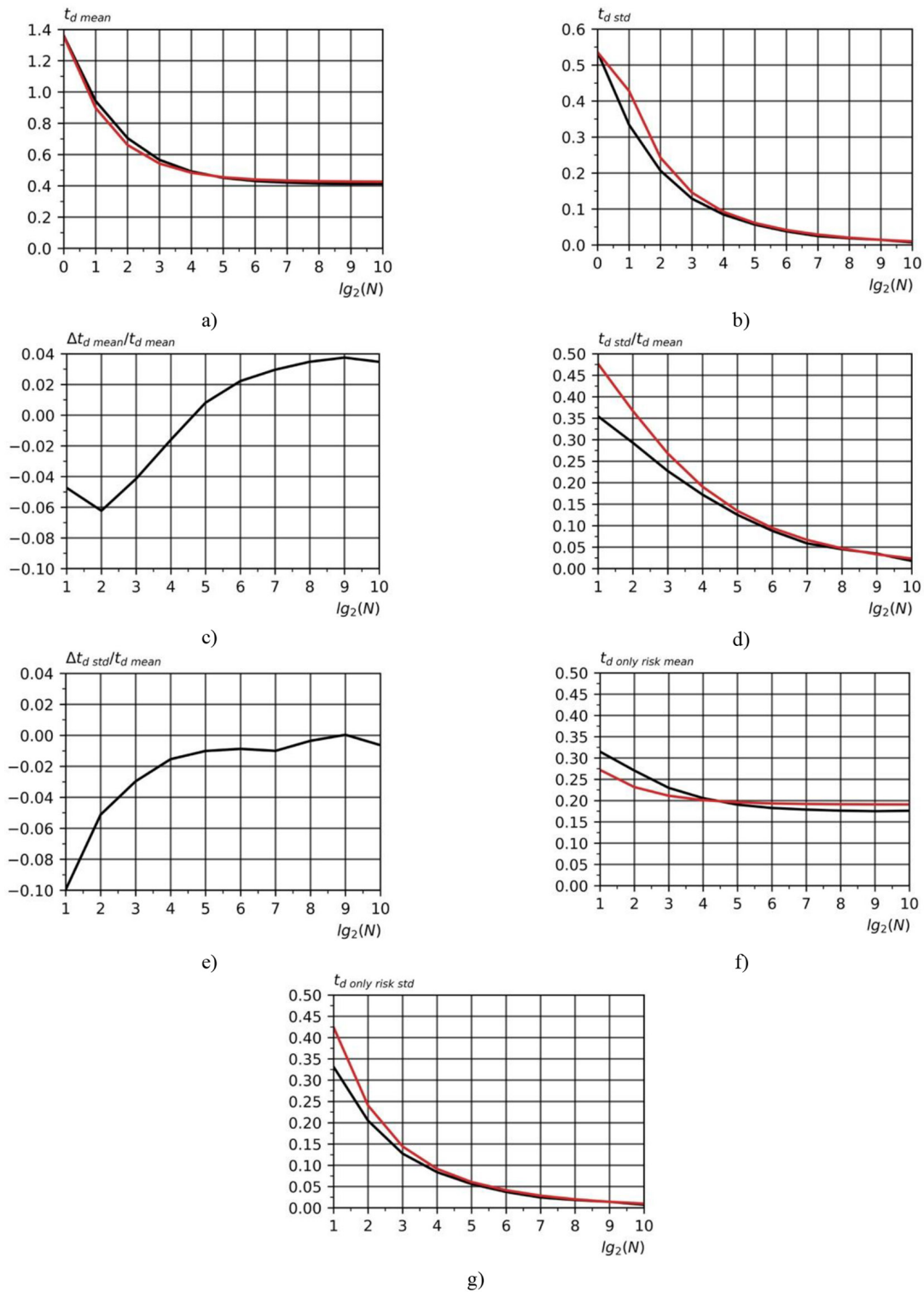


Figure 6. Statistical characteristics of the processing time of parts in the present of production risks: black line – Monte Carlo method; red line – approximation model. a) mathematical expectation, $\tau_{d \text{ mean}}$; b) standard deviation, $\tau_{d \text{ std}}$; c) relative error, $\Delta \tau_{d \text{ mean}} / \tau_{d \text{ mean}}$; d) relative standard deviation, $\tau_{d \text{ std}} / \tau_{d \text{ mean}}$; e) relative error, $\tau_{d \text{ std}} / \tau_{d \text{ mean}}$; f) risk-related mathematical expectation of operating, $\tau_{d \text{ only risk mean}}$; g) risk-related standard deviation of operating time, $\tau_{d \text{ only risk std}}$

the statistical characteristics of batch production times τ_d under real conditions. At the same time, the graphs clearly demonstrate the systematic error of the approximate model when calculating the mathematical expectation $\tau_{d\ mean}$. The error in calculating $\tau_{d\ mean}$ in the presence of production risks increases compared to the risk-free case, which is especially noticeable when comparing Figures 5c and 6c. This phenomenon is most pronounced for large-scale and mass production. However, as can be seen from the graphs, even under risk conditions, the relative error remains within the 5% range, which is quite acceptable accuracy for practical forecasting of the production time of a batch of parts in modern production systems. A comparative analysis of Figures 5b and 6b demonstrates the significant impact of production risks on the standard deviation of part processing time $\tau_{d\ std}$. The presence of risks leads to a significant increase in variance in both small-batch and large-batch production. The presence of risks leads to a significant increase in variance in both small-scale and large-scale production. As the batch size increases, the value of $\tau_{d\ std}$ tends to zero (Figure 6g), but production risks significantly increase this indicator, as confirmed by a comparison of Figures 5d and 6d. This increase in dispersion is due to the stochastic nature of the risks themselves, which introduce additional uncertainty into the processing process.

DISCUSSION

The results demonstrate that the proposed approximate model for calculating statistical parameters of the processing time τ_d of a part from a batch of N parts, taking into account production risks, provides a consistent and practically applicable framework for analyzing production systems with sequential process operations. This fundamental advantage distinguishes the model from the classical stochastic flowshop models, where risk is not interpreted as the probability of failure to fulfill the plan. These models, typically do not fully account for the probabilistic nature of the process, require significantly more computational resources, and produce less accurate results when estimating processing time variance in real-world production lines with 4M risk categories. A key finding of the study is the confirmation that production risks significantly impact not only the expected processing time but, to a greater extent,

its variability. Unlike the article [13], where only the expected execution time is minimized, the developed model provides analytical expressions for the variance that take into account 4M categories of risk. This underscores the importance of considering higher-order statistical characteristics when analyzing production systems under uncertainty.

From the perspective of risk analysis, this result is particularly important because the variability of batch completion time can be directly related to the probability of failure to meet planned production targets. In practical manufacturing systems, production risk is not only associated with the occurrence of individual disturbances, but also with their cumulative effect on process flow stability, delivery reliability, and the ability to complete a batch within the planned time horizon. Therefore, the proposed model provides a quantitative basis for assessing the operational consequences of risk factors, especially when the disturbances caused by equipment failures, material problems, human-related factors, or organizational deviations lead to increased makespan variability. This interpretation is consistent with the current approaches to reliability assessment and uncertainty quantification in manufacturing systems [25, 28, 29].

The applied modification of the FMEA method, using the normalized WJRPN index, enables a transition from qualitative expert assessment to a quantitative, probabilistic interpretation of risk. The novelty of the proposed approach lies in modifying the standardized WIRPN index to account for the stochastic nature of a part's occurrence in one of the risk states (standard or risk). This allows establishing a direct quantitative relationship between the probability of a part in a risk state and the probability of failure to meet the overall production plan. This probabilistic interpretation of risk, integrated directly into the analytical stochastic model of the production line, fundamentally distinguishes this work from all previously published studies on stochastic planning and is absent from the classical models of expected waiting time minimization. In this formulation, risk can be interpreted as the probability of failure to meet planned production goals, allowing for the direct integration of the model into reliability analysis and production planning systems. This distinguishes this approach from the classical RPN-based methods, which lack normalization and probabilistic meaning. An

additional advantage of the proposed approach is that it extends the role of FMEA from a static ranking tool to a component of a stochastic risk assessment model. In classical FMEA, the identified risk factors are usually prioritized, but their influence on batch completion time, inter-operational delays, and production plan reliability is not explicitly quantified. In the proposed model, the normalized WJRPN values relate to the probability of risk states and with the statistical characteristics of operation times. As a result, the contribution of each 4M category to the expected value and dispersion of production time can be analyzed. This makes it possible to identify not only the most severe risks, but also those risks that have the greatest influence on the uncertainty and instability of the production process.

From a practical perspective, the developed model makes it possible to identify the technological operations that contribute most significantly to the overall production risk. This creates a basis for targeted managerial decisions aimed at reducing variability and improving system reliability, for example through equipment modernization, workforce stabilization, or improvement of material supply processes. The proposed model may also support risk-oriented decision-making in production planning. For example, if a given technological operation significantly increases the variance of batch completion time, this operation may be treated as a critical point of the production route, even if its average processing time is not the highest. Such information can support preventive maintenance planning, improvement of material supply reliability, better allocation of operators, or modification of technological procedures. Thus, the proposed model can be used not only for estimating batch processing time, but also for prioritizing the risk mitigation actions in manufacturing lines.

The proposed analytical model is intended for production systems in which technological operations are performed in a predetermined sequence, and the occurrence of production risks is represented by the stochastic events affecting the processing time of technological operations. Based on these assumptions, the model is applicable to a wide range of batch production systems characterized by clearly defined technological processes. Since the mathematical formulation of the proposed model depends only on the probabilistic characteristics of process operations and associated production risks, the proposed approach can be

adapted to various manufacturing industries. For practical implementation, based on production data, it is necessary to evaluate only the statistical characteristics of operation durations and equipment downtime associated with a specific production risk. The necessary model parameters can be obtained from manufacturing execution systems (MES), enterprise resource planning (ERP) systems, equipment monitoring systems, and historical production databases. Production logs, maintenance records, quality control reports, and equipment diagnostics provide sufficient statistical information to estimate processing times, identify failures, and evaluate the probability distributions of production delays. This allows the model to be integrated into existing decision support systems without requiring significant changes to the manufacturing information system architecture. Compared with the existing methodologies for assessing production risks and analyzing uncertainty, the proposed analytical approximation occupies an intermediate position between qualitative engineering methods and computationally expensive stochastic modeling methods. Unlike classical failure mode and effects analysis (FMEA) [5], which provides a systematic approach for identifying and prioritizing potential failure modes using qualitative risk metrics, the proposed approach quantifies the probability distribution and statistical characteristics of the overall production completion time, enabling probabilistic assessment and optimization of production plans under stochastic production risks. Fault tree analysis (FTA) and event tree analysis (ETA) focus on modeling logical failure paths and accident propagation scenarios in manufacturing systems and are well suited for assessing equipment reliability and safety [51]. However, these methods are significantly less suitable for predicting the distribution of production order lead times and production schedule variability under stochastic disturbances. At the same time, the proposed analytical approach allows for a quantitative assessment of the cumulative probabilistic impact of 4M risks on batch completion times. Models based on queuing theory represent another important class of analytical methods widely used in manufacturing systems. In modern research, where queuing theory is integrated with Markov chains [52], these approaches provide useful estimates of throughput, equipment utilization, bottleneck behavior, and waiting times, primarily under steady-state operating conditions. However, such models are primarily focused on

analyzing average system characteristics and do not consider constraints, such as those associated with the availability of interoperational bunnies of a given capacity. In contrast, the proposed approach quantifies the cumulative probabilistic impact of 4M risks on production planning, taking these constraints into account. Discrete event simulation (DES) and Petri net-based models provide a high-level description of the parallel execution of process operations, resource allocation, synchronization constraints, and dynamic production behavior. However, their high modeling flexibility results in model complexity, labor intensity, and computational costs, especially when multiple evaluations of a large number of production processes are required during planning and optimization [53]. The Monte Carlo method remains one of the most versatile approaches to analyzing stochastic production systems and is a powerful tool for risk assessment. However, obtaining reliable estimates requires a very large number of simulation iterations, resulting in significant computational costs, limiting its use in optimization problems and real-time decision support systems. In contrast, the proposed approach reduces computational complexity while maintaining acceptable accuracy. Thus, the proposed analytical approximation combines several complementary advantages. It preserves an explicit probabilistic interpretation of production risks, directly estimates the statistical characteristics of the overall production completion time without repetitive modeling, requires only a limited number of statistically significant input parameters, and demonstrates computational efficiency suitable for production planning and operational decision support. The proposed model is considered a complementary analytical tool, occupying an intermediate position between qualitative engineering risk assessment and computationally expensive stochastic modeling. It provides an effective compromise between computational efficiency, modeling accuracy, and practical applicability for risk-based production planning. The above comparison should be interpreted with respect to the objectives and application domains of the considered methodologies. The proposed model addresses an important engineering challenge by providing an analytical link between production risk assessment and probabilistic forecasting of production completion times. The comparative analysis presented in this study demonstrates that, for the manufacturing system under consideration, the proposed approximation

reproduces the probability distribution and statistical characteristics of production completion times with close agreement with Monte Carlo simulation, while requiring significantly lower computational costs. At the same time, unlike the RPN-based methods, the primary goal of which is qualitative or semi-quantitative prioritization of production risks, the proposed analytical framework enables direct quantitative assessment of production completion time distributions. The proposed model's contribution should be viewed as a complement to existing methodologies, providing an effective method for risk-based production planning, rather than as a replacement for established modeling or engineering risk assessment methods.

At the same time, the proposed approximate model has several limitations. The current formulation assumes that risk events affect processing time only through additional delays and does not account for clearly irreversible defects that lead to part rejection. Furthermore, using a normal approximation for total delays can lead to inaccuracies in the presence of highly skewed or heavy-tailed distributions. Further research could focus on expanding the model to account for defect formation and absorbing states, incorporating more complex probability distributions, and using real-world production data to refine the model parameters. Another promising direction is integrating the proposed approach with the methods for modeling and adaptive control of real-time manufacturing.

The impact of the adopted modeling assumptions on the obtained results should be interpreted in light of the intended scope of the proposed analytical model. The assumption of a normal distribution of manufacturing delays is appropriate for the manufacturing processes in which the cumulative impact of numerous independent random factors dominates the observed variability. However, for the manufacturing environments characterized by highly asymmetric, multimodal, or "heavy-tailed" delay distributions, the accuracy of the approximation may decrease, and alternative probability distributions may provide a more adequate representation of manufacturing delays. Similarly, the exclusion of irreversible defects, failures occurring during process operations, and absorbing states implies that the proposed model is primarily intended for the manufacturing systems in which production failures primarily result in temporary delays rather than permanent interruptions to the production process. Therefore, the

analytical model estimates the impact of stochastic production delays on production completion time, but does not explicitly account for permanent product defects, catastrophic equipment failures requiring process shutdown, or other irreversible production conditions. A thorough quantitative assessment of the individual impact of each modeling assumption on the analytical approximation is a separate research problem beyond the scope of this study. Instead, this paper evaluates their combined impact through a comparative analysis of the proposed analytical model and Monte Carlo simulation. The close agreement between the obtained statistical characteristics demonstrates that, for the manufacturing system under consideration, the adopted simplifying assumptions introduce only limited approximation errors while providing a significant reduction in computational complexity. At the same time, manufacturing environments with high defect rates, catastrophic equipment failures, absorbing states, or significantly non-Gaussian stochastic behavior may require further extension of the proposed analytical framework.

Although the validation presented in this study demonstrates good agreement between the proposed analytical approximation and Monte Carlo simulation for the considered production scenario, the model's robustness to changes in production risk parameters has not been thoroughly investigated. Specifically, a systematic sensitivity analysis to changes in the probabilities of production risk states and the statistical characteristics of work delays would provide additional information on the stability of the proposed approximation and its applicability to various production conditions. Such an analysis is a logical extension of the present study and will be the subject of future work aimed at extending the proposed analytical model to a wider range of production conditions.

CONCLUSIONS

This study proposed a stochastic modeling approach for analyzing production risks in manufacturing systems with sequential process operations. The developed approximate model is based on a probabilistic interpretation of risk and integrates a modified FMEA method using the normalized WJRPN index, allowing risks to be considered as probabilities in the interval [0,

1]. Analytical expressions were obtained for key statistical characteristics of the manufacturing process, including the expected value and standard deviation of inter-operational delays. Based on these results, an approximate analytical model was developed for estimating the processing time of a part in a batch. The model significantly reduces computational complexity compared to a stochastic model based on trajectories while maintaining acceptable accuracy. The results of numerical experiments confirmed the adequacy of the proposed approach. The relative error of the approximate model does not exceed 5%, which is sufficient for practical application in production planning and forecasting. It was shown that production risks have a limited impact on the expected processing time, especially in small-scale production. A key contribution of the study is the demonstration that the normalized WJRPN index enables a transition from qualitative expert-based risk assessment to a quantitative probabilistic framework. This allows direct integration of risk evaluation into reliability analysis and production planning systems, as well as the identification of technological operations that contribute most significantly to overall production risk. From a practical perspective, the proposed model can be used to support decision-making aimed at increasing the reliability of a production system and optimizing process parameters. Specifically, it provides a basis for prioritizing risk mitigation measures within process operations.

The obtained results also show that production risk should be evaluated not only through the expected value of processing time, but also through its variability. In real manufacturing systems, an increase in the standard deviation of batch completion time may significantly reduce the reliability of production plans, even when the average processing time remains acceptable. Therefore, the proposed model enables a more complete interpretation of production risk, including the probability of delay, instability of material flow, and uncertainty related to the realization of 4M risk factors. This strengthens the practical value of the model as a tool for risk-aware production planning and reliability-oriented decision support. The proposed approach may be particularly useful for identifying the operations and risk categories that have the greatest influence on makespan variability and the probability of failing to meet the planned production time. In this sense, the model supports the

transition from descriptive risk identification to quantitative risk assessment. It can help production managers select appropriate preventive actions, such as reducing equipment downtime, improving material supply stability, limiting human-related disruptions, or improving technological procedures. As a result, the model can be applied not only to predict batch completion time, but also to support risk mitigation strategies in manufacturing systems.

Future research should focus on extending the model to analyze the effects of inter-operation cell size, defect formation, and part rejection, and to include more complex probability distributions of the part processing time statistics for each process operation for a sequential production line. Further research should also include broader risk-related extensions of the proposed model. It would be valuable to consider machine failures occurring during operation execution, stochastic processing time disturbances, material shortages, operator unavailability, rework loops, and dynamic propagation of delays between workstations. Another important direction is the development of risk indicators, such as the probability of exceeding the planned batch completion time, expected lateness, sensitivity of makespan to individual 4M factors, and confidence intervals for production completion time. These extensions would make it possible to integrate the proposed model with manufacturing execution systems, digital twins, and real-time production monitoring tools, thereby supporting adaptive and risk-aware decision-making in modern manufacturing environments.

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