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Maximum material requirement visualisation, which illustrates the roots of this concept and encourages the use of maximum material requirement

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ABSTRACT

This article emphasises that effectively teaching the application of geometrical tolerances poses a significant challenge for university lecturers and vocational training staff. The concise review of modern tools designed to promote the principles and rules of geometrical tolerances, thereby facilitating their successful implementation, is presented. A new series of recently developed animations is presented, aiming to show the roots of the maximum material requirement (MMR) concept, which enables functional tolerancing that ensures assemblability. It is demonstrated that MMR is the specification tool that introduces an interdependency between dimensions and geometry. This allows mating parts with exceeded geometrical tolerances for their derived features, when the actual sizes of the features that have to be mated do not reach their maximum material sizes. Developed dynamic animations significantly improved the effectiveness of geometrical tolerancing lectures at the university and vocational education in industry. The establishment and movement of the tolerance zones and datum features, as well as the visualisations of the actual toleranced features and datums enable the clear and effective communication of complex geometrical tolerancing rules in a form readily understood by students.

Keywords: geometrical tolerancing, geometrical deviations, maximum material requirement, MMR, tolerancing by zone, tolerancing by gauge, ISO 1101, ISO 2692.

INTRODUCTION

The basic problem in any measurement is the correct and unambiguous definition of the measurand, i.e., "the quantity intended to be measured" [1, 2]. The size deviations and the geometrical deviations of components are the measurands and key factors that determine the possibility of assembly and the proper functioning of all products and machines. The Technical Committee ISO/TC 213 "Dimensional and geometrical product specifications and verification" aims to develop standards that enable the industry to specify measurands uniquely and verify mechanical components most effectively [3]. According to the Definitive drawing principle: "The drawing is definitive. All specifications shall be indicated on the drawing using geometrical specification symbology (with or without specification modifiers), associated default rules or special rules and references to related documentation, e.g. regional, national or company standards. Consequently, requirements not specified on the drawing cannot be enforced" [4].

It is underlined in [5] that in the new generation of the ISO Geometrical Product Specification (GPS) standards, the uncertainty concept includes: total uncertainty, correlation uncertainty, conformance uncertainty, specification uncertainty, measurement uncertainty, method uncertainty and implementation uncertainty. This paper presents the latest developments in the

Application Geometrical Tolerancing. The tool, which is being implemented at universities and in industry to disseminate knowledge about the provisions of the ISO GPS system standards. Effective training in the ISO GPS system concepts, principles, rules and indications can significantly reduce the specification uncertainty in technical product documentation.

Let us revisit the Definitive drawing principle. The implications of this principle are significant, particularly in light of surveys conducted by the co-author [6] and other researchers in Germany and Italy [7, 8]. The drawing is a part of the contract that describes the part to be delivered. The lack of indication of geometrical tolerances or misuse of indications in technical product documentation can result in customer-supplier misunderstandings, superfluous disputes, delivery delays, and significant financial losses. All referred surveys showed that practical understanding of visible annotations and default geometrical tolerancing rules in technical product documentation remains insufficient across the industry. Several underlying causes of this deficiency were identified, including shortcomings in university education and insufficient professional training in industry. This motivated us to prepare the Application Geometrical Tolerancing, which contains animations designed to demonstrate rules for complete specification annotations and clarify the meaning of individual tolerance indications. The majority of the presented cases are enriched through animations illustrating how deviations are assessed for specific tolerances. The first animations were created a few years ago [9]. More than one hundred animations for various geometrical tolerance cases have been created so far to demonstrate the use of specific annotations. The Application Geometrical Tolerancing has been well received both at universities and in the industry. In the beginning, the fully interactive animations were developed using Adobe Flash; however, since the end of 2021, Adobe Inc. has ceased support for Flash Player, so we had to switch to the markup language HTML5. The homogenous graphical interface has been retained in the Application Geometrical Tolerancing. This facilitates user navigation across application windows and enables focus on the windows' technical content. Users can quickly familiarise themselves with the application's interface and then focus on the rules governing a particular tolerance specification and the meaning of each specification.

Many scientists and industry experts point out the need for more effective teaching of geometrical tolerances to eliminate or at least significantly reduce the number of technical drawings with tolerances that raise doubt. The need to improve the educational offer of the ISO GPS system and to develop more effective utilisation practices is highlighted in [8]. This may influence future strategies for the development of the ISO GPS standards. The concept of creating a digital teaching and learning platform to teach the ISO GPS principles and rules [3, 4, 10] with the support of a virtual tutor is presented in [11]. The flipped learning method combining video training materials with face-to-face training for the ISO GPS education is proposed in [12]. In the flipped learning method, the learners acquire knowledge at home and deepen their understanding of the ISO GPS system through face-to-face training. We share the following opinion: "the latest version of GPS empowers an engineer to describe requirements precisely and unambiguously; however, it also considerably increases the difficulty of the ISO GPS education", expressed in [13]. Other attempts at development and implementation of computer-based teaching of dimensional metrology are also reported [14].

The primary objective of this paper is to demonstrate a novel educational methodology for explaining the roots and functional meaning of maximum material requirement (MMR) [15]. This methodology allows the transformation of the ISO GPS system principles, generally formulated in the standards, into dynamic visualisations, enabling intuitive understanding of the functionality and benefits of the MMR specifications. It takes into account the interdependencies between size deviations and geometrical deviations. An innovative educational concept is presented below, focusing specifically on demonstrating the transition from "tolerancing by zone" to "tolerancing by gauge". This educational approach introduces students (or engineers in training) to geometrical tolerancing not as an abstract activity but as a functional decision-making process linked to verification. It is clearly shown that MMR enables functional tolerancing that guarantees assemblability while simultaneously decreasing parts accuracy requirements, which is economically beneficial.

The originality of the presented work is particularly visible in the newly developed animation sequence that explains the origins and functional interpretation of the MMR concept. The developed animations do not merely present tolerance indications, but dynamically demonstrate the mechanism by which assemblability depends on the combined effect of geometrical and dimensional deviations.

THE ROOTS AND ADVANTAGES OF MMR

In functional tolerancing, maximum or least material specifications should be used systematically for all connections because they define strictly necessary requirements; however, in practice, many designers remain insufficiently familiar with these specifications [16]. The authors of this paper completely agree with such a diagnosis. The industrial reality regarding the application and understanding of the MMR indicated in technical product documentation is quite often obscure. This encouraged us to develop a series of animations that show the roots of MMR, demonstrate equations that should be used to calculate maximum material virtual size (MMVS) and are able to highlight the benefits of MMR specifications. MMR can be regarded as a revolutionary approach in geometrical tolerancing. The tolerancing of derived feature (derived axis, derived median plane), commonly referred to as tolerancing by zone, is substituted by functional tolerancing (tolerancing by gauge) that is specified by the application of MMR modifier.

To maintain logical and graphical consistency in the Application Geometrical Tolerancing, the layout of the windows used for MMR is consistent with the windows used in previously developed animations [9]. However, the messages displayed in the windows and the content of the newly developed animations designed to illustrate the roots and benefits of MMR are completely different. The initial Definition Window of the new series of animations that aims to explain the concept of MMR is shown in Figure 1. The position tolerance for the hole with MMR with respect to the datum system established by three planes is considered. The MMR modifier is specified after the tolerance value in the second compartment of the tolerance indicator. In this window, in the caption below the part drawing, it is underlined that MMR provides functional and economical assembling of products. Next, it is pointed out that MMR combines two independent requirements – the size tolerance of the external or internal feature of size and the geometrical tolerance of its derived feature (axis, symmetry plane) into one combined requirement. Thus, in the Definition window (Figure 1), the purpose of

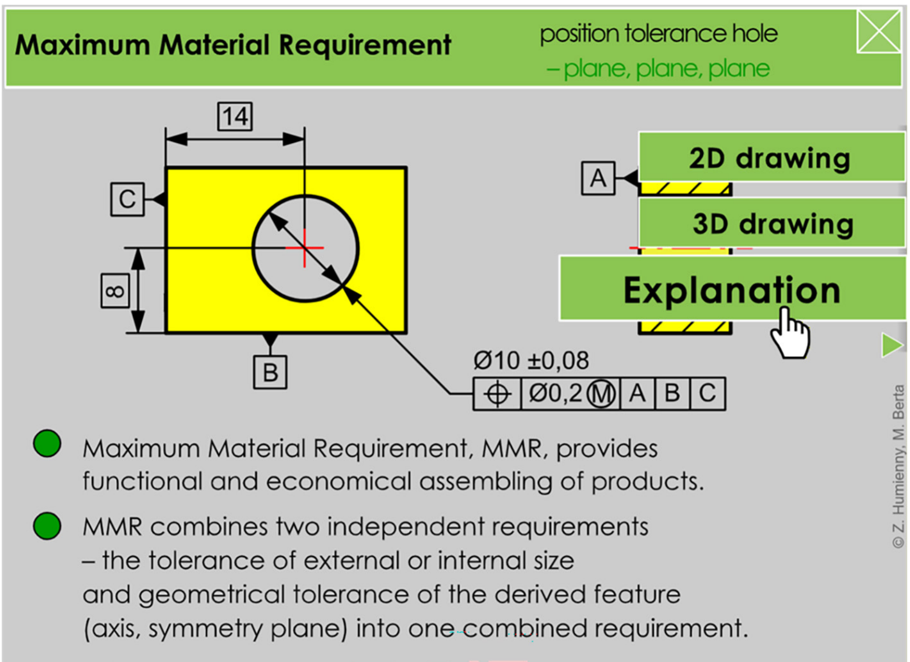

Figure 1. Definition window – the first window of the new module of the application geometrical tolerancing that is devoted to the presentation of MMR concept and its roots. The Explanation button is activated by the hand cursor

using MMR is emphasised, and the concept to achieve this purpose is formulated.

The objective of the next windows in this new series of animations is to show the source of the idea behind introducing MMR as the geometrical product specification tool. The Explanation Window (Figure 2) is released by the Click Event on the Explanation button. The Explanation button (Figure 1) is accessible in the drop-down menu available as the expanded menu activated while the mouse cursor is dragged and held over the small triangle available in the middle of the right edge of the Definition Window. In the top left corner of the first scene of the Explanation Window, the position tolerance indicator with crossed through encircled M is shown (Figure 2). This means that the discussion in this new animation series will start with the specification of the position tolerance for the hole axis without any modifier with respect to the datum system established by three planes (Figure 3). It is assumed that the user of the Application Geometrical Tolerancing is already familiar with the position tolerance specification and its interpretation that have been discussed earlier. The functional requirement is that the plate with the hole shall be located on the plate with the pin, with full contact of the large base surfaces and smooth transition between their edges (without a step).

The command Show the manufactured plate displayed in the Prompt line in the Explanation Window 1st scene (Figure 2) informs that the actual plate with imperfect sides will be shown in the next scene. This scene is triggered by the Click event on the Forward button located in the bottom right corner of each Explanation Window. The caption in the Explanation window 2nd scene (not shown in the paper) clarifies how the datum coordinate system, composed of three planes, is established for the position tolerance of the hole axis. The next Click event on the Forward button produces the Explanation window 3rd scene (Figure 4) with the plate located with respect to the datum system A|B|C. Two commands, Show the tolerance zone for the axis and Show the hole of 9.92 diameter displayed in the Prompt line in the Explanation Window 3rd scene (Figure 4), announce that in the Explanation Window 4th scene (Figure 5) the cylindrical tolerance zone for the hole axis will be visualised. According to the Independency principle [4], the diameter of the tolerance zone is constant regardless of the actual hole size.

Another Click event on the Forward button opens the next scene of the Explanation window (Figure 6), which aims to show possible positions of the hole with a diameter of 9.92 mm. The visualisation in the 5th scene of the Explanation window is dynamic, which is a great advantage for the Application Geometrical Tolerancing. It enables students to discover that there is a space for a pin with a diameter of 9.72 mm inside the hole with a diameter equal to the Maximum Material Size, MMS = 9.92 mm, and the position deviation varies from 0 to 0.2 mm. Unfortunately, it is

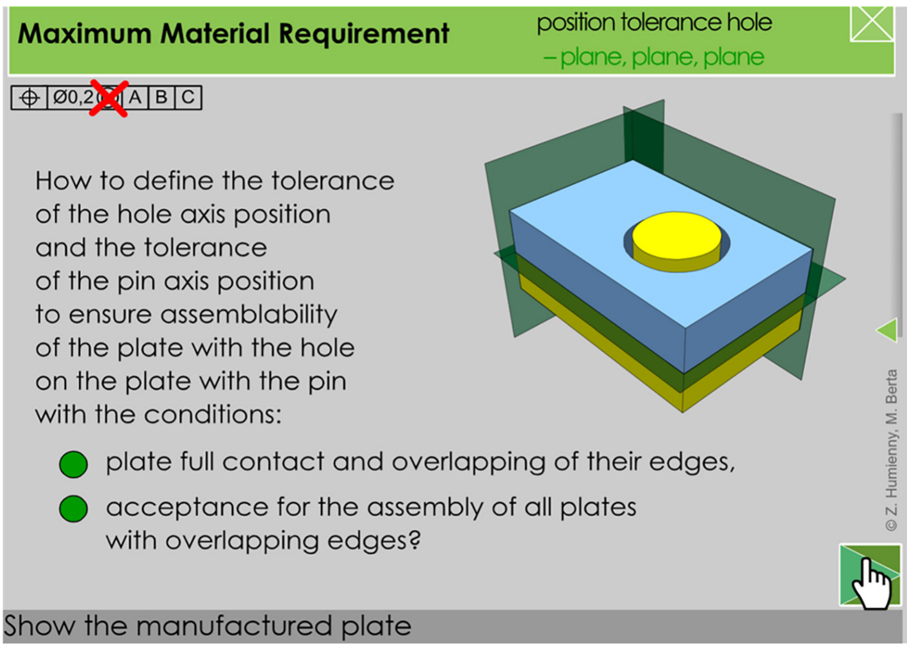

Figure 2. Explanation window – the 1st scene. The requirements for the assembly of the plates are defined

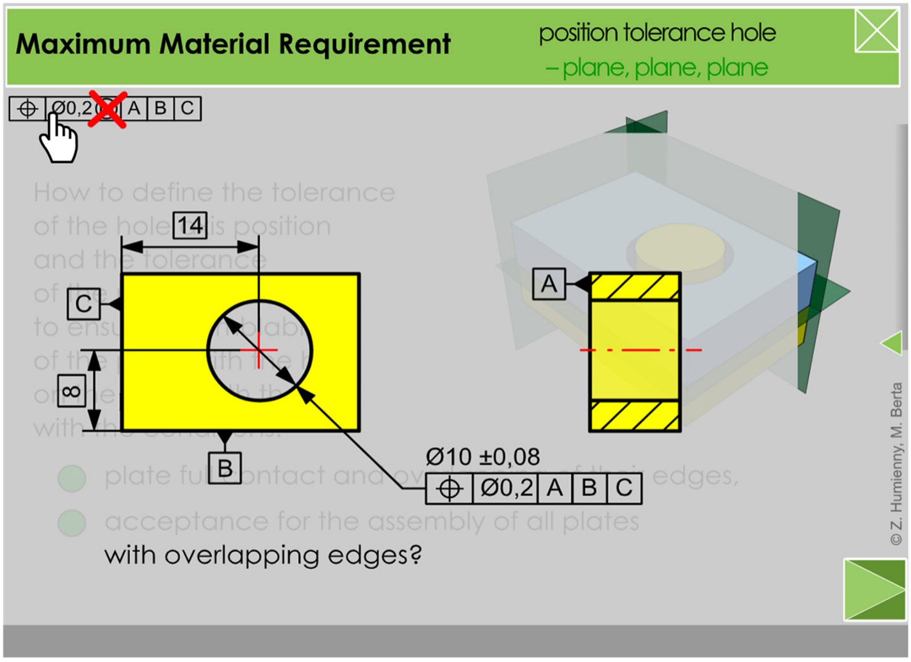

Figure 3. Explanation window – the expanded 1st scene. This specification is displayed (pulled down) while the mouse cursor is dragged and held over the tolerance indicator shown in the top left corner of the Explanation window. Please note that the position tolerance is specified without a modifier

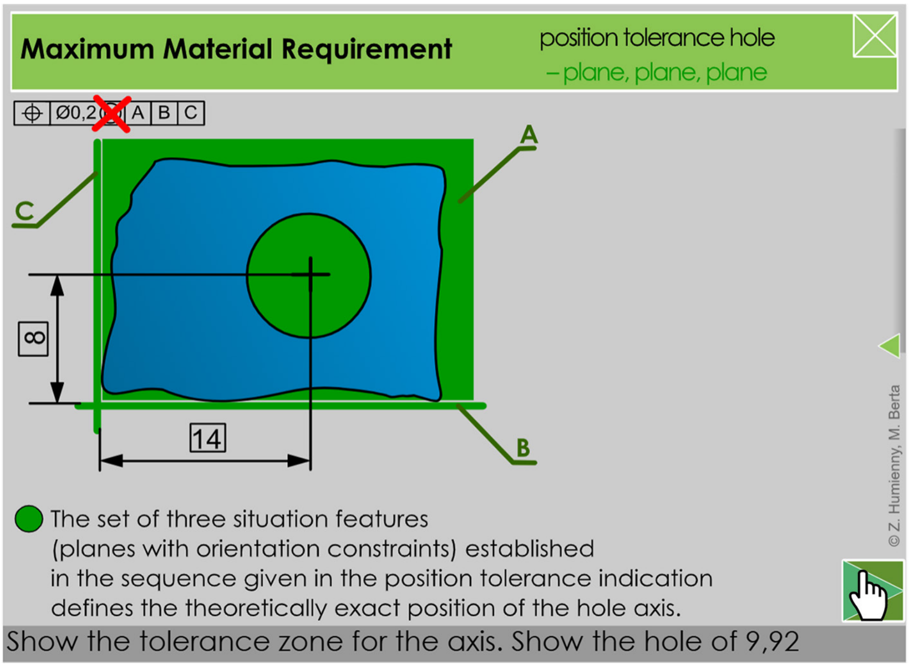

Figure 4. Explanation window – the 3rd scene. The theoretically exact position of the hole axis is shown for the actual plate in the coordinate system based on the datum features

impossible to show the movement of the 9.92 mm diameter hole in the static drawing included in this paper.

The Click event on the Forward button in the Explanation Window 5th scene displays the 6th scene (skipped in the paper) with the hole of diameter 10.08 mm as indicated in the Prompt line. The next Click event on the Forward button releases the 7th scene in the Explanation Window (Figure 7). The visualisation in the 7th scene of the Explanation window is also dynamic. It can be observed that the point representing the actual

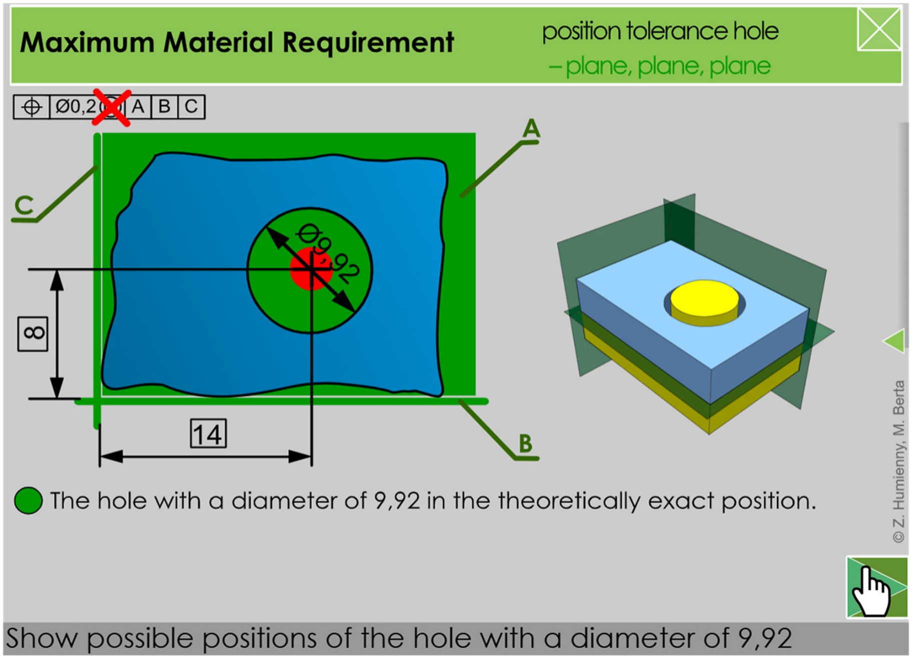

Figure 5. Explanation window – the 4th scene. The cylindrical tolerance zone in which the actual axis of the toleranced hole shall be contained is shown

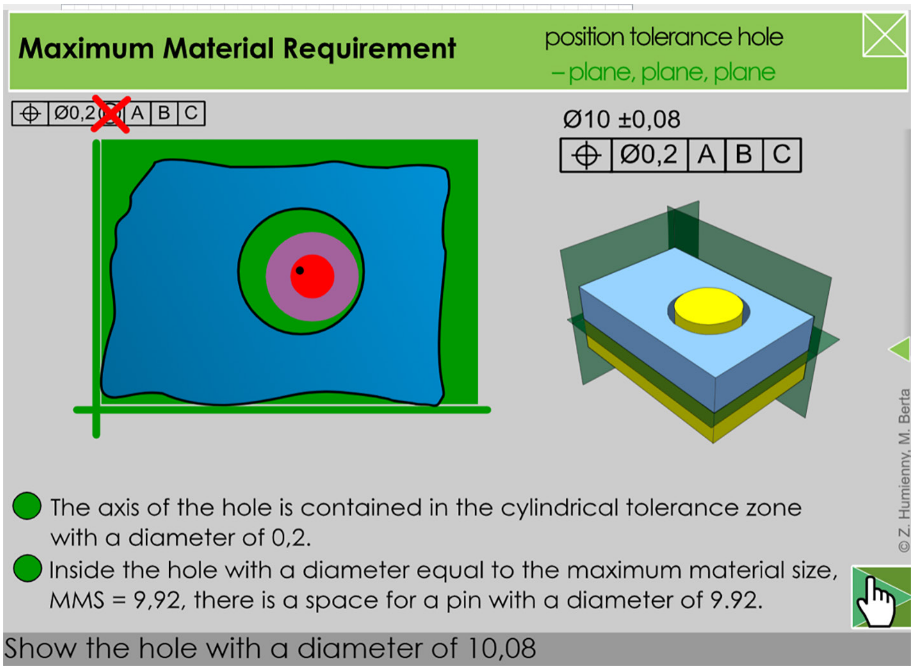

Figure 6. Explanation window – the 5th scene. One of an infinite number of positions of the 9.92 mm hole is shown in this screenshot. Due to the allowed position deviation up to 0.2 mm, the hole surface does not violate the boundary with a diameter of 9.72 mm

position of the toleranced hole axis is quite frequently outside the 0.2 mm diameter tolerance zone, which means that the plate does not fulfil the specified requirement (element NOK). However, the boundary with the diameter of 9.72 mm is still not violated. This implies that although according to the specification the part should be rejected, it can be released for successful assembly on the counterpart with the pin on the plate and the condition that the plate's edges overlap. The displayed visualisation clearly shows that assemblability results from the combined effects

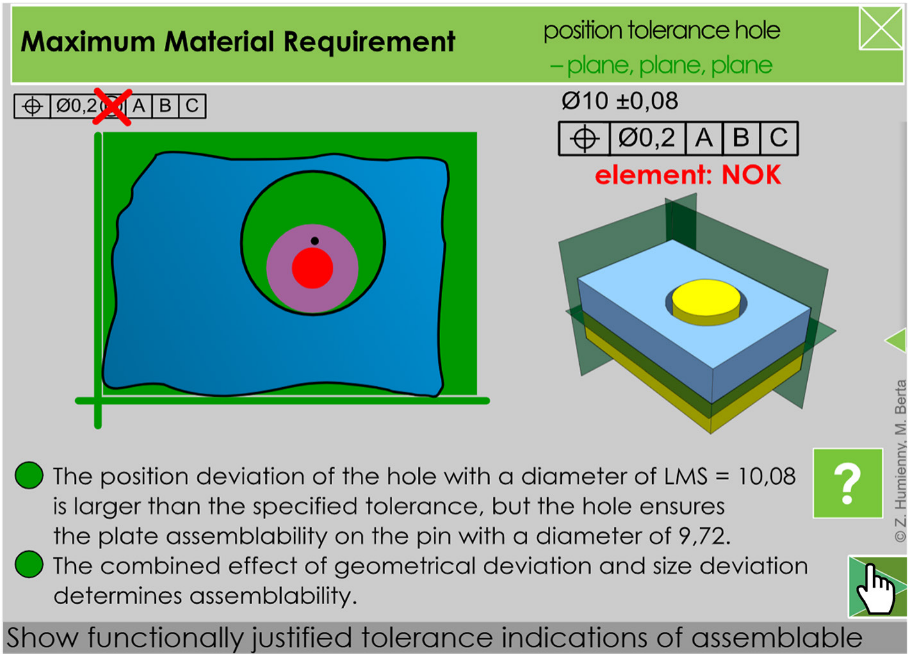

Figure 7. Explanation window – the 7th scene. One of an infinite number of positions of the 10.08 mm hole is shown in this screenshot. The actual hole axis is outside the tolerance zone, but the hole surface does not violate the boundary with the diameter of 9.72 mm. The part fulfils the functional requirement – the assembly on the pin with clearance, unfortunately it violates the requirement specified by the tolerance indicator, therefore the caption "element NOK" is displayed

of geometrical deviations and size deviations. The conclusion: The surface interpretation (verification by gauge) is functionally superior to the axis interpretation (by zone) whenever the two interpretations do not produce the equivalent results, based on the presented two extreme cases of the size deviations is available under the Question mark button (Figure 8).

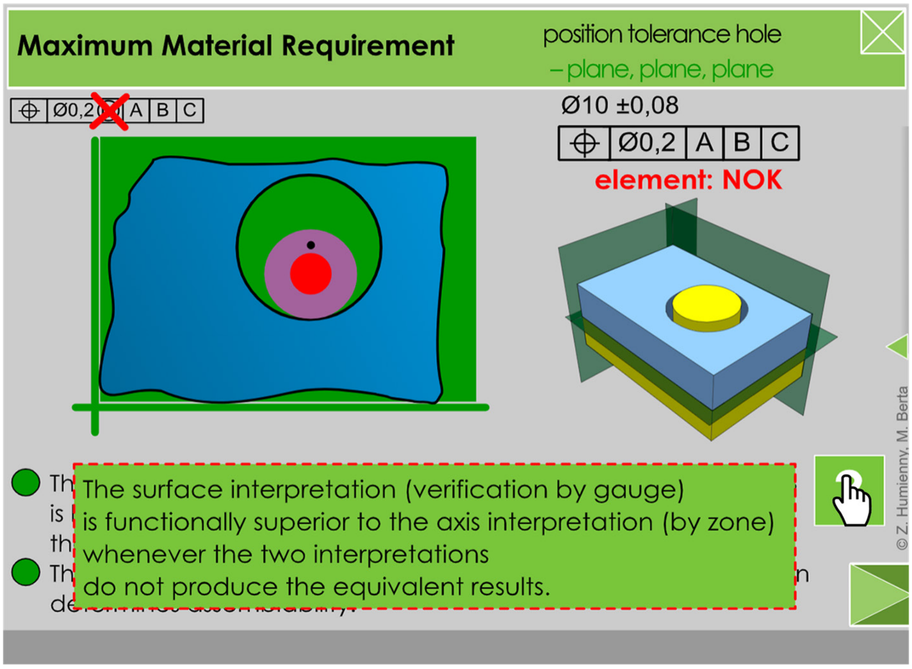

Figure 8. Explanation window – the expanded 7th scene. The conclusion based on the presented animation series is displayed (pulled down) while the mouse cursor is dragged and held over the button with the question mark sign

The Click event on the Forward button in the Explanation Window 7th scene releases the 8th scene of this animation series, which can serve as the alternative Definition Window (Figure 9). This is the breakthrough scene in the discussed series of animations. Based on the presented observations, it was found that the 9.72 mm diameter boundary is not violated in cases where the hole is larger, even if the position deviation exceeds the specified tolerance T = 0.2 mm. It is therefore necessary to provide an indication that allows acceptance of such parts. To mark this, the encircled M is introduced in the ISO GPS system. The MMR modifier is specified after the tolerance

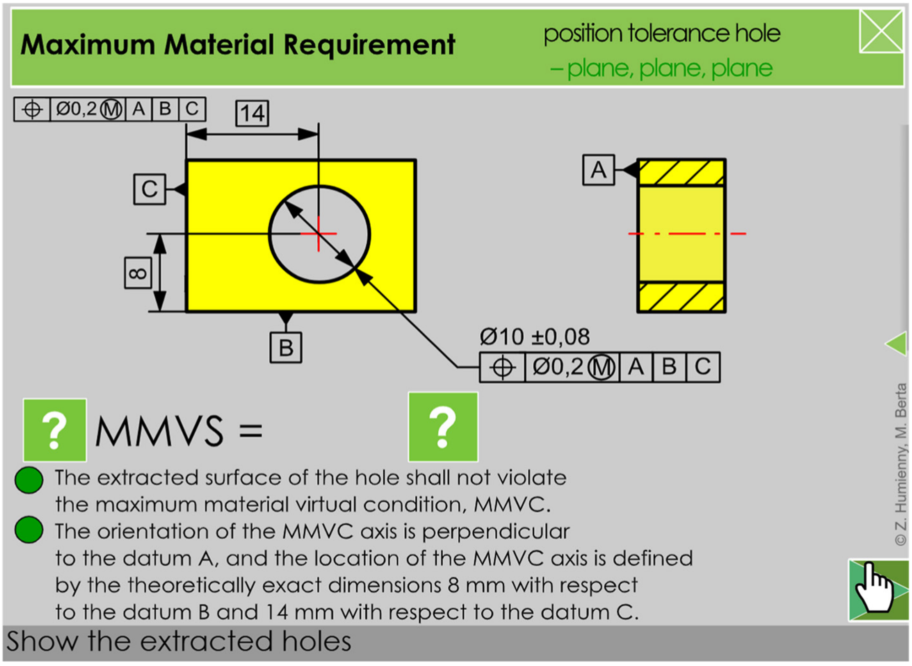

Figure 9. Explanation window – the 8th scene. Specification with MMR allows acceptance of all parts that can be assembled. It does not matter for the assembly which part of the allowable deviation is consumed by the size deviation and which by the geometrical deviation

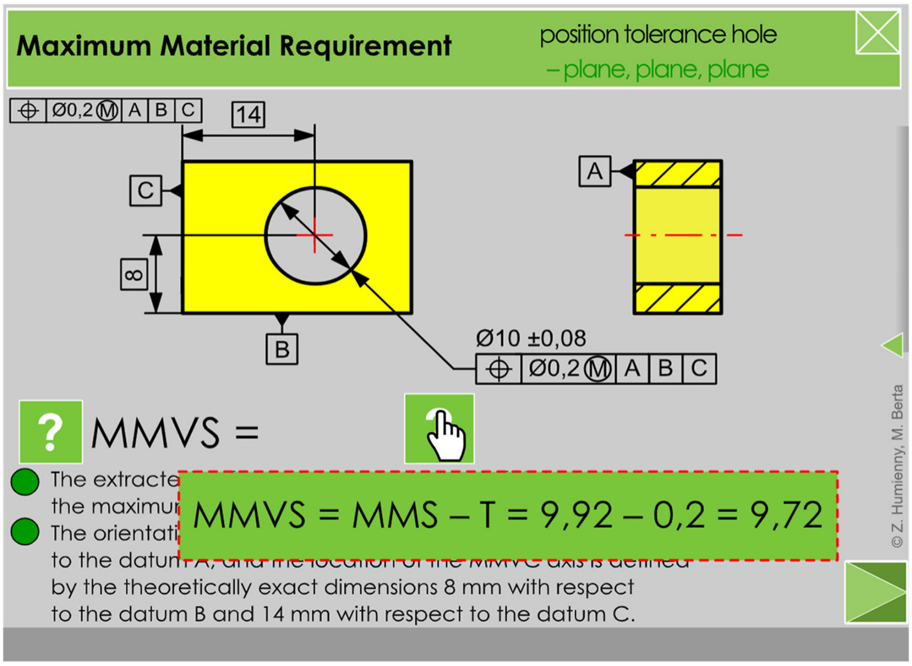

Figure 10. Explanation window – the expanded 8th scene. The maximum material virtual size formula is displayed while the mouse cursor is dragged and held over the button with the question mark

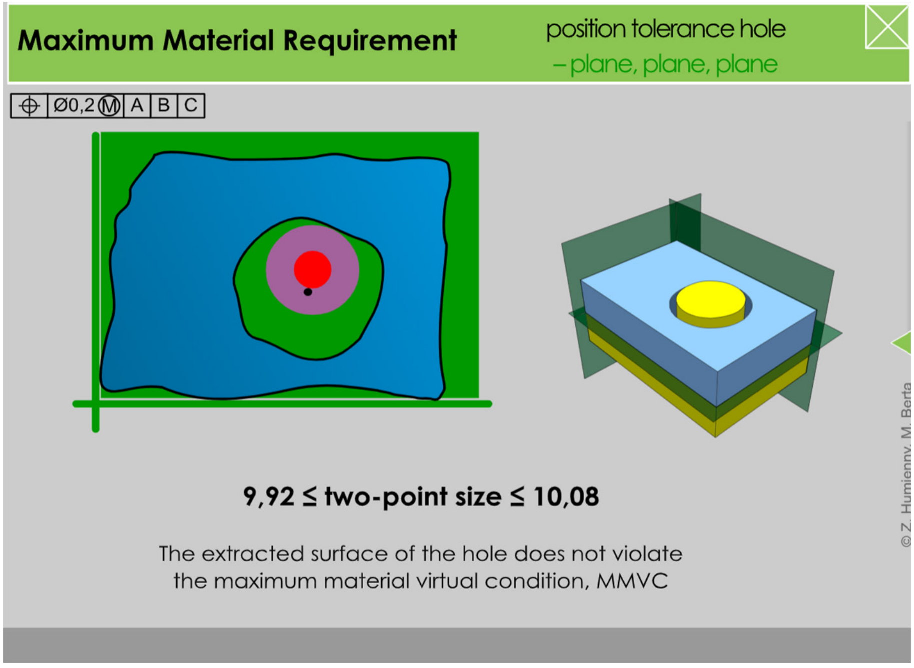

Figure 11. Explanation window – the 9th scene. The surfaces of the various holes with variable sizes and geometrical deviations are outside the boundary defined by MMVS = 9.72

value in the second compartment of the tolerance indicator (Figure 9). In this 8th scene, below the part drawing, it is underlined that, according to the given specification, the extracted surface shall be outside the boundary defined by the cylindrical surface with maximum material virtual size. The state of the associated feature with size equal to the MMVS is defined as the maximum material virtual condition (MMVC) [15]. The Application user may display the numerical value of the MMVS in the expanded 8th scene of the Explanation window (Figure 10) to verify he/she understands MMR concept.

The last two Explanation windows are added to reinforce and consolidate the knowledge of the MMR concept among users of the Application Geometrical Tolerancing. The designer, by specification of MMR, defines the geometrical feature of the same type as the toleranced feature and of perfect form with size equal to MMVS [15], which limits the non-ideal (actual) feature on the outside of the material. The actual feature of the produced part shall not violate the boundary defined by MMVS, which has a diameter of 9.72 mm (Figure 11).

It is shown (Figure 6 and Figure 7) that the MMVS is the size created by the collective effect of the MMS and the geometrical tolerance specified for the derived feature of the same feature of size. When the MMR is not indicated, the position tolerance value is constant regardless of the diameter of the actual hole due to the default Independency principle [4]. The MMR is the collective requirement pertaining to the combination of the size deviation and the geometrical deviation of the feature of size. Due to MMR indication for the hole, the intended function of the hole is ensured i.e. the clearance fit with the pin of the same length protruding from the plate, with the plates in full contact and their edges overlapping (Figure 2).

CONCLUSIONS

It should be emphasised that the implementation of Application Geometrical Tolerancing in academic teaching and industrial vocational training confirmed its usefulness. Developed dynamic animations significantly improved the effectiveness of geometrical tolerancing courses at universities and in industry. The establishment and movement of the tolerance zones and datum features, along with the visualisations of the actual toleranced and datum features facilitate clear and effective communication of complex geometrical tolerancing rules in a student-friendly manner.

The recently developed animations discussed in this paper clearly demonstrate that specified geometrical tolerances can be increased in practice, thereby ensuring assemblability when the actual sizes of mating features differ from their maximum material sizes. This observation is the root of the maximum material requirement concept indicated in a drawing by the encircled M inserted after the tolerance value in the second compartment in the tolerance indicator (Figure 9). The animations discussed in this paper show that, through the specification of the MMR, full functionality and interchangeability of the manufactured parts are ensured at minimal cost, since all mating parts are accepted without individual consideration of their size and geometrical deviations.

The ISO GPS system comprises 14 geometrical tolerances, complemented by over 20 additional modifiers that refine tolerance specifications in technical product documentation. We have developed over 100 animations illustrating the application of all 14 tolerances to workpieces with cylindrical and planar features, as well as to workpieces with more complex geometry. The meaning of a few modifiers is also explained in the Application Geometrical Tolerancing. The next challenge is to develop animations that illustrate the functions of other modifiers.

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