

Analysis on predictive functional controller performance and tuning rule based on a first-order plus dead time model

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ABSTRACT

This work presents the results of both simulation and analytical investigations on tuning of predictive functional control (PFC) algorithm for first-order plus dead time (FOPDT) system. The controller's characteristics were analyzed to determine the influence of the main control parameters on its performance. The PFC equations are transformed and an alternative surrogate parameter representing the algorithm's effective gain was proposed. Most common control quality factors are analyzed to find a trade-off between response speed and controller aggressiveness. Combined control quality factor was proposed and investigated to determine optimal tuning parameters. That analysis resulted in tuning formulas for main parameters of the PFC algorithm such as desired response time t_{rbf} and prediction horizon H_p . The proposed tuning approach was verified both on a higher-order dynamic system and in an experimental heating process setup.

Keywords: predictive control, controller tuning, control quality, tuning rule, predictive functional control.

INTRODUCTION

One of the first advanced control methodologies that was developed and applied in industry was model based predictive control [1, 2]. The idea of process model application for the synthesis of the controller is simple and generally promises better control performance in terms of the selected quality factors. There are many control techniques based on the simple input-output models, e.g. model algorithmic control (MAC) [1], predictive functional control (PFC) [3, 4] dynamic matrix control (DMC) [5] and internal model control (IMC) [6]. In these cases, process nonlinearities can be accounted for through nonstationary modeling, resulting in control that exhibits the properties of adaptive control.

PFC is one of the oldest [7], and at the same time one of the simplest among various MPC algorithms [8, 9]. The simplicity of final form makes it possible to implement this algorithm in standard automatic control equipment like PLCs [10–13]. It should be noted that this feature

is highly desirable for industrial implementation. This algorithm was identified as one of the advanced controls with the greatest industrial success. Recent publications indicate that this algorithm is still used in different practical applications: motion [14, 15], energy systems [16, 17], chemical processes [18]. It is evident that PFC remains a subject of analysis, research, and algorithm extensions [19–21]. Despite this, it is difficult to find in the literature any strict tuning rules for PFC algorithm. Main inventor of this algorithm (J. Richalet) claimed that tuning is intuitive because of basic assumptions of such control. That statement is somewhat true, since the most relevant tuning parameter of this controller t_{rbf} is closed loop response time which is equal to triple time constant of exponential reference trajectory. However, this parameter is not the only one to be selected in this control algorithm. Prospective users that are implementing this algorithm need to select also two other parameters: controller cycle t_{rbf} and prediction horizon h . In the most recent

book describing PFC [22] some hints on how to tune the PFC controller can be found, but those hints are very general without any strict formulas. Some further PFC tuning hints might be found in a small number of publications describing other research on this topic [23–26] with respect to integral error index. In general, this issue is briefly discussed and open for further research.

Any research on controller tuning requires certain assumptions about the dynamic properties of controlled process. These dynamic properties can be determined based on internal models when designing tuning rules for MPC algorithms. Without doubt, any tuning rules yield controller parameter values closer to the optimal values (for a chosen control quality factor) when the assumed model dynamic is closer to real plant dynamics. In practice, however, it is nearly impossible to create perfect model of a given technological process. Thus, any tuning rule can give only controller parameters close to optimal ones and afterwards the process' operator must subsequently correct them manually to achieve satisfactory results. Nevertheless, such tuning rules are necessary to obtain initial values of controller parameters that significantly shorten the control loop startup procedure. Over many decades, numerous tuning rules were developed for the standard PID algorithm [27]. Some of these rules are based on closed loop performance results, while others require an analysis of the dynamic properties of a given controlled plant. In the second case, different model forms are considered for approximation of controlled process dynamics. The most common dynamics approximation used for this purpose is FOPDT (first-order plus dead time), which is also considered in this research for PFC algorithm tuning.

PFC ALGORITHM FOR FOPDT PROCESS

The PFC control algorithm has only four main assumptions:

- discrete internal model,
- coincidence horizon at time $h \cdot t_s$ (where: t_s – sampling time),
- constant future set point – y_{set} ,
- exponential reference trajectory with decrement λ .

Reference trajectory binds the signal of controlled variable y_p and the setpoint value y_{set} . Setpoint y_{set} is assumed to be constant value for

simplicity when forming control equations. However, this algorithm performance, as any other control algorithm is tested for setpoint change. The reference trajectory was chosen as the simplest one – exponential function. Thus, it is defined by its decrement $\lambda = \exp(t_s/t_R)$, where: $t_R = t_{rbf}/3$ and t_{rbf} is the desired response time of the closed loop system. In this case the error signal $e(.) = y_{set} - y_p(.)$ at discrete time $n + h$ is expressed by very simple relation: $e(n + h) = e(n) \cdot \lambda^h$. There is no assumption that model output y_m is perfectly following process output y_p , but in general y_m increments should follow increments of y_p . (Figure 1)

In this research dynamics of controlled process is assumed as FOPDT (Equation 1), which is one of the most frequently used dynamics approximation for controller tuning purposes [27–29].

$$\frac{y_m(s)}{u(s)} = \frac{G}{1 + \tau \cdot s} \cdot e^{-s\tau_d} \quad (1)$$

where: τ – time constant, G – gain,
 τ_d – time delay, u – process input value,
 y_m – model output value.

The discrete form of the model expressed by Equation 1 is:

$$y_m(n) = y_m(n-1) \cdot \alpha + (1 - \alpha) \cdot u(n-1) \cdot G \quad (2)$$

where: $\alpha = e^{-t_s/\tau}$, t_s – sampling time, n – discrete instance of time.

Controller equations for FOPDT model have the following form:

$$\hat{y}_{Pred}(n) = y_p(n) + y_m(n) - y_m(n-D) \quad (3)$$

$$u(n) = \frac{(1-\lambda^h)[y_{set}(n) - \hat{y}_{Pred}(n)]}{G(1-\alpha^h)} + \frac{1}{G} y_m(n), \quad (4)$$

where: $\hat{y}_{Pred}$ – predicted process output (without delay), y_p – process output value, D – discrete delay, h – prediction horizon, y_{set} – setpoint value, λ – reference trajectory increment.

A series of simulation experiments were performed for the PFC control loop using Equations 2–4, assuming that the process model was identical to the controller's internal model. In the first stage, the influence of the controller cycle time on

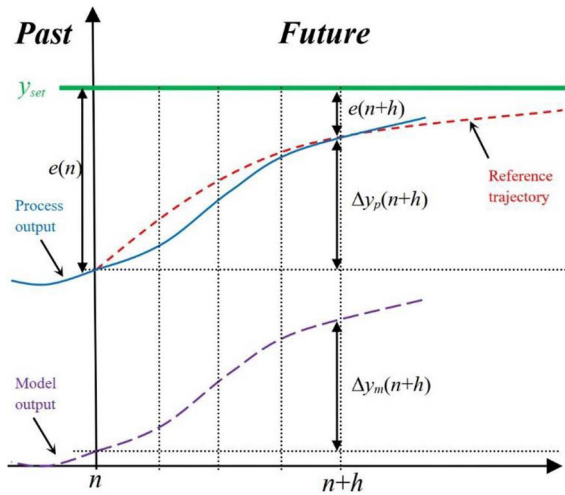


Figure 1. General time chart for PFC algorithm

closed-loop stability was investigated. The experiments were carried out using illustrative process parameters ($G = 1$, $\tau = 2$, $\tau_d = 1$). It was observed that for constant value of discrete prediction horizon $h = 1$ ($H_p = t_s$, where: H_p is the prediction horizon expressed in time units) stability depends on both sampling time t_s and tuning parameter t_{rbf} . It is obvious that numerical calculations errors are proportional to calculation time interval; therefore, stability of control loop for different t_s values mainly depends on numerical errors. The shorter t_s value is, the stability of control loop is stronger. Stability area was investigated for process model of above given parameters with respect to various values of t_s and t_{rbf} with constant value of discrete prediction horizon $h = 1$. Results showed that the longer desired response time t_{rbf} the system tends to behave more stable. The obvious is that the longer sampling time (controller cycle) t_s the system is less stable but even if controller cycle is half of process time constant the system is still stable. (Figure 2)

The experiments showed that the control loop is unstable for small values of t_{rbf} when the controller cycle approaches the process time constant τ . However, control loop stability with respect to controller cycle t_s does not guarantee satisfactory control performance. Excessively large values of t_s lead to periodically dumped oscillations in time evolution of process variable (Figure 3a) and rapid variations in the manipulated variable (Figure 3b). Therefore, a reasonable design choice is to select the controller cycle significantly smaller than dominant process time constant.

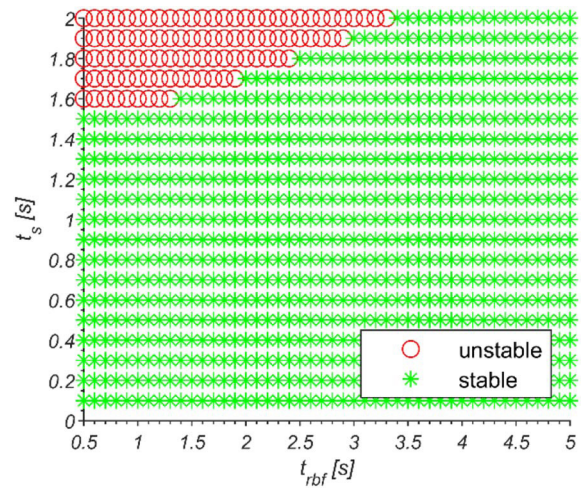


Figure 2. Stability range of PFC control for FOPDT dynamics

PFC CONTROL QUALITY INVESTIGATIONS

Results of these experiments analysis gave the conclusion that some arbitrary selected value of controller cycle should be selected with relation to FOPDT model time constant. It could be assumed the relation which is often used in such cases [29]:

$$t_s \leq \frac{\tau}{10} \quad (5)$$

In practice, determination of this value is related to computational efficiency of automatic control equipment. Thus, controller cycle should be selected longer than the time in which the controller is capable of computing all required algorithm equations. Other parameters of the PFC controller have a major impact on the selected and commonly used control quality factors (ISE , IAE , $ISdU$, t_{setl}). Simulation research resulted in conclusion that ISE , IAE and settling time t_{setl} have very similar shape in respect to PFC algorithm tuning parameters. Thus, in this research two of the factors mentioned were selected for further investigations:

$$ISE = \int_0^{t_k} e^2(t) dt \quad (6)$$

$$ISdU = \int_0^{t_k} (u(t) - u_\infty)^2 dt \quad (7)$$

where: e – control error, t_k – time end of experimental data, u_∞ – final value of controller output after step change of setpoint

or disturbance, Δe – assumed control error margin (usually 5% of setpoint step change).

A set of experiments performed using PFC controller for the FOPDT dynamic object with selected parameters ($G = 1$, $\tau = 2$, $\tau_d = 1$) and sampling time $t_s = 0.1$ showed that both parameters, t_{rbf} and H_p , significantly influence the behavior of the manipulated and controlled variable responses. When $H_p = t_s$, small values of t_{rbf} result in aggressive changes of the manipulated value (Figure 4). As t_{rbf} increases, smoother changes in the manipulated variable are observed; however, this also leads to a longer settling time. A substantial influence of the prediction horizon H_p on the time evolution of both manipulated and controlled variables was also observed (Figure 5) for a constant value of $t_{rbf} = \tau$. The results indicate that this parameter affects both the settling time and the aggressiveness of the controlled variable response.

It can be stated that both aforementioned parameters have a substantial influence on the time responses of the controlled and manipulated variables and consequently affect considered quality factors. This influence was investigated, and the results are presented in Figure 6. The results were obtained from simulation experiments performed for a PFC controller applied to FOPDT process model for different values of the parameters t_{rbf} and H_p .

The results presented in Figure 6 might also be obtained analytically, based on formulas describing the time evolution of manipulated and control variables. The form of PFC given in Equation 4

for FOPDT process model could be simplified in following way:

$$u(s) = \frac{K_{PFC}}{G} [y_{set}(s) - \hat{y}_{Pred}(s)] + \frac{1}{G} y_m(s) \quad (8)$$

where: K_{PFC} represents the effective gain of PFC controller is given by:

$$K_{PFC}(\tau_m, h, t_{rbf}, t_s) = \frac{(1 - \lambda^h)}{(1 - \alpha^h)} = \frac{1 - \exp\left(-\frac{3 \cdot H_p}{t_{rbf}}\right)}{1 - \exp\left(-\frac{H_p}{\tau_m}\right)} \quad (9)$$

In this research the assumed internal model of the process has the following form:

$$\frac{y_m(s)}{u(s)} = \frac{G_m}{1 + \tau_m \cdot s} \cdot e^{-s \cdot \tau_d} \quad (10)$$

Predicted process dynamics without time delay can be expressed as:

$$\hat{y}_{Pred}(s) = \frac{G}{1 + \tau \cdot s} \cdot u(s) \quad (11)$$

Thus, process output is represented by:

$$y_P(s) = e^{-s \tau_d} \cdot \hat{y}_{Pred}(s) \quad (12)$$

Transforming the above equations and assuming equivalence between the model and the

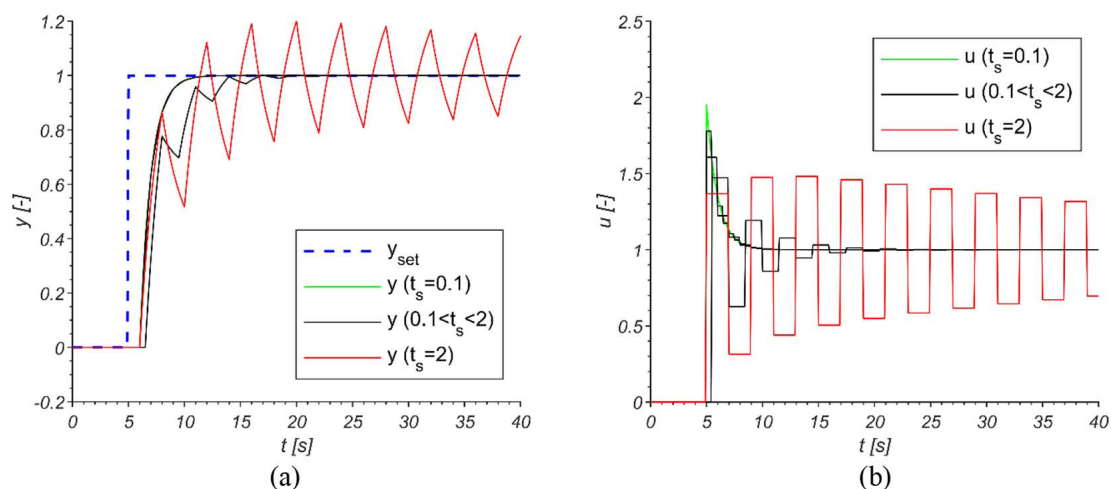


Figure 3. Time evolution of controlled (a) and manipulated (b) variables for PFC ($t_{rbf} = 3$, $h = 1$) control of FOPDT process ($G = 1$, $\tau = 2$, $\tau_d = 1$) with different controller cycle time t_s

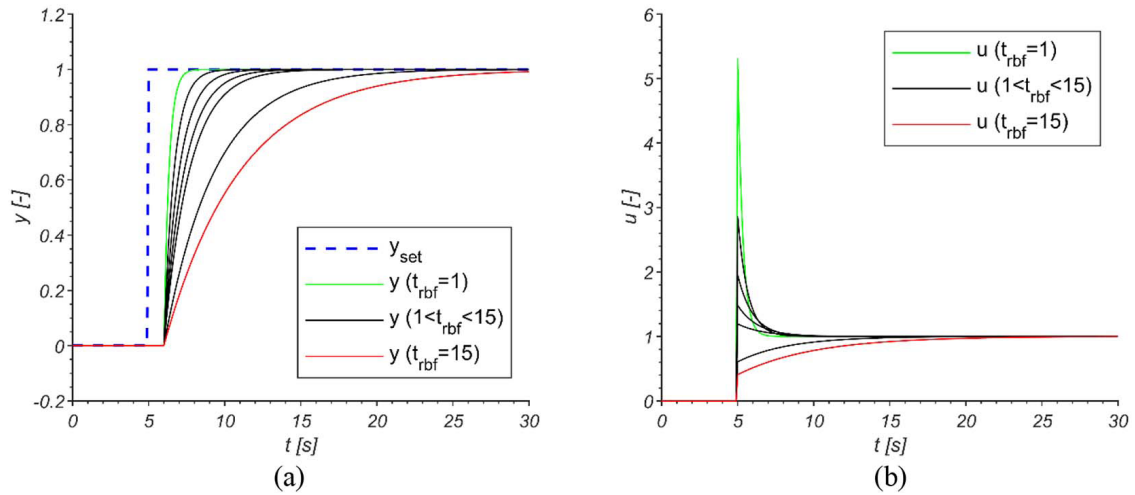


Figure 4. Time evolution of controlled (a) and manipulated (b) variables for PFC control of FOPDT process with different values of t_{rbf} and constant value of $H_p = t_s = 0.1$

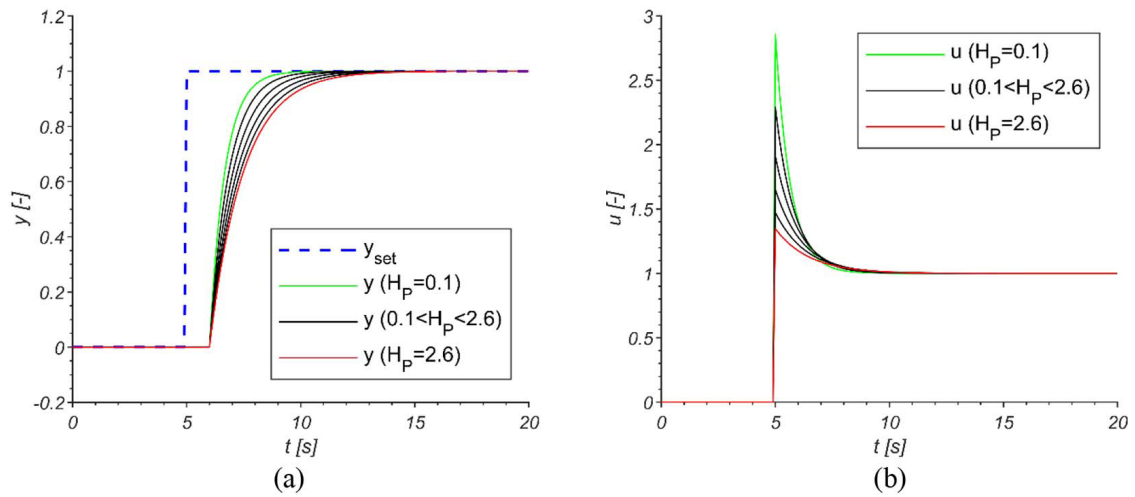


Figure 5. Time evolution of controlled (a) and manipulated (b) variables for PFC control of FOPDT process with different values of H_p and constant value of $t_{rbf} = \tau = 2$

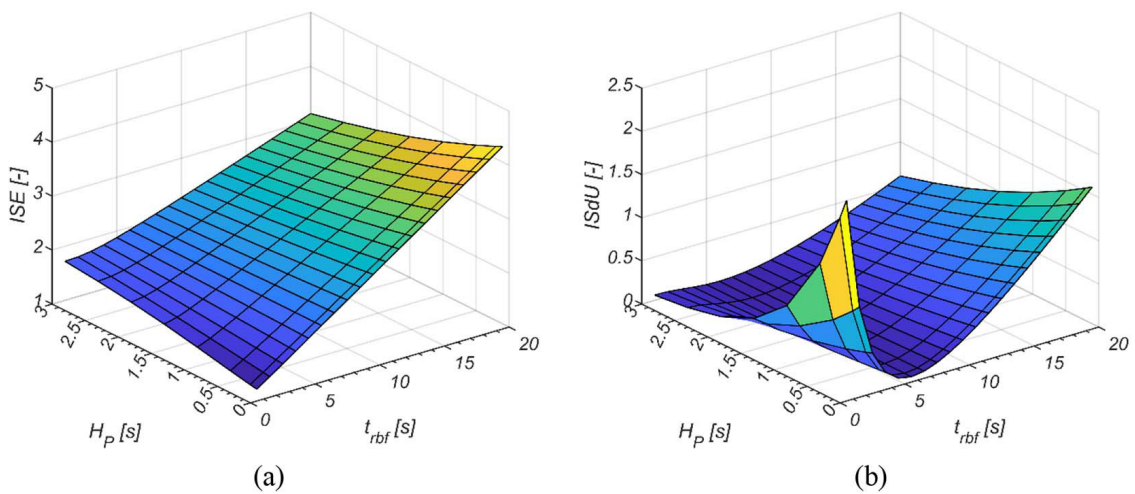


Figure 6. Relation of control quality factors ISE (a) ISdU (b) to PFC controller parameters t_{rbf} and H_p for FOPDT dynamics process

process, the following relations for the manipulated and the controlled variables are obtained:

$$u(s) = \frac{1}{G} \cdot \frac{s \cdot \tau + 1}{s \cdot \frac{\tau}{K_{PFC}} + 1} \cdot y_{set}(s) \quad (13)$$

$$y(s) = \frac{1}{s \cdot \frac{\tau}{K_{PFC}} + 1} \cdot e^{-s\tau_d} \cdot y_{set}(s) \quad (14)$$

Above relations Equations 13–14 take the following form in the time domain:

$$u(t) = \left[1 + (K_{PFC} - 1) \cdot e^{-\frac{K_{PFC}}{\tau} t} \right] \cdot \frac{\Delta y_{set}}{G} \cdot 1(t) + u(0) \quad (15)$$

$$y(t) = \left[1 - e^{-\frac{K_{PFC}}{\tau} (t - \tau_d)} \right] \cdot \Delta y_{set} \cdot 1(t - \tau_d) + y(0) \quad (16)$$

hence control error is expressed as:

$$e(t) = \Delta y_{set} \cdot 1(t) - \left[1 - e^{-\frac{K_{PFC}}{\tau} (t - \tau_d)} \right] \cdot \Delta y_{set} \cdot 1(t - \tau_d) \quad (17)$$

The values of the f ISE and $ISdU$ quality factors can be calculated based on Equations 15 and 17:

$$ISE = \int_0^\infty e^2(t) dt = (\Delta y_{set})^2 \cdot \left(\tau_d + \frac{\tau}{2 \cdot K_{PFC}} \right) \quad (18)$$

$$ISdU = \int_0^\infty (u(t) - u_\infty)^2 dt = (\Delta y_{set})^2 \cdot \frac{\tau \cdot (K_{PFC} - 1)^2}{2 \cdot G^2 \cdot K_{PFC}} \quad (19)$$

PFC ALGORITHM TUNING RULE

There are many possible ways to define combined quality factors that account for both effective control error reduction and controller aggressiveness. One of such factor considered in this work, is defined as:

$$Q_F = \frac{1}{(\Delta y_{set})^2} (ISE + G^2 \cdot ISdU) \quad (20)$$

Multiplying the $ISdU$ by the squared process gain ensures that both elements in the sum are expressed in the same units, which is essential for formal correctness of any calculations. Dividing this sum by squared step change of setpoint makes analysis results independent of the setpoint magnitude value and dependent only on the PFC controller parameters and the process dynamics parameters. The influence of PFC controller parameters on control quality factor defined in Equation 20 for a FOPDT process with selected parameters ($G = 1$, $\tau = 2$, $\tau_d = 1$) is presented in Figure 7.

Comparable results are obtained for different values of the FOPDT parameters (gain, time constant and delay). In each case minimum-level curve is observed originating on axis at around twice the process model time constant.

Substituting Equations 18–19 into Equation 20 yields the following expression for quality factor:

$$Q_F = \tau_d + \tau \cdot \frac{(K_{PFC}^2 - 2 \cdot K_{PFC} + 2)}{2 \cdot K_{PFC}} \quad (21)$$

Determining the optimal value of the effective gain K_{PFC} is a straightforward task in this case since the considered function

Q_F has only a single minimum. Finding the minimum of this function through differentiation gives the result $K_{PFC} = \sqrt{2}$. It should be noted that this value was derived under the assumption that the model and the process have equivalent dynamics which is rarely case in practical applications. Nevertheless, this result may be used as an initial estimate for tuning procedure. It can be also observed that increasing K_{PFC} results in more aggressive controller performance whereas decreasing K_{PFC} results in smoother time response of the controlled variable.

The tuning procedure can be initiated by determining the prediction horizon H_p , regarding fact that H_p value is an integer multiple of the sampling time t_s ($H_p = h \cdot t_s$). The inventor of PFC algorithm, Jacques Richalet, stated that for processes with simple dynamics, the prediction horizon might even be equal to the controller cycle. The appropriate value of H_p depends on mismatch between the model and the process dynamics and should therefore be determined individually and increased if necessary. In the next stage of the tuning procedure

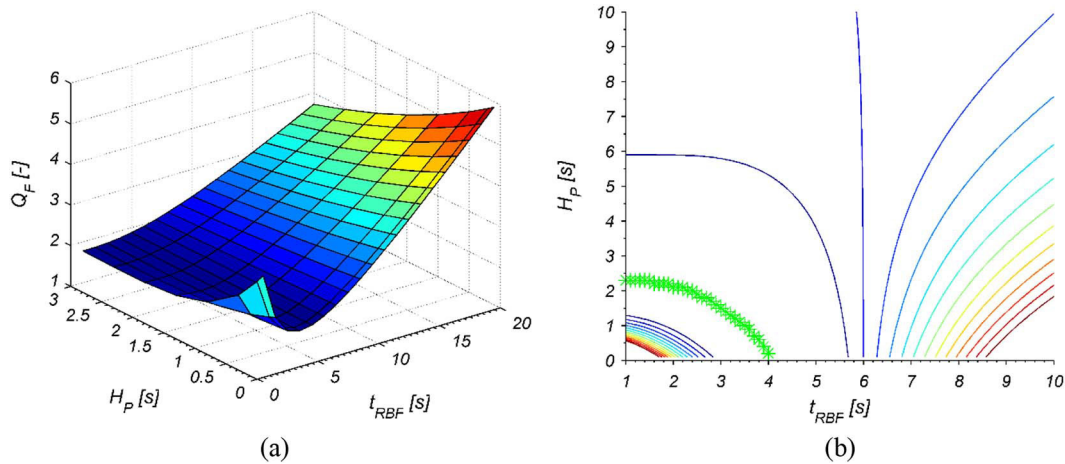


Figure 7. Relation between Q_F factor to PFC algorithm parameters (a). The plot (b) presents contour of this function, where green stars represent minimal level curve

value of t_{rbf} should be calculated using the formula obtained by transforming Equation 9:

$$t_{rbf} = \frac{-3 \cdot H_P}{\ln \left(1 - K_{PFC0} \cdot \left(1 - e^{-\frac{H_P}{\tau}} \right) \right)} \quad (22)$$

$$= \frac{-3 \cdot H_P}{\ln \left(1 - \sqrt{2} \cdot \left(1 - e^{-\frac{H_P}{\tau}} \right) \right)}$$

The value of the parameter t_{rbf} determined using the above formula represents a response time, thus must be a positive real number. Consequently, the argument of the logarithm in the denominator must also be positive. This requirement results in the following condition:

$$H_P < -\tau \cdot \ln \left(1 - \frac{1}{K_{PFC0}} \right) \quad (23)$$

In practice, tuning the PFC controller for FOPDT process can be performed solely by manipulating the value of K_{PFC} , without the need to determine formal controller parameters explicitly. This approach is motivated by the fact that all controller parameters are embedded in the equation defining K_{PFC} . Therefore, this substitute tuning parameter may be interpreted as an aggressiveness factor. When the parameter is equal to 1, a setpoint step change causes the controller to immediately set the manipulated variable to its final value. In this case the process variable response corresponds to the free response of assumed dynamics. Values of K_{PFC} greater than 1 cause the manipulated variable to overshoot the

final value, while values below 1 result in an initial change of manipulated variable lower than its final value. The plots in Figure 8 present the time evolution of the manipulated and the controlled variable for a PFC controller applied to FOPDT process with different values of $K_{PFC} = 2; \sqrt{2}; 1; 1/\sqrt{2}; 0.5$. Analysis of these results shows that the value of K_{PFC} is equal to the ratio of initial change of the manipulated variable to its final value when control action becomes zero.

PFC TUNING EXPERIMENTS

Since the proposed methodology was designed for FOPDT dynamics its applicability should also be verified for processes exhibiting another type of dynamics. This verification procedure was performed for 3rd order dynamics process model [30] with equal time constants:

$$\frac{y(s)}{u(s)} = \frac{1}{(1+s)^3} \quad (24)$$

This type of dynamics was approximated with FOPDT using least-squares method applied to the free response. This procedure resulted in the following values of parameters: $G = 1$, $\tau = 2$, $\tau_d = 1.1$. The plot in Figure 9 presents comparison of assumed process dynamics Equation 24 and its FOPDT approximation. The results of quality factor Q_F (Equation 20) analysis with 3rd order dynamics as the process one and FOPDT dynamics used as the internal model in PFC controller are presented in Figure 10. In this case

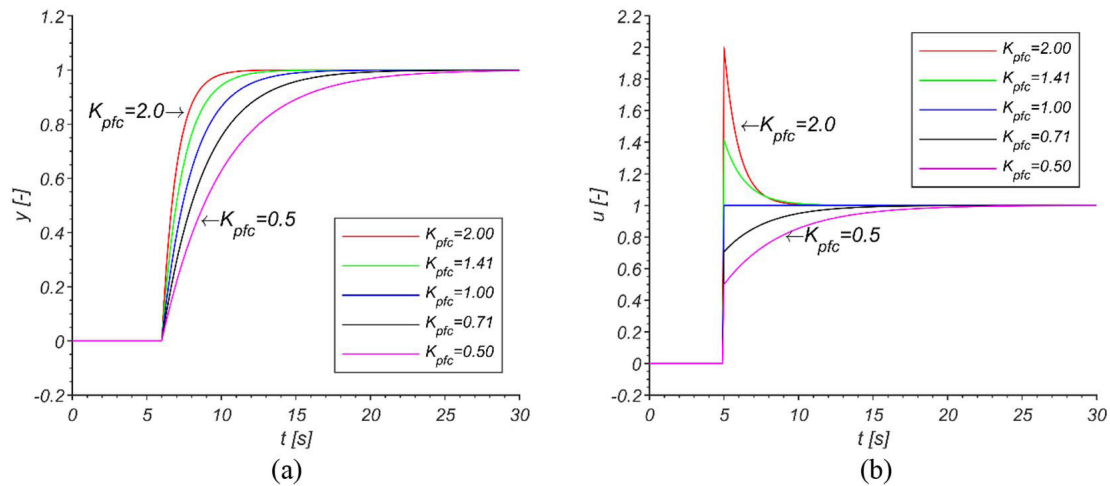


Figure 8. Setpoint step change response of process (a) and manipulated (b) variables for various values of substitute parameter K_{pfc}

minimum-level curve of Q_F value starts near twice the approximated time constant on the t_{rbf} axis. Analysis of time evolution of the manipulated and controlled variables showed that inconsistency of process and internal model dynamics results in oscillations in the responses of both the process and manipulated variables. This effect becomes less visible for lower values of K_{pfc} (Figure 11). In this case the value of the t_{rbf} parameter that gives the optimal results is compatible with the one obtained from Equation 22.

Similar experiments were conducted using a real experimental heating plant [31] (Figure 12). The source of heat energy in the experimental plant was an electric heater with a rated power of $P = 5$ kW. Water entering the tank at temperature

T_{in} is heated to the outlet temperature T_{out} . During the experiments the flow rate F was stabilized at 2.5 L/min. The heating power was manipulated using PWM (Pulse Width Modulation) technique. The measured signals together with the manipulated variable are connected to an NI-cRIO device which enable implementation of PFC controller in the LabView environment.

At constant flow rate, the FOPDT dynamic parameters of the system were approximated using multidimensional unconstrained nonlinear minimization. This resulted in following parameters:

$G = 0.27$ °C/%, $\tau = 16$ s, $\tau_d = 15$ s. When dealing with real dynamic systems experiments should be repeated to avoid nonrepresentative results. Therefore, each experiment consisted of a four setpoint changes as illustrated in Figure 13.

A set of 15 experiments were performed to investigate the influence of PFC parameters (H_p , t_{rbf}). In each experiment the sampling time (controller cycle) was set to $t_s = 1$ s, which satisfies the requirement expressed by Equation 5. The value of t_{rbf} was selected within the range from 30 s to 90 s while H_p was varied from 1 s to 5 s. For each experiment quality factor Q_F was calculated as a mean value obtained for all setpoint changes of T_{set} . It can be observed that the final value of the manipulated variable P is not stable at the end of each setpoint change experiment. This effect is mainly a result of fluctuations of flow rate F , which was varied around 2.5 L/min. Therefore, the final value of manipulated variable for calculation of $ISdU$ factor was determined as the average of last 100 samples. Figure 14 presents the results of Q_F calculation for all experiments.

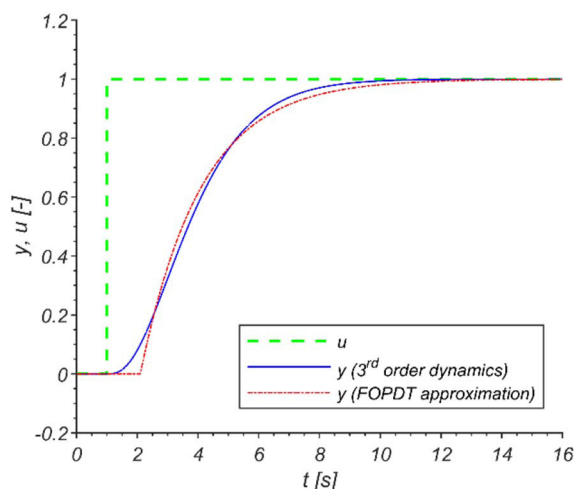


Figure 9. Setpoint step change response of process and manipulated variables for various values of substitute parameter K_{pfc}

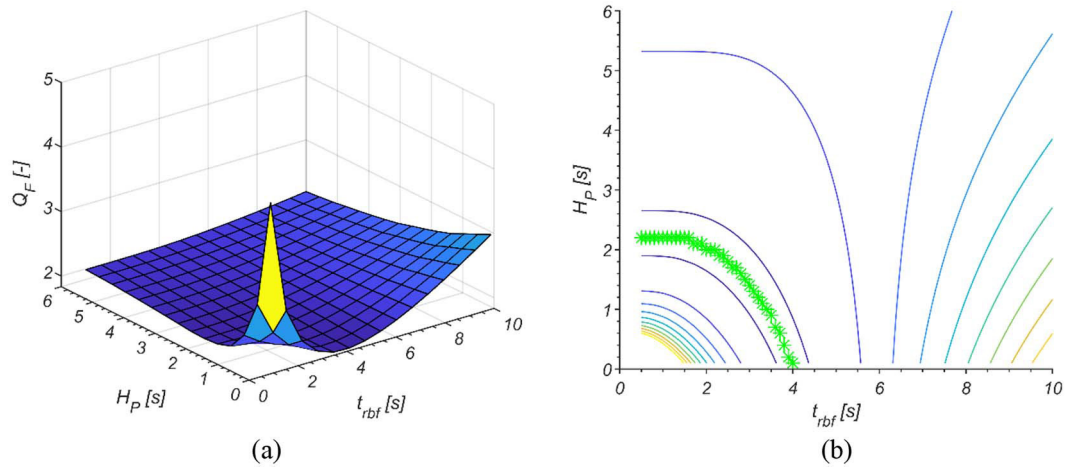


Figure 10. Relation between Q_F factor and parameters of PFC algorithm with FOPDT internal model used for control of process with 3rd order dynamics (a). Plot (b) presents contour of this function, where green stars represent minimal level curve

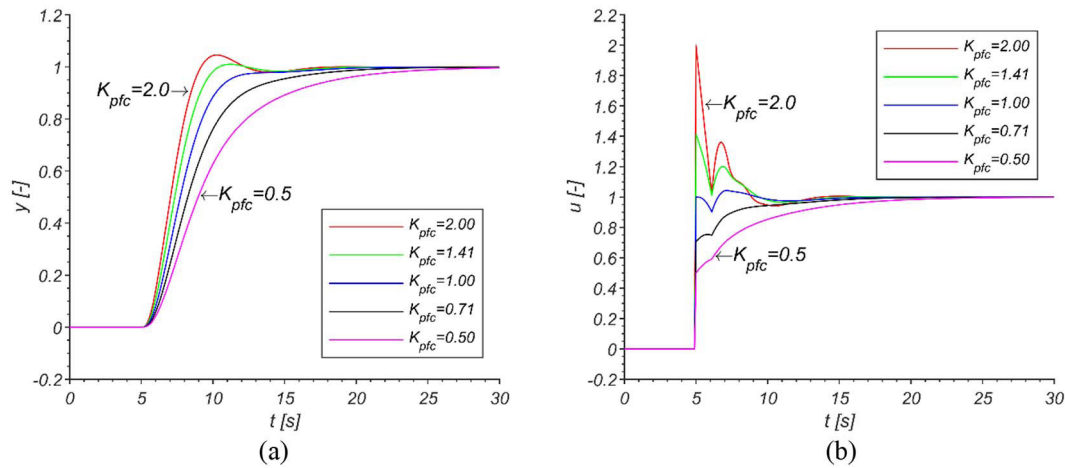


Figure 11. Setpoint step change response of process (a) and manipulated (b) variables for various values of substitute parameter K_{PFC} in PFC control loop with FOPDT internal model used for control of 3rd order dynamics process

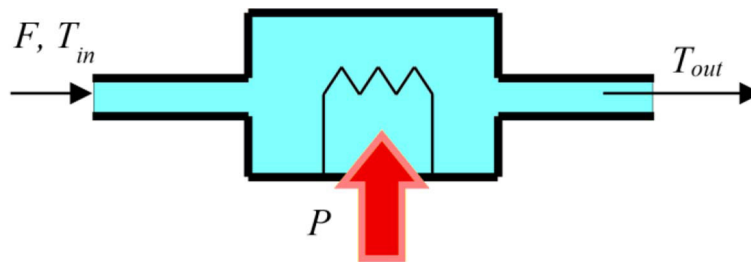


Figure 12. Simplified scheme of heating process

It can be observed that in this case the influence of H_p parameter on controller performance is marginally visible. The dominant influence is associated with the value of t_{rbf} , while values over 45s gives only minor improvement in the

discussed quality factor. The value calculated of t_{rbf} parameter calculated using Equation 22 is 33.5 s for $H_p = 1$ s. Therefore, it should be increased by approximately 10 s to obtain satisfactory results.

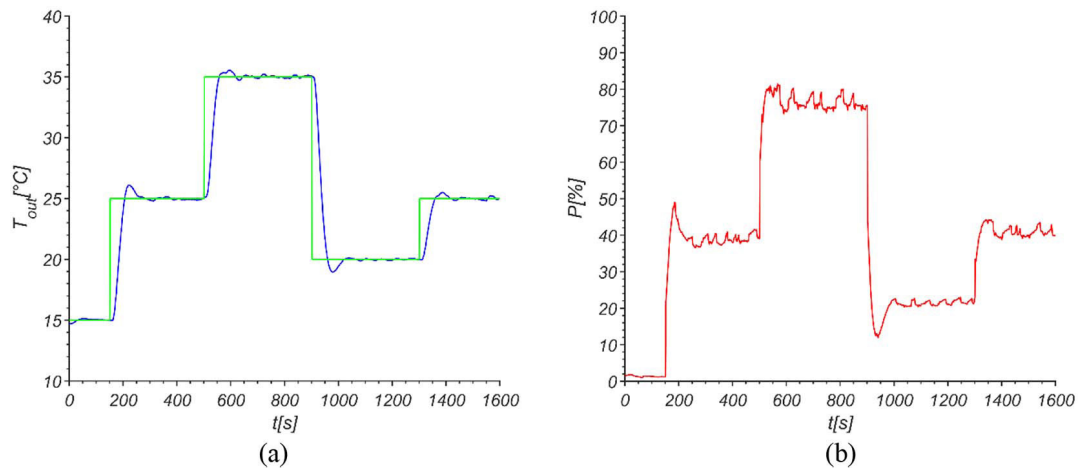


Figure 13. Exemplary result with PFC algorithm applied to experimental plant (process value T_{out} – green line, setpoint T_{set} (plot a) – blue line, manipulated value (plot b) P – red line)

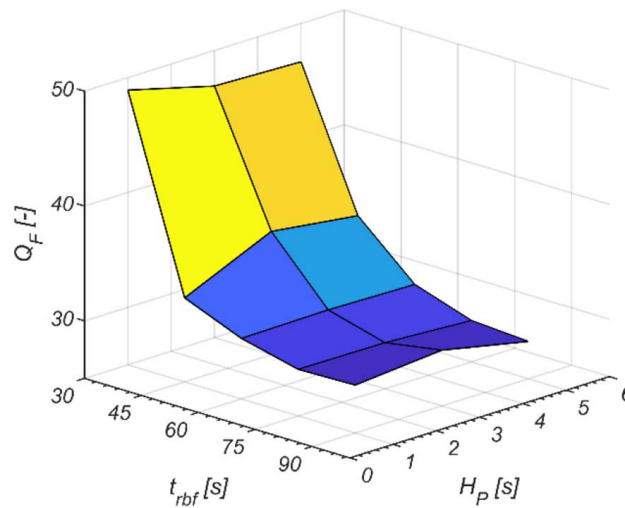


Figure 14. Quality factor Q_F values for experimental heating process control using PFC algorithm

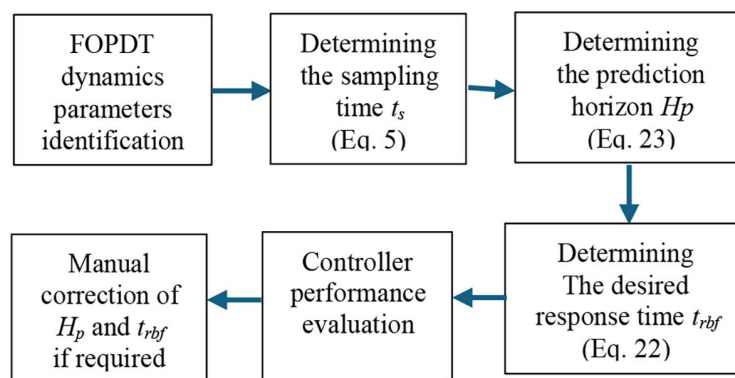


Figure 15. Diagram for the proposed controller tuning procedure

CONCLUSIONS

In this paper, a simple tuning rule for the PFC algorithm applied to a FOPDT process was

proposed. The tuning rule was developed using a combined quality factor that assesses both the process and the manipulated variables time evolutions. The proposed tuning methodology can

be extended to higher-order dynamics using the same procedure. The PFC algorithm as one of MPC group is capable of handling nonlinearities while feedforward action is naturally embedded when disturbances are measurable.

However, the experiments showed that for a constant value of desired response time t_{rbf} PFC controller performance is almost unchanged. Verification of the proposed tuning rule using simulation experiments confirmed its validity and demonstrated an almost identical shape of the discussed quality factor function. The same verification procedure conducted using real plant exhibited that main controller parameter time t_{rbf} should be increased compared with the value obtained from the proposed tuning rule. That result is a consequence of non-perfect dynamics approximation and additional disturbances (here: flow rate fluctuations). However, it should be noted that any tuning rule for a control algorithm provides only initial estimates of the controller parameters, which are subsequently adjusted manually by an experienced process engineer. The formally derived tuning rule proposed in this paper provides the opportunity to introduce an additional level of adaptiveness to this algorithm for processes with varying gain, time constant and time delay. However, the application of such adaptive PFC controller should be performed with additional analysis since the FOPDT model represents only the approximation of real process dynamics. Another challenge associated with the implementation and performance analysis of the PFC algorithm is the presence of disturbances, which may be difficult to eliminate in real industrial plants. Therefore, future research on this subject should also take into account the influence of disturbance fluctuations on controller performance.

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