

Design-oriented stability assessment of hybrid timber beams under uniformly distributed load: Serviceability and strength limit-state criteria

Le Thuy Nguyen^{1*} , Son Nguyen Hong¹, Hung Pham Thanh¹, Thanh Le Huu¹

¹ Hanoi Architectural University, 129 Tran Phu, Hanoi, Vietnam

* Corresponding author's e-mail: sonnh@hau.edu.vn

ABSTRACT

This study presents a design-oriented computational framework for the nonlinear stability assessment of hybrid three-layer timber beams under a uniformly distributed transverse load, composed of high-stiffness face layers and a compliant core, with particular emphasis on the influence of geometric imperfections and interface slip on structural performance. The formulation is developed within a variational energy framework based on the principle of minimum total potential energy, incorporating bending, membrane, and interlayer slip effects under large geometric imperfections, and is solved directly from the governing equilibrium equation rather than through a load-amplification approximation. Extending a previously validated point-load formulation to this practically relevant loading case, an energy-based indicator is employed to interpret the evolution of structural response from bending-dominated behavior to membrane-influenced behavior under increasing deformation. The numerical implementation is carried out using an incremental–iterative procedure, enabling efficient parametric investigation of imperfection amplitudes and interface stiffness levels. The results demonstrate that geometric imperfections trigger early membrane (catenary) engagement and a pronounced hardening of the tangent stiffness from the onset of loading, while simultaneously producing a knock-down in the load capacity available before a fixed serviceability deflection limit is reached. In parallel, a strength-governed demand-capacity criterion reveals an opposite trend, in which pre-curved beams sustain a higher load before reaching the material strength limit, highlighting a genuine design trade-off between serviceability and strength limit states. In addition, large deflections promote the increasing contribution of membrane action, which influences the post-critical structural response. Parametric analyses are conducted to establish relationships between imperfection magnitude, interface stiffness, and stability limits. The obtained results provide design-oriented insights by identifying practical stability thresholds beyond which conventional linear design approaches may become unconservative. The proposed framework therefore offers a computationally efficient tool for the preliminary stability assessment and practical evaluation of hybrid timber members.

Keywords: nonlinear stability, hybrid timber beams, geometric imperfections, mechanism-based energy approach, membrane effect, interlayer slip, stability assessment, design-oriented analysis.

INTRODUCTION

The increasing demand for sustainable and high-performance structural systems has led to the rapid development of engineered timber solutions in modern construction. Among these, hybrid timber beams composed of high-stiffness outer layers and a relatively compliant core have emerged as an efficient structural configuration for medium- and long-span applications [1–3]. By combining

materials with contrasting elastic properties, such systems can optimize stress distribution and enhance load-carrying capacity while maintaining reduced self-weight [4]. However, the improved structural efficiency is often accompanied by increased slenderness, which makes these members particularly sensitive to geometric nonlinearities and stability-related phenomena [5].

The mechanical behavior of hybrid timber beams is strongly governed by interlayer

interaction and the associated slip mechanisms at the interfaces. Conventional design approaches, such as the γ -method adopted in Eurocode 5, provide a simplified means of evaluating effective bending stiffness under linear-elastic assumptions [6, 7]. While these formulations are suitable for routine design, they neglect second-order geometric effects that become significant as deflections increase. Classical studies in structural mechanics have demonstrated that the coupling between axial strains and curvature may substantially alter the load-carrying mechanism in slender members [8, 9]. In hybrid systems, this effect is further complicated by partial interaction between layers, which introduces additional nonlinearities that cannot be captured by simplified analytical models [10]. Recent studies have further emphasized the importance of incorporating geometric nonlinearity and interlayer slip in the analysis of timber composite members, particularly under large deformation regimes [11, 12].

Geometric imperfections represent another critical factor influencing the structural reliability of timber members. In practical applications, deviations from the ideal geometry arise from manufacturing tolerances, assembly inaccuracies, and environmental effects such as moisture-induced warping [13]. Even relatively small initial imperfections may trigger premature stiffness degradation and reduce the critical load capacity of slender structures, a phenomenon commonly described through knock-down effects in stability theory [14, 15]. Recent numerical and experimental investigations have demonstrated that imperfection sensitivity in timber and composite beam systems is strongly coupled with interface behavior and nonlinear load redistribution mechanisms [11, 12]. As the deformation increases, the structural response may transition from a bending-dominated regime to a membrane-activated state, in which axial tension contributes significantly to the overall resistance [16, 17].

From a computational perspective, high-fidelity finite element models can accurately capture the complex interaction between bending, membrane forces, and interlayer slip. Nevertheless, such approaches are often computationally demanding and less suitable for extensive parametric studies or practical design applications. Variational energy methods, based on the principle of minimum total potential energy, provide an efficient alternative for describing nonlinear structural behavior while retaining clear physical

interpretation [18]. Despite their potential, existing studies rarely establish a direct link between energy-based formulations and design-relevant stability limits for hybrid timber systems.

To address these limitations, this study proposes a computational framework for evaluating the nonlinear stability behavior of hybrid timber beams under realistic geometric imperfections. The formulation incorporates bending, membrane, and interlayer slip effects within a unified energy-based approach, enabling the assessment of tangent-stiffness evolution and serviceability-related stability response under increasing deformation. An energy-based indicator is further utilized to interpret the evolution of structural behavior from bending-dominated response toward membrane-influenced behavior.

Unlike existing approaches, which primarily focus on simplified numerical prediction, the present study establishes practical relationships between imperfection amplitude, interface stiffness, and stability performance relevant to design-oriented assessment. Through systematic parametric analyses, critical thresholds associated with stiffness reduction and stability deterioration are identified, providing practical insight into the imperfection-sensitive behavior of hybrid timber members. The proposed framework therefore offers both engineering relevance and computational efficiency for the preliminary stability assessment of slender hybrid timber systems.

Building upon our research group's recent investigations into engineered timber structures, spanning from the experimental evaluation of their mechanical properties and connection performance [18] to the assessment of their combustion characteristics under thermal exposure [5], as well as our newly developed computational framework for evaluating geometric nonlinearity using the γ -method [19], the present study builds on the same mechanism-based energy formulation (bending, membrane and interlayer-slip energy terms, and the mechanism index η) developed and validated for a concentrated-load reference case in our companion study [20]. Here this theoretical basis is extended to a beam under uniformly distributed load, solved directly from the governing equilibrium equation rather than through a load-amplification approximation, and reformulated around design-oriented stability criteria (Figs. 10–11) rather than the imperfection-sensitivity mapping of [20]. The present study thus

extends these efforts toward the design-oriented stability evaluation of imperfect hybrid timber beams. By doing so, it provides a computationally efficient alternative to extensive three-dimensional finite element simulations during the preliminary design stage.

THEORETICAL FRAMEWORK AND FORMULATION

Kinematic description and geometric nonlinearity

The nonlinear response of hybrid timber beams is formulated within the framework of geometrically nonlinear beam theory. To account for moderate-to-large deflections, the von Kármán strain–displacement relations are adopted, allowing the coupling between axial strains and transverse displacements to be captured [14, 15].

The total transverse displacement $w(x)$ is expressed as the superposition of the initial geometric imperfection $w_0(x)$ and the load-induced displacement $w_a(x)$, following the geometrically nonlinear beam kinematics of [14, 15]:

$$w(x) = w_0(x) + w_a(x) = e_0 \sin\left(\frac{\pi x}{L}\right) + w_a(x) \quad (1)$$

where: $w_0(x)$ represents the initial out-of-straightness, typically assumed in sinusoidal form with amplitude e_0 .

The geometric and kinematic configuration of the hybrid timber beam is illustrated in Figure 1. The figure defines the three-layer arrangement, the initial imperfection profile, the total transverse displacement, and the interlayer slip considered in the present formulation.

To account for the initial geometric imperfection, the axial strain is defined relative to the imperfect reference configuration, consistent with the imperfect-beam kinematics of [14, 15], as:

$$\varepsilon_x = \frac{du}{dx} + \frac{1}{2} \left[\left(\frac{dw}{dx} \right)^2 - \left(\frac{dw_0}{dx} \right)^2 \right] \quad (2)$$

This formulation enables the representation of membrane effects arising from large deflections, which are essential for capturing the nonlinear behavior of slender hybrid members.

Effective flexural stiffness with partial interaction

The hybrid beam is composed of stiff outer layers and a compliant core, connected through mechanical or continuous interfaces. The effective flexural stiffness EI_{eff} is evaluated using an enhanced γ -method that accounts for partial interaction between layers [6–8]:

$$EI_{eff} = \sum_i E_i I_i + \sum_i E_i I_i a_i^2 \cdot \gamma_i \quad (3)$$

where: a_i represents the distance from the centroid of each layer to the neutral axis of the composite section; γ_i represents the interaction coefficient associated with the slip stiffness of the interface, and a_{i0} denotes the distance from the centroid of each layer to the neutral axis.

This formulation allows the reduction of stiffness due to interlayer slip to be incorporated in a consistent manner.

Variational energy functional

The equilibrium state is determined by minimizing the total potential energy functional Π , which represents the internal strain energy and the potential of external loads, following the mechanism-based energy formulation of [20]:

$$\Pi = U_b + U_m + U_s - W \quad (4)$$

where: U_b – bending energy; U_m – membrane energy; U_s – interlayer slip energy; W – external work.

For the uniformly distributed load q considered in this study, W is obtained by integrating q against the assumed single-mode deflection shape $w_a(x) = (w_m - e_0) \sin(\pi x/L)$ over the span, following [20]: $W = q(w_m - e_0)(2L/\pi)$. The bending energy is defined as:

$$U_b = \frac{1}{2} \int_0^L EI_{eff} \left(\frac{d^2 w_a}{dx^2} \right)^2 dx = \frac{\pi^4 EI_{eff}}{4L^3} (w_m - e_0)^2 \quad (5)$$

The membrane energy, associated with axial stretching induced by transverse deformation and formulated following [20], is expressed as:

$$U_m = \int_0^L \frac{N_{mem}^2}{2EA} dx, \quad N_{mem} = \frac{EA}{2L} (w_m^2 - e_0^2) \cdot \frac{\pi^2}{L} \quad (6)$$

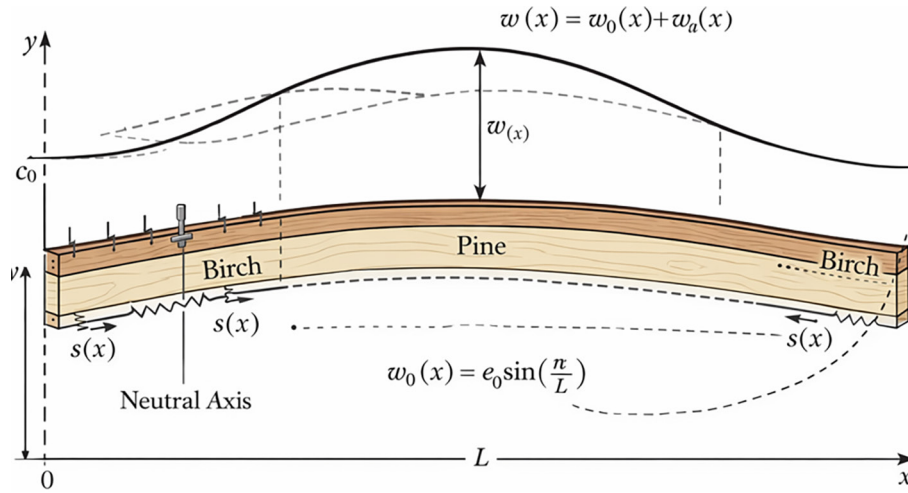


Figure 1. Geometric and kinematic configuration of the hybrid timber beam, including initial imperfection profile, transverse displacement, and interlayer slip

The slip energy accounts for the relative displacement between layers and is governed by the interface stiffness k_s :

$$U_s = \frac{1}{2} \int_0^L k_s \cdot s(x)^2 dx \quad (7)$$

It is important to clarify the fundamental methodological assumptions underpinning this energy formulation. First, both the timber layers and the mechanical fasteners are assumed to remain within the linear elastic range, meaning that energy dissipation due to material plasticity or local crushing is not considered. Second, the kinematic description strictly follows the von Kármán approximation, which is mathematically valid for moderate rotations and large transverse deflections. Finally, the interface stiffness k_s is assumed to be continuously smeared along the beam length. By explicitly coupling the bending (U_b), membrane (U_m), and slip (U_s) energy terms,

this formulation captures the nonlinear redistribution of internal forces that simplified linear design methods inherently neglect. This rigorous energy basis ensures that the subsequent stability assessment is physically consistent and robust.

Energy-based stability indicator

To quantify the transition between different load-resisting mechanisms, a dimensionless energy index η , defined following [20], is introduced:

$$\eta(P) = \frac{U_m}{U_b + U_m + U_s} \quad (8)$$

This index represents the relative contribution of membrane energy to the total internal energy of the system.

- $\eta \approx 0$: bending-dominated behavior;
- $\eta \rightarrow 1$: membrane-activated regime.

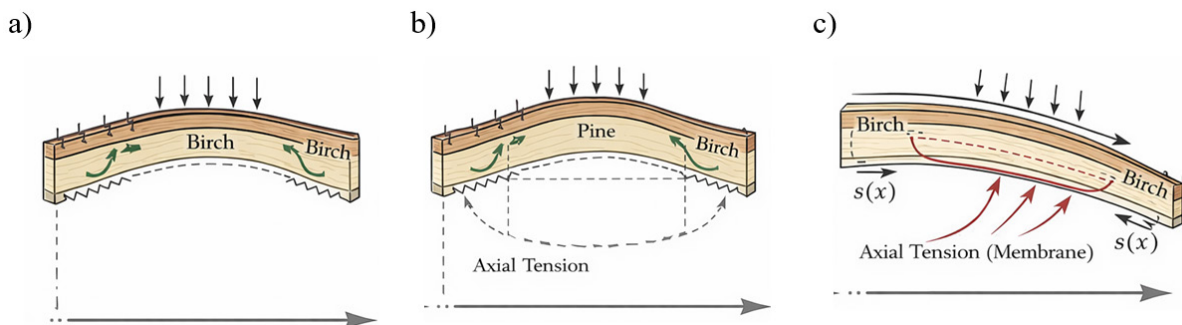


Figure 2. Schematic illustration of structural response regimes: (a) bending-dominated, (b) coupled bending–membrane, (c) membrane-activated behavior

This index serves as a key indicator for identifying the transition between structural response regimes and forms the basis for the subsequent design-oriented interpretation.

The physical interpretation of the mechanism index is schematically illustrated in Figure 2, which shows the transition from a bending-dominated response to a coupled bending–membrane mechanism and finally to a membrane-activated regime.

The evolution of η provides a physically interpretable indicator of structural response, enabling the identification of critical transition points along the nonlinear equilibrium path.

Governing equations and solution strategy

The nonlinear equilibrium equations are obtained by minimizing the total potential energy functional, following standard variational principles [16]:

$$\frac{\partial \Pi}{\partial \omega} = 0 \quad (9)$$

The resulting system of nonlinear equations is solved using an incremental–iterative procedure. At each load step, the tangent stiffness is evaluated, following [16], as:

$$K_{\tan} = \frac{\partial^2 \Pi}{\partial^2 \omega_a^2} \quad (10)$$

The solution is obtained through a Newton–Raphson scheme, ensuring convergence under large geometric nonlinearities. This approach provides a robust and computationally efficient framework for evaluating the nonlinear stability of hybrid timber beams.

NUMERICAL ANALYSIS

Reference configuration and input parameters

The numerical analysis is conducted on a reference hybrid timber beam composed of two high-stiffness face layers (Birch) and a compliant core layer (Pine). The beam is simply supported and subjected to a uniformly distributed transverse load as shown in Figure 3.

The geometric and material properties adopted in the analysis are defined as follows:

- Beam length: $L = 5.0$ m;
- Face layers (Birch): Young’s modulus $E_f = 22$ GPa, thickness $t_f = 0.02$ m;
- Core layer (Pine): Young’s modulus $E_c = 7$ GPa, thickness $t_c = 0.10$ m;
- Interlayer connection: Slip stiffness: $k_s = 0.8 \cdot 10^8$ N/m².

The beam is discretized along its span into a sufficient number of nodes to ensure convergence of displacement and stress fields. The nonlinear response is evaluated under incremental loading conditions. To investigate imperfection sensitivity, the initial geometric imperfection is assumed in sinusoidal form $w_0(x) = e_0 \cdot \sin(\pi x/L)$, where e_0 is the imperfection amplitude.

To systematically investigate the imperfection sensitivity, the following values of initial imperfection amplitude are considered: $e_0 = 2$ mm, 50 mm, and 100 mm. The selection of these specific values is grounded in practical construction tolerances and theoretical sensitivity limits for engineered timber structures. Specifically, $e_0 = 2$ mm represents a nearly perfect geometric configuration, serving as a baseline for the fundamental equilibrium path. The value of $e_0 = 50$ mm corresponds to an out-of-straightness

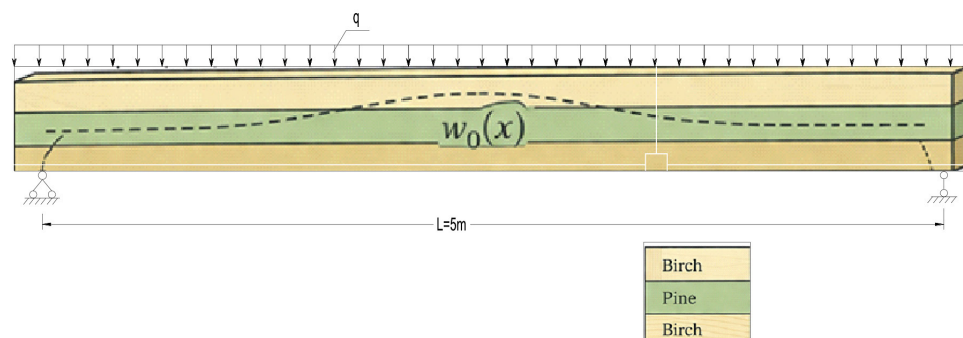


Figure 3. Reference configuration of the hybrid timber beam, including geometry, material layers, supports, loading, and initial imperfection

of $L/100$, which aligns with typical manufacturing, assembly tolerances, and long-term moisture-induced distortions commonly observed in long-span timber members. Finally, $e_0 = 100$ mm ($L/50$) is deliberately chosen to represent an extreme geometric deviation, allowing for a comprehensive investigation of the structure's stability knock-down effect and post-critical mechanisms under severe conditions.

Load–deflection response and imperfection sensitivity

The nonlinear load–deflection response of the reference hybrid timber beam is shown in Figure 4 for different imperfection amplitudes.

The results indicate that all cases follow a common hardening trend, in which the applied load increases faster than the additional deflection once membrane (catenary) action is engaged. A clear separation between the curves is observed as the imperfection amplitude increases: for a given additional deflection, beams with larger initial imperfections sustain a higher applied load, because a pre-curved member mobilizes membrane tension earlier and therefore behaves as an effectively stiffer system from the onset of loading.

In particular, the beam with $e_0 = 100$ mm reaches the reference load range within the smallest additional deflection, while the nominally straight beam ($e_0 = 2$ mm) requires substantially larger additional deflections to mobilize a comparable load-carrying capacity. This trend confirms that the nonlinear response of hybrid timber beams is highly sensitive to initial geometric deviations, even though all cases follow the same qualitative hardening pattern.

This does not imply that imperfect beams are structurally superior: because part of their total deflection budget is already consumed by the initial curvature itself, the same beams reach a fixed total-deflection serviceability limit at a correspondingly lower applied load, as quantified separately through the serviceability-governed load capacity in Figure 5. From a design perspective, the effect of geometric imperfections should therefore be assessed jointly through both the tangent-stiffness evolution (Fig. 6) and the serviceability load capacity (Fig. 5), rather than through the stiffness trend alone.

Using a representative total-deflection serviceability limit $\delta_{lim} = L/50$ (matching the paper's own definition of the extreme imperfection

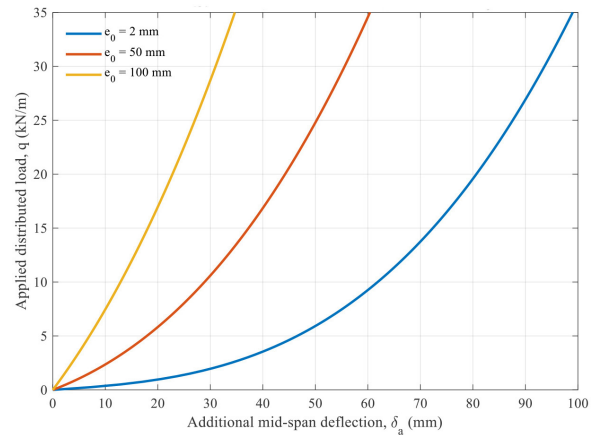


Figure 4. Load–deflection response for different imperfection amplitudes

case, $e_0 = 100$ mm), the load required to reach this limit decreases from about 34 kN/m for the near-perfect beam ($e_0 = 2$ mm) to about 25 kN/m for $e_0 = 50$ mm, and to zero for $e_0 = 100$ mm at this imperfection level, the initial curvature alone already consumes the entire serviceability deflection budget before any load is applied. This serviceability-governed trend is opposite to the strength-governed (ULS) trend shown in Figure 10, revealing a genuine design trade-off between the two limit states.

Tangent stiffness evolution and mechanism transition

The evolution of structural stiffness is evaluated through the normalized tangent stiffness ratio, as shown in Figure 5.

For small imperfections, the stiffness remains relatively stable during the initial loading stage,

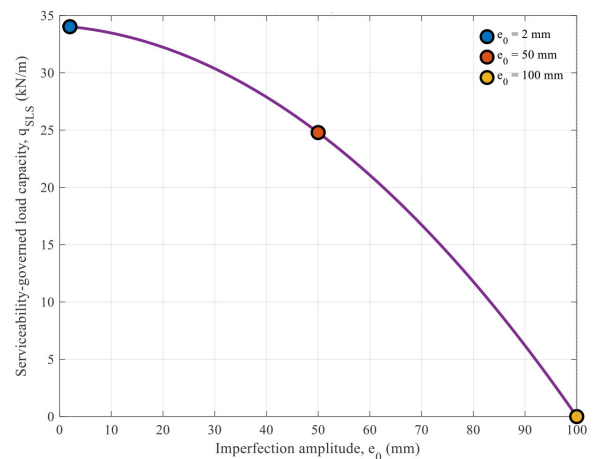


Figure 5. Serviceability-governed load capacity versus imperfection amplitude

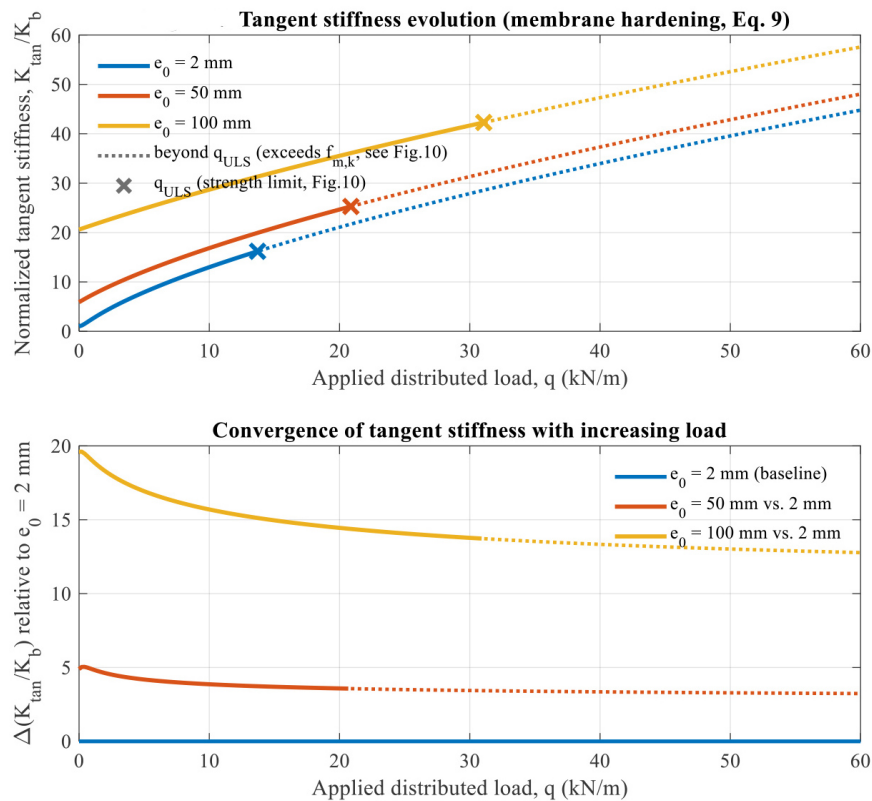


Figure 6. Normalized tangent stiffness evolution under increasing load (membrane hardening)

followed by a sharp reduction near the critical load. In contrast, large imperfections lead to an immediate and substantial decrease in stiffness, with reductions exceeding 60% observed at relatively low load levels.

The transition in load-resisting mechanisms is further characterized using the mechanism index η , as illustrated in Figure 7. For the near-perfect beam ($e_0 = 2$ mm), η starts close to zero and rises progressively as deformation increases, reflecting

the growing contribution of membrane forces. For beams with larger initial imperfections, η is already substantial at the onset of loading because the pre-existing curvature immediately mobilizes part of the membrane energy.

For large imperfection amplitudes, membrane energy becomes the dominant component before reaching the ultimate load, confirming the transition to a membrane-influenced structural regime. This behavior highlights the

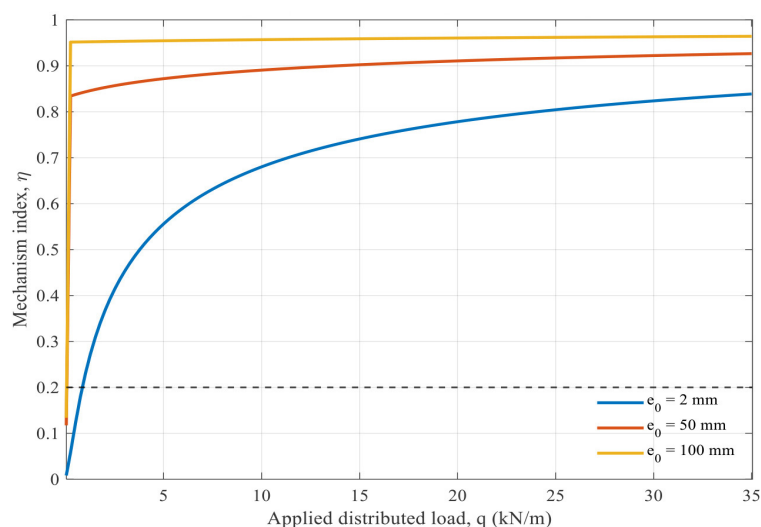


Figure 7. Evolution of mechanism index η under increasing load

importance of accounting for geometric nonlinearity in stability assessment. From a structural stability perspective, the evolution of the mechanism index η provides critical insights into the system's capacity to resist premature collapse. When $\eta \approx 0$, the structure relies almost entirely on its effective flexural stiffness, making it highly susceptible to geometric destabilization under large initial imperfections. However, as η approaches 1.0, the beam geometrically stiffens due to the activation of axial tension. This transition implies that the timber composite beam possesses a secondary post-critical reserve. For practical engineering design, monitoring η allows for the identification of the exact load threshold where conventional linear bending theory ceases to be conservative. It reveals that structures with lower interface stiffness or larger initial geometric deviations (e_0) are forced to activate this membrane mechanism earlier to maintain equilibrium, thereby accelerating the degradation of overall stability limits.

Interface behavior and stability limits

The interaction between layers is analyzed through the distribution of shear stress and interlayer slip, as shown in Figure 8 and Figure 9.

Figure 9 presents the distribution of interlayer slip along the beam. The results show a smooth

variation with maximum values near the supports and a zero crossing at mid-span, consistent with the distribution of shear force under uniform loading.

It is also observed that the magnitude of slip increases with the imperfection amplitude, indicating that larger initial geometric deviations lead to higher interfacial deformation demands.

The results indicate that:

- shear stress exhibits a peak near mid-span;
- stress gradients become steeper with increasing imperfection amplitude;
- interlayer slip increases significantly under large deformation;

These effects may lead to localized damage or delamination at the interfaces. The strength-governed load capacity is quantified independently through the demand-capacity ratio (DCR), as presented in Figure 10. In contrast to the serviceability-governed trend of Figure 5, the results show that the strength-governed load capacity increases with increasing imperfection amplitude, because a pre-curved beam requires less incremental curvature and therefore develops lower incremental bending stress to reach a given deflection, even though its membrane stress is higher. From a design perspective, this finding emphasizes that:

- allowable imperfection limits should be carefully controlled;

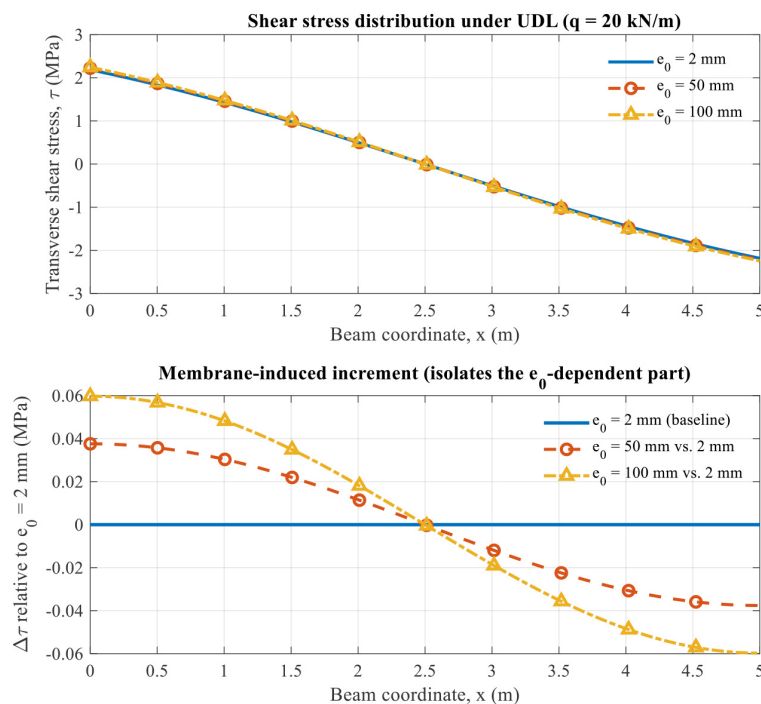


Figure 8. Transverse shear stress distribution along the beam

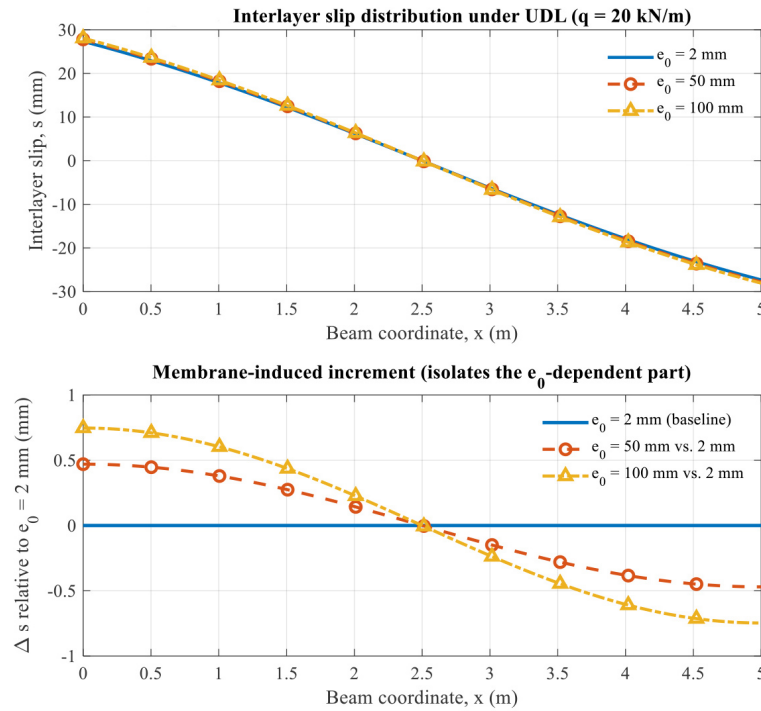


Figure 9. Interlayer slip distribution along the beam

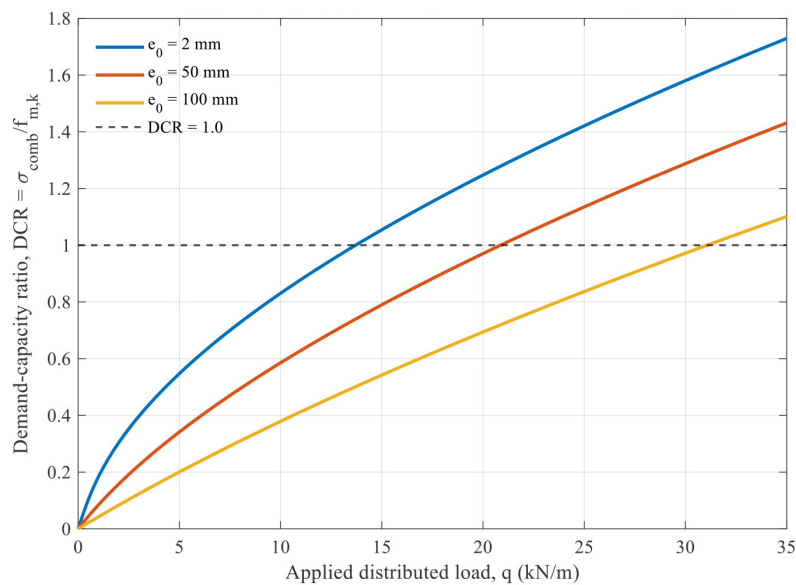


Figure 10. Demand-capacity ratio (DCR) versus applied load for varying imperfection amplitude

- interface stiffness plays a critical role in maintaining structural stability.

To further investigate the combined influence of geometric imperfections and interlayer stiffness on structural stability, a parametric analysis is conducted by varying both imperfection amplitude and slip stiffness. The resulting design-oriented stability response surface is presented in Figure 11. It illustrates the variation of the strength-governed load capacity q_{ULS} as a

function of imperfection amplitude and interface stiffness level. The results indicate a progressive increase in strength-governed load capacity with increasing imperfection amplitude, and higher interface stiffness further improves this capacity.

The combined effect demonstrates that both geometric imperfections and interface stiffness significantly influence the stability capacity of hybrid timber beams. From a practical engineering perspective, the results suggest that improving

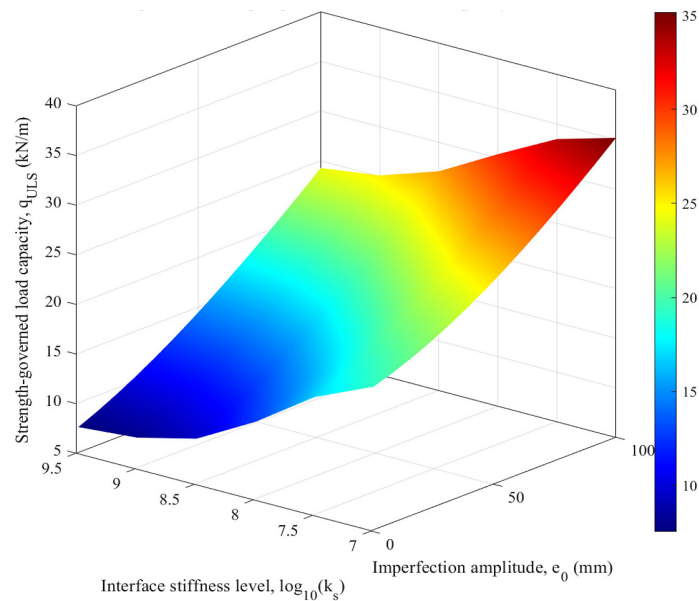


Figure 11. Design-oriented stability response surface considering imperfection amplitude and interface stiffness level

interface stiffness can partially mitigate the adverse effects associated with geometric imperfections and delay the onset of stability deterioration.

Significance and limitations of the proposed framework

The primary scientific contribution of the proposed framework lies in its ability to combine geometric nonlinearity and interlayer slip within a unified energy-based formulation. This approach overcomes several limitations of conventional linear analytical methods while providing a physically interpretable basis for stability assessment. From a practical engineering perspective, this mechanism-based approach serves as a highly efficient design-oriented tool. It enables engineers to conduct rapid parametric assessments and establish critical imperfection thresholds without resorting to computationally expensive three-dimensional continuum finite element simulations. However, the present study has certain limitations that should be acknowledged. The current formulation assumes linear elastic material behavior and focuses exclusively on short-term static loading. Complex phenomena such as long-term creep, local material plasticity, time-dependent moisture variations, and dynamic load effects are not considered. These aspects represent important boundaries for the application of the proposed model and highlight necessary directions for future comprehensive investigations.

CONCLUSIONS

This study presents a computational framework for evaluating the nonlinear stability behavior of hybrid timber beams, with particular emphasis on the combined influence of geometric imperfections and interlayer slip on structural performance. Based on the variational energy formulation and parametric analyses, the following key conclusions are drawn:

- The energy-based indicator η effectively characterizes the progressive evolution of structural behavior from bending-dominated response toward increasing membrane contribution under large deformation conditions. Large geometric imperfections (e.g., $e_0 = L/50$) accelerate this behavioral transition and significantly influence the post-critical response of the system.
- Initial geometric imperfections strongly influence the tangent stiffness from the onset of loading. Beams with larger imperfections start from a substantially higher stiffness ratio (up to about twenty times the pure-bending value for $e_0 = 100$ mm) because pre-existing curvature immediately mobilizes membrane tension, and all configurations converge toward a common membrane-dominated stiffness at higher loads. This early hardening does not remove the serviceability penalty of geometric imperfections: because part of the total deflection budget is already consumed by e_0 itself, the load required to reach a fixed total-deflection

serviceability limit ($L/50$) drops from about 34 kN/m for the near-perfect beam to zero for $e_0 = 100$ mm, the imperfection alone consumes the entire serviceability margin.

- Transverse shear stresses and interlayer slips increase nonlinearly under large geometric deviations. Nevertheless, higher interface stiffness partially mitigates these adverse effects and contributes to improved structural stability performance.

REFERENCES

1. Khorsandnia N., et al. Comparative study of analytical modelling of short and long-term behaviour in timber-concrete composite systems. World Conference on Timber Engineering, Auckland, New Zealand, 2012, 304–312.
2. Khorsandnia N., et al, Structural response of timber-concrete composite beams predicted by finite element models and manual calculations. *Advances in Structural Engineering*, 2014, 17(11), 1601–1615.
3. Brandner R., Schickhofer G., Glued laminated timber in bending: thoughts, experiments, models and verification. World Conference Timber Engineering WCTE, 2010, 1, 11.
4. Fragiaco M. Hybrid wood-based systems: State-of-the-art and future developments. Technical Report, University of Sassari, Italy, 2017, 1–45.
5. Do, T.T., et al. Experimental study on the combustion characteristics of timber structures. *Lecture Notes in Civil Engineering*. Vol. 747, Proc.of the 4th International Conference on Structural Health Monitoring and Engineering Structures, Springer Nature, 2025, 791–799. https://doi.org/10.1007/978-3-032-04645-1_92
6. Supe P., Iejavs J., Bending properties of mechanically jointed solid timber beams. *Drewno/Wood*, 2017, 60(199), 151–159 <https://doi.org/10.12841/wood.1644-3985.152.11>
7. CEN (2004). EN 1995-1-1: Eurocode 5 – Design of timber structures – Part 1-1: General – Common rules and rules for buildings. European Committee for Standardization, Brussels.
8. Marjanović M. et al. Three-dimensional stress analysis and design of cross-laminated timber panels using full-layerwise-theory-based finite element method. *Thin-Walled Structures*, 2020, 157. <https://doi.org/10.1016/j.tws.2020.107156>
9. Silva G.S. et al. Proposed correction to the Gamma method for estimating the bending stiffness of CLT panels. *Structures*, October 2024, 68, <https://doi.org/10.1016/j.istruc.2024.107214>
10. Timoshenko S.P. On the correction for shear of the differential equation for transverse vibrations of bars. *Philosophical Magazine, Series 1921*, 6, 41(245), 744–746.
11. Derikvand, M., Hosseinzadeh, S., Fink, G. Mechanical properties of dowel laminated timber beams with connectors made of salvaged wooden materials. *J. Archit. Eng.* 2021, 27. [https://doi.org/10.1061/\(ASCE\)AE.1943-5568.0000513](https://doi.org/10.1061/(ASCE)AE.1943-5568.0000513)
12. Ramage, M.H., Burrige, H., Busse-Wicher, M., Fereday, G., Reynolds, T., Shah, D.U., Wu, G., Yu, L., Fleming, P., Densley-Tingley, D., et al. The wood from the trees: The use of timber in construction. *Renew. Sustain. Energy Rev.*, 2017, 68, 333–359. <https://doi.org/10.1016/j.rser.2016.09.107>
13. Xu, B.-H., Zhang, S.-D., Zhao, Y.-H., Bouchaïr, A., Zhang, B. Mechanical properties of adhesive-free cross-laminated timber. *J. Struct. Eng.*, 2022, 148(9). [https://doi.org/10.1061/\(ASCE\)ST.1943-541X.0003446](https://doi.org/10.1061/(ASCE)ST.1943-541X.0003446)
14. Vlasov V.Z. Thin-walled elastic beams (2nd ed.). Israel Program for Scientific Translations, Jerusalem, 1961.
15. Mayo J., Domínguez J. Geometrically nonlinear formulation of flexible multibody systems in terms of beam elements: Geometric stiffness. *Computers & Structures* 1996, 59(6), 1039–1050. [https://doi.org/10.1016/0045-7949\(96\)00352-5](https://doi.org/10.1016/0045-7949(96)00352-5)
16. Crisfield M.A. Non-linear finite element analysis of solids and structures: Vol. 1 – Essentials. John Wiley & Sons, Chichester, 1991, 1–345.
17. Nguyen L.T., et al. Numerical modelling of densified wooden nails in timber assemblies using Abaqus. *Lecture Notes in Civil Engineering*, Vol. 747, Proceedings of the 4th International Conference on Structural Health Monitoring and Engineering Structures, Springer Nature, 2025, 871–879. https://doi.org/10.1007/978-3-032-04645-1_101
18. Do T.T., et al. Study on the mechanical properties of glued laminated timber members and performance of beam-column connections. *Lecture Notes in Civil Engineering*, Vol. 747, Proceedings of the 4th International Conference on Structural Health Monitoring and Engineering Structures, Springer Nature, 2025, 881–891. https://doi.org/10.1007/978-3-032-04645-1_102
19. Nguyen, L.T., Nguyen, H.S., Tran Thi, T.V. Geometric nonlinear analysis of two-layer timber composite beams with elastic connections using the γ -method in MATLAB. *International Journal of GEOMATE*, 2026, 30(140), 35–43. <https://doi.org/10.21660/2026.140.5324>
20. Nguyen, L.T., Nguyen, H.S. Nonlinear stability and imperfection sensitivity analysis of hybrid timber beams formulated via a mechanism-based energy approach. *International Journal for Computational Civil and Structural Engineering*, 22(2), 2026, 149–159. <https://doi.org/10.22337/2587-9618-2026-22-2-149-159>