

# Comparison of confidence interval models for long-term road noise indicators with a semivariance-based approach

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## ABSTRACT

This study compares confidence interval models for long-term road traffic noise indicators  $L_d$ ,  $L_e$ ,  $L_n$ , and  $L_{den}$ , including classical approaches and the author's method for constructing asymmetric intervals based on semivariance. The analysis covers intervals derived from the arithmetic mean, logarithmic mean, energy mean, Taylor series approximation, and semivariance-based constructions. The comparison was performed using daily equivalent sound level data from continuous road noise monitoring in Gdańsk over a full calendar year. Method performance was evaluated through empirical coverage of the true annual indicator values for sample sizes ranging from 5 to 50 observations using repeated random sampling. The results indicate substantial differences among the analyzed methods. Semivariance-based intervals provided more stable coverage, particularly for small samples and distributions exhibiting asymmetry or heavy tails. These findings suggest that semivariance-based confidence intervals may offer a useful tool for uncertainty assessment of long-term environmental noise indicators.

**Keywords:** road traffic noise, semivariance, uncertainty assessment, confidence interval models, long-term noise indicators, environmental acoustics, distribution asymmetry, empirical coverage.

## INTRODUCTION

The main indicators used in environmental noise protection are the long-term noise indicators  $L_d$ ,  $L_e$ ,  $L_n$ , and  $L_{den}$ , defined as equivalent sound levels determined over the entire calendar year for the daytime (6:00 a.m.–6:00 p.m.), evening (6:00 p.m.–10:00 p.m.), and night-time (10:00 p.m.–6:00 a.m.) periods, respectively [1]. The indicator  $L_{den}$  is expressed as a weighted logarithmic mean of  $L_d$ ,  $L_e$ , and  $L_n$ , according to Equation 1:

$$L_{den} = 10 \log \left( \frac{\frac{12}{24} 10^{0.1L_d} + \frac{4}{24} 10^{0.1(L_e+5)} + \frac{8}{24} 10^{0.1(L_n+10)}}{\frac{8}{24} 10^{0.1(L_n+10)}} \right) \quad (1)$$

When complete measurement data are not available, or when long-term indicators are calculated on the basis of small measurement samples, it is necessary to report not only the estimated values of the indicators but also their uncertainty.

According to GUM [2], an uncertainty interval is defined as an interval around a measurement result that is expected to encompass a large portion of the distribution of values that could reasonably be attributed to the measurand. In practice, this interval is usually expressed as a statistical confidence interval in the sense introduced by Sława-Neyman [3]. The construction of such an interval assumes either that the measurement sample follows a normal distribution or that the sample is sufficiently large for the probability distribution of the estimated mean to converge rapidly to normality. The rate of this convergence depends on several factors, most notably the skewness and kurtosis of the sample and, consequently, of the entire distribution of the measured variable.

Since sound levels are measured in decibels, the determination of uncertainty intervals for noise indicators, including long-term noise indicators, is problematic due to the specific nature of acoustic phenomena.

In environmental acoustics, in accordance with ISO 1999 [4], which concerns occupational noise, the acoustic pressure expressed in pascals is assumed to follow a log-normal distribution [5,6]. This implies a normal distribution for the random variable describing the A-weighted equivalent sound pressure level  $L_{Aeq,T}$ . Similar assumptions are also applied in road traffic noise modelling [7, 2]. In practice, however, the normal distribution adequately describes only selected cases, such as aircraft noise [8], point sources under laboratory conditions [9], or homogeneous traffic consisting of vehicles of the same type [10].

In road traffic noise measurements, deviations from distribution normality are frequently observed, resulting both from the superposition of multiple sound sources and from variability in traffic conditions over the calendar year, including fluctuations in traffic volume, changes in vehicle composition, and periodic differences in traffic flow [12–15]. Under such conditions, empirical distributions are typically asymmetric [10–12, 16, 17], which may lead to substantial discrepancies in the estimation of logarithmic means and uncertainty intervals when approximations based on the assumption of normality are applied [18].

In practice, however, it is often assumed that for sufficiently large samples the energy mean, understood as the arithmetic mean of energy levels, follows a distribution close to normal in accordance with the central limit theorem. It should be emphasized, however, that the concept of a “large sample” is not absolute and depends on the properties of the analyzed distribution, particularly its skewness and kurtosis. Studies on road traffic noise have shown that asymmetry may substantially slow the convergence of the distribution of the energy mean to normality [17].

One of the methods used to estimate the uncertainty of the energy mean is the application of Type A uncertainty, preceded by an analysis of the acoustic signal and the selection of representative events [2,19–21]. However, this procedure requires considerable effort and a detailed analysis of signal variability [1]. If the energy mean is assumed to be normally distributed, it is possible to determine a Type A uncertainty interval and then transform it logarithmically.

When the distribution of the analyzed variable exhibits significant asymmetry, classical confidence intervals based on the standard deviation may fail to provide an adequate level of coverage of the true value. An alternative approach is

the use of semivariance, that is, the separate consideration of variability on the left and right sides of the mean. This approach is based on a generalized form of Chebyshev’s theorem and allows the construction of asymmetric uncertainty intervals that reflect the characteristics of empirical distributions. As shown in [23] intervals determined using semivariance better reproduce the actual coverage of the noise indicators  $L_d$ ,  $L_e$ ,  $L_n$ , and  $L_{den}$ , particularly for small measurement samples. In contrast to classical intervals, the semivariance-based approach does not require the assumption of distribution normality or a large sample size.

From the perspective of environmental noise management, reliable estimation of uncertainty intervals is important for planning mitigation measures, allocating monitoring resources, and assessing compliance with regulatory requirements. Underestimating this uncertainty may lead to inappropriate management decisions, particularly when such decisions are based on limited or incomplete measurement data.

This paper presents an analysis of seven variants of confidence intervals for long-term noise indicators, including classical confidence intervals for energy levels, the arithmetic mean and the logarithmic mean of sound levels, a confidence interval based on Taylor series approximation, and three semivariance-based confidence intervals. In contrast to the construction proposed in [23], the semivariance-based approach was extended to three variants: energy levels, the arithmetic mean of sound levels, and the logarithmic mean. This framework enabled a comprehensive comparison of the analyzed methods under conditions of asymmetric distributions and limited sample sizes. The comparison was carried out using sound level measurement data obtained from the acoustic monitoring system of the city of Gdańsk.

## METODOLOGY

Models and methods used in practice for estimating the uncertainty of noise indicators are based on specific assumptions concerning the probabilistic nature of the measurement sample. Depending on the adopted approach, one may assume normality of the distribution of individual sound levels  $L_{A,i}$ , normality of daily noise indicators such as  $L_{d,i}$ ,  $L_{e,i}$ ,  $L_{n,i}$  and  $L_{den,i}$ , or normality of energy levels  $e_i = 10^{0.1L_{A,i}}$ . In many methods, it is also assumed that:

- the energy mean  $\bar{e} = \frac{1}{n} \sum_{i=1}^n e_i$
- the arithmetic mean of sound levels  $\bar{L} = \frac{1}{n} \sum_{i=1}^n L_{A,i}$
- and the logarithmic mean  $\bar{L}^{log} = 10 \log \left( \frac{1}{n} \sum_{i=1}^n e_i \right)$  follow a normal distribution.

Adopting any of the above assumptions leads to different constructions of confidence intervals used in the assessment of uncertainty for long-term noise indicators. The following section presents the forms of these intervals and introduces the notation necessary for their formulation and comparison.

To present the analyzed confidence interval constructions, the following notation is introduced. Long-term noise indicators are determined on the basis of daily equivalent sound levels recorded over multiple measurement days, separately for the daytime, evening, and night-time periods. Accordingly, three measurement samples are considered:

$$\{L_{d,1}, L_{d,2}, \dots, L_{d,k_d}\} \quad (2)$$

for the daytime period,

$$\{L_{w,1}, L_{w,2}, \dots, L_{w,k_w}\} \quad (3)$$

for the evening period,

$$\{L_{n,1}, L_{n,2}, \dots, L_{n,k_n}\} \quad (4)$$

for the night-time period.

On the basis of these samples, the corresponding logarithmic means, denoted by  $\bar{e}_d, \bar{e}_w, \bar{e}_n$ :

$$\begin{aligned} \bar{e}_d &= \frac{1}{k_d} \sum_{i=1}^{k_d} 10^{0,1L_{d,i}}, \bar{e}_w = \\ &= \frac{1}{k_w} \sum_{i=1}^{k_w} 10^{0,1L_{w,i}}, \bar{e}_n = \\ &= \frac{1}{k_n} \sum_{i=1}^{k_n} 10^{0,1L_{n,i}}. \end{aligned} \quad (5)$$

These quantities describe the average acoustic impact during the day, evening, and night, respectively, while the energy means are calculated as arithmetic means of the energy values corresponding to the individual sound levels.

For each of the analyzed samples, measures of variability are also determined. In the classical approach, these are variances and standard deviations of energy levels.

$$\begin{aligned} s_{\bar{e}_d}^2 &= \frac{1}{k_d - 1} \sum_{i=1}^{k_d} (e_{d,i} - \bar{e}_d)^2, \\ s_{\bar{e}_w}^2 &= \frac{1}{k_w - 1} \sum_{i=1}^{k_w} (e_{w,i} - \bar{e}_w)^2, \\ s_{\bar{e}_n}^2 &= \frac{1}{k_n - 1} \sum_{i=1}^{k_n} (e_{n,i} - \bar{e}_n)^2. \end{aligned} \quad (6)$$

In the semivariance-based approach, variability is considered separately on the left and right sides of the mean, which leads to the definitions of the lower semivariance

$$S_-^2 = \frac{1}{(n-1)^2} \sum_{x_i < \bar{x}} (x_i - \bar{x})^2 \quad (7)$$

and the upper semivariance

$$S_+^2 = \frac{1}{(n-1)^2} \sum_{x_i > \bar{x}} (x_i - \bar{x})^2 \quad (8)$$

Semivariances may be calculated from different types of samples, including sound level samples, energy level samples, or other derived quantities, depending on the adopted interval construction. This way of describing variability makes it possible to capture distribution asymmetry and forms the basis for constructing asymmetric uncertainty intervals. The following section presents the formulations of uncertainty intervals for the respective constructs.

### Classical confidence interval for the arithmetic mean of sound levels

The following construction corresponds to the assumption that the long-term noise indicator is normally distributed. Under this assumption, the noise indicator is represented by its arithmetic mean [22].

$$\left( \bar{L}_{Aeq} - t_{1-\frac{\gamma}{2}; n-1} \frac{s_{Aeq}}{\sqrt{n-1}}, \bar{L}_{Aeq} + t_{1-\frac{\gamma}{2}; n-1} \frac{s_{Aeq}}{\sqrt{n-1}} \right) \quad (9)$$

When the distribution of the noise indicator is asymmetric, an alternative uncertainty interval may also be adopted [22], due to the properties of decibel arithmetic and the difference between the logarithmic mean and the arithmetic mean:

$$\left( \begin{array}{c} \bar{L}_{Aeq}^{log} - t_{1-\frac{\gamma}{2};n-1} \frac{S_{Aeq}}{\sqrt{n-1}} \\ \bar{L}_{Aeq}^{log} + t_{1-\frac{\gamma}{2};n-1} \frac{S_{Aeq}}{\sqrt{n-1}} \end{array} \right) \quad (10)$$

where the coverage factor  $t_{1-\frac{\gamma}{2};n-1}$  is determined by the quantile of Student's t-distribution.

### Classical confidence interval for energy levels

It is also assumed in the literature that the mean of energy levels converges to a normal distribution due to the additivity of energy levels [20]. In this case, the uncertainty interval is given by [22]:

$$\left( \begin{array}{c} 10\log \left( \bar{e} - t_{1-\frac{\gamma}{2};n-1} \frac{S_{\bar{e}}}{\sqrt{n-1}} \right) \\ 10\log \left( \bar{e} + t_{1-\frac{\gamma}{2};n-1} \frac{S_{\bar{e}}}{\sqrt{n-1}} \right) \end{array} \right) \quad (11)$$

where: the coverage factor is again given by the quantile of Student's t-distribution,  $t_{1-\frac{\gamma}{2};n-1}$ .

In Equation 12, both the mean energy levels and the corresponding standard deviations for the noise indicators are given by Equations 6 and 7, respectively.

### Confidence interval based on the Taylor series approximation

Assuming normality of the sample of sound levels, the uncertainty can be determined by approximating the logarithmic mean with the arithmetic mean [18]:

$$\begin{aligned} L_{Aeq} &= 10\log \left( \frac{1}{n} \sum_{i=1}^n 10^{0.1L_{A,i}} \right) \approx \\ &\approx \bar{L} + 0.115S_L^2 \end{aligned} \quad (12)$$

where:

$$\bar{L} = \frac{1}{n} \sum_{i=1}^n L_{A,i}, \quad (13)$$

$$S_L = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (L_{A,i} - \bar{L})^2} \quad (14)$$

The expanded uncertainty for  $L_{Aeq}$  is then determined from the following expression:

$$U = \pm \sqrt{\frac{S_L^2}{n} + \frac{0.026S_L^4}{n-1}} t_{1-\frac{\gamma}{2};n-1} \quad (15)$$

### Semivariance-based confidence interval for energy levels

If, in uncertainty interval (12), the standard deviations are replaced by their equivalents in the form of two semivariances, a semivariance-based confidence interval for energy levels is obtained [23]. The construction of this interval does not require prior knowledge of the sample probability distribution, since by its nature it accounts for the asymmetry of the measurement sample. In this case, the confidence interval is expressed as follows:

$$\left( \begin{array}{c} 10\log \left( \bar{e} - t_{1-\frac{\gamma}{2};n-1} \frac{2S_{Aeq,-}}{\sqrt{n-1}} \right) \\ 10\log \left( \bar{e} + t_{1-\frac{\gamma}{2};n-1} \frac{2S_{Aeq,+}}{\sqrt{n-1}} \right) \end{array} \right) \quad (16)$$

where:  $t_{1-\frac{\gamma}{2};n-1}$  is again given by the quantile of Student's t-distribution.

### Semivariance-based confidence interval for decibel levels

Analogously to previous Sections, if the standard deviations in the uncertainty intervals are replaced by their semivariance-based counterparts, the following uncertainty interval constructions are obtained:

$$\left( \begin{array}{c} \bar{L}_{Aeq} - t_{1-\frac{\gamma}{2};n-1} \frac{2S_{Aeq,-}}{\sqrt{n-1}} \\ \bar{L}_{Aeq} + t_{1-\frac{\gamma}{2};n-1} \frac{2S_{Aeq,+}}{\sqrt{n-1}} \end{array} \right) \quad (17)$$

$$\left( \begin{array}{c} \bar{L}_{Aeq}^{log} - t_{1-\frac{\gamma}{2};n-1} \frac{2S_{Aeq,-}}{\sqrt{n-1}} \\ \bar{L}_{Aeq}^{log} + t_{1-\frac{\gamma}{2};n-1} \frac{2S_{Aeq,+}}{\sqrt{n-1}} \end{array} \right) \quad (18)$$

### Measurement data

The measurement data consisted of daily noise indicator values collected in 2016 from two

monitoring stations of the Gdańsk urban noise monitoring system. The first station, numbered 117, is located at 2–3 Wincentego Pola Street. It is mounted on the façade of a building situated along a dual-carriageway road separated by a green belt with a tram track (Figure 1). Details of this monitoring station can be found in [24] (Tables 1, 2).

The second monitoring station, numbered 145, is located at 3 Maja 3 Street. This station is also mounted on the façade of a building situated at the intersection of a dual-carriageway road separated by a tram track and another dual-carriageway road (Figure 2).

## Experimental procedure

Based on data obtained from road traffic noise measurements carried out in the city of Gdańsk, a simulation experiment was conducted to evaluate the performance of the analyzed confidence intervals. For each of the considered indicators, namely  $L_d$ ,  $L_e$ ,  $L_n$ , and  $L_{den}$ , a reference value was determined on the basis of the complete set of annual data obtained for the two monitoring stations numbered 117 and 145.

In the next stage, samples of size 5, 7, 10, 20, 30, and 50 observations were randomly drawn from the available data. For each sample,



**Figure 1.** Street view of the location of monitoring station No. 117 (source: Google Street View)

**Table 1.** Descriptive statistics of the noise indicators for the full calendar year for station No. 117

| Parameter     | Min/max value [dB] | Mean [dB] | Standard deviation [dB] | Median [dB] | Skewness | Kurtosis |
|---------------|--------------------|-----------|-------------------------|-------------|----------|----------|
| $L_d$ 117     | 73.2/91.8          | 79.6      | 2.1                     | 79.4        | 1.1      | 6.5      |
| $L_e$ 117     | 73.1/86.8          | 79.0      | 1.3                     | 78.8        | 0.5      | 3.9      |
| $L_n$ 117     | 69.8/79.4          | 73.8      | 1.4                     | 73.7        | 0.3      | 1.3      |
| $L_{den}$ 117 | 77.9/90.4          | 82.3      | 1.4                     | 82.2        | 1.2      | 6.8      |

**Table 2.** Descriptive statistics of the noise indicators for the full calendar year for station No. 145

| Parameter     | Min/max value [dB] | Mean [dB] | Standard deviation [dB] | Median [dB] | Skewness | Kurtosis |
|---------------|--------------------|-----------|-------------------------|-------------|----------|----------|
| $L_d$ 145     | 65.6/79.6          | 72.0      | 2.3                     | 71.8        | 0.1      | -0.2     |
| $L_e$ 145     | 64.6/79.4          | 71.1      | 2.5                     | 71.2        | 0.0      | -0.7     |
| $L_n$ 145     | 60.1/74.3          | 66.0      | 2.5                     | 65.9        | 0.4      | 0.1      |
| $L_{den}$ 145 | 69.6/81.7          | 74.6      | 2.2                     | 74.5        | 0.2      | -0.4     |





**Figure 2.** Street view of the location of monitoring station No. 145 (source: Google Street View)

uncertainty intervals were constructed according to the corresponding formulas, i.e. (10), (11), (12), (16), (17), (18), and (19). The sampling and interval construction procedure was repeated 10,000 times, and in each repetition it was checked whether the resulting interval contained the reference value of the given indicator.

On this basis, the empirical coverage level was determined for the individual methods and for all analyzed sample sizes, namely 5, 7, 10, 20, 30, and 50 observations. The obtained results made it possible to compare the performance of the analyzed methods as a function of sample size and to assess their ability to achieve coverage of the true value close to the nominal confidence level. This, in turn, allowed the identification of those confidence interval constructions that most accurately reflect uncertainty in terms of containing the true value, particularly for asymmetric distributions and limited sample sizes.

## RESULTS AND DISCUSSION

As a result of the conducted experiment, the following coverage values of the true value by the confidence intervals were obtained. For the presentation of the results, the following notation is introduced:

- CE – classical confidence interval for energy levels, defined by Equation 11
- SE – semivariance-based confidence interval for energy levels, defined by Equation 12

- T – confidence interval based on the Taylor series approximation, defined by Equation 16
- CA – classical confidence interval for the arithmetic mean, defined by Equation 10
- CL – classical confidence interval for the logarithmic mean, defined by Equation 17
- SA – semivariance-based confidence interval for the arithmetic mean, defined by Equation 18
- SL – semivariance-based confidence interval for the logarithmic mean, defined by Equation 19

For monitoring station No. 117, the results are presented in Tables 3–6. The results presented in Tables 3–9 are expressed in decibels [dB].

For monitoring station No. 145, the obtained results are presented in Tables 7–10.

The analysis shows that the classical confidence intervals for the arithmetic mean (CA) and for energy levels (CE) exhibit a strong tendency to underestimate the level of uncertainty, particularly for larger sample sizes and/or asymmetric data distributions. Their coverage often deviates substantially from the expected 95% confidence level, falling below 80% and, in some cases, even below 60%. This effect results from the shift between the logarithmic mean and the arithmetic mean, which leads to a progressive decrease in coverage as the sample size increases. A similar phenomenon was reported in [25]. Likewise, the confidence interval based on the Taylor series approximation (T) shows a decline in performance with increasing sample size. Although the coverage remains relatively high for  $n=5$  or  $n=7$ , it decreases sharply for  $n=50$ , often reaching only

**Table 3.** Coverage values of the true value of the Ld indicator by uncertainty intervals for station No. 117

| n  | Ld    |       |       |       |        |       |       |
|----|-------|-------|-------|-------|--------|-------|-------|
|    | Ce    | Se    | T     | CA    | CL     | SA    | SL    |
| 5  | 84.45 | 90.7  | 87.25 | 87.49 | 99.92  | 88.35 | 94.65 |
| 7  | 85.51 | 90.91 | 86.14 | 85.00 | 99.96  | 87.23 | 95.21 |
| 10 | 83.23 | 89.53 | 80.75 | 78.04 | 99.99  | 82.60 | 95.37 |
| 20 | 79.30 | 86.6  | 66.51 | 62.94 | 100.00 | 72.59 | 94.23 |
| 50 | 77.19 | 82.97 | 41.28 | 35.59 | 100.00 | 65.46 | 90.68 |

**Table 4.** Coverage values of the true value of the Le indicator by uncertainty intervals for station No. 117

| n  | Le    |       |       |       |       |       |       |
|----|-------|-------|-------|-------|-------|-------|-------|
|    | Ce    | Se    | T     | CA    | CL    | SA    | SL    |
| 5  | 92.32 | 95.45 | 92.31 | 92.37 | 96.3  | 92.79 | 96.37 |
| 7  | 92.44 | 95.94 | 92.77 | 91.71 | 96.43 | 93.71 | 97.12 |
| 10 | 92.96 | 96.34 | 91.13 | 91.32 | 96.45 | 92.77 | 97.36 |
| 20 | 93.97 | 97.24 | 88.60 | 88.07 | 96.02 | 92.37 | 98.13 |
| 50 | 94.35 | 97.78 | 78.45 | 77.82 | 93.15 | 91.84 | 98.67 |

**Table 5.** Coverage values of the true value of the Ln indicator by uncertainty intervals for station No. 117

| n  | Ln    |       |       |       |       |       |       |
|----|-------|-------|-------|-------|-------|-------|-------|
|    | Ce    | Se    | T     | CA    | CL    | SA    | SL    |
| 5  | 93.63 | 97.31 | 94.03 | 94.68 | 98.25 | 94.32 | 98.10 |
| 7  | 94.13 | 97.98 | 94.05 | 93.76 | 98.16 | 94.49 | 98.46 |
| 10 | 93.8  | 98.09 | 93.18 | 93.32 | 98.35 | 94.61 | 98.94 |
| 20 | 94.58 | 98.46 | 90.61 | 90.18 | 98.1  | 94.18 | 99.11 |
| 50 | 95.69 | 98.76 | 84.31 | 82.99 | 96.62 | 95.23 | 99.53 |

**Table 6.** Coverage values of the true value of the Lden indicator by uncertainty intervals for station No. 117

| n  | Lden  |       |       |       |       |       |       |
|----|-------|-------|-------|-------|-------|-------|-------|
|    | Ce    | Se    | T     | CA    | CL    | SA    | SL    |
| 5  | 91.64 | 95.77 | 92.73 | 92.26 | 96.55 | 93.39 | 97.02 |
| 7  | 91.34 | 96.21 | 91.8  | 91.23 | 96.88 | 92.94 | 97.58 |
| 10 | 91.6  | 96.32 | 90.15 | 89.09 | 96.41 | 91.90 | 97.75 |
| 20 | 89.78 | 95.52 | 83.44 | 82.18 | 95.02 | 88.79 | 97.95 |
| 50 | 88.4  | 94.9  | 70.62 | 70.18 | 89.33 | 85.95 | 97.94 |

**Table 7.** Coverage values of the true value of the Ld indicator by uncertainty intervals for station No. 145

| n  | Ld    |       |       |       |       |       |       |
|----|-------|-------|-------|-------|-------|-------|-------|
|    | Ce    | Se    | T     | CA    | CL    | SA    | SL    |
| 5  | 90.55 | 94.29 | 92.4  | 91.38 | 99.55 | 92.91 | 96.62 |
| 7  | 92.07 | 94.75 | 91.84 | 90.43 | 99.71 | 93.62 | 97.02 |
| 10 | 92.7  | 95.35 | 89.88 | 88.27 | 99.93 | 93.61 | 97.49 |
| 20 | 93.71 | 96.29 | 83.13 | 79.96 | 100   | 93.97 | 98.49 |
| 50 | 94.88 | 97.23 | 59.43 | 53.08 | 100   | 94.39 | 99.03 |

**Table 8.** Coverage values of the true value of the Le indicator by uncertainty intervals for station No. 145

| n  | Le    |       |       |       |       |       |       |
|----|-------|-------|-------|-------|-------|-------|-------|
|    | Ce    | Se    | T     | CA    | CL    | SA    | SL    |
| 5  | 91.86 | 94.85 | 93.30 | 92.52 | 96.04 | 93.83 | 96.53 |
| 7  | 92.90 | 95.41 | 92.34 | 91.04 | 95.81 | 94.29 | 97.17 |
| 10 | 93.22 | 95.87 | 90.69 | 87.91 | 95.41 | 94.57 | 98.06 |
| 20 | 94.17 | 96.87 | 82.25 | 77.72 | 92.67 | 94.63 | 98.8  |
| 50 | 95.51 | 97.89 | 55.64 | 48.95 | 79.3  | 95.59 | 99.45 |

**Table 9.** Coverage values of the true value of the Ln indicator by uncertainty intervals for station No. 145

| n  | Ln    |       |       |       |       |       |       |
|----|-------|-------|-------|-------|-------|-------|-------|
|    | Ce    | Se    | T     | CA    | CL    | SA    | SL    |
| 5  | 89.82 | 93.23 | 91.85 | 90.80 | 94.95 | 92.49 | 95.88 |
| 7  | 90.31 | 93.49 | 89.64 | 88.40 | 94.62 | 92.07 | 96.34 |
| 10 | 90.83 | 94.07 | 87.52 | 84.35 | 93.23 | 92.32 | 96.91 |
| 20 | 90.84 | 94.34 | 77.51 | 73.61 | 88.70 | 91.01 | 97.31 |
| 50 | 92.84 | 95.59 | 49.29 | 43.88 | 70.17 | 90.90 | 98.03 |

**Table 10.** Coverage values of the true value of the Lden indicator by uncertainty intervals for station No. 145

| n  | Lden  |       |       |       |       |       |       |
|----|-------|-------|-------|-------|-------|-------|-------|
|    | Ce    | Se    | T     | CA    | CL    | SA    | SL    |
| 5  | 92.42 | 94.78 | 92.84 | 91.86 | 95.35 | 93.27 | 96.03 |
| 7  | 92.91 | 95.02 | 92.43 | 90.88 | 95.34 | 94.19 | 96.98 |
| 10 | 93.05 | 95.52 | 90.8  | 89.07 | 94.97 | 94.37 | 97.59 |
| 20 | 94.08 | 96.88 | 83.96 | 80.61 | 92.41 | 94.04 | 98.31 |
| 50 | 95.24 | 97.91 | 61.68 | 56.9  | 81.35 | 94.29 | 98.91 |

40–60%. This method appears to be particularly sensitive to discrepancies between the arithmetic and logarithmic means.

Classical confidence intervals for the logarithmic mean (CL) provide very high coverage, often close to 100% for all sample sizes considered.

The semivariance-based intervals, SE, SA, and SL, are characterized by a very high and stable level of coverage regardless of sample size. In particular, the SL and SE intervals exhibit an optimal balance between interval width and accuracy, with coverage fluctuating around the assumed 95% level, even for small samples ( $n=5$ ).

Special attention should be paid to the relationship between the coverage levels of the CL and SL intervals. In the case of the Ld indicator for station No. 117, the classical interval for the CL achieved coverage almost equal to 100% for all analyzed sample sizes, thereby exceeding

the semivariance-based interval SL. This phenomenon should not be interpreted as evidence of higher accuracy of the classical method, but rather as a consequence of its excessively conservative nature. For this indicator, the empirical distribution exhibited marked right-skewness (skewness 1.1) and elevated kurtosis (6.5), which increased the classical standard deviation due to the influence of observations from the right tail of the distribution. As a result, the CL interval, although centered around the logarithmic mean, became excessively wide and therefore provided coverage substantially exceeding the nominal 95% confidence level.

A similar, although weaker, effect was also observed for station No. 145. In the case of the Ld indicator, the coverage values for the CL interval were also very high and, for larger sample sizes, approached 100%, despite the fact that the



empirical distribution in this case was much less asymmetric (skewness 0.1, kurtosis -0.2). This indicates that, also for this station, the classical logarithmic interval tended to yield a conservative uncertainty estimate, although in this case the effect was no longer driven by such a strong influence of extreme observations as for station No. 117.

At the same time, it should be emphasized that skewness and kurtosis alone do not provide a sufficient explanation for the observed relationships between the CL and SL intervals. This is confirmed by the example of the  $L_{den}$  indicator for station No. 117, which exhibited even greater skewness (1.2) and kurtosis (6.8) than  $L_d$ , yet in this case the SL interval, rather than CL, achieved higher coverage. This indicates that the relationship between the performance of the two methods is determined not only by the general shape of the distribution, but also by the position of the true value relative to the estimator, the effect of logarithmic transformation on the interval width, and the behavior of the estimators under small-sample conditions. It also means that classical descriptive measures of distribution shape, although useful for interpretation, should not be treated as the sole determinants of coverage level.

## CONCLUSIONS

The aim of the analysis was to evaluate the effectiveness of different methods for constructing uncertainty intervals for long-term noise indicators ( $L_d$ ,  $L_e$ ,  $L_n$ ,  $L_{den}$ ) based on measurement samples of varying sizes, ranging from 5 to 50 observations.

Among all the analyzed approaches, the semivariance-based intervals SE, SA, and SL demonstrated the highest effectiveness. These methods maintained a high and stable coverage level, close to the nominal 95% confidence level, regardless of sample size, including very small samples ( $n=5$ ). Their main advantage lies in the ability to account for asymmetry and heavy-tailed distributions without requiring assumptions of normality or large sample size. The semivariance-based uncertainty intervals presented in this study represent a new approach to interval construction.

The analysis also shows that the classical confidence intervals for the arithmetic mean (CA) and for CE, although commonly used in practice, tended to systematically underestimate uncertainty, especially for larger samples.

This suggests that relying solely on the standard deviation and the assumption of normality may lead to incorrect conclusions in the assessment of long-term noise indicators.

From a management perspective, the proposed approach is important because it supports more reliable decision-making in environmental noise assessment. More accurate uncertainty intervals reduce the risk of underestimating or overestimating noise exposure, which may directly affect the prioritization of mitigation measures, allocation of public resources, planning of monitoring campaigns, and evaluation of compliance with environmental requirements. As a result, the method may provide managers and decision-makers with a more robust basis for planning, risk assessment, and evidence-based environmental policy.

The results clearly support the application of the new semivariance-based methods proposed by the author for constructing uncertainty intervals for environmental noise indicators. These methods not only provide higher coverage levels but also better reflect the actual empirical properties of acoustic data and do not require assumptions regarding the form of the probability distribution.

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