Advances in Science and Technology Research Journal

Advances in Science and Technology Research Journal, 2026, 20(8), 201–212
https://doi.org/10.12913/22998624/220804
ISSN 2299-8624, License CC-BY 4.0

Received: 2026.03.07
Accepted: 2026.06.01
Published: 2026.07.01

A theoretical model of a collision between a car and a car-trailer set

Piotr Jan Aleksandrowicz[1*], Jarosław Zalewski[2], Iwo Aleksandrowicz[3]

[1] Bydgoszcz University of Science and Technology, Faculty of Mechanical Engineering; Al. prof. S. Kaliskiego 7, 85-796, Bydgoszcz, Poland
[2] Warsaw University of Technology, Pl. Politechniki 1, 00-661 Warsaw, Poland
[3] ALMOTIVE, Zakole 9, 86-022, Borówno, Poland
* Corresponding author e-mail: p.aleksandrowicz@pbs.edu.pl

ABSTRACT

In this paper, a theoretical model of a collision between a car and a car with a trailer was considered. To properly model the phenomena occurring in this case, the principles of mechanics of bodies with variable mass were used, specifically in case of trailer unhooking and disconnecting from the car. When such a set as a car-trailer is decomposed into two bodies, the loss of mass can be assumed. In the given case, the loss of mass related to the car after the trailer separated from it because of the collision. The mass of the trailer was considered as a part of the mass disconnecting from the body taking part in a collision. Of course, the mass of the trailer was assumed to be much lower than the mass of the car. The aim of this paper was to analyze the potential applicability of collision mechanics with variable mass to motor vehicle collision analysis in general. From the mechanical point of view, e.g. the Meschersky equation can be used as one of the main tools to solve the problem discussed.

Keywords: car accidents, mechanics of bodies with variable mass, motor vehicle collision.

INTRODUCTION

Multiple attempts to analyze the phenomena of a crash between the vehicles in road traffic have been made so far. One of the concerns was the ability to create a mathematical model of the phenomena occurring during a road collision. As the potential scenarios in road traffic are varied, a mathematical description seems essential to, for example, properly reconstruct a road accident to point out the cause of it. In addition, a mathematical model of a collision does not need to be universal, since each road scenario can be different. Therefore, such a mathematical model should be easily transformable or adaptable to a specific case.

The reviewed bibliography shows that the research on road traffic accidents covers different areas, such as public health, social aspects, technical factors, methodological approaches, and more. For this paper, an analysis of the up-to-date selected works combining the state of knowledge was prepared and divided into the following scopes.

One of them is the ability to scale the problem of road accidents and fit it into a perspective of public health issues.

The works [1] and [2] highlight road traffic injuries and fatalities, which are often belittled, as a global health issue. Especially in [1], the authors emphasized the impact of effects of road accidents on both the low- and the middle-developed countries. In [2] a review of causes of the road accident along with their measures was prepared, including selected factors based on human, environmental, and vehicle dependencies.

A comparative view of both accident risks and safety challenges in high developed and low developed countries were presented in [3] where the authors, among others, compared the accident prediction techniques and congestion management for the discussed types of countries.

The research of this kind conducted by selected authors provides the global approach towards the importance of road accidents as a problem requiring a multidisciplinary approach.

The second major issue is the causes and the main factors of road accidents. Here, selected works dealt with the so-called infrastructure and environmental factors, such as the blackspots [4] and their influence on accidents, with the infrastructure, human behavior, and traffic conditions as additional factors. Other issues related to road accidents are the weather conditions.

The forecasts regarding the number of accidents in Poland with the use of the weather condition forecast models were presented in [5], showing the relations between the weather conditions and frequency of road accidents.

Accident duration and spatial distribution on the example of Houston was analyzed in [6]. The results showed how location and statistical patterns may influence both traffic recovery and congestion. On the other hand, urban accident behavior was examined in [7], considering spatial and temporal patterns of road accidents and their potential connection to dynamics in urban traffic.

Third main area included in this analysis of knowledge considers the causes related to vehicle motion.

In [8, 9], a review of the potential vehicle systems failures and the influence of the technical condition of braking systems as causes of road accidents was presented, while a vehicle-trailer stability in multiple configurations was examined in [10], showing the potential dependence of an accident scenario on the altering spread of load distribution in a trailer.

Crashworthiness is one of the main features of road vehicles in terms of passive safety. Therefore, for example, in [11, 12] mathematical models for assessment of vehicle crashworthiness were reviewed, with the advances in vehicle safety modeling and simulation considered.

Structural damage in thin-walled components during vehicle collisions were analyzed in [13] with implications for design of more crash protective structures which, combined with the scopes presented in this section indicates the complex nature of road accidents and the interaction of the components involved, i.e., the road infrastructure, the environment, vehicles and behavior of road users.

The next important issue is mathematical modeling of road collisions, presented e.g., in [14] (variable mass dynamics), [15] (equation for generalized variable mass systems) and [16] (equation of motion for variable mass systems) from the theoretical point of view. Also essential in this paper, as one of the main approaches, will be to treat the trailer as a mass separating during a collision.

Potential post-impact vehicle motion control was discussed in [17] with stability and safety as the general direction of approach to post collision analysis. The above-mentioned works combine fundamental mechanics with the applied vehicle dynamics.

Although the vehicle collision was properly described before, there is still a trend to implement more advanced dynamics, especially for mathematical modeling, to better understand road collisions and, above all post collision behavior of the vehicles involved. Therefore, in [18] some explainable machine learning models for accident prediction were used, while the authors of [19] applied deep learning for crash detection possibility.

Coordinated control systems which could be used for active collision avoidance were proposed in [20], and in [21] a study of the effective reduction of operational accidents, including case studies on vehicle safety management was presented. This approach shows the attempts to use AI support and data-driven methods for collision analysis and prevention.

Finally, in the general approach towards road collisions is the mechanics of injuries and the resulting conclusions for further safety development. For example, in [22] driver injuries in rear collisions with various impact conditions were examined. Such an approach relates biomechanics and crash severity to road accident modeling.

Various attempts are either easy or complex and sometimes require knowing the additional factors, such as the trigonometrical dependencies or even a combination of several factors that may have influenced the course of a specific road collision and its consequences, e.g. the influence on traffic safety, even if momentarily [23].

The aim of this paper was to analyze whether it is possible to include the equation of variable mass (in this case the Meschersky equation) into a collision model specifying the scenario of car to car with trailer crash. The specific of this approach is to assume that the trailer disconnected during the discussed collision. Such attempt can become a starting point to analyze such examples as potentially life threatening.

The originality of this approach lies in mixing the variable mass mechanics with the theory of road collision analysis. It is based on its simplicity which can be useful especially when evaluating the simulation results with the use of analytical calculations. At the same time, such approach can be modified and complicated to obtain more specific results, for example when three-dimensional motion would be considered.

MATERIALS AND METHODS

General assumptions

To describe a mathematical model of an impact between two vehicles, one of which is towing a trailer, the following assumptions were made. The collision occurred on a flat road without any slants and is regarded as a two-dimensional model with vehicles moving on a road plane. The collision was frontal, oblique and eccentric. Both vehicles involved were passenger cars and one of them was towing a trailer which detached from it due to the collision. At first the vehicle and the trailer were considered one body having a mass consisting of a vehicle and a trailer combined. The mass of a vehicle was assumed as greater than that of a trailer. The discussed collision is presented in Figure 1.

When the trailer detached, its mass was considered as a particle (smaller amount of mass) that separated from the vehicle (greater amount of mass) which allowed using the equation for the variable mass systems. The vehicles were regarded as quasi stiff bodies that could deform but could not lose their parts, except for the detaching trailer. The suspension of both vehicles and the trailer did not play any role in the discussed phenomena, because the collision took place on a flat road and no vibrations were assumed. The collision took place on a clean and dry road surface with a coefficient of adhesion 0.8. The global *Oxy* coordinate system was adopted as an inertial system with respect to which moved the *O'nt* system located in the so-called geometric center of a collision, i.e. a point of the initial contact between two vehicles *O'* (Figure 2). The non-inertial coordinate system *O'nt* two consisted of two coordinates: *n* (normal) and *t* (tangential) along the axes *O'n* and *O't* respectively, as the vehicles were moving on a road plane (Figure 2). The *O'nt* system was selected in an easy way, so that the tangential axis *O't* would mark the mutual plane of a collision and the normal axis *O'n* would pass through the center of mass of the vehicle no 1. The components of the impulse of the collision forces were assumed to lay along the normal *O'n* and the tangential *O't* directions, separately for each vehicle. Normal impulse components S_n were oriented contrary to the normal velocities, while the tangential impulse components S_t were oriented positively for the vehicle no. 1, and negatively for the vehicle no. 2 (Figure 3). The external forces and moments resulting from a towed trailer were not considered, because during the collision the vehicle-trailer set was considered as one body having a common mass. The pitch angles of both the wheels and the body as well as any impulses of the additional momentary forces that might have occurred during the collision were neglected.

The presented model of road collision has its limitations resulting from the fact that momentary phenomena have been discussed and the use of a variable mass equation was the primary aim of this paper. However, to specify the problem discussed here, these limitations include:

- lack of the suspension effect which, in the authors' opinion, would be of a crucial matter if the post impact phenomena and the effect of such collision were discussed;
- the constant value of a coefficient of friction which, for the momentary nature of the discussed problem, does not play an essential role, but in a matter of a collision duration, i.e. pre-collision and post-collision motion, could

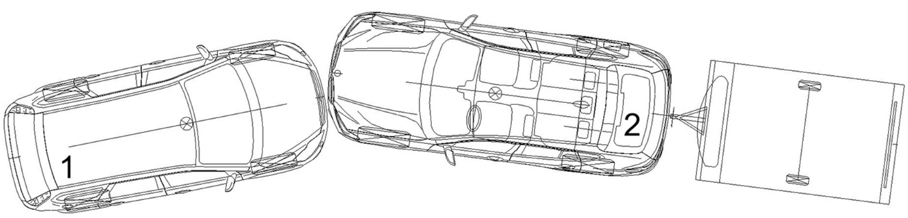

Figure 1. The discussed example

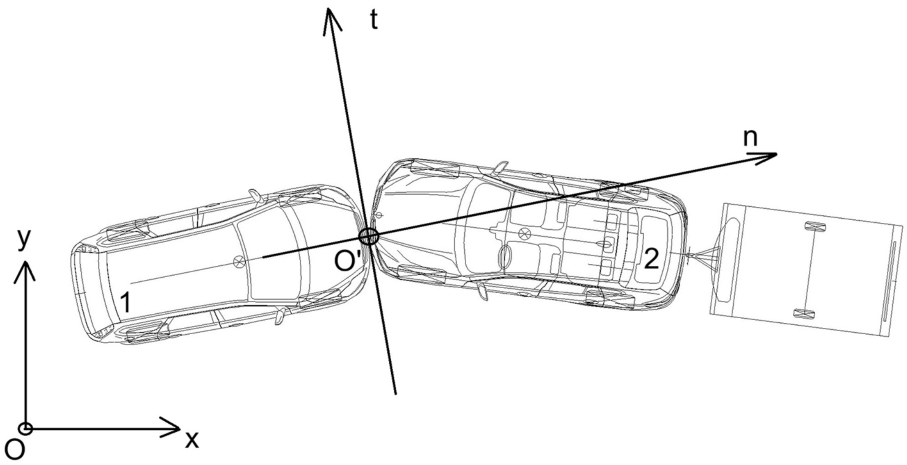

Figure 2. The adopted coordinate systems

be an important factor, for example altering certain parameters of the collision;

- lack of deformation of the vehicles. In terms of the presented example, deformations of the vehicles as a result of the collision are not as important as the phenomenon of a trailer detaching from one of the vehicles.

In Figure 3, the centers of mass of both vehicles (C1 and C2 respectively) are marked. Following [23], basic equations of motion for the discussed example may be presented. The translational and angular velocities after the collision were marked with an apostrophe, which was also used in the further parts of the paper:

$$m_{\mathrm{i}}(v'_{\mathrm{i}n} - v_{\mathrm{i}n}) = \pm S_n, i = 1.2$$
$$m_{\mathrm{i}}(v'_{\mathrm{i}t} - v_{\mathrm{i}t}) = \pm S_t, i = 1.2 \quad (1)$$
$$I_{\mathrm{i}}(\omega'_{\mathrm{i}} - \omega_{\mathrm{i}}) = \pm S_t n_{\mathrm{i}} \pm S_n t_{\mathrm{i}}, i = 1.2$$

where: m_i – the mass of the i-th vehicle (i.e. vehicles 1 or 2), v_{it} – velocity of the i-th vehicle tangential to the plane of collision, v_{in} – velocity of the i-th vehicle normal to the plane of collision, ω_i – angular velocity of the i-th vehicle, I_i – the moment of inertia around the vertical axis passing through the center of mass of the i-th vehicle, S_n, S_t – the normal and the tangential components of the resultant collision force impulse.

The presented assumptions relate to the discussed example and do not influence the generality of the presented approach. They mainly depend on the analyzed example, but can be adjusted to obtain conclusions for other cases, which makes them universal. First, the adopted coordinate system *O'nt* can be altered by changing direction of its axes with its origin (O') always located at the

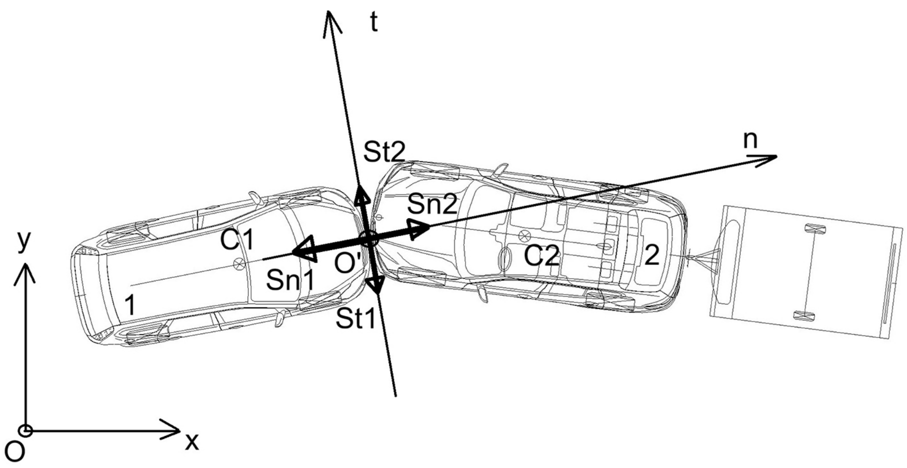

Figure 3. The components of the impulse used in the discussed example

point of an initial contact. Moreover, including suspension response to, e.g. the random irregularities of the road may require adding additional impulses resulting from the normal reactions at the tire-road contact. This would of course complicate the collision model, but the methodology will be the same. Perhaps the coefficient of adhesion between the tires and the road could also matter in terms of the additional force impulses, but it should be clear that such analyses are usually conducted for the momentary phenomena.

All in all, the equations of collision would have additional factors which would make it more difficult to solve, requiring the knowledge of the adopted forces complicating these equations.

The simplified collision model

In general, the equations of both vehicles involved in a collision are quite simple, if no additional factors are included that could complicate the collision model. Such situation is presented in Figure 4.

To complete this simple model of collision the components of the longitudinal velocities of both vehicles will be necessary as well as marking of the positive angular velocities. In Figure 5, these parameters are marked along with the adopted positive rotation, according to which the proper equations could be composed.

In the case of this simplified model of collision, i.e. without moving the center of mass towards the rear of vehicle 2, such equations could be used in accordance with the positive axes of the local *O'nt* system (Figure 5 – the positive rotation is marked as well):

$$\begin{gathered} m_1(v_{1n} - v'_{1n}) = -S_{n1} \\ m_2(v'_{2n} - v_{2n}) = S_{n2} \\ m_1(v'_{1t} - 0) = -S_{t1} \\ m_2(v_{2t} - v'_{2t}) = S_{t2} \\ I_1(\omega'_1 - \omega_1) = -S_{t1}n_1 \\ I_2(\omega_2 - \omega'_2) = -S_{t2}n_2 - S_{n2}t_2 \end{gathered} \tag{2}$$

where: m_1, m_2 – the mass of vehicles 1 and 2, I_1, I_2 – the moment of inertia around the vertical axis passing through the center of mass of vehicles 1 and 2, v_{1t}, v_{2t}, v'_{1t}, v'_{2t} – velocity of vehicles 1 and 2 tangential to the plane of collision before and after the crash, v_{1n}, v_{2n}, v'_{1n}, v'_{2n} – velocity of vehicles 1 and 2 normal to the plane of collision before and after the crash, ω_1, ω_2, ω'_1, ω'_2 – angular velocity of vehicles 1 and 2 before and after the crash, S_{n1}, S_{n2}, S_{t1}, S_{t2} – the normal and the tangential components of the crash impulse of vehicles 1 and 2.

If the loss of mass through the detaching trailer were not included, such simple approach would be enough to solve this problem, with the knowledge of the angles between the colliding vehicles, of course. These angles are marked in Figure 6, from which the necessary trigonometric dependencies can be drawn. In Figure 6, the α angle is between the forward velocity of vehicle 1 and the *Ox* axis of the *Oxy* system, while the β angle is between the forward velocity of vehicle 2 and the *Ox* axis of the *Oxy* system.

However, in this simplified approach it was assumed that the center of mass of vehicle 2 (i.e. the one with the trailer) is located incorrectly, yet

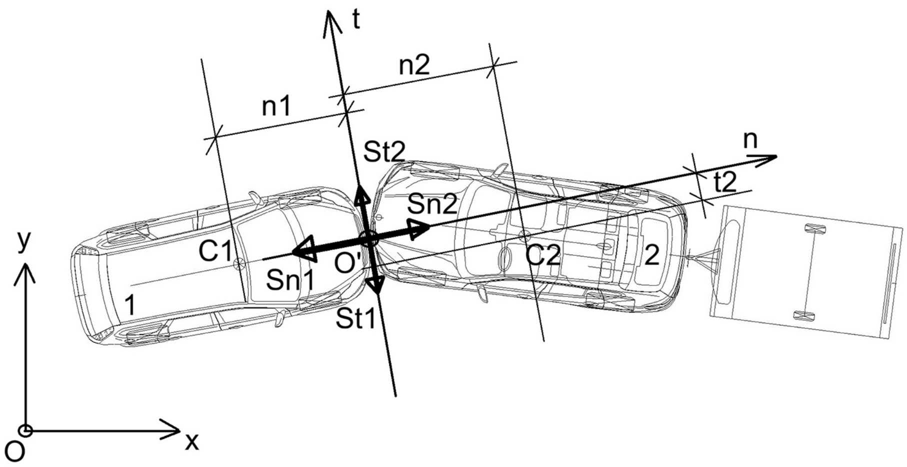

Figure 4. The basic collision model

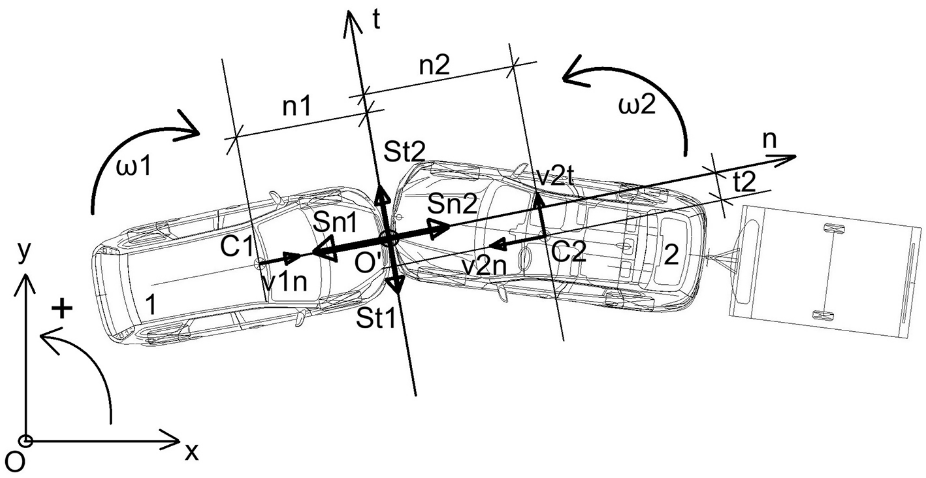

Figure 5. The positive directions of the forward and angular velocity

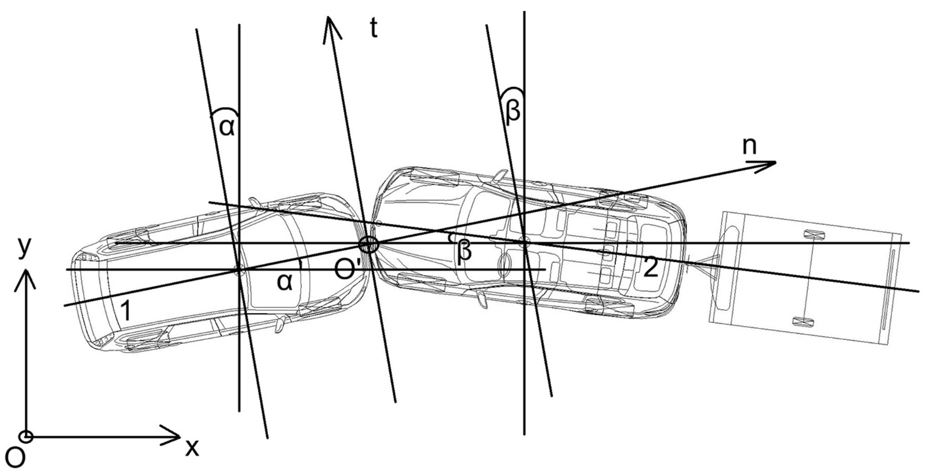

Figure 6. The angles between the colliding vehicles

close to the initial position. Its location marked with C2 (the center of mass of vehicle 2 alone). This is a result of an assumption that the mass of the trailer was a lot lower than that of vehicle 2. If the vehicle-trailer set is to be regarded as one body, then placing its center of mass in C2 may seem wrong. Therefore, for this simplified model of collision the center of vehicle 2 should be moved towards the rear of this vehicle due to an extra mass in the form of a trailer. In the next chapter this approach will be considered.

Summing up, this example has two major faults. One, this case did not include the detachment of the trailer. Two, the center of mass of the vehicle-trailer set was located at the same point as the center of mass of vehicle 2 alone, which can be used only in simplified cases. Therefore, these two faults were reconsidered in the next chapter.

Collision model with the detaching trailer

Since the problem requires complication and the use of the selected principles of variable mass mechanics, some modifications and additions to the equations (2) will be necessary. Detaching requires adding certain essential components to develop the collision model. Let us assume that two phases of the collision were considered:

a) before detaching when the trailer remains attached to the vehicle and can be considered as one mass,
b) after detaching when the trailer moves along its own way and is the mass that was lost by the vehicle-trailer set.

Phase a)

First, in Figure 7 the location of the center of mass of vehicle 2 with trailer is marked. It was moved towards the rear of vehicle 2 so that it reflects the actual location of this special point for the vehicle-trailer set. Let us mark this point as C2a, because its location was altered. At the same time, both C1 and the C2a were marked with black dots. The mass of the trailer was marked with m_t.

Now, it seems important to include every necessary parameter and component used in the mathematical model, which was presented in Figure 8. Also, the initial angular velocities were presented in Figure 8 along with the positive direction of rotation.

Let us assume two additional things important to solve the given problem. First, let us assume that the mass of the vehicle-trailer set will consist of the mass of vehicle 2 alone (let us call it m_{2v}) and that of the trailer (m_t). Hence, the mass marked m2 here will consist of 2 parts joined together before the detachment, i.e. $m_2 = m_{2v} + m_t$. That is why, in Figures 7 and 8 the center of mass of the vehicle-trailer set was moved to the back versus the initial location.

Second assumption can be related to the variable mass problem. Let us assume that in the middle of the collision, i.e. when the impulse gains the greatest value, the vehicles move with a common velocity which can be marked as v_c. Let the first phase end with this common velocity v_c which can be decomposed into two parts (Figure 9):

- the normal common velocity v_{cn},
- the tangential common velocity v_{ct},
- the angular common velocity $\omega_{c,}$

The second phase will begin with this common velocity.

In this case, Equations 2 can be modified to reflect the discussed phase:

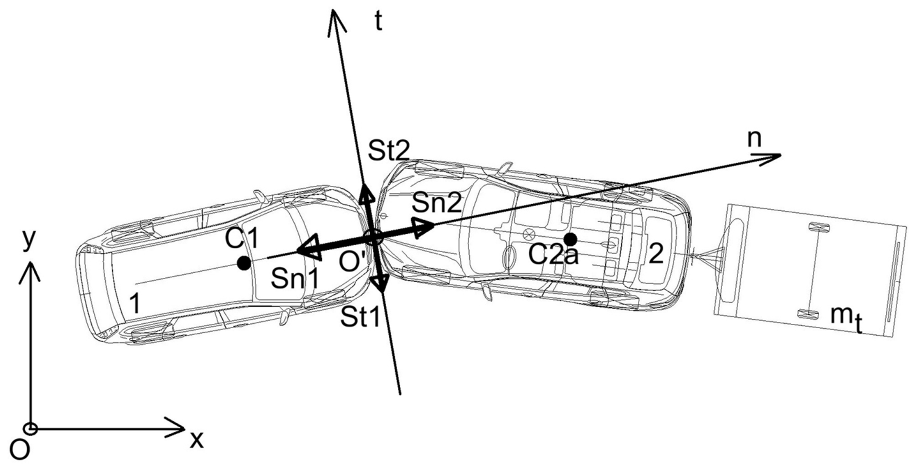

Figure 7. Accident – the components necessary for the completion of the mathematical model of the discussed collision with the altered location of the vehicle-trailer set

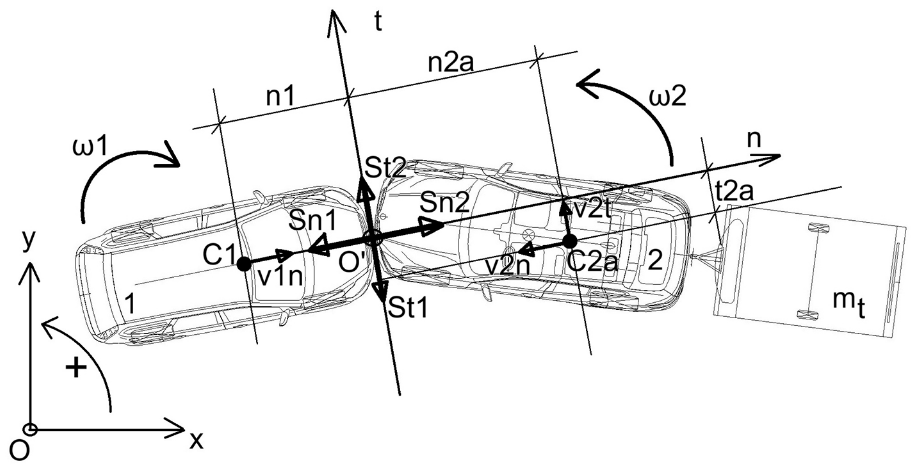

Figure 8. The phase of the collision before detaching of the trailer

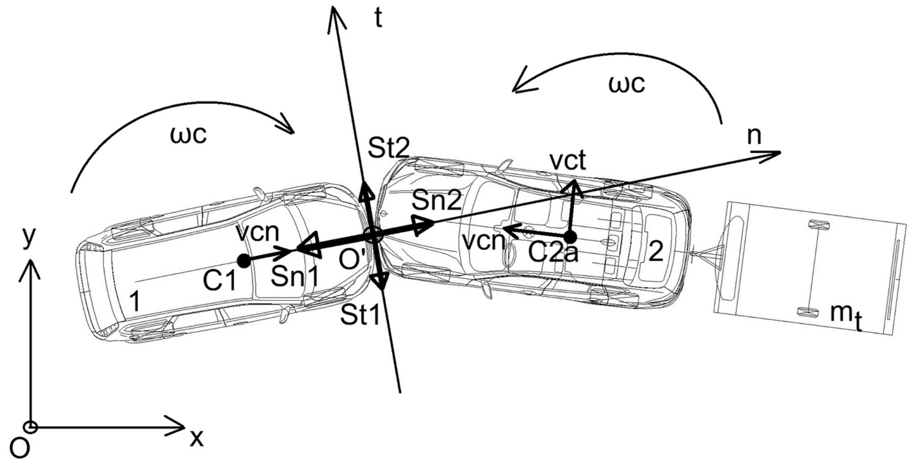

Figure 9. The common velocity in the middle of the collision

$$\begin{gathered} m_1(v_{1n} - v_{cn}) = -S_{n1} \\ (m_{2v} + m_t)(v_{cn} - v_{2n}) = S_{n2} \\ m_1(v_{ct} - 0) = -S_{t1} \\ (m_{2v} + m_t)(v_{2t} - v_{ct}) = S_{t2} \\ I_1(\omega_c - \omega_1) = -S_{t1}n_1 \\ I_{2a}(\omega_2 - \omega_c) = -S_{t2}n_{2a} - S_{n2}t_{2a} \end{gathered} \tag{3}$$

where: m_1, m_2 – the mass of vehicles 1 and 2, with $m_2 = m_{2v} + m_t$, I_1, I_{2a} – the moment of inertia around the vertical axis passing through the center of mass of vehicles 1 and 2, while the moment of inertia I_{2a} has been altered by moving the center of mass to the C2a point, v_{1t}, v_{2t}, v_{ct} – velocity of vehicles 1 and 2 tangential to the plane of collision before the crash and the common tangential velocity at the middle of the crash, v_{1n}, v_{2n}, v_{cn} – velocity of vehicles 1 and 2 normal to the plane of collision before the crash and the common normal velocity at the middle of the crash, ω_1, ω_2, ω_c – angular velocity of vehicles 1 and 2 before the crash and the common angular velocity at the middle of the crash, S_{n1}, S_{n2}, S_{t1}, S_{t2}, – te noal and the tangential components of the crash impulse of vehicles 1 and 2, n_1, t_1 n_{2a}, t_{2a} – the coordinates of center of mass of each vehicle with respect to the geometric center of the impact O'.

The kinematic state of the colliding vehicles can be presented to determine the mutual velocities at the time of switching from phase a) to phase b), determining the common velocity v_c:

$$\begin{gathered} v_{cn} = \frac{S_{n1}}{m_1} + v_{1n} \\ v_{cn} = \frac{S_{n2}}{(m_{2v} + m_t)} + v_{2n} \\ v_{ct} = \frac{S_{t1}}{m_1} \\ v_{ct} = v_{2t} - \frac{S_{t2}}{(m_{2v} + m_t)} \\ \omega_c = \omega_1 - \frac{S_{t1}n_1}{I_1} \\ \omega_c = \omega_2 + \frac{S_{t2}n_{2a}}{I_{2a}} + \frac{S_{n2}t_{2a}}{I_{2a}} \end{gathered} \tag{4}$$

Phase b)

The second phase of the discussed collision involves the phenomenon of detaching the trailer from vehicle 2. The common velocity v_c was also used here. In this phase the trailer was regarded as the mass separating from the vehicle 2 – trailer set. It is also necessary to assume that the components of impulses S_{n1}, S_{n2}, S_{t1}, S_{t2} will not change in phase b) because the duration of this collision was assumed as short as in each collision, i.e. less than 0.1 s.

In Figure 10, the second phase of the discussed example was presented, where the trailer has just detached from the vehicle and this allows some certain phenomena to be discussed.

In Figure 10, the velocity of the trailer was divided into the normal (v'_{tn}) and the tangential (v'_{tt}) velocity, relative to the $O'nt$ coordinate system and it was in the center of mass of the trailer. It was assumed that the location of the center of mass in vehicle 2 moved back to its initial location.

The distances between the O' point and the center of mass of vehicle 2 adopted the same values as in Figure 5. This allows assuming that at the beginning of phase b), when the trailer detached, the situation switched back to the simple example from Figure 5.

It is necessary to mention that the returns of the normal and tangential velocities in Figure 10 are opposite to those from Figure 9, which depicts the kinematic state of both vehicles after the middle of the collision. It is more of a schematic than the actual meaning due to the fact that the equations of collision describe the momentary state, rather than the long lasting phenomena.

Still, the variable mass equation must be used here, because vehicle 2 – trailer set lost the mass of the trailer (m_t).

First, let us call in the variable mass equation, based on, e.g. [24] where the Meschersky equation was used for the dynamics of the variable mass systems, such as rockets. In its simplest form, this equation is as follows:

$$m\frac{dv}{dt} = F + \mu u \tag{5}$$

where: m – mass of the object, $\frac{dv}{dt}$ – acceleration of the object, F – the force acting on the object, μ – mass detaching from the object, v – velocity of the detaching mass.

Let us transform this equation a little to obtain the more applicable form for the discussed example:

$$m_2\frac{dv_2}{dt} = F \pm m_t v_t' \tag{6}$$

where: m_2 – mass of vehicle 2 – trailer set, $\frac{dv_2}{dt}$ – acceleration of vehicle 2 – trailer set, F – the force acting on the vehicle 2 – trailer set, m_t – mass of the detaching trailer, v_t – velocity of the detaching trailer.

The ± sign means that the mass can be added or subtracted (detached). The apostrophe at the trailer velocity (v_t') means the velocity obtained after detachment, because in phase a) the trailer moved with a car as a combined set. Such an attempt will enable using the equations of collision with the inclusion of Equation 6 which in this case should be modified by multiplying both sides by dt, as follows:

$$m_2 dv_2 = Fdt \pm m_t v_t' dt \tag{7}$$

which will enable us to use it in the equations of motion for phase b) of the discussed example in this final form:

$$m_2(v_c - v_2') = S_2 \pm m_t(v_t' - v_c) \tag{8}$$

where: S_2 – the impulse of the second phase of collision for vehicle 2, v_c – the common velocity at the end of phase a).

Of course, this equation should be divided into two components: normal and tangential to the common plane of collision.

On the basis of Figure 10, Equation 8 and the common velocity of both vehicles introduced in Equations 3 such a set can be provided:

$$\begin{gathered} m_1(v_{cn} - v_{1n}') = -S_{n1} \\ m_2(v_{2n}' - v_{cn}) = S_{n2} - m_t v_{tn}' - m_t v_{cn} \\ m_1(v_{1t}' - v_{ct}) = -S_{t1} \\ m_2(v_{ct} - v_{2t}') = S_{t2} - m_t v_{tt}' - m_t v_{ct} \\ I_1(\omega_1' - \omega_c) = -S_{t1} n_1 \\ I_2(\omega_c - \omega_2') = -S_{t2} n_2 - S_{n2} t_2 \end{gathered} \tag{9}$$

where: m_1, m_2 – the mass of vehicles 1 and 2, I_1, I_2 – the moment of inertia around the vertical axis passing through the center of mass of vehicles 1 and 2 without the trailer attached to vehicle 2. Of course, it is fair to assume that the moment of inertia of vehicle 2 will not change noticeably due to the collision, v_{1t}', v_{2t}', v_{ct} – velocity of vehicles 1 and 2 tangential to the plane of collision after the crash and the common tangential velocity at the middle of the crash, v_{1n}', v_{2n}', v_{cn} – velocity of vehicles 1 and 2 normal to the plane of collision after the crash and the common normal velocity at the middle of the crash, v_{tn}', v_{tt}' – velocity of the detaching trailer in the normal and the tangential direction versus the adopted common plane of collision, $\omega_1', \omega_2', \omega_c$ – angular velocity of vehicles 1 and 2 after the crash and the common angular velocity at the middle of the crash, S_{n1}, S_{n2}, S_{t1}, S_{t2} – the normal and the tangential components of the crash impulse of vehicles 1 and 2, n_1, t_1, n_2, t_2 – the coordinates of center of mass of each vehicle with respect to the geometric center of the impact O' without the trailer.

This approach allows using the obtained mutual velocities to determine the post-collision velocities for both vehicles involved. The kinematic state of the colliding vehicles can be presented to determine the mutual velocities at the

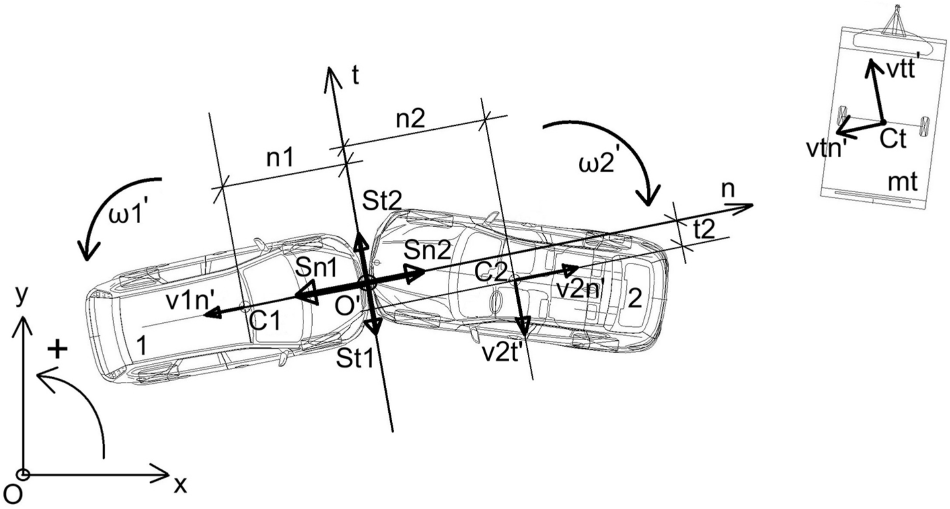

Figure 10. The phase of the collision after the trailer detaching

time of switching from phase a) to phase b), determining the common velocity v_c:

$$= \frac{m_1 v_{cn} + S_{n1}}{m_1}$$

$$v'_{2n} = v_{cn} + \frac{S_{n2} - m_t v'_{tn} - m_t v_{cn}}{m_2}$$

$$v'_{1t} = \frac{m_1 v_{ct} - S_{t1}}{m_1}$$

$$v'_{2t} = v_{ct} - \frac{S_{t2} - m_t v'_{tt} - m_t v_{ct}}{m_2} \qquad (10)$$

$$\omega'_1 = \omega_c - \frac{S_{t1} n_1}{I_1}$$

$$\omega'_2 = \omega_c + \frac{S_{t2} n_2 + S_{n2} t_2}{I_2}$$

As it can be observed in Equations 10, the final velocities in both the normal and the tangential direction require knowing the common velocity v_c in these directions along with the components of impulses S_{n1}, S_{n2}, S_{t1}, S_{t2} and the mass of the detaching trailer as well as its velocity in both adopted directions (v'_{tn}, v'_{tt}).

In order to simplify this problem, it could be assumed that the velocity of the trailer at the beginning of phase b) is equal to the common velocity in the middle of the crash, i.e.:

$$v'_{tn} = v_{cn}, v'_{tt} = v_{ct} \qquad (11)$$

which would make Equations 10 easier to solve. Nevertheless, solving these equations requires some additional factors which will be discussed in the next chapter.

Potential applicability

The presented example relates the specific case of a road traffic collision where the trailer is detached from the towing car. Such types of road collisions are more seldom than the typical ones between vehicles with no trailers. However, the potential consequences of such may be more harmful for other road users. This example allowed the authors to use the simplified method including variable mass mechanics which was the main aim of this paper. Although Equation 5 was simplified, it seems that it can be used in such cases. In addition, the mass of the trailer detached only once, rather than continuously like in the case of the fuel in a rocket. The remaining question is whether such attempt towards analyzing such a road accident can be applicable in any solution, apart from a pure scientific problem. For a forensic expert, it would seem difficult because there may not be enough data to calculate the pre-crash velocities, i.e. at the beginning of collision, especially when the trailer detachment is included. Moreover, the coefficient of restitution would be necessary to specify, e.g. the impulses in phase b) in relation to phase a). For the discussed problem two coefficients of restitution are required: one in the normal (let us call it R) and one in the tangential direction (let us call it θ). They can be described by formula (12), assuming that the collision is non-slip, i.e. the bodies of the vehicles remain in contact for the whole duration of the collision, and they do not slide on each other:

$$R = \frac{S_{n2}}{S_{n1}}, \theta = \frac{S_{t2}}{S_{t1}}, \qquad (12)$$

where: R – coefficient of restitution in the normal direction versus the plane of collision,
θ – coefficient of restitution in the tangential direction versus the plane of collision,
S_{n1}, S_{n2} – components of normal impulse in phase a) and b), respectively,
S_{t1}, S_{t2} – components of tangential impulse in phase a) and b), respectively.

The impulses introduced here were used in chapter 3 for phase a) and phase b). Of course, ability to calculate those from phase b) requires knowing the ones from phase a). In the case of the R coefficient, it is necessary that $S_{n1} > S_{n2}$ because the normal coefficient of restitution R takes values between 0 and 1. Knowing the tangential coefficient of restitution θ is more difficult, although there have been works, mainly by Japanese scientists, providing some details on this subject.

As for dividing the impulse into the normal and the tangential component, a simple trigonometric dependency stemming from angular calculations can be used as in Figure 6. Of course, this would deal with only half the problem, allowing calculation of the S_{n1}, S_{t1} components. However, knowing them and being able to specify the R and θ coefficients may lead to specify the S_{n2}, S_{t2} components.

For a forensic expert this may be troublesome due to inability to determine some essential parameters to solve it. However, for certain algorithms allowing approximations, this would be possible.

The discussed model of a collision in which a trailer detaches form one of the vehicles involved could be validated with the use of proper software, e.g. PC-Crash or V-SIM where such a scenario is possible. The main problem would be analyzing to what extent a trailer could alter the motion of a vehicle, from which it detached, especially in the post-collision phase, where the vehicles move after impact. Of course, there are many real-world accident data, for example collected by the Volvo experts to improve the work of the Safety Centre. This could also be useful as a mathematical model validation platform. Maybe one of the gravest questions is whether a trailer could cause the 3D motion instead of a planar which has been assumed here.

CONCLUSIONS

The present study provided tangible results and conclusions which offer an application value. The aim of this paper was to implement the basic principles of variable mass mechanics to a model of road collision. This was possible owing to simplifying the Meschersky equation so that the mass detached at once instead of continuously. Modeling road traffic events such as collisions is a complicated process which requires knowing not only the basic factors but also those that may complicate a collision model or enable it to provide greater realism reflecting the actual occurrence that may have happened on the road.

As long as people drive, the unexpected road traffic collisions seem inevitable, but the ability to describe and understand them may lead to predicting the potential consequences and introducing the procedures, policies or even technical factors which will provide greater road safety, especially in the areas of the more common road accident occurrence.

Modeling road accidents with the use of some additional factors, such as variable mass mechanics as in this case, can also help improve the software dedicated to simulation and reconstruction of road accidents which will be also an added value to, e.g. forensics.

In the next step of research, the authors would like to present the possible application of such an approach as presented here, mainly through validating the proposed model of collision with the use of V-SIM simulations and the actual data, e.g. the masses and velocities of the vehicles selected for the simulations. Furthermore, this research would be completed with the real-life accident data as a reference point to use the presented collision model.

REFERENCES

1. Sirwan S., et al. Road traffic accidental injuries and deaths: A neglected global health issue. Health Sci Rep. 2023; 6(5): 1–6. https://doi.org/10.1002/hsr2.1240
2. Sharma V., et al. A systematic review on road traffic accident: Causes and control measures. IJCRT. 2023; 11(4). https://doi.org/10.1729/Journal.33667
3. Berhanu Y., et al. Examining car accident prediction techniques and road traffic congestion: a comparative analysis of road safety and prevention of world challenges in low-income and high-income countries. Journal of Advanced Transportation. 2023; 1: 6643412. https://doi.org/10.1155/2023/6643412
4. Athiappan K., et al. Identifying influencing factors of road accidents in emerging road accident blackspots. Advances in Civil Engineering. 2022; 9474323. https://doi.org/10.1155/2022/9474323

5. Gorzelańczyk P. Forecasting the number of road accidents in Poland using weather-dependent trend models. Technical Sciences, University of Warmia and Mazury in Olsztyn. 2023; 26(1):57–76. https://doi.org/10.31648/ts.8289
6. Sun X., et al. Study on the impact of road traffic accident duration based on statistical analysis and spatial distribution characteristics: An empirical analysis of Houston. Sustainability. 2022; 14(22): 14982. https://doi.org/10.3390/su142214982
7. Cabrera-Arnau C., et al. Uncovering the behaviour of road accidents in urban areas. R. Soc. Open Sci. 2020; 7(4): 191739. http://doi.org/10.1098/rsos.191739
8. Achiri A., et al. Automobile crash investigation based on vehicle system related causes: Systematic literature review. World Journal of Engineering and Technology. 2022; 10(2): 139–157. https://doi.org/10.4236/wjet.2022.102008
9. Aleksandrowicz P. The impact of a vehicle braking system state on safe driving - part one. In: Proc. of 17th International Conference on Diagnostics of Machines and Vehicles, Pieczyska, Poland 2018. https://doi.org/10.1051/matecconf/201818201025
10. Dižo, J., et al. Investigation of driving stability of a vehicle–trailer combination depending on the load's position within the trailer. Acta Mechanica et Automatica. 2023; 17(1): 60–67. https://doi.org/10.2478/ama-2023-0007
11. Noorsumar, G., et al. Mathematical models for assessment of vehicle crashworthiness: a review. International Journal of Crashworthiness. 2021; 27(5): 1545–1559, https://doi.org/10.1080/13588265.2021.1929760
12. Aleksandrowicz P. Modeling Head-on collisions: The problem of identifying collision parameters. Applied Sciences-Basel. 2020; 10(18): 6212. https://doi.org/10.3390/app10186212
13. Zhi Z., et al. Research on damage mechanism and protection of the collision penetration of the thin-walled long rod structure. Sci Prog. 2020; 103(1): 36850419874216. https://doi.org/10.1177/00368504198742
14. Buśkiewicz J. Dynamics of a mass variable drum winding a heavy rope. Arch. Mech. 2021; 73(5–6): 557–582. https://doi.org/10.24423/aom.3860
15. Biswal S.K. An equation for generalized variable mass systems and its consequences. International Journal of Scientific Research in Physics and Applied Sciences. 2021; 9(6): 19–22. https://doi.org/10.26438/ijsrpas/v9i6.1922
16. Vippala, V. A brief review on equation of motion of variable mass system. Journal of Physics Education and Science. 2025; 2(5). https://doi.org/10.47134/physics.v2i3.1723
17. Yang, D., et al. Post-impact motion control of road vehicles. Vehicle System Dynamics. 2025; 63(7): 1300–1357. https://doi.org/10.1080/00423114.2025.2507772
18. Shakil A., et al. A study on road accident prediction and contributing factors using explainable machine learning models: analysis and performance. Transportation Research Interdisciplinary Perspectives. 2023; 19: 100814. https://doi.org/10.1016/j.trip.2023.100814
19. Sahana S., et al. Car crash detection system using machine learning and deep learning algorithm. ICDSIS. 2022; 1–6. https://doi.org/10.1109/ICDSIS55133.2022.9915889
20. Yuan C., et al. Research on active collision avoidance and hysteresis reduction of intelligent vehicle based on multi-agent coordinated control system. World Electr. 2023; 14(1), 16. https://doi.org/10.3390/wevj14010016
21. Zhang X., et al. Research on the effective reduction of accidents on operating vehicles with fsQCA method—Case studies. Appl. Sci. 2022; 12: 12737. https://doi.org/10.3390/app122412737
22. Liu H., et al. Research on driver injuries in vehicle rear-end collisions considering different overlap rate and speed difference. Sci Rep. 2025; 15: 18286. https://doi.org/10.1038/s41598-025-02620-9
23. Kisilowski J. and Zalewski J. Modeling of Road Traffic Events. Cham: Springer Nature Switzerland AG. 2022. https://doi.org/10.1007/978-3-030-91398-4
24. Wolny J., et al. Momentum in the dynamics of variable-mass systems: Classical and relativistic case. Acta Physica Polonica A. 2019; 3(135). https://doi.org/10.12693/APhysPolA.135.475