Advances in Science and Technology Research Journal

Advances in Science and Technology Research Journal, 2026, 20(7), 335–351
https://doi.org/10.12913/22998624/220064
ISSN 2299-8624, License CC-BY 4.0

Received: 2026.02.28
Accepted: 2026.05.15
Published: 2026.06.01

Verification-based modelling framework for dynamic analysis of beam systems using LS-DYNA

Thuy Van Tran Thi[1], Ilya Ravodin[2], Sergey Gridnev[2], Le Thuy Nguyen[1*]

[1] Hanoi Architectural University, 129 Tran Phu, Hanoi, Vietnam
[2] Department of Structural Mechanics, Faculty of Civil Engineering, Voronezh State Technical University, Moskovsky Ave., 14, Voronezh, 394026, Russian Federation
* Corresponding author's e-mail: thuynl@hau.edu.vn

ABSTRACT

This study presents a structured verification framework for assessing the applicability of the LS-DYNA finite element solver to the dynamic analysis of beam-type structural systems. The reliability of LS-DYNA is evaluated through representative benchmark problems involving free vibration, forced harmonic response, and moving load scenarios. Modal characteristics of straight beams with various boundary conditions are first analysed and compared with classical analytical solutions. The accuracy of the finite element modelling strategy is further examined for a simply supported beam subjected to a harmonic load applied at midspan. A moving harmonic load is implemented using a nodal force approach along a predefined travel path and validated against solutions obtained via the Bubnov–Galerkin method in Wolfram Mathematica. Additionally, a coupled model of a sprung moving mass traversing a beam with a localized surface irregularity is developed using contact algorithms to capture inertial interaction effects. Numerical results show close agreement with analytical solutions and published reference data, confirming modelling accuracy and numerical stability. The study establishes a verification-based modelling procedure and supports the broader application of LS-DYNA in structural dynamic analysis.

Keywords: structural dynamics, beam vibration, finite element modelling, LS-DYNA, moving load analysis, contact interaction, verification framework.

INTRODUCTION

Dynamic loading represents one of the most demanding aspects of structural engineering, particularly in the evaluation of vibration behaviour, transient response, and stability of beam-type structural systems widely employed in bridges, building frames, industrial facilities, and transportation infrastructure. Despite decades of research in beam dynamics, the transition from classical analytical benchmark solutions to robust nonlinear numerical modelling frameworks suitable for practical engineering applications remains an ongoing challenge. Accurate prediction of structural response under dynamic excitation is essential to ensure serviceability, safety, and long-term structural performance.

Classical analytical formulations based on Euler–Bernoulli and Timoshenko beam theories provide the theoretical foundation for structural dynamics and remain indispensable reference solutions for idealized systems and validation studies [1,2]. These formulations enable closed-form solutions for modal characteristics and forced vibration problems under well-defined boundary conditions. However, real engineering structures frequently involve complex support configurations, nonlinear material behaviour, damping mechanisms, geometric imperfections, and realistic loading scenarios that cannot be adequately captured by simplified analytical models alone. Consequently, advanced numerical methods capable of representing such complexities are increasingly required in engineering practice [3,4].

The rapid development of computational capabilities has led to the widespread adoption of the finite element method (FEM) as the principal framework for structural dynamic analysis. FEM enables detailed modelling of complex geometries, heterogeneous materials, and nonlinear response mechanisms, thereby allowing more realistic simulation of engineering structures [5,6]. Over recent decades, general-purpose commercial software packages such as ANSYS, ABAQUS, MIDAS, and LS-DYNA have significantly expanded the range of dynamic problems addressed in practice. Among these tools, LS-DYNA has attracted particular attention as a robust nonlinear dynamic solver capable of handling contact interaction, large deformations, high strain-rate effects, and complex transient loading conditions [7,8]. Originally developed for impact and blast simulations, LS-DYNA has evolved into a versatile multiphysics platform applied in crashworthiness analysis, seismic simulations, and fluid–structure interaction problems.

In parallel, extensive research has been conducted on dynamic behaviour of beam systems under moving and time-dependent loads, including vehicle–bridge interaction models, accelerating moving masses, and coupled train–structure dynamics [9–13]. Advanced formulations accounting for shear deformation, inertial effects, and damping have been proposed to improve modelling fidelity [14–17]. Reduced-order modelling approaches and specialised finite element formulations have also been introduced to balance computational efficiency and engineering accuracy [18,19]. Despite these advances, practical dynamic analysis of beam systems in civil engineering is still frequently performed using simplified analytical tools or dedicated structural software with limited modelling flexibility.

One of the primary reasons for this situation is the absence of systematic evaluation and structured verification of general-purpose nonlinear solvers against well-established beam dynamics benchmark problems. Without rigorous validation against analytical solutions and recognised reference cases, uncertainties remain regarding modelling assumptions, parameter selection, numerical stability, and expected accuracy. As a result, engineers may be reluctant to employ advanced nonlinear solvers for conventional structural problems that could benefit from their enhanced modelling capabilities.

Although several studies have applied nonlinear dynamic solvers to structural systems [20–23], and recent studies have further investigated the dynamic behaviour of beam-type structures subjected to moving loads and related numerical modelling issues [28–30], a unified verification framework addressing multiple representative beam-type dynamic scenarios remains largely lacking.

From an engineering practice perspective, establishing reliable numerical modelling procedures for beam-type structures subjected to dynamic loading is essential for improving design efficiency and broadening the applicability of advanced analysis tools. Many practical problems including vibration assessment of bridge girders, dynamic response of floor systems, and evaluation of structures subjected to moving loads require modelling strategies that combine accuracy, robustness, and practical usability [24–30].

Developing validated modelling workflows using general-purpose nonlinear solvers can reduce dependence on overly simplified assumptions while providing greater flexibility in capturing complex structural behaviour. Despite the availability of advanced solvers, structured verification linking analytical benchmark problems with practical finite element implementation strategies remains limited in the literature.

Although the present study does not introduce a new numerical formulation, its novelty lies in the development of a structured verification-oriented modelling workflow linking analytical benchmark problems with practical finite element implementation strategies in LS-DYNA.

The objective of this study is therefore to evaluate the applicability and reliability of LS-DYNA for modelling the dynamic behaviour of beam systems through a structured verification framework. Several representative benchmark cases are considered, including modal vibration analysis, forced vibration response, and moving load scenarios. Numerical results are systematically compared with analytical solutions and reference data reported in the literature in order to assess modelling accuracy, numerical stability, and practical implementation aspects. Rather than proposing a new numerical formulation, this work provides a scientifically grounded assessment of modelling strategies and establishes practical recommendations to support the use of LS-DYNA as a versatile tool for structural dynamic analysis.

The main contributions of this study can be summarised as follows:

- Systematic verification of LS-DYNA for classical beam dynamic problems using analytical benchmark solutions;
- Comprehensive evaluation of modelling assumptions and numerical performance under multiple dynamic loading scenarios;
- Critical assessment of the applicability of a general-purpose nonlinear solver to structural engineering problems traditionally addressed using simplified approaches;
- Development of practical modelling guidance and verification-oriented workflows to facilitate broader adoption of LS-DYNA in structural dynamics applications.

PROBLEM STATEMENT AND VERIFICATION FRAMEWORK

Reference beam system

To establish a consistent verification framework, a reference beam-type structural system is adopted as the fundamental modelling object throughout this study. The structure is idealised as a straight prismatic beam with uniform material and geometric properties, characterised by span length L, flexural rigidity EI, mass per unit length ρA, and structural damping parameters where applicable. The beam behaviour is described using classical beam theory assumptions, which provide the analytical basis for verification against numerical simulations.

A unified reference beam model is maintained while different boundary conditions and dynamic loading scenarios are introduced to represent typical structural configurations encountered in engineering practice. The investigated cases include a cantilever beam with elastic end restraint, a beam with intermediate support at midspan, and simply supported beams subjected to stationary harmonic excitation and moving harmonic loads.

These configurations are illustrated schematically in Figure 1(a–d) and serve as the reference structural and loading models for all dynamic verification cases presented in subsequent sections. By adopting a unified reference system, variations in dynamic response can be attributed primarily to loading conditions and modelling assumptions rather than changes in structural definition.

Dynamic loading scenarios and verification stages

To evaluate the applicability of the numerical modelling approach under different dynamic conditions, a series of representative loading scenarios is considered within a structured verification framework. The selected cases introduce progressively increasing levels of modelling complexity while maintaining the unified reference beam system defined.

The verification process begins with modal analysis of the beam system, which provides fundamental validation of discretisation accuracy and numerical implementation through comparison of natural frequencies and mode shapes with analytical solutions. This initial stage establishes

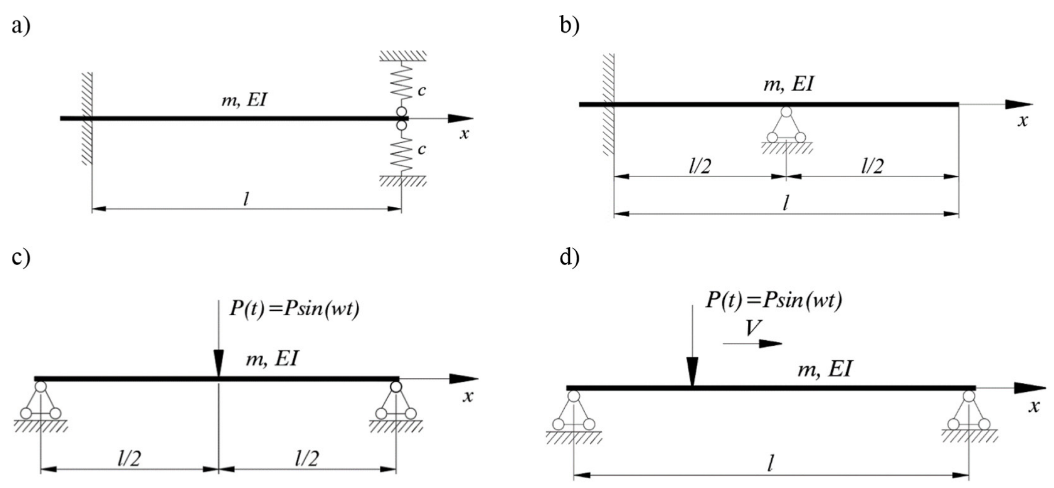

Figure 1. Reference beam configurations: (a) cantilever beam with elastic end restraint; (b) beam with intermediate support at midspan; (c) simply supported beam subjected to harmonic excitation; (d) simply supported beam under moving harmonic load

a baseline assessment of the finite element formulation and numerical stability.

Subsequently, forced vibration under harmonic excitation is investigated. A stationary sinusoidal load is applied to the beam in order to assess the solver's capability to capture steady-state dynamic response and resonance behaviour under time-dependent loading.

The third stage considers a moving harmonic load travelling along the beam span. This scenario introduces coupled spatial–temporal effects and allows evaluation of the numerical implementation of moving loads, time integration schemes, and dynamic response accuracy.

Finally, a higher level of modelling complexity is introduced through dynamic interaction effects associated with moving loads, enabling assessment of solver performance under conditions representative of practical engineering applications.

Together, these loading stages establish a structured pathway for systematic verification of modelling assumptions and numerical performance.

Analytical reference solutions

Verification of the numerical modelling approach is performed through systematic comparison with established analytical solutions and reference formulations available in the literature. These analytical models provide benchmark results for assessing the accuracy of modal characteristics, forced vibration response, and dynamic behaviour under moving loads.

For free vibration analysis, classical solutions derived from Euler–Bernoulli beam theory are employed to determine natural frequencies and mode shapes under different boundary conditions. These solutions serve as reference values for evaluating discretisation accuracy and eigenvalue extraction procedures in the numerical model.

In the case of forced harmonic excitation, analytical expressions describing steady-state response under sinusoidal loading are used to assess amplitude and phase characteristics of the dynamic response. Comparison with these solutions allows validation of time integration accuracy and damping representation.

The moving harmonic load scenario is evaluated using analytical formulations based on modal decomposition methods. In particular, the Bubnov–Galerkin method is adopted as an independent analytical approach for obtaining reference solutions implemented outside the finite element environment.

Together, these analytical benchmarks provide a consistent basis for structured verification of the numerical modelling framework across different levels of dynamic complexity.

The overall structured verification procedure adopted in this study is summarised in Figure 2.

Numerical discretisation considerations

In explicit dynamic solvers such as LS-DYNA, both spatial discretisation (mesh density) and time-step size may influence the accuracy of predicted dynamic response.

In the present study, the mesh density was selected based on preliminary numerical tests to ensure stable modal characteristics and consistent dynamic response. The adopted element sizes provide a reasonable balance between computational efficiency and numerical accuracy.

The explicit central difference integration scheme determines a stable time step according to the Courant stability condition. Although a systematic mesh convergence and time-step sensitivity study is beyond the scope of the present verification-oriented work, the good agreement observed between numerical results and analytical solutions indicates that the adopted discretisation strategy is adequate for the considered benchmark problems.

More detailed convergence investigations for complex structural configurations represent an important direction for future research.

MODAL VERIFICATION

The first stage of the verification framework focuses on modal analysis of the reference beam system. This stage provides a fundamental assessment of the finite element formulation by comparing numerical predictions of natural frequencies and mode shapes with analytical solutions derived from classical beam theory.

Modal verification is essential for ensuring that discretisation strategies, element formulations, and boundary condition implementations are correctly defined prior to introducing external dynamic loading. Accurate prediction of modal characteristics establishes confidence in the numerical model and serves as a baseline for subsequent forced and moving-load analyses.

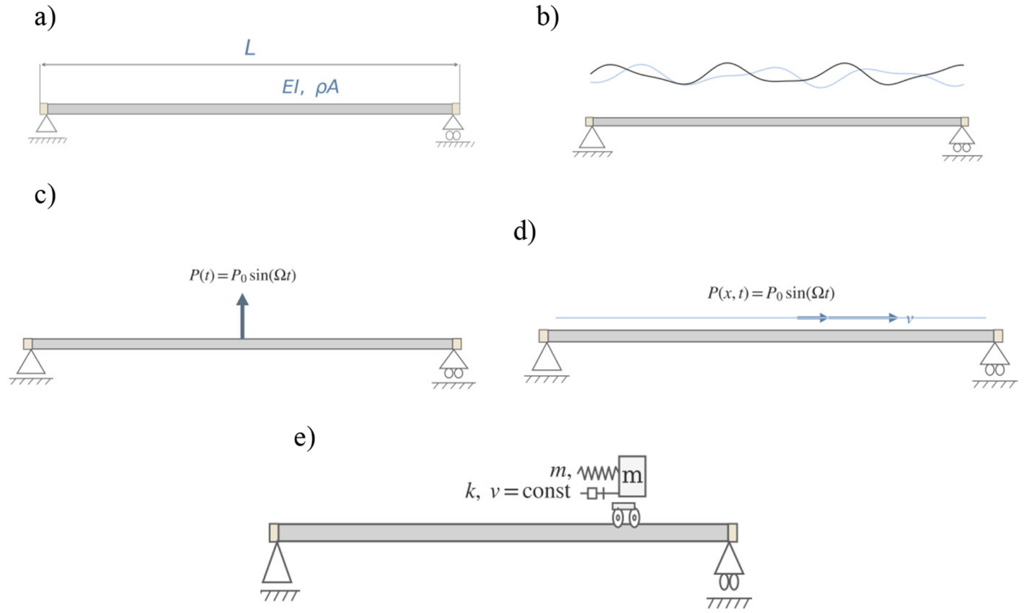

Figure 2. Schematic representation of the structured verification framework adopted in this study, including modal analysis, harmonic excitation, moving load, and moving mass interaction cases

Several representative beam configurations described in Section 2 are considered, including cantilever beams with elastic restraint and beams with intermediate supports. Natural frequencies and corresponding mode shapes are extracted using the explicit finite element formulation and compared with analytical reference solutions.

Finite element formulations for modal verification

The modal verification stage begins with evaluation of the finite element formulations adopted for modelling the reference beam system. Analytical solutions for natural frequencies and mode shapes of beam structures with different boundary conditions are available in the literature and are used as benchmark references for validating numerical results. In particular, the analytical formulations presented by Babkov [2] are employed to establish reference modal characteristics.

The beam system considered in this study has a span length L=1.0 m, cross-sectional width b=4.0 cm, height h=0.4 cm, mass per unit length m=1.256 kg/m, and flexural rigidity EI=42.667 N·m^2. Boundary conditions are varied according to the configurations defined in Section 2.

Two alternative finite element discretisation approaches are implemented in LS-DYNA in order to evaluate modelling sensitivity and numerical consistency:

- a fully integrated shell element formulation based on Mindlin–Reissner theory, accounting for transverse shear deformation effects;
- a two-node Hughes–Liu beam element with six degrees of freedom per node.

The use of different element formulations allows evaluation of the modelling flexibility of LS-DYNA for beam-type structures. Beam elements provide computational efficiency for slender structural members, while shell elements allow a more detailed representation of cross-sectional behaviour and shear deformation effects. Solid elements are employed in cases where local geometry or contact interaction must be explicitly represented.

The shell element formulation represents the structural member as a thin-walled continuum with five degrees of freedom per node, including translational and rotational components. The beam element formulation models the structure as a one-dimensional member with additional integration points along the cross-section to capture bending and shear effects.

The finite element formulations adopted for modal verification are illustrated in Figure 3. Comparison between these discretisation strategies enables assessment of modelling accuracy and provides insight into the suitability of different element types for dynamic beam analysis within LS-DYNA.

The shell elements used in the numerical model are four-node quadrilateral elements with five

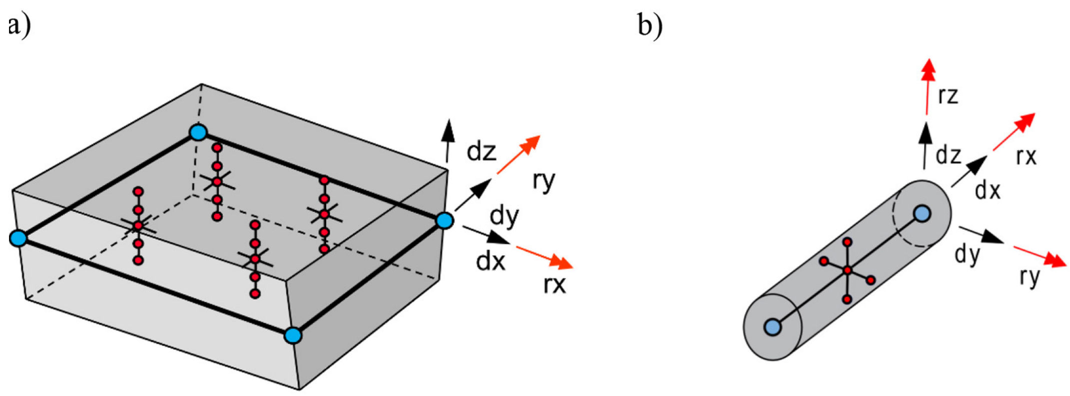

Figure 3. Finite element formulations adopted for modal verification: (a) fully integrated shell element; (b) Hughes–Liu beam element

degrees of freedom per node, including three translational components and two rotational components. The formulation of this element is based on the Mindlin–Reissner assumption, allowing transverse shear deformation through the thickness to be considered. Beam members are represented using two-node Hughes–Liu beam elements with six degrees of freedom per node (three translations and three rotations). Additional integration points across the beam cross-section are introduced to improve the representation of shear deformation effects.

To evaluate the accuracy of the adopted discretisation strategies, the three lowest natural frequencies of the beam are determined for each boundary condition configuration together with their corresponding mode shapes. Eigenvalue extraction is performed using the block Lanczos method with spectral shift, ensuring numerical stability and computational efficiency.

The obtained modal characteristics are compared with analytical reference solutions derived from classical beam theory. Table 1 summarises the numerical and analytical natural frequencies for the considered boundary conditions.

Modal verification of classical boundary conditions

Following the finite element modelling and modal analysis procedure described in Section 3.1, the modal behaviour of beam configurations with classical boundary conditions, namely the simply supported and cantilever cases, is examined in greater detail. These configurations

Table 1. Natural frequencies of beam vibrations

Boundary conditions	Natural frequency equation	Frequency number, *i*	Natural frequencies, Hz		
			Theoretical significance	LS-Dyna: beam model (Hughes – Liu Beam)	LS-Dyna shell model (fully integrated shell elements)
Simply supported beam	$p_i = \frac{i^2 p^2}{l^2}\sqrt{EI/m}$	1	9.151	9.157	9.158
		2	36.602	36.655	36.669
		3	82.355	82.569	82.641
Cantilever beam	$p_i = \frac{(kl)^2}{l^2}\sqrt{EI/m}$	1	3.263	3.261	3.271
		2	20.449	20.435	20.495
	$ch(kl)' \ Cos(kl)+1=0$	3	57.264	57.241	57.423
Fixed ended beam	$p_i = \frac{(kl)^2}{l^2}\sqrt{EI/m}$	1	20.764	20.779	20.899
		2	57.191	57.358	57.694
	$ch(kl)' \ Cos(kl)-1=0$	3	112.095	112.655	113.343
Over hanging beam	$p_i = \frac{(kl)^2}{l^2}\sqrt{EI/m}$	1	20.764	20.719	20.722
		2	57.191	57.080	57.109
	$ch(kl)' \ Cos(kl)-1=0$	3	112.095	111.889	112.006

provide well-established benchmark problems and allow evaluation of the consistency between analytical solutions and numerical predictions.

Representative mode shapes obtained from the numerical simulations are illustrated in Figures 4 and 5. The deformation patterns are consistent with theoretical expectations for the respective boundary conditions, with symmetric mode shapes observed for the simply supported beam and zero displacement and rotation at the fixed end for the cantilever configuration.

The close correspondence between analytical and numerical modal characteristics confirms that both the fully integrated shell element and the Hughes–Liu beam element accurately reproduce the fundamental vibration behaviour of the beam system. Minor differences observed in higher modes can be attributed to discretisation effects and variations in shear deformation representation between the two element formulations. This agreement establishes a reliable baseline for subsequent verification stages involving more complex boundary conditions and dynamic loading scenarios.

Modal verification of extended boundary configurations

To further assess the robustness of the adopted finite element formulations, modal verification is extended to beam configurations involving non-classical boundary conditions. These cases introduce additional stiffness constraints and internal supports, thereby increasing the modelling complexity while retaining analytical reference solutions for validation.

Two representative configurations are considered:
- a cantilever beam with an elastic end restraint;
- a cantilever beam with an intermediate simple support at midspan.

These cases allow evaluation of the numerical model under conditions where boundary stiffness and internal constraints modify the global vibration characteristics.

Cantilever beam with elastic end restraint

To further evaluate the capability of LS-DYNA in modelling non-classical boundary conditions, a cantilever beam with an elastic end restraint is investigated. The beam is rigidly fixed at the left end while the right end is connected to an elastic support characterised by stiffness parameter c, as shown in Figure 6. The structural parameters are identical to those defined in previous verification cases to maintain consistency.

The stiffness of the elastic support is characterised by the parameter c, which introduces an additional boundary condition coupling displacement and reaction force at the free end. The natural frequencies therefore depend on the stiffness ratio of the elastic restraint, providing a convenient benchmark for assessing numerical accuracy under varying boundary flexibility.

Analytical expressions for the natural frequencies are derived from classical beam theory

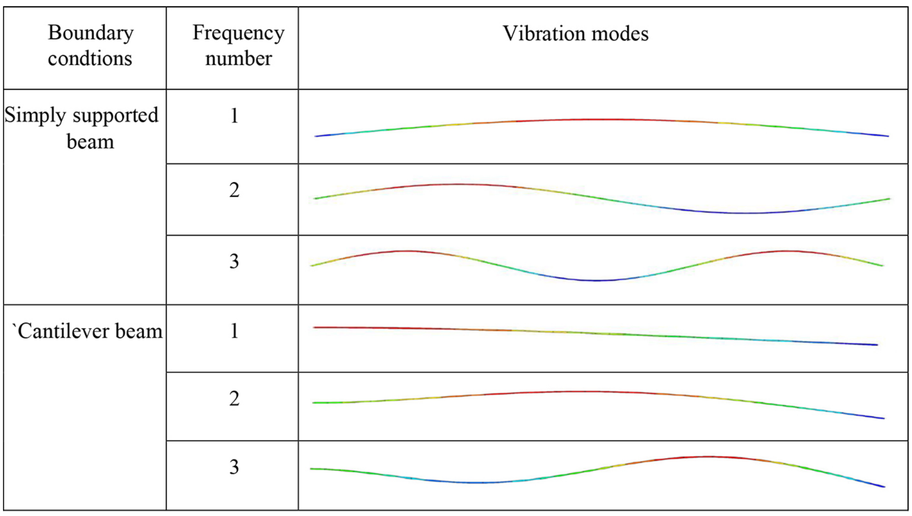

Figure. 4. Natural mode shapes of the simply supported and cantilever beam configurations obtained using LS-DYNA

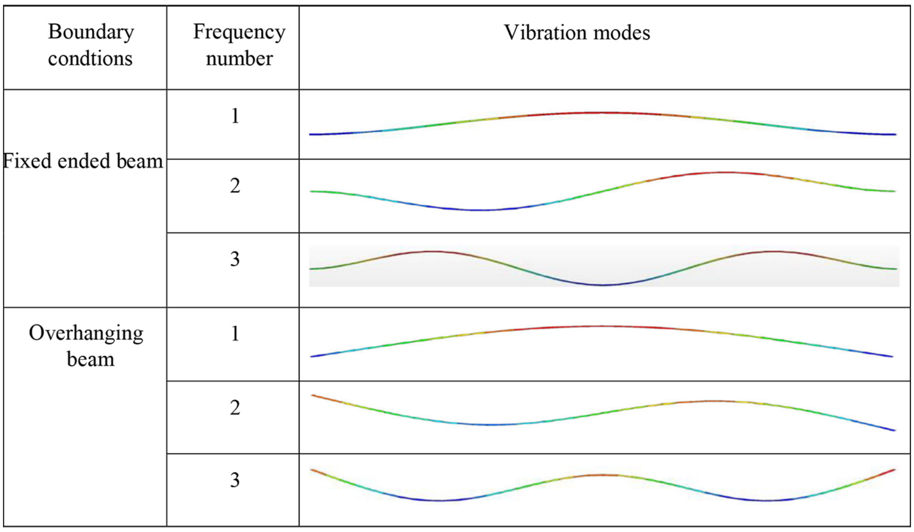

Figure 5. Numerically obtained natural mode shapes for beam configurations with fixed and free boundary conditions computed in LS-DYNA

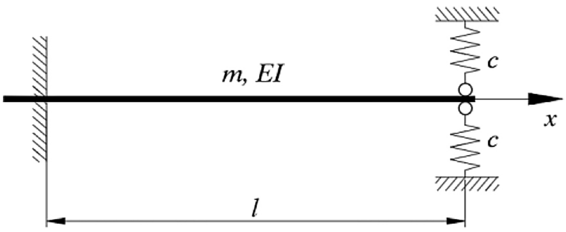

Figure 6. Computational model of a cantilever beam with an elastically restrained end

and serve as reference solutions. The first natural frequency is evaluated for several values of the dimensionless stiffness parameter $c \cdot l^3/EI$, representing different levels of elastic restraint. Modal extraction in LS-DYNA is performed using the block Lanczos method, consistent with the procedure described in Section 3.1.

The numerical results obtained using the Hughes–Liu beam element and the fully integrated shell element are compared with analytical predictions, as summarised in Table 2. The comparison shows good agreement between analytical and numerical results across the considered stiffness range, confirming that both finite element formulations accurately capture the transition between free and restrained boundary behaviour.

As expected, when $c \to 0$, the system behaviour approaches that of a classical cantilever beam with a free end. Conversely, when $c \to \infty$, the boundary condition tends towards a rigid support, resulting in modal characteristics similar to a beam with additional restraint at the right end.

The results demonstrate that LS-DYNA is capable of accurately modelling beams with elastic boundary conditions, which are frequently encountered in practical engineering applications involving semi-rigid connections and flexible supports.

Beam with intermediate support

The modal verification is further extended to a beam configuration incorporating an intermediate support. In this case, the beam is rigidly fixed at the left end, while an additional simple support is introduced at midspan, as illustrated in Figure 7. The geometric and material parameters of the beam remain identical to those defined in the preceding verification cases.

The presence of the intermediate support alters the global stiffness distribution and modifies the vibration characteristics of the system. From an analytical standpoint, the beam can be considered as consisting of two segments connected through compatibility conditions at the support location. The displacement at the intermediate support is constrained to zero, while continuity of rotation and internal forces is preserved.

By applying classical Euler–Bernoulli beam theory and enforcing the corresponding boundary and compatibility conditions, the characteristic frequency equation of the system can be expressed in transcendental form as:

$$Sin[\alpha]Cosh[\alpha] - Sinh[\alpha]Cos[\alpha] - \\ -Sinh[\alpha] + Sin[\alpha] = 0, \quad (1)$$

Table 2. Natural frequencies of a cantilever beam with an elastically restrained end

Boundary conditions	Natural frequency equation	$\frac{cl^3}{EI}$	First frequency, Hz		
			Theoretical significance	LS-Dyna: beam model (Hughes – Liu Beam)	LS-Dyna shell model (fully integrated shell elements)
Cantilever beam with elastically supported end	$p_i = \frac{(kl)^2}{l^2}\sqrt{EI/md}$; $a = kl$; $\frac{cl^3}{EI} = -a^3 \frac{cha\cos a + 1}{cha\sin a - sha\cos a}$	20	8.353	8.169	8.180
		65	11.369	11.393	11.416
		1000	14.334	14.222	14.264

where: α=k·l denotes the dimensionless frequency parameter.

For the adopted beam parameters, the first three roots of the characteristic equation are obtained numerically as: α_1=3.14, α_2=7.85, α_3=9.42 The corresponding natural frequencies are then determined using the standard frequency relation for bending vibrations of beams. Modal analysis is performed in LS-DYNA using both the Hughes–Liu beam element and the fully integrated shell element formulations described. The computed natural frequencies are compared with the analytical reference values, and the results are summarised in Table 3.

The comparison indicates very good agreement between analytical and numerical solutions for all three modes. The discrepancy between the two finite element formulations remains small and does not exceed approximately 1%. This confirms that both discretisation strategies accurately capture the stiffness redistribution induced by the intermediate support.

The corresponding mode shapes exhibit the expected behaviour, with zero transverse displacement enforced at the support location and modified curvature distribution along the span. The results demonstrate that the adopted modelling framework is capable of accurately representing beam systems with internal constraints, completing the modal verification stage prior to the introduction of external dynamic loading in subsequent sections. The preceding modal verification cases confirm the accuracy of the finite element formulations for a range of boundary conditions. The study now proceeds to dynamic response verification under external time-dependent loading.

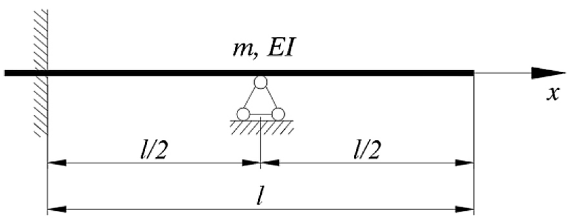

Figure 7. Analytical model of a cantilever beam with an elastic end support

Forced vibration under harmonic excitation

Following the modal verification stage, the numerical framework is further evaluated through forced vibration analysis of a simply supported beam subjected to harmonic excitation. This case introduces time-dependent loading and allows assessment of the dynamic response capabilities of the LS-DYNA solver beyond eigenvalue analysis. The investigated system consists of a simply supported beam subjected to a harmonic force applied at the midspan, as illustrated in Figure 8. The load is defined as: P(t)=P·sin(ωt), where P is the force amplitude and ω\omegaω denotes the excitation frequency. The beam parameters are defined as:

- span length L=4.35 m,
- mass per unit length m=26.3 kg/m,
- flexural rigidity EI=4.601×10^6 N/m².

The excitation force has an amplitude P=5 kN and excitation frequency f=20 Hz.

The analytical solution for the steady-state response is obtained using classical harmonic response theory, which provides reference values for displacement amplitude at midspan. According to the analytical formulation, the expected amplitude of vibration at the beam midpoint is approximately 2.82 mm. Dynamic simulations in LS-DYNA are performed using direct time integration with an explicit central difference scheme

Table 3. Comparison of natural frequencies for the beam with an intermediate support

Boundary conditions	Natural frequency equation	$\frac{cl^3}{EI}$	First frequency, Hz		
			Theoretical significance	LS-Dyna: beam model (Hughes – Liu Beam)	LS-Dyna shell model (fully integrated shell elements)
Cantilever beam with intermediate hinge support	$p_i = \frac{(kl)^2}{l^2}\sqrt{EI/m};$	1	9.151	9.153	9.212
		2	57.191	57.210	57.381
		3	82.355	82.564	83.261

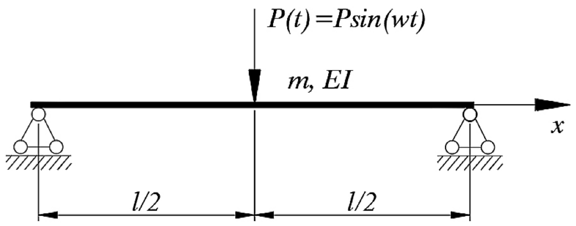

Figure 8. Computational model of a simply supported beam under harmonic loading

and automatic time stepping. Structural damping is introduced using a damping coefficient D_s=0.01 to obtain stable steady-state oscillations after the decay of transient effects.

Both beam and shell finite element formulations are employed to ensure consistency with previous verification stages. The harmonic force is applied as a nodal load at the beam midpoint with a time increment of 0.005 s. The resulting steady-state vibration response at midspan is presented in Figure 9. The numerical results show good agreement with analytical predictions, with small discrepancies attributed primarily to differences in element formulations and treatment of shear deformation effects.

Dynamic response under moving harmonic load

To further extend the dynamic verification, the beam response under a moving harmonic load is investigated. This case introduces spatial–temporal coupling between the excitation and structural response, thereby providing a more demanding benchmark for evaluating the numerical modelling capabilities of LS-DYNA. The analysed system consists of a simply supported beam subjected to a harmonic force travelling along the beam span with constant velocity v, as illustrated in Figure 10.

The load is defined as:

$$P(x,t)=P\cdot sin(\omega t)\cdot \delta(x-Vt) \qquad (2)$$

where: P is the load amplitude, ω is the excitation frequency, and δ denotes the spatial location of the moving load.

In contrast to the stationary harmonic excitation considered in Section 3.4, the moving load induces additional dynamic effects due to the interaction between load velocity and structural natural frequencies.

An analytical solution is obtained using the Bubnov–Galerkin method, which provides a reference displacement field for comparison. The numerical implementation in LS-DYNA employs the nodal force approach, where the moving harmonic load is applied sequentially along the beam nodes according to the prescribed velocity.

The displacement response is evaluated at midspan and compared with the analytical solution. The comparison, presented in Figure 11, demonstrates good agreement between the finite element predictions and the reference solution. Minor deviations are attributed to discretisation effects and the time integration scheme.

The results confirm that the adopted modelling strategy accurately reproduces the dynamic response of beam systems subjected to moving harmonic excitation, thereby extending the validation beyond stationary loading conditions.

The comparison between analytical and numerical solutions is presented in Figure 12, showing the time histories of forced vibrations at the beam midspan. The results obtained using the beam and shell finite element models in LS-DYNA demonstrate good agreement with the analytical prediction based on the Bubnov–Galerkin method.

Minor discrepancies in the predicted vibration frequencies are attributed primarily to the discrete representation of the moving load as nodal forces. Improved accuracy can be achieved by selecting an optimal load application step; however, reducing this step significantly increases the computational effort required for model construction. In

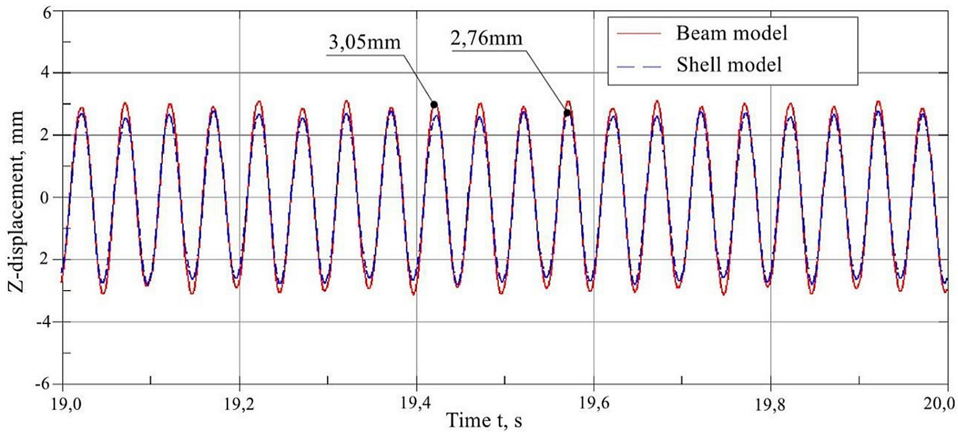

Figure 9. Steady-state forced vibrations at the beam midspan

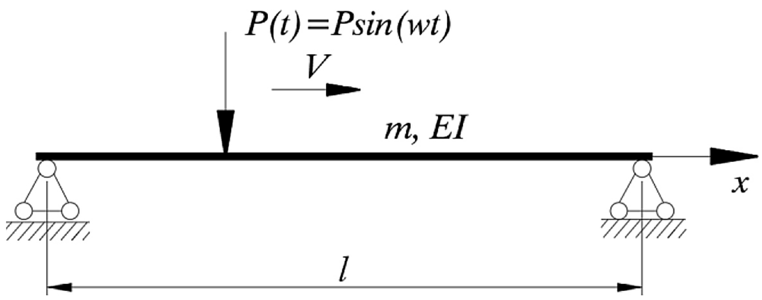

Figure 10. Simply supported beam subjected to a moving harmonic load

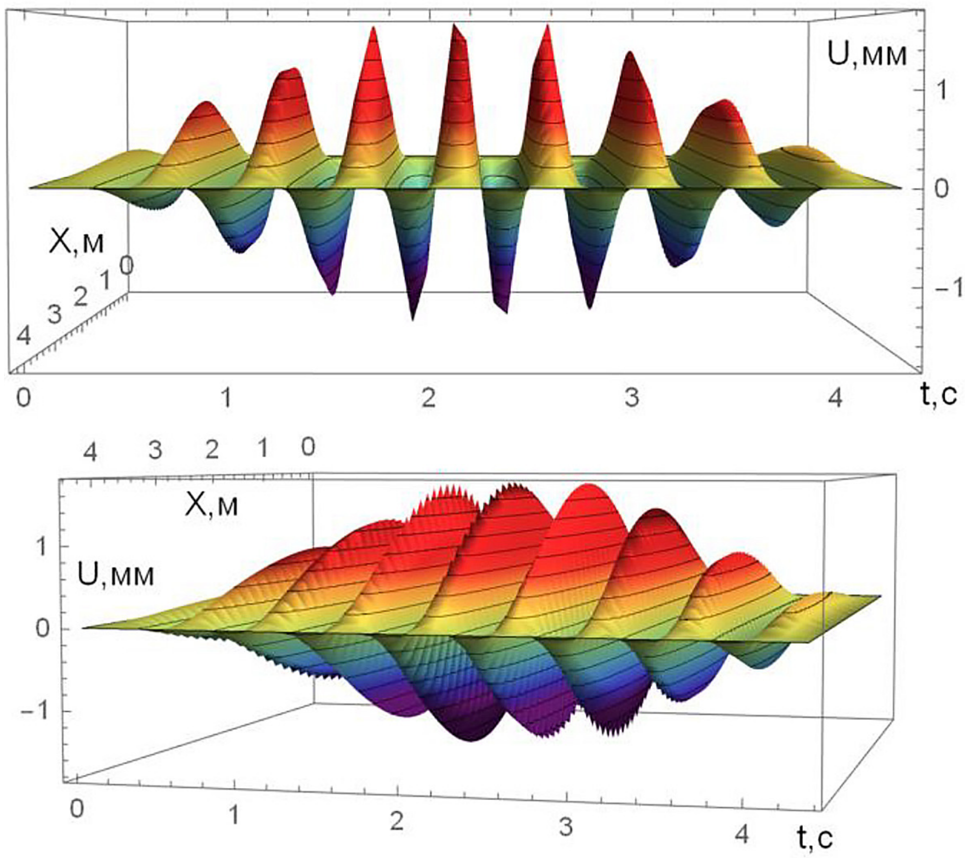

Figure 11. Comparison of midspan displacement histories under moving harmonic excitation

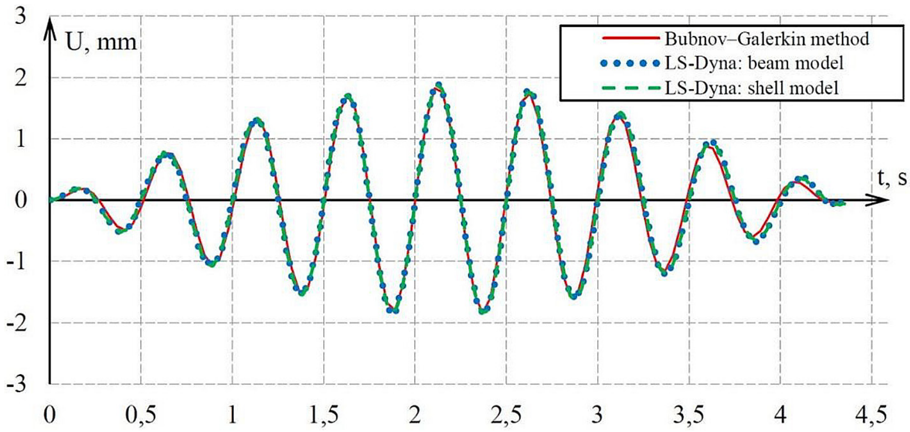

Figure 12. Comparison of midspan displacement histories under moving harmonic load obtained using the Bubnov–Galerkin method and LS-DYNA simulations

practical applications, auxiliary preprocessing tools may therefore be employed to automate the definition and assignment of nodal loads along the travel path.

In cases where load inertia and feedback effects must be considered, alternative modelling strategies based on contact algorithms may be adopted. Such approaches enable coupled simulation of moving masses and structural systems and have been implemented in previous studies using general contact formulations.

Beam response under moving sprung mass with surface irregularity

To further extend the verification framework towards coupled vehicle–structure interaction problems, a beam system subjected to a moving sprung mass travelling over a local surface irregularity is considered. This example introduces both load inertia and contact interaction effects, thereby representing a higher level of modelling complexity compared to the previous moving force case.

The structural system consists of a simply supported reinforced concrete beam with square cross-section (15 × 15 cm). The material properties adopted in the model are: Young's modulus E=30000 MPa, Poisson's ratio μ=0.2, and unit weight γ=24 kN/m³. Self-weight effects are neglected in order to isolate the dynamic response induced by the moving system.

The moving system is idealised as a single-degree-of-freedom sprung mass travelling with constant velocity V. The model includes mass M, suspension stiffness ccc, and viscous damping coefficient k, forming a simplified representation of a vehicle axle–suspension system. The adopted parameters are: M=200 kg; c=150 kN/m; k=1 kN·s/m, and velocity V=5 m/s

The description of materials and element types used in the finite element model in LS-DYNA is provided in Table 4.

The beam surface includes a localised geometric irregularity representing a single depression. The profile of the irregularity is defined analytically by:

$$\begin{cases} 0, V\cdot t < \left(\frac{l}{2}-a\right); \\ \frac{h_0}{2}\left(1-Cos\frac{2\pi\cdot t}{a}\right), \left(\frac{l}{2}-a\right) \le V\cdot t < \left(\frac{l}{2}-a\right) = 0; \\ 0, V\cdot t > \frac{l}{2}. \end{cases} \quad (3)$$

where: a=1 m is the irregularity length and h_0=0.01 m is the depression depth. The corresponding computational model is illustrated in Figure 13.

In the LS-DYNA implementation, the interaction between the sprung mass and the beam is simulated through contact-based coupling between the moving mass element and the beam surface. Time integration is performed using an explicit central difference scheme with automatic time stepping. Both beam and shell finite element formulations are considered for the structural discretisation.

The modelling concept is illustrated schematically in Figure 14. The beam is covered by an upper orthotropic layer containing a local geometric depression representing a surface irregularity. The use of three-dimensional finite elements with orthotropic material properties allows accurate representation of the irregularity geometry while maintaining realistic load transfer to the main structural beam. Owing to the anisotropic mechanical properties along and across the beam

Table 4. Description of element types and material models

Object	Element	LS-DYNA element formulation	Material model
Beam system	Beam	Hughes–Liu beam element	MAT_ELASTIC
	Upper layer with depression	8-node solid element with full integration scheme (3 DOF per node)	MAT_ORTHOTROPIC_ ELASTIC
Sprung mass system	Support part	Hughes–Liu beam element	MAT_RIGID
	Elastic spring	2-node one-dimensional element	MAT_SPRING_ ELASTIC
	Damper	2-node one-dimensional element	MAT_DAMPER_ VISCOUS
	Mass	Fully integrated shell element	MAT_RIGID

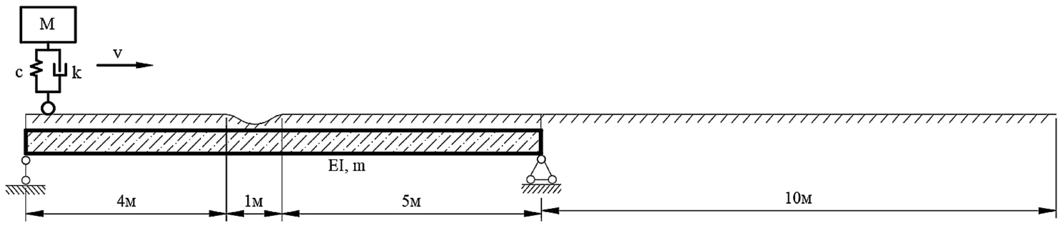

Figure 13. Computational model of a simply supported beam subjected to a moving sprung mass travelling over a local surface irregularity

axis, the irregularity layer effectively transmits the moving load without significantly altering the global stiffness of the structural system.

The translational motion of the moving mass is prescribed using the PRESCRIBED_MOTION_ RIGID option in LS-DYNA, which allows definition of velocity, acceleration, or displacement histories for rigid bodies along a specified direction. This approach is commonly applied in dynamic analyses involving moving loads or seismic excitation based on predefined motion histories.

Contact interaction between the support element of the moving load and the beam surface is modelled using the AUTOMATIC_BEAMS_ TO_SURFACE algorithm. This contact formulation provides robust numerical stability in problems involving large relative displacements between interacting bodies. During each time step, penetration between the support element and the irregularity layer is detected, and a contact force proportional to the penetration depth is applied to both bodies. A fragment of the finite element model with the moving mass is shown in Figure 15.

Material models and element formulations used in the simulation are summarised in Table 4. Structural damping is introduced using global mass-proportional damping (*DAMPING_GLOBAL), defined as a fraction of critical damping. A damping coefficient D_s=1.0 corresponds approximately to 3% damping at the fundamental vibration frequency of the beam (ω_1=14.96 rad/s). The dynamic analysis is performed using the explicit central difference integration scheme implemented in LS-DYNA with a constant time step Δt=1.78 × 10^{-6} s. The resulting midspan displacement histories are presented in Figure 16, while Figure 17 shows the evolution of dynamic contact pressure associated with the moving sprung mass. The solid red curves represent results obtained in LS-DYNA, whereas the dashed blue curves correspond to reference solutions reported in [6]. The comparison demonstrates satisfactory agreement between numerical predictions and reference data, confirming the validity of the adopted modelling approach.

The obtained dynamic response demonstrates clear amplification of beam vibrations as the sprung mass traverses the irregularity region. Compared to the moving harmonic force case, this configuration produces additional response components associated with load inertia and suspension dynamics. The results confirm that the finite element framework implemented in LS-DYNA is capable of capturing coupled vehicle–structure interaction phenomena, including transient contact effects and localised excitation due to surface imperfections.

This final verification case completes the structured hierarchy of benchmark problems, progressing from classical modal analysis to forced vibration and ultimately to coupled

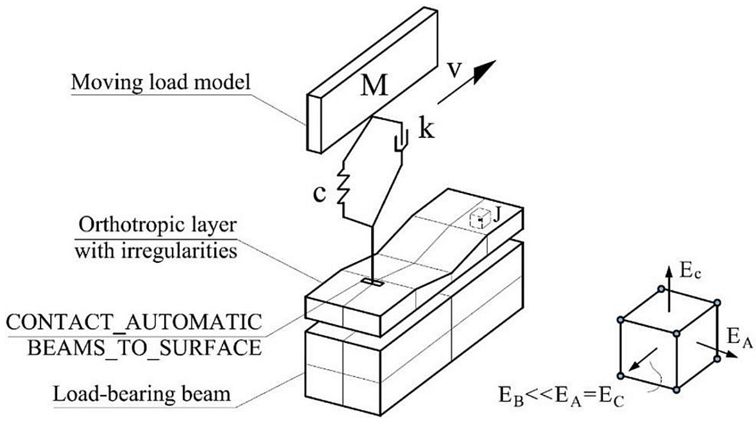

Figure 14. Conceptual model of the moving sprung mass interacting with the beam surface irregularity

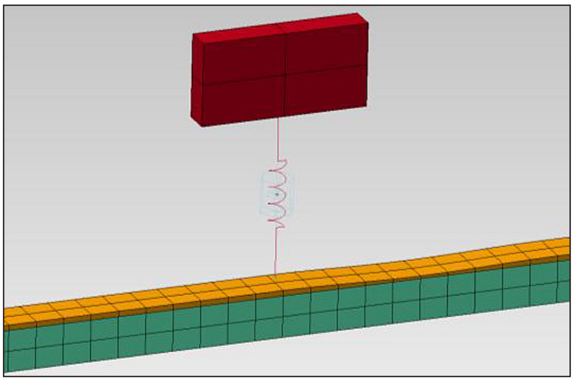

Figure 15. Finite element model of the dynamic system with moving sprung mas

moving-mass interaction. The consistent agreement observed throughout the examples demonstrates the robustness of the adopted modelling strategy for analysing beam-type structural systems under dynamic loading.

Discussion of verification results

The structured sequence of verification cases presented in this section provides a comprehensive evaluation of the numerical modelling framework across increasing levels of complexity, ranging from classical modal analysis to coupled moving-mass interaction problems. The results demonstrate that the adopted LS-DYNA implementation is capable of accurately reproducing both fundamental dynamic characteristics and more advanced structural response phenomena.

The modal verification stage confirms that the finite element formulations employed in this study, including the Hughes–Liu beam element and fully integrated shell element, yield natural frequencies and mode shapes in close agreement with analytical solutions derived from classical beam theory. The small discrepancies observed between analytical and numerical results are primarily attributed to discretisation effects and differences in element formulations, particularly regarding the treatment of shear deformation.

The extended modal cases involving elastic end restraint and intermediate support further validate the ability of the modelling framework to capture stiffness redistribution induced by non-classical boundary conditions. The results indicate that both finite element approaches remain

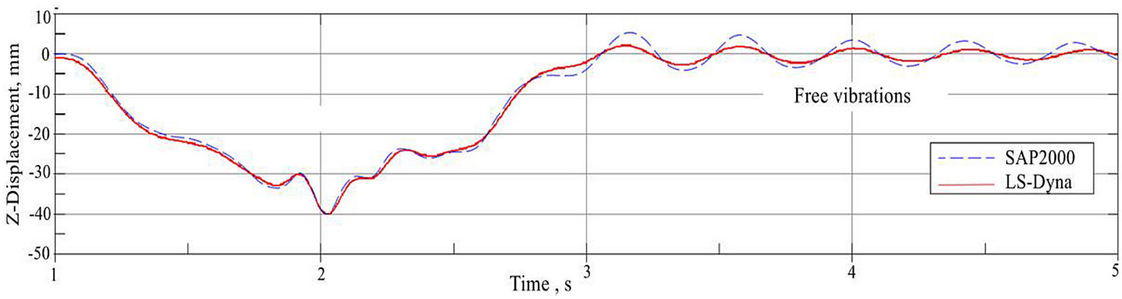

Figure 16. Midspan displacement response of the beam

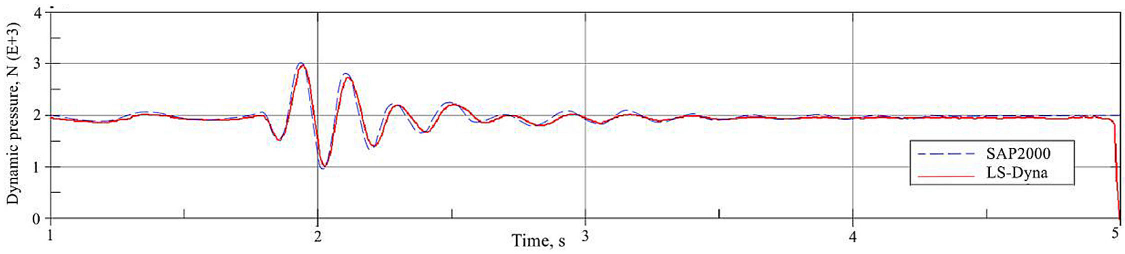

Figure 17. Time history of dynamic contact pressure

robust when internal constraints or elastic supports are introduced, with relative errors remaining within acceptable engineering limits.

The transition to forced vibration analysis demonstrates the capability of the numerical model to accurately predict steady-state response under harmonic excitation. The agreement between analytical and numerical displacement amplitudes confirms the reliability of the direct time integration approach adopted in LS-DYNA for dynamic response analysis.

More complex scenarios involving moving loads reveal additional modelling challenges associated with spatial–temporal coupling between excitation and structural response. The moving harmonic load case highlights the importance of discretisation strategy in representing travelling forces, as the accuracy of the solution depends on the selection of nodal load spacing along the load trajectory. Despite these challenges, the numerical results remain consistent with analytical reference solutions.

Finally, the moving sprung mass example introduces load inertia and contact interaction effects, representing a realistic vehicle–structure interaction scenario. The successful reproduction of transient response behaviour demonstrates the versatility of the adopted modelling framework and confirms its applicability to practical engineering problems involving moving vehicles or dynamic contact phenomena.

Overall, the progressive verification strategy adopted in this study establishes a consistent level of confidence in the numerical modelling approach. By validating the solver against benchmark problems of increasing complexity, the study provides practical guidance for applying general-purpose nonlinear dynamic solvers to beam-type structural systems encountered in civil engineering practice.

It should be noted that the present study focuses primarily on verification against analytical solutions and reference numerical results. Experimental validation is not included in the current work and represents an important direction for future research.

Further studies should also investigate mesh convergence behaviour and time-step sensitivity for more complex structural configurations and nonlinear material models.

CONCLUSIONS

The agreement between the finite element results obtained in LS-DYNA and known analytical or reference solutions for the considered beam vibration problems allows the following conclusions to be drawn:

- The eigenvalue extraction algorithm implemented in LS-DYNA, when combined with an adequate level of spatial discretisation, enables accurate determination of natural frequencies of beams with various boundary conditions, even in cases where the spectrum of eigenfrequencies is densely distributed.
- The explicit central difference method, forming the basis of dynamic analysis in LS-DYNA, can be effectively applied to the simulation of forced vibrations of beam-type structural systems.
- Numerical results obtained using fully integrated shell elements demonstrate satisfactory agreement with beam-element models for structures with the considered geometric and mechanical parameters, confirming the consistency of different finite element formulations.
- The use of contact algorithms enables reliable simulation of inertial moving loads acting on beam systems, including transient interaction effects.
- The proposed approach for modelling surface irregularities along the load trajectory in coupled moving-mass analyses may be

employed to account for the influence of road micro-profile in the dynamic analysis of highway bridge spans.
- Future research should focus on extending the proposed verification framework to more complex structural systems and investigating mesh convergence and time-step sensitivity for nonlinear dynamic problems.
- Experimental validation of moving-load interaction models also represents an important direction for further studies.

REFERENCES

1. Babakov, I.M., Theory of Vibrations, Drofa Press, Moscow, 2004. (In Russian).
2. Zienkiewicz, O., Morgan, K., Finite Elements and Approximation, Mir Publishers, Moscow, 1986.
3. Bathe, K.J., Finite Element Procedures, Prentice Hall, New Jersey, 1996.
4. Hughes, T.J.R., The Finite Element Method: Linear Static and Dynamic Finite Element Analysis, Dover Publications, New York, 2000.
5. Perelmuter, A.V., Design models of structures and the possibility of their analysis, DMK Press, Moscow, 2011.
6. Verification report of MIDAS GTS software package – Vol.1: General information and verification matrices, Podzemproekt, Moscow, 2012.
7. Hallquist, J. O., LS-DYNA Theory Manual, Livermore Software Technology Corporation (LSTC), Livermore, CA, USA, 2019.
8. Livermore Software Technology Corporation (LSTC), LS-DYNA Keyword User's Manual, Version R13.0, Livermore, CA, USA, 2021.
9. Fryba, L., Vibration of solids and structures under moving loads, Thomas Telford Publishing, London, 1999.
10. Koç, M.A., Esen, İ., Modelling and analysis of vehicle–structure–road coupled interaction considering structural flexibility, vehicle parameters and road roughness, Journal of Mechanical Science and Technology, 2017; 31(5), 2057–2074. https://doi.org/10.1007/s12206-017-0403-y
11. Jiang, L., Zhang, Y., Feng, Y., Zhou, W., Tan, Z., Dynamic response analysis of a simply supported double-beam system under successive moving loads, Applied Sciences, 2019; 9(10), 2162.
12. Li, J., Zhang, H., Chen, S., Investigation of axle-span ratio and moving load speed affecting bridge extreme response using a moving load amplitude spectrum method, Structures, 2020; 28, 2633–2645.
13. Esen, İ., Dynamic response of a beam due to an accelerating moving mass using moving finite element approximation, Mathematical and Computational Applications, 2011; 16(4), 171–182.
14. Esen, İ., Dynamic response of functionally graded Timoshenko beams subjected to accelerating load, European Journal of Mechanics A/Solids, 2019; 78, 103841.
15. Teng, X., Zhang, Y. X., Structural behavior FRP-strengthened steel-reinforced concrete slabs under moving-wheel cyclic loads, Australian Journal of Structural Engineering, 2017; 18(4), 86–94.
16. Abid, M., Ghoulbzouri, A., Ikharrazne, L., FE modelling progressive collapse assessment of steel moment frames – parametric study, Australian Journal of Structural Engineering, 2022; 23(7), 1–21.
17. Al-Mosawe, A., Al-Mahaidi, R., Alwash, D., Zhao, X.-L., Finite element simulation of unbonded retrofitting system for a steel bridge in Australia, Australian Journal of Structural Engineering, 2021; 22(1), 29–41.
18. Ravodin, I. V., Gridnev, S. Yu., Skalko, Yu. I., Safronov, V. S., Tran Thi, T. V., Verification of the modelling results of oscillations of an elastic-supported system with displacement limiters under moving load, Australian Journal of Structural Engineering, 2023. https://doi.org/10.1080/13287982.2023.2214414
19. Yildirim, E., Esen, İ., Dynamic behavior and force analysis of the full vehicle model using Newmark average acceleration method, Engineering Technology Applied Science Research, 2020; 10(1), 5330–5339.
20. Eroglu, M., Koç, M. A., Esen, İ., Kozan, R., Train–structure interaction for high-speed trains using a full 3D train model, Journal of the Brazilian Society of Mechanical Sciences and Engineering, 2022; 44(1), 1–28.
21. Koç, M. A., Esen, İ., Çay, Y., A new numerical method for analysing bridge–vehicle interaction, International Journal of Heavy Vehicle Systems, 2021; 28(1), 79–107.
22. Alimoradzadeh, M., Salehi, M., Esfarjani, S. M., Nonlinear dynamic response of AFG beam resting on nonlinear elastic foundation subjected to moving load, Nonlinear Engineering, 2018; 8, 250–260.
23. Shitikova, M., Krusser, A., Force driven vibrations of nonlinear plates on viscoelastic Winkler foundation, International Journal for Computational Civil and Structural Engineering, 2021; 17(4), 161–180.
24. Safronov, V. S., Antipov, A. V., Modern algorithms for dynamic analysis of beam systems under moving sprung load, Structural Mechanics and Structures, 2019; 1(20), 30–40. (In Russian version).
25. Gridnev, S. Yu., Ravodin, I. V., Finite element modeling of a moving load using contact conditions,

XXVII R-S-P Seminar 2018, Rostov-on-Don, Russia.

26. Gridnev, S. Yu., Skalko, Yu. I., Ravodin, I. V.,"Simulation of vibrations of elastically supported dynamical system with movement limiters, Structural Mechanics and Structures, 2017; 14, 108–117.
27. Gridnev, S. Yu., Ravodin, I. V., Experimental analysis of oscillations of a model of an elastic-supported dynamic system with displacement limiters, Structural Mechanics and Structures, 2021; 29, 92–103.
28. Abujnah, N., Ramadan, N., Dynamic analysis of a cracked beam under moving load based on the modified Adomian decomposition method, Structures, 2024. https://doi.org/10.1016/j.istruc.2024.04.086
29. Zhang, J., Wang, Y., Duan, H., Li, Y., Analyzing bridge resonance and lateral vibrations using string vibration principles, 2023. https://arxiv.org/abs/2311.11061
30. Ma, S., Chen, M., Skelton, R. E., Tensegrity system dynamics based on finite element method, 2021. https://arxiv.org/abs/2106.02176