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Modelling for dynamic matrix controller and its impact on the closed-loop control performance for higher order processes

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ABSTRACT

This paper describes the possibility of extending the functionality of the DMC controller in the application to control processes with higher-order dynamics, also with delay. Normally, in such cases, the dynamic matrix control (DMC) controller uses a predictive model in the form of a sampled step response of the process or samples of the step response generated using first-order plus dead time (FOPDT) model obtained from the process step response approximation. In this paper it is proposed to replace the FOPDT model used to generate step response samples with the second-order plus dead time (SOPDT) model, which allows for a more accurate representation of the dynamics of the controlled process in the case of processes with a higher order dynamics. The SOPDT model can be determined from the step response of the process, similarly to the FOPDT model. The analysis of the proposed solution was carried out using the simulation validation using benchmark dynamic processes and selected methods of DMC tuning. The research also takes into account the of measurement noise. The results clearly indicate that replacing the FOPDT model for generating step response samples with the SOPDT model gives significant benefits in the cases of processes with a higher order of dynamics, resulting in faster response time and smoother control action. The obtained results indicate a particularly pronounced improvement in the control effort index. In the presence of measurement noise, the improvement reaches up to 25% compared to the FOPDT-based model.

Keywords: dynamic matrix control, model predictive control, FOPDT, SOPDT, tuning.

INTRODUCTION

The proportional–integral–derivative (PID) algorithm is widely recognized as the dominant control strategy in industrial control loops. It is estimated that PID controllers are employed in over 90–95% of all industrial control systems [1]. Their widespread applications result mainly from a simple fundamental structure, which provides satisfactory performance across a broad range of applications, along with the availability of supplementary features such as autotuning algorithms [2].

However, conventional PID control is often inadequate for processes exhibiting strong nonlinearities, time-varying dynamics, or significant transport delays [3]. To circumvent these limitations, the PID controller structure can be enhanced with supplementary mechanisms, such as gain scheduling or the Smith predictor, which can significantly improve the control performance [4, 5]. Nevertheless, such extensions typically require additional identification experiments and the development of at least simplified process model. These requirements often exceed the practical competencies of most process engineers.

An alternative to PID control is offered by model predictive control (MPC) algorithms. MPC strategies utilize a mathematical model of the process to predict its future behaviour and compute an optimal control trajectory by minimizing a predefined performance index [6]. Numerous studies indicate that MPC controllers significantly outperform conventional PID controllers in terms of control performance [7, 8]. Depending on the application, MPC algorithms may employ various types of process models, including state-space models, time-series models, as well as more complex nonlinear models, neural networks or fuzzy systems [9].

One of the most popular MPC variants is the dynamic matrix control (DMC) algorithm [10]. Conventional DMC approach utilizes the sampled process step response for prediction, which makes it intuitive and relatively easy to identify in the industrial conditions. It is particularly effective in handling processes with significant transport delays, however lack of built-in adaptation which is a result of linear modelling results in poor performance in case of strongly nonlinear processes [7]. Numerous extensions of the DMC controller have been proposed in the literature [11, 12].

The performance of the DMC controller is highly dependent on the selection of its tuning parameters. Fundamental tuning rules for the DMC controller were developed by Sridhar and Cooper [13]. These rules rely on approximating the process step response using a first-order plus dead time (FOPDT) model, whose parameters are subsequently used to calculate the DMC tuning parameters. In [14], tuning rules based on reduced horizons were proposed, offering satisfactory control performance while minimizing memory requirements - a critical consideration for implementation in programmable logic controllers (PLCs). Further DMC tuning approaches are reviewed in [15–17].

Another factor that significantly influences the performance of a DMC algorithm is the accuracy of the process model used for prediction. The DMC controller applies a finite set of step response samples that constitute its internal predictive model. These samples may be obtained directly from identification experiments conducted on the real process. However, such an approach is susceptible to measurement noise, which can adversely affect the quality of the identified step response and, consequently, the closed-loop performance of the controller. From a theoretical perspective, measurement noise directly affects the quality of state or output information available to the controller and, consequently, the accuracy of model-based predictions. In predictive control algorithms such as DMC, the internal model is in a form of direct samples of the step response data; therefore, noise present during identification propagates into the model parameters and distorts future output predictions. This leads to erroneous estimation of the predicted error trajectory and may result in excessive control activity, degraded robustness, and increased control effort. Hence, the presence of measurement noise plays a critical role in predictive control performance, particularly when the internal model is directly derived from experimental data.

To mitigate the impact of noise and improve model reliability, step response samples are often generated using an identified mathematical model of the process [10, 13]. Among the available modelling approaches, the FOPDT model is most commonly used due to its simplicity and ease of identification [6, 14]. The parameters of the FOPDT model can be determined using classical two-point methods or numerical optimization techniques and are also widely applied in the tuning of PID controllers. However, despite its computational efficiency and practical relevance, the FOPDT model can provide an inadequate representation of higher-order or more complex process dynamics.

For such processes, the second-order plus dead time (SOPDT) model offers increased modelling accuracy. Previous studies have demonstrated that employing SOPDT models for the tuning of advanced control algorithms can lead to improved control performance and robustness [18, 19]. The identification of SOPDT model parameters is inherently more complex comparing to FOPDT model, as it typically requires the application of numerical techniques minimizing the modelling error. Nevertheless, the SOPDT model provides a significantly more accurate representation of higher-order process dynamics, making it a more effective choice for modelling complex industrial processes. Application of SOPDT modelling for MPC has been already investigated [20], however this approach was not used for synthesis of DMC controller.

To the best authors' knowledge, the existing literature lacks comprehensive studies that systematically investigate the impact of the method used to generate step response samples used as the internal model of the DMC controller on closed-loop control performance. To address this significant gap in the existing literature the main scientific contribution of this work lies in the systematic analysis of how the choice of internal predictive model affects the closed-loop performance of the DMC controller. In particular, step responses obtained directly from experimental data are compared with those generated using identified FOPDT and SOPDT process approximations. The use of an SOPDT model represents a compromise between modelling accuracy and the complexity of parameter identification. While more complex model structures increase the computational burden of the identification procedure, these calculations are performed only once during the controller design stage. Moreover, additional computational resources, such as fog-computing or cloud-computing platforms, can be employed to facilitate this process without impacting real-time controller operation.

The analysis is carried out through simulation studies performed on representative higher-order processes. The results clearly show that the application of the SOPDT model constitutes a practical compromise between direct use of noisy step response measurements and employing a simplified FOPDT approximation, providing improved modelling accuracy and at the same time ensuring reasonable identification computational complexity.

The remainder of this paper is organized as follows. Section 2 provides a concise derivation of the DMC control law together with a discussion of the tuning procedure and the key design parameters. Section 3 presents the simulation study, including the description of the benchmark processes and a detailed analysis of the obtained results. Finally, Section 4 concludes the paper and outlines directions for future research.

DYNAMIC MATRIX CONTROLLER

The detailed description of DMC approach can be found in literature, e.g. [6, 10]. This section presents only the fundamental principles of deriving the DMC control law and its tuning to help the readers to follow the concept presented in this paper.

Derivation of the DMC control law

The DMC algorithm represents the process dynamics using a series of sampled process step response . For the SISO processes with the output y and the manipulating input u and according to the superposition principle, the DMC approach enables the prediction of the discrete-time output response to a set of past (which corresponds to free component y^0) and future (which corresponds to forced component Δy) increments of the controlled output:

$$y(k+p \mid k) = y(k) + \\ + \sum_{j=1}^{H_P} s_j \, \Delta u(k+p-j \mid k) + \\ + \sum_{j=1}^{H_D} \left(s_{j+p} - s_j\right) \Delta u(k-j) \tag{1}$$

where: the notation $(k + p \mid k)$ refers to the value predicted for the future time instant $(k + p)$, computed at the current time instant k, H_P is the prediction horizon and H_D is the dynamics horizon.

By introducing vector representations of the predicted future process output and control increments, the DMC prediction model can be expressed in a compact matrix form. The reference trajectory, free response, forced response, and control increment vectors can be defined as

$$\mathbf{y}^{sp}(k) = [y^{sp}(k+H_W \mid k) \quad \cdots \quad y^{sp}(k+H_P \mid k)]^T \in \mathbb{R}^{H_P-H_W+1} \tag{2}$$

$$\mathbf{y}^{0}(k) = [y^{0}(k+H_W \mid k) \quad \cdots \quad y^{0}(k+H_P \mid k)]^T \in \mathbb{R}^{H_P-H_W+1} \tag{3}$$

$$\Delta\mathbf{y}(k) = [\Delta y(k+H_W \mid k) \quad \cdots \quad \Delta y(k+H_P \mid k)]^T \in \mathbb{R}^{H_P-H_W+1} \tag{4}$$

$$\Delta\mathbf{u}(k) = [\Delta u(k \mid k) \quad \cdots \quad \Delta u(k+H_C-1 \mid k)]^T \in \mathbb{R}^{H_C} \tag{5}$$

$$\Delta\mathbf{u}^{P}(k) = [\Delta u(k-1) \quad \cdots \quad \Delta u(k-H_D)]^T \in \mathbb{R}^{H_D-1} \tag{6}$$

$$\mathbf{y}(k) = [y(k) \quad \cdots \quad y(k)]^T \in \mathbb{R}^{H_P-H_W+1} \tag{7}$$

where: H_W is the number of samples representing the process time delay and y^{sp} is the closed-loop setpoint. Using the above definitions 2–7, the predicted process output can be expressed in the matrix-vector notation as:

$$\boldsymbol{y}(k) = \mathbf{y}(k) + \mathbf{M}^P \Delta \mathbf{u}^P(k) + \mathbf{M}\Delta \mathbf{u}(k) \quad (8)$$

where $\mathbf{M}^P \in \mathbb{R}^{H_P - H_W + 1 \times H_D - 1}$ is the past input matrix:

$$\mathbf{M}^P = \begin{bmatrix} s_{1+H_W} - s_1 & s_{2+H_W} - s_2 & \cdots & s_{H_D+H_W} - s_{H_D} \\ s_{2+H_W} - s_1 & s_{3+H_W} - s_2 & \cdots & s_{H_D+1+H_W} - s_{H_D} \\ \vdots & \vdots & \ddots & \vdots \\ s_{H_P+1} - s_1 & s_{H_P+2} - s_2 & \cdots & s_{H_P+H_D} - s_{H_D} \end{bmatrix} \quad (9)$$

and $\mathbf{M} \in \mathbb{R}^{H_P - H_W + 1 \times H_C}$ denotes the dynamic matrix of the DMC algorithm, defined as:

$$\mathbf{M} = \begin{bmatrix} s_{H_W} & 0 & \cdots & 0 \\ s_{H_W+1} & s_{H_W} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ s_{H_C+H_W-1} & s_{H_C+H_W-2} & \cdots & s_{H_W} \\ s_{H_C+H_W} & s_{H_C+H_W-1} & \cdots & s_{H_W+1} \\ \vdots & \vdots & \vdots & \vdots \\ s_{H_P} & s_{H_P-1} & \cdots & s_{H_P-H_C+1} \end{bmatrix} \quad (10)$$

where: H_C is the control horizon.

The objective function of the DMC algorithm is also formulated in a compact form as:

$$J(k) = \| \mathbf{y}^{sp}(k) - \mathbf{y}^0(k) - $$
$$- \mathbf{M}\Delta\mathbf{u}(k) \|^2 + \lambda \| \Delta\mathbf{u}(k) \|^2 \quad (11)$$

where: coefficient λ is the user-adjustable move suppression factor. Since the objective function is strictly convex, the necessary and sufficient condition for its minimization is given by the condition:

$$\nabla J(k) = 0 \quad (12)$$

and its analytical solution leads to:

$$(\mathbf{M}^T\mathbf{M} + \lambda\mathbf{I})\Delta\mathbf{u}(k) = \mathbf{M}^T[\mathbf{y}^{sp}(k) - \mathbf{y}^0(k)] \quad (13)$$

Solving the above equation yields the optimal control increment vector

$$\Delta\hat{\mathbf{u}}(k) = (\mathbf{M}^T\mathbf{M} + \lambda\mathbf{I})^{-1}\mathbf{M}^T[\mathbf{y}^{sp}(k) - \mathbf{y}^0(k)] =$$
$$= \mathbf{K}[\mathbf{y}^{sp}(k) - \mathbf{y}^0(k)] \quad (14)$$

where: $\mathbf{K} \in \mathbb{R}^{H_C \times H_P - H_W + 1}$ denotes the DMC gain matrix.

According to the repetition principle, at each discrete time instant k only the first element of the optimal control increment vector is applied to the process. Therefore, it is sufficient to consider the first row of the gain matrix **K**, which leads to the final DMC control law:

$$\Delta\hat{u}(k) = k^e[y^{sp}(k) - y(k)] -$$
$$- \sum_{j=1}^{H_D} k_j^u \, \Delta u(k-j) \quad (15)$$

where: $k^e = \sum_{p=H_W}^{H_P} K_{1,p-H_W+1}$, and k_j^u denotes the j-th element of the first raw of the matrix $\mathbf{KM^P}$.

The parameters k^e and k_j^u are computed offline at the controller synthesis stage. The primary factor affecting the computational complexity associated with determining the parameters k^e and k_j^u is the inversion of the matrix $(\mathbf{M}^T\mathbf{M} + \lambda\mathbf{I})$, whose dimension is $H_C \times H_C$. This operation dominates the off-line computational burden of the controller synthesis stage. The remaining calculations are based on standard matrix multiplications and summations involving step response samples, the dimensions of which depend on the selected tuning parameters. An additional aspect impacting computational complexity is the method used to generate the step response samples. In this study, the samples are obtained from FOPDT and SOPDT models and in both cases, the step responses can be computed using simple algebraic expressions.

A block diagram of the resulting control system is shown in Figure 1. It consists of DMC control law (15), which calculates the optimal control increment $\Delta\hat{u}$ based on setpoint y^{sp} and actual measurement of process value y. It is then integrated with $\frac{1}{1-z^{-1}}$ term, where z^{-1} is the delay operator resulting in control signal u, which is applied to the process. Consequently, the final computation of the DMC control signal at a given sampling instant requires only a limited number of elementary operations: two subtraction operations, (HD + 1) multiplication and addition operations. In addition, the vector of past control increments $\Delta u(k - j)$ must be shifted and updated to account for the newly applied control increment. This operation involves simple memory management and does not significantly affect the overall computational burden of the algorithm. It is a basic form of DMC control law without any constraints on control increment $\Delta\hat{u}$ and control signal u. For practical implementation, one could

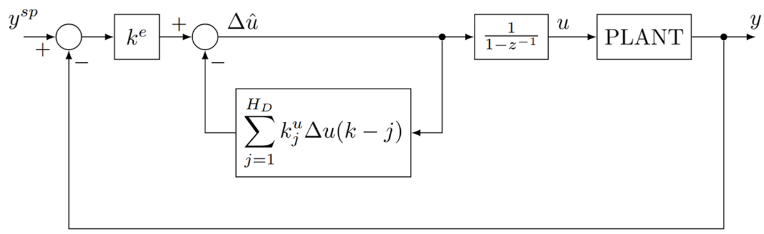

Figure 1. Block diagram of DMC control law

apply complete DMC framework including control constraints presented in [6].

Tuning of DMC

The determination of the parameters k^e and k_j^u requires appropriate tuning of the DMC algorithm, as the selection of tuning parameters has a decisive impact on the resulting closed-loop control performance. In practical applications, these parameters are most derived based on the FOPDT process model [13, 14] identified at a given operating point

$$k^e = \sum_{p=H_W}^{H_P} K_{1,p-H_W+1} \quad (16)$$

where: k denotes the steady-state gain, T is the dominant time constant, and T_0 represents the delay time.

The tuning parameters of the DMC algorithm are summarized below.

- The window horizon H_W specifies the number of initial samples of the process step response equal to zero due to the presence of the process time delay ($s_i = 0$). These samples are excluded from the evaluation of the predicted error trajectory. By omitting the first ($H_W - 1$) terms in the objective function, the computational burden is reduced without affecting the optimization result, since these terms do not affect the control decision.
- The prediction horizon H_p determines the number of future discrete time instants, specifically ($H_p - H_W + 1$), over which the output control error is predicted. This parameter should be chosen such that the objective function captures the dominant process dynamics through an adequate number of step response coefficients. Increasing H_p generally enhances closed-loop stability but may result in longer settling times.
- The control horizon H_C defines the number of future control increments used as decision variables in the optimization problem. For $H_C = 1$, the closed-loop system may exhibit oscillatory behavior. Increasing the control horizon beyond $H_C > 2$ typically results in a marginal improvements in control performance while computational complexity significantly increases.
- The dynamic horizon H_D corresponds to the number of discrete samples required for the process step response to reach steady state. Similar to the prediction horizon, it should be selected according to the process dynamics. Increasing H_D beyond the settling time does not introduce additional information about the process but increases the number of the parameters k_j^u that must be stored for computing the control law.
- Move suppression coefficient λ is also called the penalty factor for control increments. This parameter determines the relative balance between the predicted output tracking error and the variability of the control signal. Larger values of λ result in smaller control increments, leading to smoother actuator operation at the cost of increased settling time. Moreover, sufficiently large values of λ can enhance closed-loop stability in the presence of modelling uncertainty. Decreasing λ results in more aggressive control action, which can lead to oscillatory behaviour and overall decrease of robustness.

Determining the general closed-loop stability conditions is very difficult in practice, due to high number of tuning parameters and accuracy of internal process model in a form of step response samples. However, the general rules of thumb could be potentially suggested for improving stability of the closed loop system. When the prediction horizon H_P and move suppression coefficient λ are increased, it results in more accurate prediction of output trajectory and higher penalties for control increment, thus in

Table 1. Selected tuning rules for DMC

SC	RH
$H_W = \frac{T_0}{T_c} + 1$	$H_W = \frac{T_0}{T_c} + 1$
$H_C = 2$	$H_C = 2$
$H_P = \frac{5T}{T_c} + \frac{T_0}{T_c} + 1$	$H_P = \frac{T}{T_c} + \frac{T_0}{T_c}$
$H_D = \frac{5T}{T_c} + \frac{T_0}{T_c} + 1$	$H_D = \frac{3T}{T_c} + \frac{T_0}{T_c}$
$\lambda = fk^2$	$\lambda = xk^2 H_P$
$f = \frac{H_C}{500}\left(3.5\frac{T}{T_c} + 2 - \frac{H_C - 1}{2}\right)$	$x \geq \frac{0.0146}{1 + \frac{T_0}{T}}$

smoother control action and consequently more conservative closed-loop performance.

Adjusting of the above DMC tuning parameters can be made by trial and error approach or using the dedicated tuning method. In this paper, two tunings method are applied and shown in Table 1. The first method, hereafter called as the SC approach, was proposed by Shridhar and Cooper [13]. This method assumes that both the prediction horizon and the dynamic horizon should cover the entire significant portion of the process step response, defined as the time required for the output to reach 99.3% of its steady-state value. Consequently, the horizons are selected based on the relation $5T + T_0$.

The second method, denoted as the RH approach, was introduced by Klopot *et. al*. [14]. In this case, reduced prediction and dynamic horizons are employed. Such a reduction facilitates the implementation of the DMC algorithm on platforms with limited computational capabilities, such as programmable logic controllers (PLCs), while avoiding noticeable degradation of control performance. This tuning strategy assumes that the essential information about the process dynamics is contained within the portion of the step response corresponding to 95% of the steady-state value, which is approximated by $3T + T_0$.

In both tuning approaches considered in this study, the controller sampling time was set based on the time constant of the process FOPDT approximation:

$$T_C = 0.1T \tag{17}$$

In practical process automation systems, the sampling time achievable with PLC controllers can be significantly shorter than the values theoretically required and even assumed in this study. Therefore, it does not constitute a practical limitation. It should be emphasized, however, that the choice of the sampling time directly affects both the prediction and dynamic horizons. The latter, in turn, determines the number of parameters that must be stored in the PLC memory, which may impact the overall implementation requirements.

SIMULATION STUDIES

This section presents the results of simulation studies that illustrate how the selection of the model used for prediction in DMC approach affects the quality of closed loop behavior for selected systems with different higher-order dynamics.

Selection of dynamical systems

Three dynamical systems with different complex dynamics included in the set of benchmark dynamical systems [21], were selected and used in this study to represent the dynamics of the controlled processes. The selected systems enable a comprehensive evaluation of the closed-loop DMC controller performance for processes exhibiting both higher-order dynamics and significant transport delays:

- the Fourth-order system with multiple equal poles (MEP):

$$K(s) = \frac{1}{(s+1)^4} \tag{18}$$

which represents a high-order lag process with uniform time constants,

- the Fourth-Order System with exponentially decaying (FOS) with ($\alpha = 0.5$):

$$K(s) = \frac{1}{(s+1)(1+\alpha s)(1+\alpha^2 s)(1+\alpha^3 s)} \quad (19)$$

representing a process with distributed time constants that exhibits more gradual dynamic behaviour compared to the MEP system,

- the third-order plus dead time (TOPDT) system:

$$K(s) = \frac{1}{(s+1)^3} e^{-0.5s} \quad (20)$$

representing processes with both higher-order dynamics and a measurable delay time.

In the further part of this work, the FOPDT and SOPDT approximations are also used for each of the considered dynamic processes (MEP, FOS and TOPDT) where the SOPDT model is described by

$$K(s) = \frac{k}{(sT_1+1)(sT_2+1)} e^{-sT_0} \quad (21)$$

where: k denoting process gain, T_1, T_2 are the time constants, and T_0 is the transport delay (dead time).

The parameters of both approximating models were computed numerically using the *fmincon()* optimization solver in the MATLAB environment. The identification procedure consisted of minimizing an objective function defined as the sum of squared errors between the discrete step response samples of the respective process and those of the approximating model. The applied solver requires initial set of the decision variables (k_{init}, $T_{j,int}$, $T_{0,init}$), with $j = 1$ for FOPDT and $j = 1, 2$ for SOPDT approximation models and the their predefined constraints. As a rule of thumb, step response samples of the process could be initially approximated by FOPDT model e.g. by two-points method resulting in set of parameters (k_{tpm}, T_{tpm}, $T_{0,tpm}$). Then, obtained parameters could be used as an initial set for *fmincon()* solver for FOPDT ($k_{init} = k_{tpm}$, $T_{1,init} = T_{tpm}$, $T_{0,init} = T_{0,tpm}$) and SOPDT ($k_{init} = k_{tpm}$, $T_{1,init} = 0.8 \cdot T_{tpm}$, $T_{2,init} = 0.8 \cdot T_{tpm}$, $T_{0,init} = 0.7 \cdot T_{0,tpm}$). For constraints of the decision variables, the values greater by 100% than their initial values could be suggested as the upper constraint. The lower constraints can be adjusted to zero assuming positive process gain.

Finally, the parameters of these approximations with optimal values of modelling error are presented in Table 2 and used unchanged in the rest of this work. The quality of such approximation can be visually assessed in Figure 2 for the example of the MEP process.

Performance indices for quantitative evaluation of the closed loop behaviour

To provide a quantitative and objective comparison of the control performance, several numerical performance indices frequently used for closed-loop performance validation are used in this study.

the settling time t_r defines the time between the moment of applying the disturbance to the closed-loop system and the moment when the

Table 2. FOPDT and SOPDT parameters for considered benchmark processes

FODPT		SOPDT	
Parameters	Modelling error	Parameters	Modelling error
MEP			
$k = 1$ $T = 2.36$ $T_0 = 1.83$	1.176	$k = 1$ $T_{1,2} = 1.5$ $T_0 = 1.08$	0.184
FOS			
$k = 1$ $T = 0.64$ $T_0 = 0.33$	0.00251	$k = 1$ $T_{1,2} = 0.4$ $T_0 = 0.14$	0.00243
TOPDT			
$k = 1$ $T = 2.01$ $T_0 = 1.63$	0.765	$k = 1$ $T_{1,2} = 1.27$ $T_0 = 1.0$	0.0653

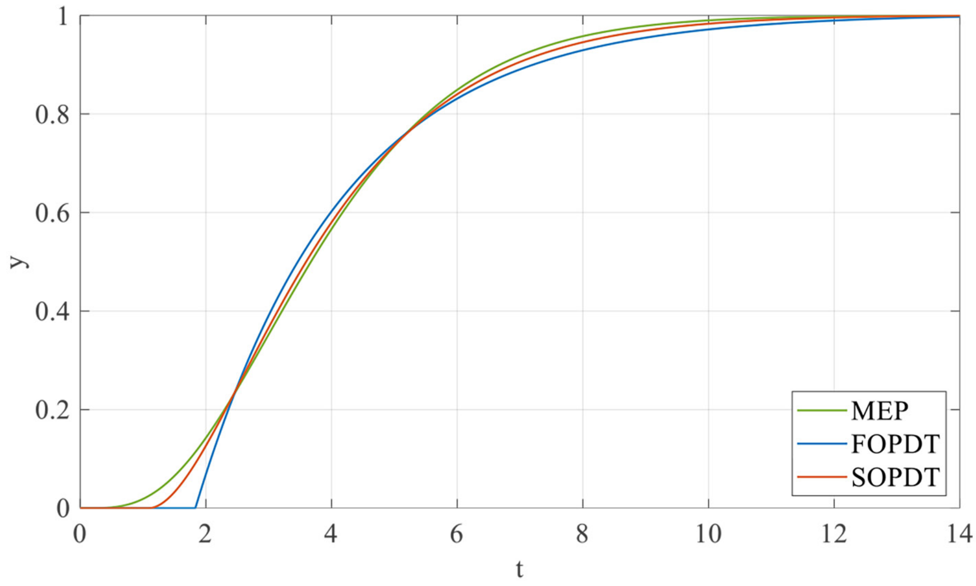

Figure 2. Step response of MEP benchmark process with its FOPDT and SOPDT approximations

output signal permanently returns to the interval defined as:

$$y^{sp} \pm 0.02 \cdot |y^{sp} - y(0)| \tag{22}$$

where: $y(0)$ denotes the initial output value.

The rising time t_n characterizes the speed of the closed loop response and is defined as:

$$t_n = t_{90\%} - t_{10\%} \tag{23}$$

where: $t_{10\%}$ and $t_{90\%}$ correspond to the moments of time when the output respectively reaches

$$y(t_{10\%}) = y(0) + 0.1 \cdot |y^{sp} - y(0) \tag{24}$$

$$y(t_{90\%}) = y(0) + 0.9 \cdot |y^{sp} - y(0)| \tag{25}$$

the maximum overshoot (*max0S*) quantifies the largest absolute deviation between the process output and the setpoint.

the Integral of absolute error (*IAE*) criterion computed as:

$$IAE = \sum |y^{sp} - y| \cdot T_c \tag{26}$$

where: T_s denotes the sampling period.

the control effort (*CE*) represents the variability of the applied control actions and is defined as:

$$CE = \sum |\Delta u| \tag{27}$$

These performance indices provide a comprehensive assessment of the closed-loop control system, capturing both the speed and accuracy of the output response, as well as the aggressivity of the control actions.

Impact of quality of the internal predictive model for the conventional DMC formulation

In its classical formulation, the DMC algorithm relies on the model of the process represented as the samples of the process step response obtained directly from the real process. One factor that may significantly affect the resulting control performance of this approach is the presence of the measurement noise in the collected samples. Consequently, the initial stage of the analysis focuses on assessing how different noise levels affect the closed-loop performance of the conventional DMC controller.

For this purpose, for each considered dynamical processes MEP, FOS and TOPDT, three different approaches of deriving DMC controller are investigated. The first approach is based on the simulated sampled process step responses without measurement noise and this case is called as "reference DMC" (REF). It represents the perfect case that is not realistic from the practical viewpoint but should be considered as the case representing reference closed-loop performance. Then, two levels of the measurement noise are added to the samples of the simulated step responses of each considered dynamical process. In both cases the samples of the measurement noise are generated as the normally distributed random signals: $N(0, \sigma^2 = 0.0002)$ and $N(0, \sigma^2 = 0.0006)$. Finally, the noiseless step responses for each MEP, FOS and TOPDT systems were approximated by FOPDT models (see Table 2) and these models were used for tuning of the corresponding DMC (namely, adjusting the respective values of the parameters k^e and k_j^u). Tuning was made using SC tuning method.

Closed-loop simulation experiments were obtained for three closed-loop systems, each of which consisted of a different process MEP, FOS or TOPDT and the corresponding DMC controller. These experiments involved tracking without any measurement noise additionally introduced in the simulation of the considered closed-loop systems. One should be aware that such simulations experiments are not realistic but they were carried out this way on purpose. This procedure enables for separate assessment how the inaccuracy of the internal DMC prediction model (here the presence of the measurement noise) represented as the series of the samples of the process step response impacts the closed loop performance. The results of such experiments are shown in Figures 3–5 and

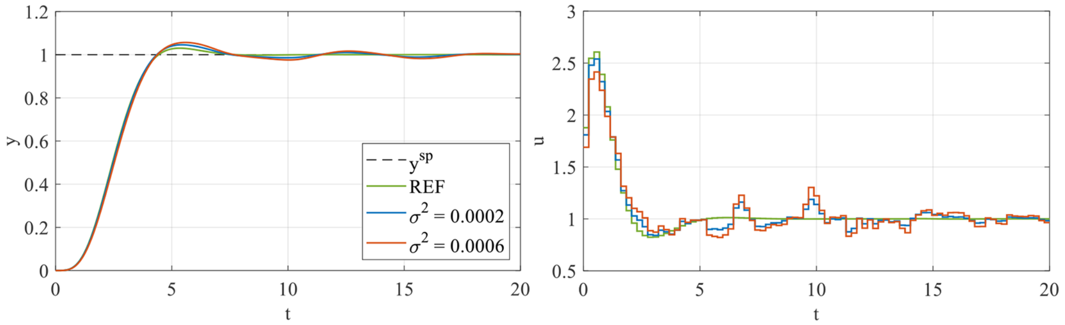

Figure 3. Comparison of DMC tuned based on SC tuning rules and process response samples for selected noise levels for step setpoint change – process value (left plot) and manipulated value (right plot) for MEP benchmark process

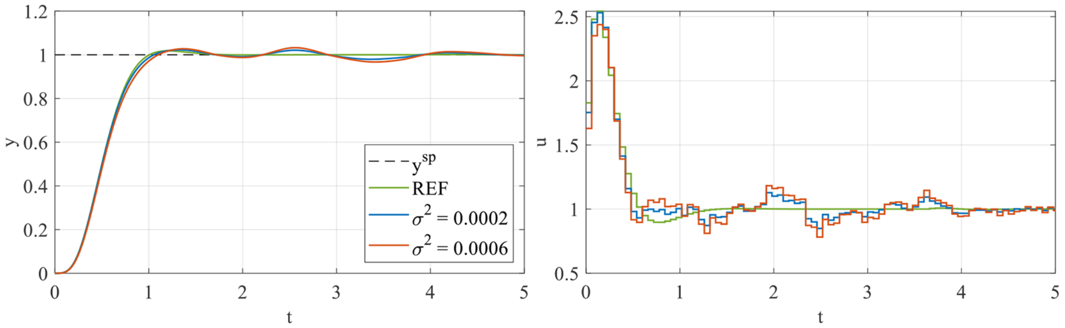

Figure 4. Comparison of DMC tuned based on SC tuning rules and process response samples for selected noise levels for step setpoint change – process value (left plot) and manipulated value (right plot) for FOS benchmark process

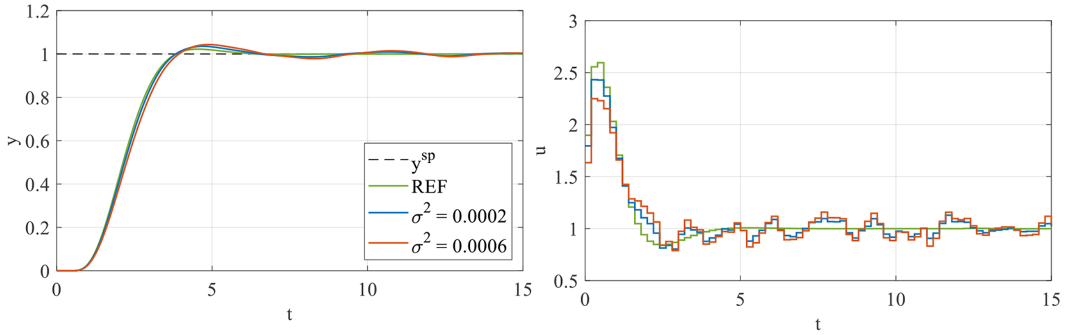

Figure 5. Comparison of DMC tuned based on SC tuning rules and process response samples for selected noise levels for step setpoint change – process value (left plot) and manipulated value (right plot) for TOPDT benchmark process

Table 3. Performance indices calculated based on step response of closed loop systems, where the DMC algorithm operated with the noise-corrupted step response samples as the internal predictive model

σ^2	t_r	t_n	*maxOS*	*IAE*	*CE*
MEP					
0 (REF)	6.19	2.40	0.030	2.54	4.60
0.0002	6.75	2.39	0.046	2.67	6.58
0.0006	10.57	2.42	0.057	2.79	7.75
FOS					
0 (REF)	0.94	0.54	0.018	0.53	4.32
0.0002	3.44	0.56	0.023	0.57	6.09
0.0006	3.73	0.59	0.033	0.60	7.09
TOPDT					
0 (REF)	4.88	1.88	0.022	2.22	4.53
0.0002	5.62	1.92	0.036	2.34	6.78
0.0006	8.55	2.00	0.044	2.44	7.78

their qualitative assessment is presented in Table 3. For the strict analysis of the impact of measurement noise, a rigorous statistical evaluation may be relevant. However, in practical applications such an analysis is often highly challenging. Nevertheless, for the purposes of this study, the primary focus is placed on the observable trends in the individual performance indices rather than on detailed statistical inference.

The analysis of the obtained results indicates that the presence of noise in the step response samples directly used as the internal DMC prediction model significantly affects the control performance, regardless of the dynamics of the controlled process

An increase in the noise level leads to a deterioration of all considered performance indices. In particular, the control signal exhibits increased variability, forcing the controller to compensate for inconsistencies in the predictive model resulting only from the presence of the measurement noise in the sampled step response. As a consequence, the control effort measure *CE* increases significantly and, for higher noise levels, it can exceed twice the *CE* value obtained for the DMC reference case (REF) representing DMC tuning based on the noiseless process step responses.

The increased variability of the control signal directly results in the increased variability of the closed-loop process output response, which oscillates around the setpoint what is shown by higher values of the *IAE* index and increased maximum overshoot *maxOS*. Contrary, the rise time t_n is only marginally affected by the presence of noise, indicating that the initial tracking capability of the DMC controller is practically preserved. The negative effect is also observed in the settling time t_r, which increases significantly with the noise level for all considered processes. This deterioration occurs due to the presence of oscillations in the closed-loop responses.

Impact of using FOPDT and SOPDT process models for prediction on the DMC control performance

The results presented in the previous section clearly confirm that the quality of the internal model used for DMC synthesis significantly affects the closed-loop behaviour. The subsequent stage of the study investigates how this behaviour is affected by substituting the model in the form of the sampled process step response by the samples of the step response generated using the FOPDT and SOPDT approximations of the process dynamics. Potentially, such an approach should reduce the negative impact of the measurement noise and thus improve the closed-loop performance.

The experiments were conducted for three different DMC designs. One is the “reference DMC” denoted as REF and identical with the one used in section III.c. Two others were designed based on the samples of the step response of FOPDT and SOPDT approximations of the process step response and are respectively denoted as FOPDT and SOPDT. All three DMC design approaches were derived separately for each considered process MEP, FOS and TOPDT. To give the common ground for the conducted experiments, the same two DMC tuning approaches (SC and RH for x =

0.1 methods) were used and for each process, tunings of the corresponding three DMC controllers were computed based on the FOPDT approximation of the noiseless step responses of the respective process. These tunings remained unchanged during all subsequent experiments. The only exception is the window horizon H_W, which was selected individually for FOPDT and SOPDT approximating models.

Figures 6 and 7 present the closed-loop DMC performance for three considered internal predictive models and for MEP benchmark process. Results of the same experiments conducted for two other considered benchmark processes are presented in Appendix I. The results are also quantified by the performance indices defined in section III.a and their values are presented in Table 4.

In general, the analysis of the obtained results indicates the obvious conclusion that the best closed-loop performance was obtained for reference controller (REF) but once again readers should note that this reference case is not realistic in the practice. Using FOPDT model for generating the samples for the internal DMC predictive model deteriorates the closed-loop performance significantly. Using SOPDT for the same purpose allows to partially preserve the DMC performance that was lost by using FOPDT-based sampled internal model.

Focusing on comparison of two considered realistic approaches to DMC designed (FOPDT and SOPDT), it can be seen that the application of the SOPDT-based step response samples for DMC design improves control performance for all considered processes MEP, FOS and TOPDT comparing to FOPDT approach. All control error related performance indices are improved as it can be seen in Table 4 and in the closed-loop results presented in Figures 6–7 and Figures A1–A4 in Appendix I. In some cases, the control error related performance indices for SOPDT almost reach the values obtained for the REF case. This property clearly results from the fact that the SOPDT-based internal predictive model is able to approximate the dominant dynamics of higher-order processes much more accurately, especially when the process exhibits a clear set of dominant poles.

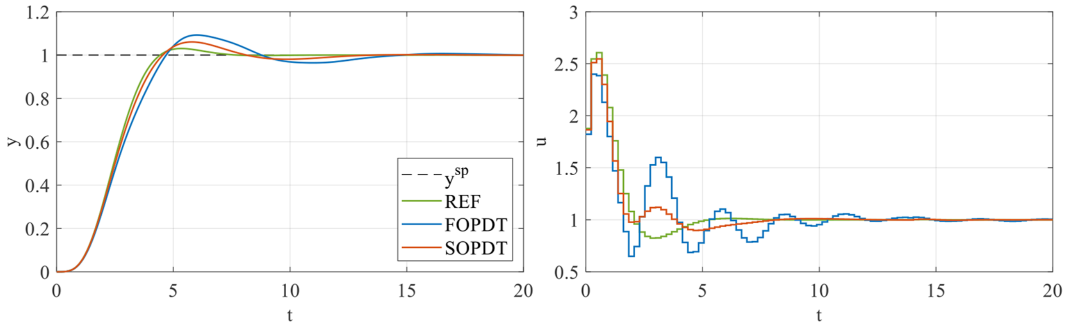

Figure 6. Comparison of DMC tuned based on SC tuning rules and original process response samples (REF) and its FOPDT and SOPDT approximations for tracking – process value (left plot) and manipulated value (right plot) for MEP benchmark process

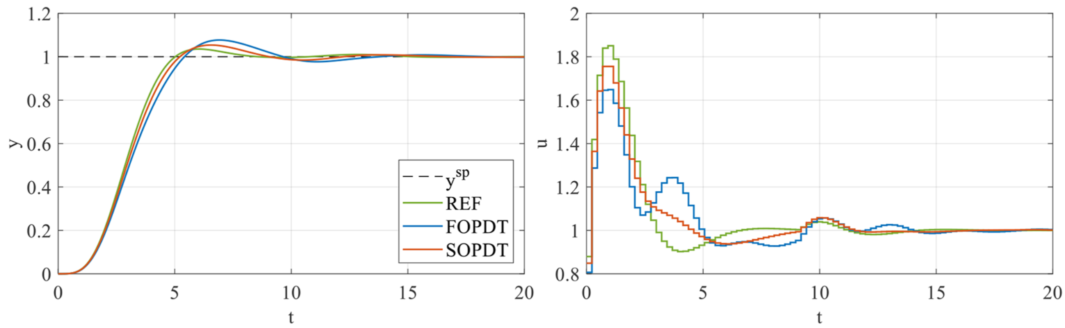

Figure 7. Comparison of DMC tuned based on RH (x=0.1) tuning rules and original process response samples (REF) and its FOPDT and SOPDT approximations for tracking – process value (left plot) and manipulated value (right plot) for MEP benchmark process

Table 4. Performance indices calculated for tracking for the closed loop systems. Best values of indices in terms of comparison between FOPDT and SOPDT cases are highlighted with green colour

Tuning method	Internal predictive model	t_r	t_n	*maxOS*	*IAE*	*CE*
MEP						
SC	REF	6.19	2.40	0.030	2.54	4.60
	FOPDT	12.76	2.79	0.092	3.06	7.49
	SOPDT	7.43	2.56	0.060	2.75	4.63
RH, x = 0.1	REF	7.16	2.70	0.035	3.00	3.04
	FOPDT	11.70	3.10	0.077	3.39	3.06
	SOPDT	8.23	2.88	0.053	3.17	2.78
FOS						
SC	REF	0.94	0.54	0.018	0.52	4.32
	FOPDT	2.57	0.62	0.065	0.61	4.78
	SOPDT	0.96	0.56	0.017	0.53	4.27
RH, x = 0.1	REF	1.15	0.65	0.017	0.65	2.84
	FOPDT	2.00	0.73	0.050	0.71	2.60
	SOPDT	1.16	0.66	0.016	0.65	2.83
TOPDT						
SC	REF	4.86	1.88	0.022	2.22	4.53
	FOPDT	10.79	2.26	0.092	2.62	7.20
	SOPDT	6.10	2.01	0.043	2.33	4.56
RH, x = 0.1	REF	5.96	2.21	0.028	2.65	2.93
	FOPDT	7.71	2.52	0.071	2.92	2.98
	SOPDT	6.81	2.31	0.040	2.73	2.77

The very similar tendency can be observed for the *CE* performance index that quantifies aggressivity of the control action. In all cases (apart from FOS process with RH, x = 0.1 DMC tuning) application of SOPDT case leads to reducing *CE* measure comparing to FOPDT case. In some cases, SOPDT approach allows for reducing *CE* measure even comparing to REF approach, which takes place for all process with RH, x = 0.1 DMC tuning and for FOS process with both DMC tuning methods. This fact shows that using the proposed SOPDT approach potentially allows for partial preserving the reference closed-loop DMC performance with a similar or even less aggressive control actions.

At the final stage it was decided to investigate how the presence of the measurement noise added in the closed loop simulations affects the results presented above. For this purpose, the same simulation experiments were conducted with the only difference that the measurement noise was added to each closed-loop simulation. As previously, two levels of noise were generated as two normally distributed random signals: $N(0, \sigma^2 = 0.0002)$ and $N(0, \sigma^2 = 0.0006)$ and they were separately added to each process input. In this case, the noisy reference DMC controller (REFn) is derived using the noisy step response samples of each process as the internal predictive model, which corresponds to a realistic scenario, as noiseless samples cannot be obtained without additional filtering or modelling. The closed-loop performance can be visually assessed for MEP process in Figures 8–11 and for two other processes (FOS and TOPDT) in Figures B1-B8 in Appendix 2. The qualitative assessment of all noisy results is presented in Table 5. In this table, the measure representing settling time t_r was removed on purpose because, for the previously assumed threshold, calculating the value of this indicator is useless in the presence of the measurement noise.

In general, the error-based performance indices show little deterioration of the closed-loop performance resulting from the presence of the measurement noise, in the comparison to the noiseless cases. However, the same tendency is generally preserved. The higher deterioration can be observed when FOPDT model is used for generating the internal DMC model.

Table 5. Performance indices calculated for tracking for the closed loop systems. Best values of indices in terms of comparison between FOPDT and SOPDT cases are highlighted with green colour

σ^2	Tuning method	Internal predictive model	t_n	*maxOS*	*IAE*	*CE*
MEP						
0.0002	SC	REFn	2.47	0.087	2.76	7.60
		FOPDT	2.81	0.13	3.12	8.52
		SOPDT	2.57	0.096	2.85	6.35
	RH, *x* = 0.1	REFn	2.69	0.079	3.15	3.58
		FOPDT	3.10	0.11	3.46	3.47
		SOPDT	2.87	0.089	3.25	3.26
0.0006	SC	REFn	2.47	0.13	2.95	9.52
		FOPDT	2.79	0.15	3.20	9.58
		SOPDT	2.57	0.12	2.96	7.78
	RH, *x* = 0.1	REFn	2.60	0.11	3.29	4.20
		FOPDT	2.93	0.14	3.54	3.92
		SOPDT	2.83	0.12	3.36	3.80
FOS						
0.0002	SC	REFn	0.55	0.067	0.60	6.74
		FOPDT	0.60	0.10	0.63	6.21
		SOPDT	0.54	0.056	0.58	5.77
	RH, *x* = 0.1	REFn	0.65	0.076	0.69	3.60
		FOPDT	0.70	0.077	0.73	3.11
		SOPDT	0.65	0.058	0.69	3.37
0.0006	SC	REFn	0.55	0.11	0.65	8.30
		FOPDT	0.60	0.13	0.66	7.34
		SOPDT	0.52	0.095	0.61	7.00
	RH, *x* = 0.1	REFn	0.59	0.13	0.73	4.42
		FOPDT	0.68	0.097	0.76	3.64
		SOPDT	0.62	0.097	0.72	3.91
TOPDT						
0.0002	SC	REFn	1.85	0.075	2.41	7.14
		FOPDT	2.26	0.13	2.66	7.91
		SOPDT	1.97	0.084	2.42	5.98
	RH, *x* = 0.1	REFn	2.20	0.080	2.77	3.41
		FOPDT	2.52	0.11	2.96	3.37
		SOPDT	2.20	0.080	2.79	3.30
0.0006	SC	REFn	1.85	0.11	2.58	8.83
		FOPDT	2.18	0.16	2.72	8.82
		SOPDT	1.85	0.11	2.50	7.29
	RH, *x* = 0.1	REFn	2.30	0.12	2.87	4.01
		FOPDT	2.48	0.13	3.02	3.83
		SOPDT	2.30	0.11	2.86	3.83

After substituting FOPDT by SOPDT, the deterioration is lower comparing to REFn case, which justifies the use of SOPDT model for the considered higher order processes.

The most significant impact of the measurement noise can be seen for *CE* measure quantifying the aggressivity of the control signal. It can be seen that for REFn case, its value increased very significantly comparing to the noiseless cases regardless of the dynamics of the controlled process, tuning method and noise level. Using FOPDT-based samples as the

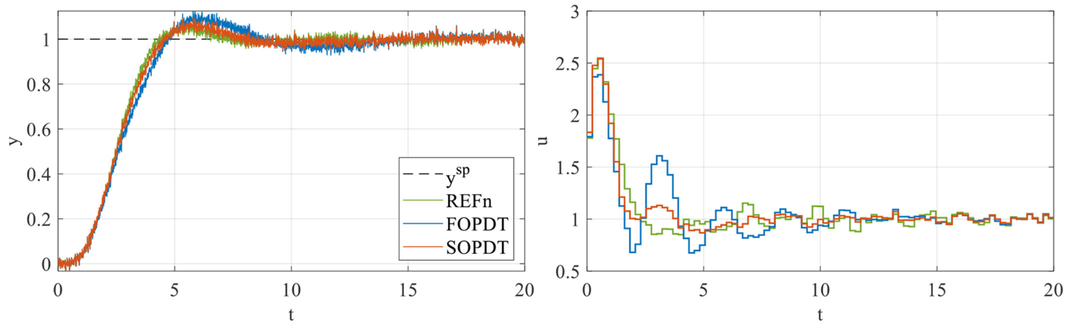

Figure 8. Comparison of DMC tuned based on SC tuning rules for original, noisy process response samples (REFn) and its FOPDT and SOPDT approximations for tracking with included measurement noise ($\sigma^2 = 0.0002$) – process value (left plot) and manipulated value (right plot) for MEP benchmark process

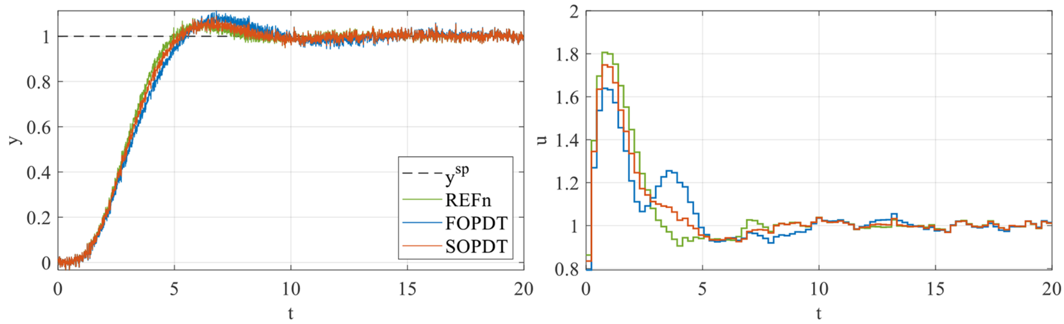

Figure 9. Comparison of DMC tuned based on RH (x=0.1) tuning rules for original, noisy process response samples (REFn) and its FOPDT and SOPDT approximations tracking with included measurement noise ($\sigma^2 = 0.0002$) – process value (left plot) and manipulated value (right plot) for MEP benchmark process

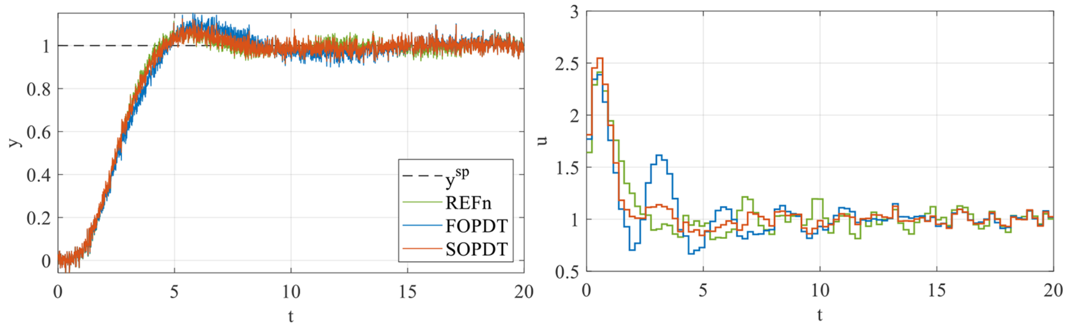

Figure 10. Comparison of DMC tuned based on SC tuning rules for original, noisy process response samples (REFn) and its FOPDT and SOPDT approximations for tracking with included measurement noise ($\sigma^2 = 0.0006$) – process value (left plot) and manipulated value (right plot) for MEP benchmark process

internal DMC model additionally deteriorates this aggressivity. The results show that when FOPDT case is substituted by SOPDT modelling for generating samples of the internal DMC model, it ensures significant decrease in *CE* value not only comparing to FOPDT case but also to REFn case. Namely, in majority of cases, application of SOPDT-based DMC internal model provides the smallest value of *CE* and in the cases when *CE* is not the smallest, its value is very close to the smallest case indicated for certain process, tuning method and noise level.

CONCLUSIONS

The paper presents the concept of DMC controller synthesis based on the model in the form of step response samples of the SOPDT model determined based on the step response of the process. The results of simulation studies allowing for the evaluation of the proposed solution are presented, taking into account the impact of the measurement noise. The research was conducted for three example benchmark dynamic processes and using two DMC tuning methods.

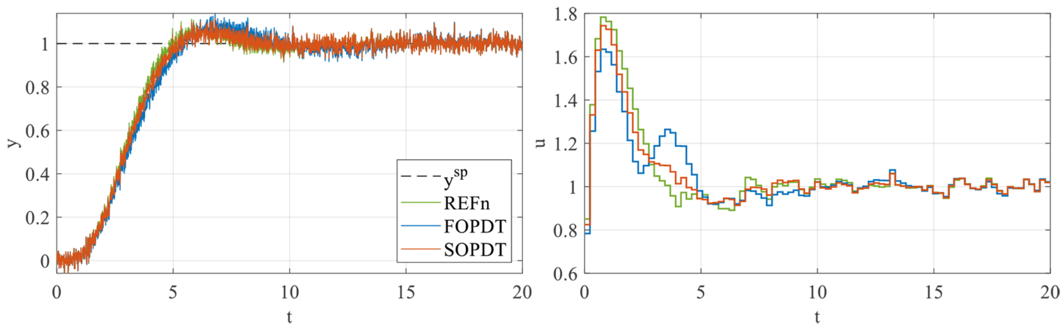

Figure 11. Comparison of DMC tuned based on RH (x=0.1) tuning rules for original, noisy process response samples (REFn) and its FOPDT and SOPDT approximations for tracking with included measurement noise (σ^2 = 0.0006) – process value (left plot) and manipulated value (right plot) for MEP benchmark process

The analysis of the results of the conducted research leads to a clear conclusion that the use of the SOPDT model to generate samples of the internal DMC model provides measurable benefits in the case when the regulated process has higher-order dynamics. This is due to the fact that the SOPDT model is able to approximate such dynamics more accurately compared to the FOPDT model. In some cases, the closed-loop performance even approaches the ideal case where noise-free step response samples of the process are used directly as the DMC internal model.

The greatest benefit from using the SOPDT model can be obtained when the priority is to reduce the variability of the control signal resulting from the presence of the measurement noise. In this case, the results clearly indicate that the lowest (or close to the lowest) variability is obtained using the SOPDT model. The obtained results indicate that the improvement in the *CE* index reaches up to 18% compared to the reference model–based approach and up to 25% compared to the FOPDT-based model. This result has the practical importance when an actuator that requires energy to change its position (e.g. a control valve) is operating in the control system. In this case, limiting the unnecessary variability of the control signal allows to significantly reduce the energy consumption for unnecessary changes of the position and the mechanical wear of the actuator itself. Apart from the *CE* index, nearly all analysed performance indices demonstrate that employing the SOPDT model as the internal model yields noticeable improvement compared to the use of the FOPDT model. This confirms that, for higher-order processes, the application of the SOPDT model improves control performance compared to the use of FOPDT model, even in comparison to the direct use of the noisy process step response samples.

In industrial practice, the DMC control algorithm is typically implemented in programmable logic controllers (PLCs). The most computationally demanding stages are the off-line identification of the FOPDT and SOPDT models used for generating the step response coefficients and then determination of the controller parameters k^e and k_j^u. Given the limited computational resources of standard PLC platforms, advanced numerical optimization libraries are usually unavailable. Instead, parameter estimation can be carried out using computationally efficient methods based on simple algebraic calculations, such as the Leapfrogging approach [22, 23]. It should be emphasized that these calculations are performed only once during the controller synthesis stage, under the assumption of linear process dynamics at the selected operating point. Alternatively, external computational resources—such as fog-computing or cloud-computing infrastructures—may be employed to perform model identification and controller synthesis. Such an approach significantly facilitates the practical implementation of the complete DMC framework while preserving real-time execution capabilities at the PLC level. Within the scope of this study, the DMC controller was evaluated on self-regulating, minimum-phase processes. Future research will investigate the impact of the selected internal models on the closed-loop performance for other classes of processes, including non-minimum-phase and integrating systems.

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