Advances in Science and Technology Research Journal

Advances in Science and Technology Research Journal, 2026, 20(8), 282–294
https://doi.org/10.12913/22998624/217002
ISSN 2299-8624, License CC-BY 4.0

Received: 2026.02.02
Accepted: 2026.06.01
Published: 2026.07.01

Kinematic synthesis of unsymmetric straight-line and dwell linkage mechanisms using kinematic differential geometry

Viacheslav Kharzhevskyi[1], Galyna Kalda[2,3], Mykhaylo Pashechko[4*], Olha Kharzhevska[1]

[1] Department of Industrial and Agricultural Engineering, Khmelnytskyi National University, Instytutska str. 11, 29016, Khmelnytskyi, Ukraine
[2] Department of Water Supply and Sewage Systems, Rzeszow University of Technology, 12, al. Powstańców Warszawy, 35-959, Rzeszow, Poland
[3] Department of Construction and Civil Security, Khmelnitskiy National University, 11, Instytutska street, 29016 Khmelnitskiy, Ukraine
[4] Department of Information Technology, Lublin University of Technology, Nadbystrzycka 38, 20-618, Lublin, Poland
* Corresponding author's e-mail: mpashechko@hotmail.com

ABSTRACT

This paper is dedicated to the problem of optimal synthesis of linkage mechanisms that can provide straight-line movement and periodic dwells of the working bodies, according to the technological operation of the machine. For that purpose, different types of mechanisms can be used and it is known that the usage of linkage mechanisms has a number of advantages: in particular – reliability of operation and load capacity. But the main problem is their complicated kinematic synthesis by the given design parameters, besides in many cases the actual dwell values differ from theoretical ones, which caused a delayed exit of the output link from the dwell phase; it also increases the kinematic parameters, which is undesirable. For the synthesis of such mechanisms the theoretical grounds of kinematic differential geometry can be used. Thus, the aim of the study is the development of a combined numerical–analytical method that enables the synthesis of mechanisms using Ball points with prescribed dwell duration and a prescribed straight-line segment of the coupler curve. Mechanisms that are synthesized using Ball points can archive dwells within the range of 30–120 degrees of crank rotation, with deviations of 0.001–0.01 (the unit of length is mechanism's interaxial distance). The usage of proposed method allows to obtain linkage mechanisms by given dwell values without slow exit from the dwell phase, using numerical optimization to meet various design criteria.

Keywords: ball point, dwell linkages, path generating, numerical optimization, coupler curves.

INTRODUCTION

In many cases during the design process of machines it is an important task to synthesize a mechanism which actuator has to move along a prescribed path, or according to the cyclogram of a machine, it must have a periodic dwell of the output link with a prescribed duration, during the continuous rotation of the input link [1, 2]. As known, linkage path generating mechanisms that can reproduce a given shape of the coupler curve in their certain part may be used independently (for example, as actuators for reproducing complex profiles) or when a certain part of the coupler curve approaches a straight line or arc of a circle, such mechanisms can also be used as basic mechanisms for designing mechanisms with a dwell of the output link [3]. During that dwell, a machine's technological operation can be performed or in that way the movement of several working bodies of high-speed automatic machine may be aligned according to the cyclogram. For that purpose, the mechanisms with higher kinematic pairs are often used, but it is known that linkage mechanisms due to the geometric closure of the links and absence of the higher kinematic pairs are more durable and reliable, have more load capacity [4]. But the main problem of the usage of linkage path generating mechanisms and dwell linkages in practice is their complicated kinematic synthesis, the known methods usually allow to obtain just partial and not always the best solution.

It is obvious that in order to obtain a mechanism that would plot a coupler curve with a straight-line part, i.e., to synthesize a basic straight-line mechanism, it is necessary to select its geometric parameters in a certain way. That problem is called kinematic synthesis of mechanisms [3].

The significant disadvantage of known analytical methods of the kinematic synthesis of linkages is the fact that is not always possible to accurately predict the value of the approximation section, and existing methods often generate mechanisms with slow exit of the output link from the dwell phase, which simply provides larger dwells compared to the theoretical values that negatively affects both the technological purpose of the machine and its kinematic characteristics. Thus, this paper considers an improved method that does not have such disadvantages.

Known methods that are used to synthesize straight-line and dwell linkages can be divided into several groups. Since the kinematic synthesis of path generating and dwell linkage mechanisms is a multi-criteria task, one of effective methods is to use numerical optimization for that purpose, which allows to carry out kinematic synthesis using various conditions, as it is shown in [5, 6]. Besides, for the synthesis of four-bar linkages nonlinear regression analysis can be used [7]. However, as practice shows, it is more effective to use analytical and numerical methods concurrently, when the analytical methods are used to determine the areas of existence of mechanisms, while numerical methods enable to identify the best ones according to certain criteria. The first group of analytical methods includes Chebyshev's algebraic methods [8]: for a given structural scheme of a mechanism such relationships between its parameters can be found that provide the best Chebyshev's approximation, i.e., ensure the minimum possible deviation from the given function, and the value of this deviation reaches its limit value the maximum number of times. Algebraic synthesis methods were further developed in the works of Blokh et al. [9]. In particular, the analytical methods of Chebyshev were combined with numerical methods in order to solve the problem of the optimization of the Chebyshev lambda mechanism, however, the problem of determining the actual approximation part was not considered in that work.

The second group of kinematic synthesis methods includes theoretical grounds of kinematic differential geometry of infinitesimally close positions of the plane figure [3]. According to these methods, for the synthesis of the path generating mechanisms, multiple interpolation nodes can be used. The first fundamental work in this area belongs to Burmester, whose ideas were developed in the works of Grübler, Alt, Lichtenheldt, Beyer, and further improved in the works of Wang [3], Yin [10]. The solution of Alt–Burmester synthesis problems for the four-bar linkages was considered by Brake [11]. Such methods were also used for the kinematic analysis of slider-crank mechanisms via the Bresse and Jerk's circles, as described by Figliolini [12]. The usage of Burmester points for the synthesis of circular path generating mechanism was also discussed in the work [13], but the usage of Ball points that can be also used for the synthesis of dwell mechanisms was not considered in the article. Besides, when the radius of approximation circle of the Burmester point's mechanism leads to infinity, it is possible to synthesize a straight-line mechanism with high accuracy, as shown in the paper [14], but it is the partial case when Burmester points coincide with Balls points. The usage of curvature theory for the approximate motion generation of linkage mechanisms is considered in the paper [15], dimensional synthesis of these mechanisms on the basis on cognate mechanisms was shown in the work [16]. However, the described methods do not allow to carry out optimal synthesis of dwell mechanisms according to specified criteria.

Lanni [17] considered an example of 10-bar long dwell mechanism and showed that multilink mechanisms can be assembled in different way by using the dead-points superposition method. Despite the possibility to obtain the long dwell, the usage of 6-bar linkages is more convenient for practice. A method to solve the optimal dimensional synthesis for planar and spherical Stephenson III six-bar linkages, both for path generation and to obtain dwells, was developed by Peón-Escalante [18]. The task of synthesizing the mechanism may be not just a guiding a link along a certain trajectory, but also in positioning the link in specified poses, as it is shown in the paper [19]. In particular, the kinetic-geometric three-position synthesis of linkage mechanisms was considered by van der Wijk [20, 21]. Although the mentioned methods can be used for the synthesis, by means of kinematic differential geometry methods a much greater variety of kinematic schemes can be obtained. Another important aspect is ensuring the operability of the designed mechanisms and the appropriate conditions for force transmission, for which the transmission angles of the mechanism are determined, as it is shown in [22].

In the work of Sanchez-Marin [4], a procedure to search for 6-bar dwell linkages on the basis on four-bar linkage has been considered using the developed multi-criteria evaluation system. It was used to search for optimal dwell linkage mechanisms structural exploration, gradient and genetic optimization, and the existence of a high number of 6-bar linkages with long output dwells was proved. But in several cases considered in the paper, the actual dwell values were larger than theoretical ones, which leads to delayed exit of the output link from the dwell phase, and as a result – higher maximum velocities and accelerations.

Since exact synthesis methods do not allow to fully satisfy the additional requirements of the designer in the most cases, therefore, the usage of approximate methods of synthesis by means of theoretical grounds of kinematic differential geometry allows to obtain a much greater variety of mechanisms that can satisfy additional requirements of designer.

Despite the large number of publications dedicated to the problem of synthesis of linkage path generating mechanisms and dwell mechanisms on their basis, the topical issue is to solve the problem of optimal kinematic synthesis of such mechanisms taking into account additional requirements, which is important from the point of view of practical application of these mechanisms. To solve this problem, it is necessary to determine the areas of existence of these mechanisms' parameters, which can be done by combining numerical analysis with analytical methods of kinematic differential geometry, i.e., to combine the accuracy of analytical methods with the multiple variations of numerical optimization.

However, the main disadvantage of the analytical method is a problem to precisely define the length of approximated part of the coupler curve, which define the dwell duration of the output link. Thus, the aim of this work is to develop a numerical and analytical method for the synthesis of straight-line and dwell linkage mechanisms that are built on the basis of an unsymmetric four-bar linkage, in order to calculate of the actual values of the coupler curve's approximated part, dwell values and to carry out optimal synthesis according to different design criteria.

RESEARCH METHODOLOGY

In the Figure 1 is shown an example of six-bar linkage mechanism, which provides a dwell of the output link 5 during a continuous rotation of the input link 1.

Obviously, the main problem is the selection of appropriate dimensions for the kinematic diagram of the mechanism, which will provide the presence of a straight-line section *DD*' on the coupler curve. The dwell time of the output link is defined as a time while the coupler point D resides on the straight-line part of the coupler curve. In this paper, the theoretical basis of the developed method of such mechanisms' design is the analytical method of synthesizing path generating linkage mechanisms using Ball points, which is based on the theoretical grounds of the kinematic differential geometry of four infinitesimally close positions of a coupler plane [3, 8]. The main idea of this method is that for any given position of the mechanism it is possible to find a special point in a coupler plane, that is called Ball point: point whose coupler curve will have a contact with its circle of curvature of at least the 3rd order, and it is a straightening point of the trajectory. The coupler curve of this point will be characterized with approximated straight line in some neighborhood of it, which makes it possible to use these mechanisms as the basis for designing dwell mechanisms.

The dwell linkage mechanism works as follows (Figure 1). A structural group 2–3 of the II class of the 1st type (according to the Assur-Artobolevsky classification) is connected to the crank 1 (input link), the position of which is determined by the angle φ_1, and a certain point *D* is selected on the coupler 2, the position of which is determined by the length l_{BD} and the angle Ω. Point *D* plots coupler curve, which is approximated to a straight line in the part *DD*'. The basic four-bar mechanism *OABCD* is connected to the structural group 4–5 of the II class

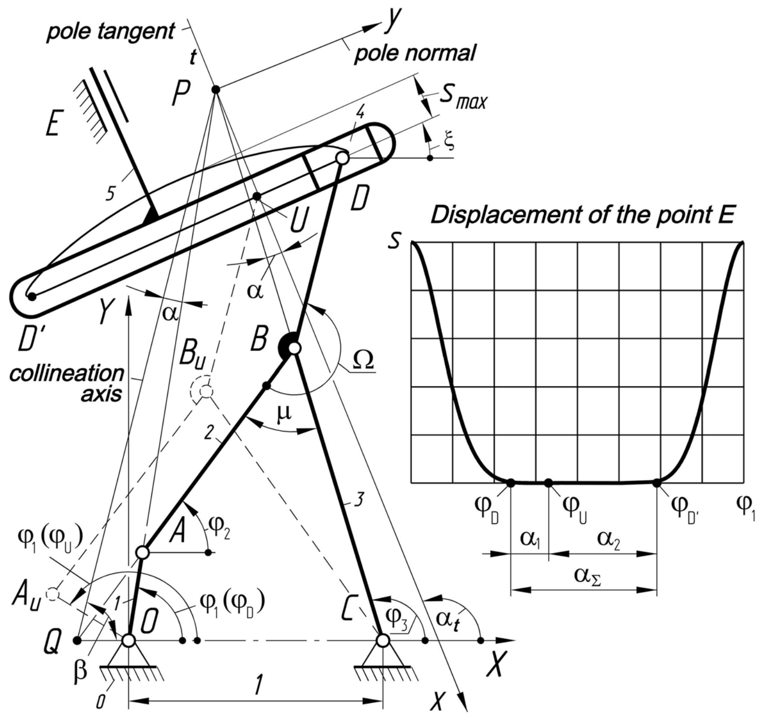

Figure 1. Six-bar dwell linkage mechanism

of the 5th type in such way that the guide of the slider 4 is parallel to the straight-line part of the coupler curve. So, when the coupler point *D* passes this part, the link 5 has an approximate dwell, the value of the theoretical deviation from the absolutely still position will be equal to the deviation with which this part of the coupler curve approximates a straight line.

The input parameters for the synthesis are the lengths of the links: crank *OA*, coupler *AB*, and rocker *BC*. To simplify the analytical dependencies, it is convenient to use relative geometric parameters, taking the interaxial distance *OC* as a unit of length ($l_{OC} = 1$), as it is done in the papers [13, 14]. Then all the linear dimensions will be expressed as the ratio of the actual length of the corresponding link to the interaxial distance l_{OC} (crank $r = l_{OA}/l_{OC}$, coupler $b = l_{AB}/l_{OC}$ rocker $c = l_{BC}/l_{OC}$, etc.). In addition, since we have to find the Ball point in the coupler plane and this point exists in any position of the mechanism, it is necessary to set the position of the crank *OA* by the angle of crank rotation φ_1. As a result of the synthesis, we can find the relative length *k* of the segment *BD* of coupler ($k = l_{BD}/l_{OC}$) and the angle Ω of the coupler *AB*, which determine the position of the coupler point *D*, as well as the value of the angle ξ of the slope of the coupler curve's straight-line section to the abscissa axis.

Analytical synthesis using Ball points by means of kinematic differential geometry

The Ball point *U* (Figure 2) is defined as the point of intersection of the inflexion circle with the circular points curve, which is the locus of the points of the moving plane whose roulettes have at least third-order contact with their tangent circles at these points, and the curve of centers is the locus of the centers of these circles. Any point of the circular points curve can be chosen to define a coupler point of the mechanism, and as a result, a path generating mechanism can be synthesized.

Moreover, the inflexion circle is the locus of the straightening points of its roulettes (coupler curves), so the Ball point, as the intersection point of these two curves, will provide the synthesis of a straight-line mechanism with high approximation accuracy.

The Ball point is characterized by the following main properties [3]:

1. In general, it is the only point of the moving plane whose roulette has at least a third-order contact with its tangent at this point. Although, as it can be seen from Figure 2, the circular points curve and the inflexion circle also have a common point at the pole *P* of the instantaneous rotation, but it is proved in the kinematic differential geometry, that the pole *P* of the coupler plane cannot be a Ball point.

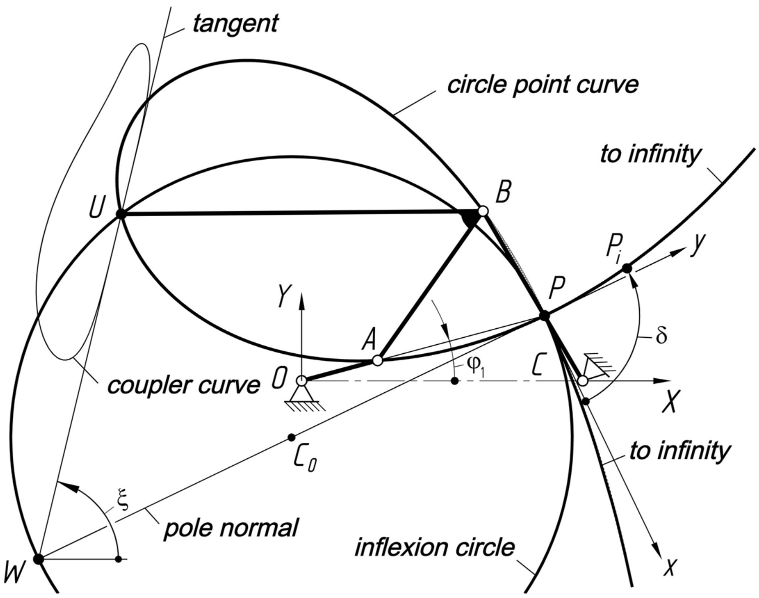

Figure 2. Ball point U for an arbitrary position of the mechanism

2. Since the Ball point belongs to inflexion circle, it is the point of straightening of its roulette, i.e. the curvature at this point is zero. It follows that the circle of curvature of the roulette of the Ball point degenerates into a straight line.
3. Ball point belongs to the focal axis of the curve of centers and is the projection of the pole of rotation W onto this axis.

Since the Ball point is a circular point, we can write the equation of the curve that is the locus of these points. The equation of the circular points curve can be written in implicit form [8]:

$$(x^2+y^2)(l_1x+l_2y)-l_3y^2-l_4xy-l_5x^2=0, \quad (1)$$

where: coefficients $l_1 \ldots l_5$ can be defined as follows:

$$\left.\begin{aligned} l_1 &= \omega x_0''' + 3\omega^2 y_0'' - 3\omega' x_0''; l_4 = 3[(y_0'')^2-(x_0'')^2]; \\ l_2 &= \omega y_0''' - 3(\omega^2 x_0'' + \omega' y_0''); l_3 = -3x_0'' y_0'',\ l_5 = 3x_0'' y_0''. \end{aligned}\right\} \quad (2)$$

where: ω – the angular velocity of rotation of the coupler plane ABD, $x_0'', y_0'', x_0''', y_0'''$ – the projections of the acceleration and the rate of change of the acceleration of the pole P of instantaneous rotation of the coupler plane, respectively.

The determination of the position of the pole P will be shown below. In order to verify the obtained results of synthesis, we will need the equation of the circular point curve (1). However, it is not possible to use this equation to plot a circular points curve, since it is written in an implicit form. For this purpose, it is more convenient to use its parametric representation [13]:

$$x=\frac{l_3\tan\delta+l_4\tan\delta+l_5}{(\tan^2\delta+1)\ (l_1+l_2\tan\delta)}, \quad y=x\tan\delta. \quad (3)$$

Thus, the position of any point in the coupler plane ABD can be determined by using a radius vector drawn from the pole P to that point. The angle δ in Equations 3 is the angle of inclination of this radius vector to the abscissa axis and varies from 0 to 2π. To find the Ball point of the coupler plane, in addition to the of circular points curve, it is necessary to determine the parameters of the inflexion circle, for which we also write its equation in implicit form [8]:

$$\omega^2(x^2+y^2)-(x_0''x+y_0''y)=0, \quad (4)$$

from which we can determine the position of its center C_0 and diameter d_k:

$$x_{C_0}=x_0''/2\omega^2,\ y_{C_0}=y_0''/2\omega^2,$$

$$d_k=\frac{1}{\omega^2}\sqrt{x_0''^2+y_0''^2} \quad (5)$$

Using Equation 3 and dependence 5, it is possible to plot a circular points curve and inflexion circle and then graphically determine the Ball point U as the point of their intersection (Figure 2). But it is more convenient to make dependencies that determine its position. Thus, let's consider the sequence of analytical determination of the Ball point in the coupler plane of the four-bar linkage mechanism. In the paper [13], the method of kinematic synthesis of linkage mechanisms by means of Burmester points was considered, which is useful to design circular path generating mechanisms, in order to synthesize mechanisms with coupler curves, certain parts of which can be approximated to an arc. Although in this work we consider the design of straight-line linkages, the kinematic parameters of the basic four-bar linkage remain the same as it was for circular path generating mechanisms. Thus, the coordinates X_A, Y_A, X_B, Y_B of the moving pivots *A*, *B*, and angles φ_2, φ_3, which determine the position of coupler 2 and rocker 3 of the mechanism, can be determined in the same way as it was shown in [13]. To synthesize the mechanism, it is also necessary to determine the position of the pole *P* of instantaneous rotation of the coupler plane. Let's some movable plane connected to the coupler 2 of the mechanism be characterized by the segment *AB* and move so that the pivots *A* and *B* describe some trajectories. Then at any time, the point *P* can be found as the intersection of the normal to the trajectories of pivots *A* and *B*. The point *P* is the instantaneous pole of motion of the coupler plane and for an infinitesimal period of time, the motion of the coupler 2 can be considered as an infinitesimal rotation around this pole [3]. Thus, for each position of the mechanism that is characterized by the angle φ_1, it is possible to find the corresponding point P_i, and all such points will belong to a certain curve called the fixed centroid. The point *P* as the intersection of segments *OA* and *BC* for any position of the mechanism can be found using the following equations [13]:

$$X_P = \tan\varphi_3 / (\tan\varphi_3 - \tan\varphi_1); Y_P = X_P \tan\varphi_1. \quad (6)$$

For a given position of the mechanism, it is possible to determine the instantaneous pole *Q* of the relative motion of the coupler 2 and rocker 3 of the mechanism, that together with the pole *P* of the instantaneous rotation of the coupler plane forms the collineation axis *PQ* [13]:

$$X_Q = (X_A X_B - X_B Y_A)/(Y_B - Y_A); Y_Q = 0; \quad (7)$$

In addition to the fixed centroid, for any four-bar linkage mechanism, a moving centroid can also be plotted, which is a purely geometric characteristic of the mechanism and is required for synthesis by geometric methods. The moving centroid is the only curve in the coupler plane and, thus, the only curve of the entire moving system that constantly rolls without slipping along the fixed centroid and it is the envelope of all its positions. Both centroids have a common point, the pole *P* of instantaneous rotation of the coupler plane, and at this point both centroids have a common tangent, called the pole tangent. The line passing through the pole *P* perpendicular to the pole tangent is the pole normal. It is important to know the direction of the pole normal, because the value of the total acceleration of the pole *P* coincides with the direction of the pole normal, and therefore, to simplify further calculations, it is convenient to move the center of the coordinate system to the pole *P* and rotate it so that the abscissa axis is directed along the pole tangent. To do this, it is necessary to determine the direction of the pole tangent, and there is no need to construct both centroids, it is much easier to use the Bobillier theorem [3, 8], according to which, if we draw normal to the trajectories of two points of a moving system, then each of them will form the same angle α with the collineation axis, as the other normal with the pole tangent. The pole tangent and the collinear axis always lie on opposite sides of the corresponding normal to the trajectories. Figure 1 shows that the angle α can be defined as $\alpha = \varphi_1 - \beta$, where $\tan\beta = Y_P/(X_P - X_Q)$. Since the acceleration vector of the pole *P* is directed along the pole normal, so the Equations 1 and 2 can be simplified, because $x_0'' = 0$ after the rotation of the coordinate system. Let's accept the angular velocity of the coupler plane's rotation ω=1. Then, Equation 1 will take the following form [13]:

$$(x^2 + y^2)(l_1 x + l_2 y) - l_4 xy = 0, \quad (8)$$

where: $l_1 = x_0''' + 3y_0''$, $l_2 = y_0'''$, $l_4 = 3(y_0'')^2$.

Note that at ω=1 the value of y_0'' in this equation is equal to the diameter of the inflexion circle that can be easily determined from the Euler-Savary equation [3]. As it can be seen from Equations 8, the unknown values are l_1 and l_2 can be determined from the condition that the circular points curve must pass through the pivots *A* and *B* of the mechanism [13]. Since the position of the pivots can be determined from the kinematic analysis of the four-bar linkage mechanism, then we can write a system of equations where we can substitute the values of x_A, y_A and x_B, y_B. From this system, we can find the coefficients l_1 and l_2:

$$l_1 = \frac{k_1 k_{22} - k_2 k_{12}}{k_{11} k_{22} - k_{12} k_{21}}, \; l_2 = \frac{k_2 k_{11} - k_1 k_{21}}{k_{11} k_{22} - k_{12} k_{21}},$$

where the coefficients:

$$\left.\begin{aligned} k_{11} = x_A\left(x_A^2 + y_A^2\right); k_{12} = y_A\left(x_A^2 + y_A^2\right); k_1 = l_4 x_A y_A, \\ k_{21} = x_B\left(x_B^2 + y_B^2\right); k_{22} = y_B\left(x_B^2 + y_B^2\right); k_2 = l_4 x_B y_B. \end{aligned}\right\}$$

Finally, the coordinates of the Ball point in the coupler plane can be calculated as follows:

$$x_U = -R\,y_0''', \quad y_U = R\,x_0''', \tag{9}$$

where: x_0''', y_0''' – the horizontal and vertical components of the rate of change of the acceleration of the pole P of the instantaneous rotation of the coupler plane, respectively; coefficient $R = x_0'''\,y_0''/(x_0'''^2 + y_0'''^2)$. Those values can be determined from Equations 9. After simple transformations related to the rotation of the coordinate system, we can calculate the coordinates of the points U and B in the coordinate system XOY. Then we can find the length of the second side of the coupler $k = l_{BD}/l_{OC}$ and the angle Ω that determine the position of the Ball point in the coupler plane:

$$\left.\begin{aligned} k = \sqrt{(X_U - X_B)^2 + (Y_U - Y_B)^2}, \\ \Omega = \pi - \varphi_2 + \arctan[(Y_U - Y_B)/(X_U - X_B)]. \end{aligned}\right\} \tag{10}$$

After the determination of the coupler point's position, it is necessary to determine the value of inclination angle ξ of straight-line part of the coupler curve. It is important in order to design a dwell mechanism, because we need to know at what angle the slider guide of the additionally attached structural group 4–5 must be placed (see Figure 1). This task can be solved by geometric methods. It is known, that all points of a moving plane plotting straight lines lie on inflexion circle, and all these lines must pass through the inflexion pole, which is the point of the moving plane whose vector velocity is geometrically equal to the vector velocity of the rolling of centroids [3, 8]. The pole of rotation W (see Figure 2) is defined as the point of intersection of the pole normal with the inflexion circle and does not coincide with the pole P of instantaneous rotation of the coupler plane. Knowing that the approximation line must pass through the pole of rotation W and the position of the Ball point U through which this line must also pass, we can determine the inclination angle of the coupler curve's straight line:

$$\xi = \arctan[(Y_W - Y_U)/(X_W - X_U)] \tag{11}$$

As it has been already mentioned, a Ball point exists for any position of a four-bar linkage mechanism. By sequentially connecting the points obtained for different positions of the crank OA of the mechanism, a curve called the Ball curve can be drawn, an example is shown in the Figure 3.

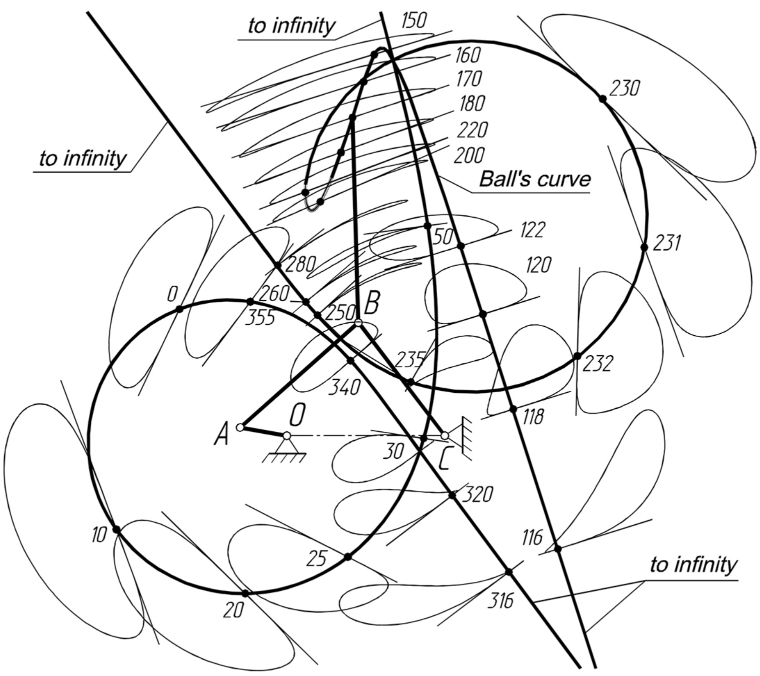

Figure 3. Ball curve of an unsymmetric four-bar linkage mechanism with coupler curves of Ball points that are defined for different crank positions

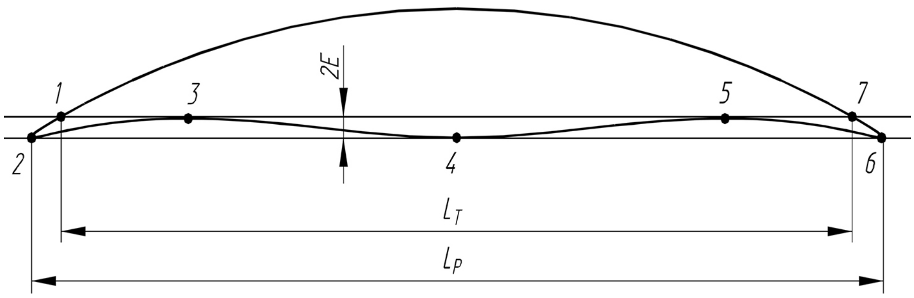

Figure 4. Coupler curve of the Chebyshev straight-line mechanism

Determination of dwell values

The analysis of existing methods for synthesizing dwell mechanisms showed that in many cases the actual dwell time is larger than predefined value and it is only guaranteed that the actual dwell value will not be less than the one given by a designer. But the problem is that in case of slow exit of the output link from the dwell phase we will have much larger maximum values of velocities and accelerations during the working stroke, besides, the dwell values must be properly aligned with the cyclogram of the machine.

For a large class of linkage path generating mechanisms, the method of the Chebyshev best approximation conditions is used for the selecting part of the coupler curve [9]. Generally, that type of approximation of the coupler curve of a four-bar linkage to a straight line or arc is defined as a uniform approximation with seven points of maximum deviations on a certain part of the curve, as shown in the Figure 4, where L_p, L – the practical and theoretical lengths of the approximation part, respectively.

The analysis and computer simulation of the dwells mechanisms that were synthesized using the Chebyshev approximation conditions showed that in the most cases the actual dwell time that is determined by the angle of rotation of the crank α_Σ, is almost always greater than the given dwell value (Figure 5), and Chebyshev mechanisms with a relatively short dwell durations practically do not exist – the actual dwell values can be larger than the theoretical ones up to 12 times.

It was established that this statement is true for Chebyshev mechanisms regardless of the peculiarities of their geometric parameters: different values of the coupler angle Ω, interaxial distance $a = l_{OC}$, as it is shown in the Figure 6.

Thus, it was established that within the dwell interval that is determined by the Chebyshev best approximation conditions, such mechanisms provide almost perfect dwell, the problem is that the movement of the output link outside this theoretical interval is also characterized by dwell, but with slightly larger deviations. A slight increase in the deviation $2E$ compared to the theoretical values does not significantly affect the actual deviation in the manufactured mechanism, taking into account the presence of gaps in kinematic pairs, elastic deformations and manufacturing errors.

To solve the problem of the proper selection of the approximation part in the circular path generating mechanisms on the basis of Burmester

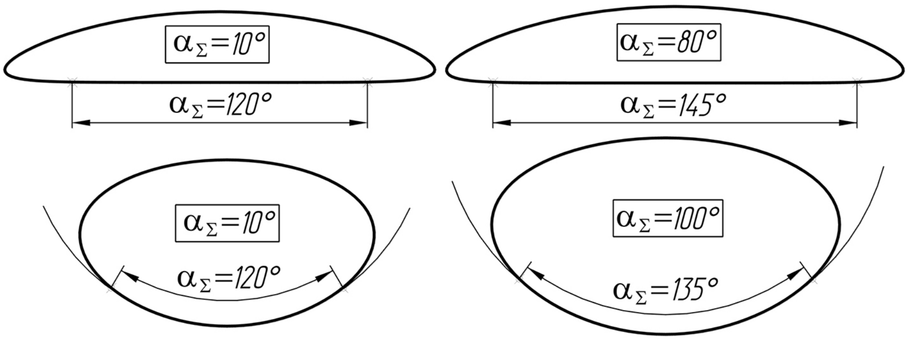

Figure 5. Examples of coupler curves of circular and straight-line linkages, where the approximation part is selected according to the Chebyshev best approximation conditions (the value in rectangle) and the actual values

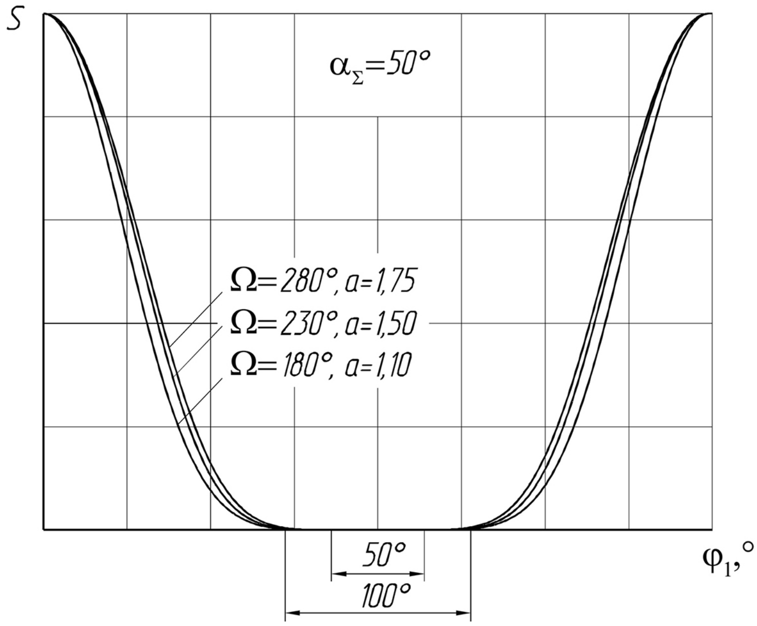

Figure 6. Diagrams of displacements of the output link of dwell mechanisms that are built on the basis of a symmetric Chebyshev lambda mechanism

point, in the paper [13] was used the numerical method by means of the unitless coefficient of maximum velocity. As it was established, this method can be also used for the synthesis of straight-line mechanisms using Ball points. This method can be used in the following sequence.

It is known [8] that to compare the laws of motion of different mechanisms, dimensionless coefficients (invariants) of kinematic quantities are used, which are numerically equal to the kinematic characteristics of the output link, recalculated per unit stroke, and they are calculated only for the period of the working stroke. Since in our case the value of the working stroke is unknown (we do not know the actual dwell value yet), it was proposed to calculate the appropriate kinematic characteristics of the output link, but for the entire period of the mechanism's movement, to compare different laws of motion. Moreover, it was proposed to set such a maximum value of the dimensionless velocity at which we will assume that the output link has moved from the dwell phase to the phase of working stroke and vice versa. Since it is not the real values of velocities that are calculated, but the dimensionless ones, it is possible to assume, and the feasibility of this assumption has been confirmed in practice [13], that different dwell mechanisms can be calculated using the same maximum value of dimensionless velocity. Thus, the procedure for the choosing the length of the approximation part of the coupler curve by using numerical method is the following:

1. We choose Ball point as the initial point in the calculations, since we know that the straight-line part of the coupler curve is in some neighborhood of this point.
2. We build a diagram of unit displacements of the output link (see Figure 1), i.e., each value of the displacement s_i is divided by the maximum stroke S_{max}. As a result, we get a diagram in which the displacements change from 0 to 1, which in turn allows us to compare the laws of motion of the output links of different mechanisms.
3. Having differentiated the diagram of unit displacements, we build a diagram of unit velocities.
4. Then we calculate the values of the velocities along this diagram to the left and right from the starting point until the maximum value of unitless velocity is reached, which is selected as it was used for the synthesis of mechanisms using Burmester's points [13]. After that the values of the maximum deviations are recorded at the points φ_L and φ_R, according to these positions, and one of these points, corresponding to a smaller deviation, changes its position so that the value of the deviation on the left and right is the same. As a result, we get the angles φ_D and $\varphi_{D'}$, which determine the approximation length *DD′* of the coupler curve (see Figure 1).
5. Calculation of the dwell value:

$$\alpha_\Sigma = \alpha_1 + \alpha_2,$$

where: $\alpha_1 = \varphi_U - \varphi_D, \alpha_2 = \varphi_{D'} - \varphi_U$

For the correct determination of the dwell value, it is important to check the condition that $\varphi_D < \varphi_U$ and $\varphi_U < \varphi_{D'}$. If that conditions are not met, the values of the corresponding angles must be changed to 2π.

It is also important how to choose the value of dimensionless maximum velocity in the proposed numerical method, in case if it is applied for the mechanisms that are synthesized using Ball points. The diversity of the fields of mechanisms' application and the requirements imposed on them may require to choose a different value of this coefficient because mechanisms may have different requirements for accuracy, velocities during the working stroke, etc. But we can recommend some average value that may be acceptable for a large number of mechanisms. As a result of numerical research, it is recommended to choose the value of this coefficient within the range $0.01 < v_\alpha < 0.05$. It is clear that larger values of this coefficient will provide longer approximation parts with a larger deviation, but it reduces the probability of obtaining a mechanism with a slow exit from the dwell phase. As it was established numerically, the value of $v_\alpha = 0.022$ is suitable for the most dwell mechanisms that are synthesized using Ball points, in the wide range of dwells: $30° < \alpha_\Sigma < 120°$ (in the most cases). It is important that for all such mechanisms, the proposed method makes it impossible for the output link to slowly exit the dwell phase, which ensures accurate implementation of the specified cyclogram of the machine, as well as lower maximum kinematic parameters during the working stroke.

As it was shown above, the method of selecting the dwell duration by the Chebyshev best approximation condition does not provide acceptable results for practice in many cases, so the task of recalculating the dwell values of the output link of such mechanisms was arisen. It was established that the proposed method is also useful to determine the actual values of the approximation part of the Chebyshev lambda mechanisms.

RESULTS

On the basis of the proposed analytical and numerical method, algorithms and corresponding software were developed that can be used to conduct the kinematic synthesis of linkage mechanisms by the given dwell values. It also allows to select the design parameters of the mechanism in accordance with the specified requirements by the designer. Moreover, the developed software uses numerical optimization by different criteria on the basis of Hooke-Jeeves method. Figure 7 shows an example of a reference map for the preliminary synthesis of dwell mechanisms based on Ball points for the lengths of the coupler *b* and rocker *c*, that can be given by designer. This diagram shows the dwells values of the Ball curve mechanisms, which was considered above (Figure 3).

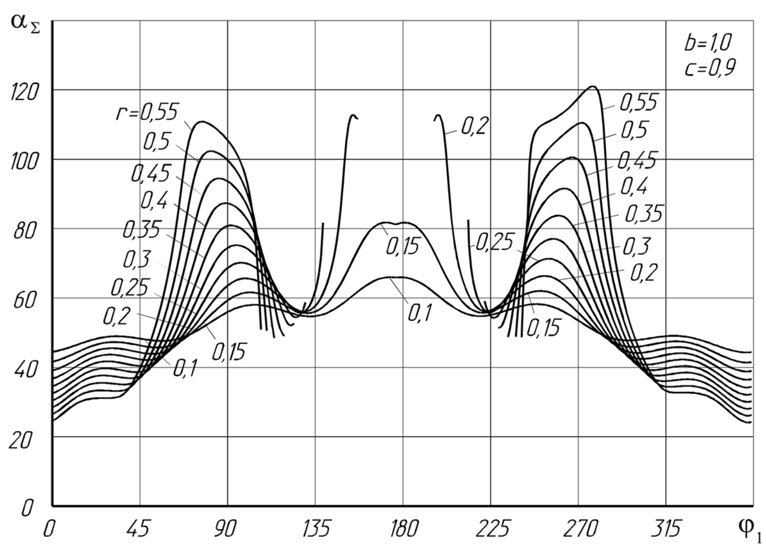

Figure 7. Diagram of the change in the output link's dwell value depending on the angle of crank rotation that determines the position of the Ball point in the coupler plane

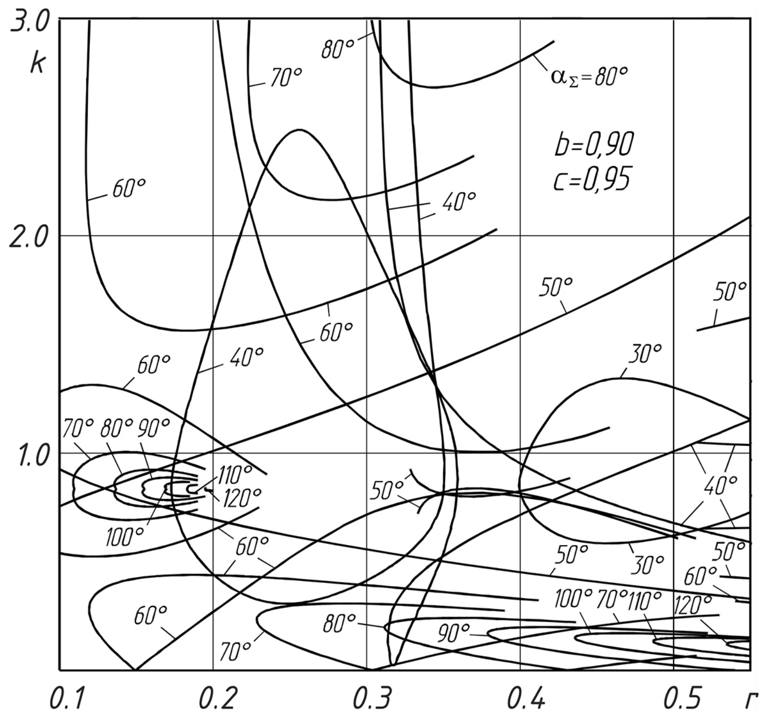

Figure 8. Example of reference card for preliminary synthesis of linkage mechanisms for a given dwell duration of the output link using Ball points, dependence $k = f(r)$

Although the dwell values α_Σ of the synthesized mechanisms can be calculated using the proposed method, it is also an important task to select all the necessary mechanism's parameters for the given dwell values. This task was also solved using numerical analysis, an example of the obtained reference card is shown in the Figure 8.

The results of the kinematic synthesis can be presented not just in the format of reference cards, but also in table form, from which the designer can easily select the appropriate numerical values of the geometric parameters. There can be several mechanisms, which were defined by numerical optimization as the best suitable solutions according to the criteria, specified by the designer. Examples of found geometric parameters of the mechanisms with different given dwell values α_Σ are shown in the Table 1, where φ_U, φ_D – angles of crank rotation φ_1, which correspond to the position of the Ball point and start of the dwell phase, respectively; E – maximum deviation in the approximated part (with the interaxial distance as a unit of length: lOC = 1); $L_P = DD'$ – length of the approximated straight line of the mechanism's coupler curve (see Figure 1); s_{max} – maximum stroke of the output link. The parameters in the Table 1 correspond to the data in the reference card (Figure 8), in particular: the length of the mechanism's coupler $b = 0.9$ and the length of the mechanism's rocker $c = 0.95$.

It should be also noted that all linear dimensions in the Table 1 are unitless, in particular: the length of the crank r and the length k of the coupler's segment BD, maximum deviation E, maximum stroke s_{max} and the length of the approximated line L_p. As it was already mentioned, the mechanism's interaxial distance OC was chosen as a unit of length, therefore, if we multiply the actual value of mechanism's interaxial distance that was set by designer (for example, 100 mm) by the linear dimensions from the Table 1, then we will obtain the actual mechanism's dimensions in millimeters. All angles in the Table 1 are given in degrees.

Calculations were made for the mechanisms where the dwell phase is located at the extreme upper or lower positions of the output link (as shown in the example in the Figure 1), but the developed method can be easily extended for all other types of mechanisms. As a result of the calculations that were carried out, reference maps were also created for all the necessary design parameters of the mechanisms, such as the dimensions of the kinematic scheme, maximum stroke of the output link, accuracy of the dwell, etc. It is also possible to take into account the kinematic parameters of the mechanisms, such as the maximum values of velocities and accelerations of mechanism's links.

Table 1. Examples of synthesized dwell linkage mechanisms

α_Σ	r	φ_U	φ_D	k	Ω	E	s_{max}	ξ	L_p
30	0.40	0.9375	345.91	0.818303385	317.1406881	0.001501	0.5204059	248.0734681	0.38777970
40	0.20	10.375	350.63	1.605015065	283.0827755	0.002025	0.1899094	241.9538516	0.39855072
50	0.45	245.1328	219.46	0.725626113	208.4386937	0.002562	0.3948169	8.9829155	0.31960863
60	0.20	88.5625	60.86	1.568639879	247.1624878	0.003415	0.1317464	209.3273451	0.12717280
70	0.25	268.9843	230.45	0.297127682	248.544962	0.004087	0.1149674	45.3993255	0.09430261
80	0.32	258.9687	215.66	0.155242321	252.4082533	0.004245	0.2058448	50.5303634	0.12044954
90	0.40	276.3593	221.79	0.197702386	251.5977634	0.006429	0.2947306	52.1103026	0.19060103
100	0.45	279.0078	215.87	0.169022356	252.9121664	0.007765	0.3743017	54.5086746	0.23800193
110	0.49	274.375	203.99	0.121613014	255.8903982	0.006541	0.4571806	57.4945671	0.26431529
120	0.54	279.5625	198.69	0.106935736	257.2566678	0.007331	0.5521122	59.7870085	0.31534599

Those parameters can be defined automatically using numerical optimization, which it implemented in the developed software program.

It is important to ensure the operability of synthesized mechanisms that is defined by the conditions for force transmission. For the most linkage mechanisms, it is important to ensure that the transmission angle μ of the basic mechanism *OABCD* (see Figure 1) is within the range between 30–150 degrees during the operation cycle.

It was also established and discussed earlier that Chebyshev's mechanisms actually do not provide small dwells of the output link (see Figure 5), where practical dwell values are much larger than theoretical ones, and Chebyshev's mechanisms with small dwells practically do not exist. In contrary, Ball point mechanisms, which are synthesized using the proposed method, provide the desired values of dwells, without slow exit from the dwell phase.

Another practical application of developed method is the ability to clarify the dwell values that are provided by Chebyshev's mechanisms, which will improve the kinematic parameters of the designed mechanisms and properly align the mechanism with the cyclogram of a machine.

CONCLUSIONS

The paper considered the problem of optimal kinematic synthesis of straight-line and dwell linkage mechanisms, and there was proposed a new approach to use the methods of kinematic differential geometry using Ball points along with the developed numerical method, which allows to predict the actual values of approximated part of the coupler curve. Contrary to the existing methods, it always enables to synthesize mechanisms that do not have slow exit from the dwell phase, ensures the specified dwell in accordance with the technological application of the mechanism. As shown in the paper, existing methods often ensure that the dwell phase is just not less than the specified one, but actually it can be significantly greater, which results in worse kinematic characteristics. It was established that the mechanisms that are synthesized using Ball points, can achieve dwell values from 30 to 120 degrees of crank rotation (in the most cases), with deviations of 0.001–0.01 (the unit of length is mechanism's interaxial distance). It also enables to define the actual dwell values of Chebyshev's mechanisms, which can be significantly larger from the theoretical ones (up to 12 times). The developed algorithm and appropriate software can perform numerical optimization on the basis of Hooke-Jeeves method by the different design criteria.

The further research is planned to be carried out to determine the main dynamic characteristics of the synthesized mechanisms, including optimization by additional parameters. Besides, the proposed method can be used not just for the mechanisms on the basis of four-bar linkage, but also for other types of linkage mechanisms.

REFERENCES

1. Waldron, K.J., Kinzel, G.L., Agrawal, S.K.: Kinematics, Dynamics, and Design of Machinery, 3rd ed. Wiley, Chichester, 2016.
2. McCarthy, J., Soh G. Geometric Design of Linkages, 2nd edition. – Springer-Verlag, New York, 2011. https://doi.org/10.1007/978-1-4419-7892-9

3. Wang D., Wang W. Kinematic Differential Geometry and Saddle Synthesis of Linkages – John Wiley & Sons Singapore Pte. Ltd., 2015.
4. Sanchez-Marin F., Roda-Casanova V. An approach for the global search for top-quality six-bar dwell linkages. Mechanism and Machine Theory 2022; 176. https://doi.org/10.1016/j.mechmachtheory.2022.104974
5. Hills C., Baskar A., Plecnik M., Hauenstein J., Computing complete solution sets for approximate four-bar path synthesis. Mechanism and Machine Theory, 2024; 196, 105628, https://doi.org/10.1016/j.mechmachtheory.2024.105628.
6. Ao M., Yu G., Wang L., Sun L., Ye B., Integrated optimization synthesis of linkage mechanism structures and dimensions free from kinematic defects. Mechanism and Machine Theory, 2025; 214, 106121, https://doi.org/10.1016/j.mechmachtheory.2025.106121
7. Akay O.E., Catalkaya M., Synthesis of a four-bar mechanism using nonlinear regression analysis. Journal of Mechanical Science and Technology, 2025; 39, 1427–1434, https://doi.org/10.1007/s12206-025-0234-1
8. Kharzhevskyi V.O. Synthesis of the linkage mechanisms with dwell of the output links using kinematic geometry methods: monography. Khmelnytskyi RVC KhNU, 2015 (in Ukrainian).
9. Miler D., Birt D., Hoić M. Multi-objective optimization of the Chebyshev lambda mechanism. Strojniški vestnik. Journal of Mechanical Engineering. 2022; 68(12), 725–734. https://doi.org/10.5545/sv-jme.2022.349
10. Yin, L., Huang, L., Huang, J., Xu, P., Peng, X., and Zhang, P. Synthesis theory and optimum design of four-bar linkage with given angle parameters, Mech. Sci., 2019; 10, 545–552. https://doi.org/10.5194/ms-10-545-2019
11. Brake D. A., Hauenstein J. D., Murray A. P., Myszka D. H., and Wampler C. W. The complete solution of alt–Burmester synthesis problems for four-bar linkages, Journal of Mechanisms and Robotics, 2016; 8041018. https://doi.org/10.1115/1.4033251
12. Figliolini G., Lanni C., Tomassi L. Kinematic analysis of slider – crank mechanisms via the bresse and jerk's circles. In: Carcaterra, A., Paolone, A., Graziani, G. (Eds) Proceedings of XXIV AIMETA Conference 2019. Lecture Notes in Mechanical Engineering, 2020. Springer, Cham. https://doi.org/10.1007/978-3-030-41057-5_23
13. Kharzhevskyi V.O. Kinematic synthesis of linkage mechanisms using Burmester points at the given dwell duration of the output link. Advances in Science and Technology Research Journal, 2017; 2(11), 139–145. https://doi.org/10.12913/22998624/68465
14. Kharzhevskyi V., Pashechko M., Tatsenko O., Marchenko M., Nosko P. The synthesis of dwell mechanisms on the basis of straight-line linkages with fivefold interpolation nodes Advances in Science and Technology Research Journal, 2021; 15(1), 18–25. https://doi.org/10.12913/22998624/128817
15. Diab N. Using instantaneous invariants of the curvature theory in motion generation synthesis of 4R mechanisms with higher-order coupler point kinematics, Mechanism and Machine Theory, 2024; 195, 105596, https://doi.org/10.1016/j.mechmachtheory.2024.105596
16. Soriano-Heras E., Pérez-Carrera C., Rubio H., Mathematical dimensional synthesis of four-bar linkages based on cognate mechanisms, Mathematics, 2025; 13(1), 11–35, https://doi.org/10.3390/math13010011
17. Lanni C., Figliolini. G., Tomassi L. Higher order kinematic analysis of long-dwell mechanisms. Proceedings of ASME 2023 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference, IDETC-CIE2023. 2023; 8. https://doi.org/ 10.1115/DETC2023-116699
18. Peón-Escalante R., Cuenca Jiménez F., Escalante Soberanis M.A., Peñuñuri F. Path generation with dwells in the optimum dimensional synthesis of Stephenson III six-bar mechanisms. Mechanism and Machine Theory, 2020; 144, https://doi.org/10.1016/j.mechmachtheory.2019.103650
19. Zhou Y., Sun L., Xu H., Wang L., Ziqin X., Dimension synthesis of linkage mechanism with approximate poses and velocity constraints, Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science, 2025; 239(10), 3657–3668, https://doi.org/10.1177/09544062251313688
20. Van der Wijk V. Kinetic-geometric three-position synthesis of a balanced 4R four-bar linkage. In Proc. Advances in Robot Kinematics 2024 (ARK 2024), Cham: Springer, 2024; 31, 204–212, https://doi.org/10.1007/978-3-031-64057-5_24
21. Van der Wijk V. Kinetic-Geometric Synthesis of a Reconfigurable 4R Four-bar Multitask Mechanism. In Proc. 6th International Conference on Reconfigurable Mechanisms and Robots, IEEE, 2024, pp. 368-372, https://doi.org/10.1109/ReMAR61031.2024.10617615
22. Wu X., Liu Q., Ding J., Wang C., Yu H., Bai S., Transmission angle of planar four-bar linkages applicable for different input-output links subject to external loads, Mechanism and Machine Theory, 2024; 203, 105829, https://doi.org/10.1016/j.mechmachtheory.2024.105829