

Implementation the GPS + EGNOS data for designation the accuracy of car vehicle coordinates in road transport

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ABSTRACT

The European Geostationary Navigation Overlay Service (EGNOS) augmentation system can be used in air, sea, and road transport. In the case of the latter, an important aspect is its application in the field of car navigation. The main objective of this work is to develop an algorithm and computational strategy for determining the accuracy of a car's position based on a multi-receiver GPS solution with EGNOS corrections in navigation. In particular, the article proposes determining the coordinates of a car based on a mathematical equation of the center of the segment connecting two Global Navigation Satellite System (GNSS) receivers placed on the roof of the car while driving. A total of four GNSS receivers were used for this purpose, two for each geometric segment. Thus, the center coordinates for two independent segments that intersected at a single point were finally determined, where a fifth GNSS receiver was installed to verify the coordinates obtained from the GPS/EGNOS solution. This made it possible to determine the center coordinates of the car while driving. The study used actual GPS/EGNOS kinematic data recorded by on-board GNSS receivers. The obtained results of the center coordinates of the geometric section were compared with the reference position of the movement for the purpose of calculating accuracy. Based on the tests performed, it was found that the average positioning accuracy is higher than 0.80 m for all BLH geodetic coordinates. The Kalman filter was also used in the study, reducing positioning errors by up to 40% for all BLH components. The developed methodology and calculation strategy can be used, for example, in air navigation for aircraft positioning.

Keywords: car vehicle, EGNOS, GPS, accuracy, road transport.

INTRODUCTION

The European Geostationary Navigation Overlay Service (EGNOS) assistance system, apart from its basic application in air transport [1], can also be used in road transport [2]. It is particularly prominent in the area of intelligent transport system infrastructure, i.e., in intelligent mobile applications, safety-critical applications, payment and financing-critical applications, regulated applications, and intelligent tachographs. All these applications make SBAS systems extremely important for road transport. In addition, smart mobile applications can be used to find information on navigation applications, improve Global Positioning System (GPS) positioning,

and increase the accuracy of determining the coordinates of motor vehicles [3, 4]. At this point, it is also worth mentioning the phenomenon of vehicle fleet management and traffic monitoring [5]. According to safety requirements [6], the use of the EGNOS system in road transport allows for the location of a motor vehicle with an accuracy of 5–20 m. Other criteria and requirements for road transport include: alarm systems, road transport management, traffic control, collision avoidance, road accident data collection, and road infrastructure management [6]. In addition to the general requirements, road transport also involves an increase in the number of motor vehicle users in the European Union due to the urgent need to use Global Navigation Satellite System (GNSS)

satellite systems [7], including the EGNOS support system [8]. This is, of course, possible thanks to the basic service of the EGNOS support system, i.e. the OS (Open Service) [9]. As part of the OS service, users can primarily use EGNOS corrections [10] to determine their kinematic position and increase the accuracy of coordinates obtained for car navigation in road transport. The availability, continuity, and reliability of satellite positioning [11, 12] also remain indispensable parameters thanks to the use of the EGNOS system.

BACKGROUND

The EGNOS assistance system has been used repeatedly in road transport and subsequently described in the literature on the subject of the article. It is therefore worth mentioning the following areas of research:

- GPS kinematic positioning with real-time EGNOS corrections for car navigation [13], with positioning accuracy greater than 2 m;
- analysis of the potential use of the EGNOS system in cargo transport in heavily urbanized areas [14–16], the positioning accuracy obtained drops to over 50 m in the case of heavily urbanized areas with large buildings;
- development of algorithms for simulating dynamic GPS/EGNOS positioning in the Matlab environment for car navigation [17], the positioning accuracy obtained was up to 3 m, only in single measurement epochs did the accuracy drop to about 50 m;
- determining the integrity parameter of GPS/EGNOS positioning in real time using corrections from GEO satellites and the SISNeT service [18–21], integrity values in the form of the HPL (Horizontal Protection Level) parameter do not exceed 15 m when using EGNOS corrections from GEO satellites, and when using the SISNeT service, they even fall below 20 m;
- use of data from the EDAS (EGNOS Data Access Service) to determine the location of road vehicles [22]; the study also used RAIM (Receiver autonomous integrity monitoring) algorithms for GNSS receivers, improving positioning accuracy from 2.6 m to 1.3 m;
- use of Galileo and EGNOS systems to monitor and track the transport of dangerous goods [23]; the integration of GPS+EGNOS and Galileo data significantly improved positioning in road transport;
- application of the EGNOS system for monitoring regulated road tolls [24–26];
- use of the EGNOS system for the development and deployment of the eCall system in motor vehicles within the European Union [27];
- development and implementation of the CoVel system based on EGNOS and providing information on the location of a car in relation to other motor vehicles [28];
- use of a GNSS sensor (including the EGNOS system) and other navigation sensors in the construction of an intelligent motor vehicle prototype [29];
- development of mobile, location and mapping applications using the EGNOS system [30];
- monitoring the quality of the GPS+EGNOS signal in road applications in urban areas [31];
- presenting the possible contribution of the EGNOS system in the field of ITS (Intelligent Transport System) development [32];
- using the EGNOS system to build a positioning, mapping, and communication system CVIS (co-operative vehicle infrastructure systems) for Europe [33, 34];
- studying the availability parameter of GPS+EGNOS positioning in car navigation [35, 36];
- using the EGNOS support system to monitor road transport in Africa [37];
- monitoring changes in the coordinates of local GNSS reference stations receiving the EGNOS signal for their potential use in positioning in road transport [38];
- using the EGNOS support system in multi-modal transport in the European Union [39];
- development of an integrated positioning system for car navigation based on INS, Compass, GPS, EGNOS supported by the SISNeT service [40, 41];
- concept for the development of the EGNOS system from version v2 to v3 for the purposes of transport development, including road transport [42, 43];
- analysis of the accuracy of GPS/EGNOS positioning in urban areas [44], with positioning accuracy exceeding 10 m;
- analyses of the accuracy of GPS/EGNOS positioning in relation to GPS positioning alone in road transport [45]; the use of the EGNOS system improved positioning in urban environments by 10% to 30%;

- use of the EGNOS system for mapping and determining road geometry and topology [46];
- extension of the EGNOS system to North Africa and the Middle East for road transport monitoring [47];
- development of the GINA (GNSS for Innovative Road Applications) demonstrator for calculating charges in car navigation using EGNOS and Galileo [48].

The conclusions that can be drawn from the analysis of the current state of knowledge are as follows:

- the problem of determining the kinematic position of a motor vehicle has often been discussed in scientific publications [13–23, 35, 36, 44–47],
- the problem of determining the accuracy of kinematic positioning has also been presented in publications [13–23, 35, 36, 44–47],
- the EGNOS system has been used in conjunction with other navigation systems such as GPS and Galileo [23, 48],
- the EGNOS system was very often used in payment applications [24–27, 48],
- the EGNOS system was very often used in mobile, location and navigation applications [23, 28, 30, 33, 34],
- the EGNOS system was used extensively to test the quality of GNSS positioning in road transport [18–21, 31, 35, 36].
- the EGNOS system was actively used in various branches of road transport [14–16, 32, 37, 39].

RESEARCH PROBLEM

As the analysis of the state of knowledge shows, most studies focused on GPS+EGNOS positioning itself and the accuracy parameter. In this case, it is worth noting that both GPS alone and GPS with EGNOS corrections were used for GNSS positioning. What is more, the European Galileo system was also used in road transport in the literature on the subject. The satellite positioning process itself was based mainly on the use of a single GNSS receiver in car navigation. Of course, navigation applications also used mobile, location, and map data from other sensors. However, ultimately, it was the GPS and EGNOS systems that formed the basis for determining the kinematic position. This raises the question of how to determine the position of a motor vehicle in which several GNSS receivers with GPS/

EGNOS positioning options are installed. Additionally, what mathematical model should be proposed to determine the kinematic coordinates of a motor vehicle? This study used an algorithm to determine the coordinates of the center of the segment connecting two GNSS receivers placed on the roof of a car while driving. A total of four GNSS receivers were used for this purpose, two for each geometric segment. From there, the coordinates of the center point for two independent sections that intersected at a single location were determined, where a fifth GNSS receiver was installed to verify the coordinates obtained from the GPS/EGNOS solution. This made it possible to determine the coordinates of the center of the car while driving. The aim of the work was to determine the accuracy of GPS/EGNOS positioning for the proposed algorithm and calculation strategy. The most important contribution of the author to the presented publication is:

- the development of a new algorithm for determining the kinematic position of a motor vehicle from the GPS/EGNOS solution,
- the use of an algorithm for determining the coordinates of the center of a section for the purpose of determining the kinematic position of a motor vehicle,
- the development of a computational strategy for determining the kinematic position of a motor vehicle based on a multi-receiver GPS/EGNOS solution.
- determination of the accuracy characteristics of GPS/EGNOS kinematic positioning in road transport.

MATERIALS AND METHODS

A computational strategy based on multi-receiver GPS/EGNOS positioning in car navigation was used to solve the research problem. Specifically, four GNSS receivers were placed on the roof of a car during the research. Two pairs of GNSS receivers were created to form geometric segments, as shown in Figure 1. Receiver No. 1 forms a segment with receiver No. 2, and receiver No. 3 with receiver No. 4. Receivers No. 1–4 are Topcon HiperPro receivers [49]. In addition, a Septentrio Asterx2i GNSS receiver [50], marked as No. 0, was mounted in the middle of the beam. GNSS receivers were installed on a Subaru Forster 2.0. The car was manufactured in 2003 with

a 4-cylinder Boxer gasoline engine. The Subaru model belongs to the Forester II generation, which was produced between 2002 and 2008. The car has an engine capacity of 1994 cm³ and a power output of 130 kW. The maximum weight of the car is 1880 kg, and it has 5 passenger seats [51, 52]. The Subaru can be used for urban driving, longer journeys, and family trips.

Each of the GNSS receivers No. 1–4 collected GPS+EGNOS data. On this basis, the kinematic coordinates of the car's movement were determined every 1 s. The kinematic position was expressed in BLh ellipsoidal coordinates [53], and the calculations were performed in RTKLIB v.2.4.3 [54]. In turn, the coordinates of the central receiver No. 0 were determined with an accuracy greater than 0.05 m in Emlid Studio v.1.9 [55] using a precise DD (Double Difference) phase solution in post-processing mode [56]. Receiver No. 0 is the reference for the determined kinematic coordinates from the GPS/EGNOS solution within the developed research methodology. In addition, for each GNSS receiver, the characteristics of the antenna phase center were determined based on the ANTEX format from IGS service [57]. With the GPS/EGNOS kinematic coordinates determined for receivers No. 1–4, it is possible to create an equation of the coordinates of the center of the segments for the two pairs No. 1–No. 2 and No. 3–No. 4 in the form:

$$B_m = \left\{ \begin{array}{l} \frac{B_{nr1} + B_{nr2}}{2} \\ \frac{B_{nr3} + B_{nr4}}{2} \end{array} \right. \quad (1)$$

$$L_m = \left\{ \begin{array}{l} \frac{L_{nr1} + L_{nr2}}{2} \\ \frac{L_{nr3} + L_{nr4}}{2} \end{array} \right. \quad (2)$$

$$h_m = \left\{ \begin{array}{l} \frac{h_{nr1} + h_{nr2}}{2} \\ \frac{h_{nr3} + h_{nr4}}{2} \end{array} \right. \quad (3)$$

where: (B_m, L_m, h_m) – estimated central coordinates of car vehicle for two pairs nr 1–nr 2 and nr 3–nr 4, (B_{nr1}, B_{nr2}) – Latitude coordinates of GNSS receiver nr 1 and nr 2, (L_{nr1}, L_{nr2}) – Longitude coordinates of GNSS receiver nr 1 and nr 2, (h_{nr1}, h_{nr2}) – ellipsoidal height coordinates of GNSS receiver nr 1 and nr 2, (B_{nr3}, B_{nr4}) – Latitude coordinates of GNSS receiver nr 3 and nr 4, (L_{nr3}, L_{nr4}) – Longitude coordinates of GNSS receiver nr 3 and nr 4, (h_{nr3}, h_{nr4}) – ellipsoidal height coordinates of GNSS receiver nr 3 and nr 4.

It is worth noting that the coordinates of the center of individual geometric segments refer to the 3D space for BLh components. On this basis, it is now possible to compare the determined coordinates of the center of individual segments with the reference position expressed by the coordinates of central receiver No. 0. Then it can be written that [58]:

$$\left\{ \begin{array}{l} \Delta B = B_m - B_{nr0} \\ \Delta L = L_m - L_{nr0} \\ \Delta h = h_m - h_{nr0} \end{array} \right. \quad (4)$$

where: $(\Delta B, \Delta L, \Delta h)$ – positioning errors of the GPS/EGNOS solution, $(B_{nr0}, L_{nr0}, h_{nr0})$ – reference coordinates of car vehicle based on DD solution.

The calculated coordinates (B_m, L_m, h_m) should in practice equal the coordinates of the vehicle's

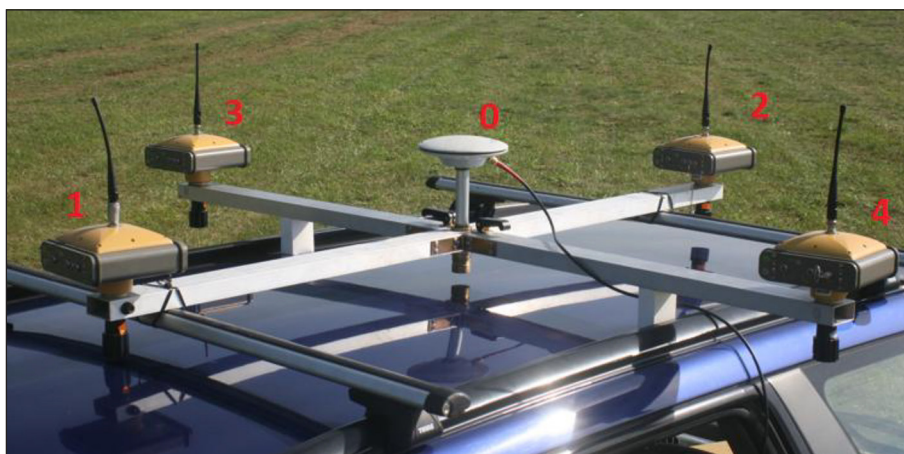


Figure 1. The scheme of location the on-board GNSS receivers

center point expressed by the position of receiver no. 0. The calculated difference (ΔB , ΔL , Δh) expresses the shift of the coordinates of the center of individual sections relative to the center point. The closer the calculated difference is to 0, the better the fit of the coordinates (B_m , L_m , h_m) to the coordinates of the center point. In addition, for the determined difference (ΔB , ΔL , Δh), a statistical measure of accuracy in the form of RMS [58, 59]:

$$\begin{cases} \text{RMS}\Delta B = \sqrt{\frac{[\Delta B^2]}{N}} \\ \text{RMS}\Delta L = \sqrt{\frac{[\Delta L^2]}{N}} \\ \text{RMS}\Delta h = \sqrt{\frac{[\Delta h^2]}{N}} \end{cases} \quad (5)$$

where: ($\text{RMS}\Delta B$, $\text{RMS}\Delta L$, $\text{RMS}\Delta h$) – RMS error for BLh coordinates, N – number of observations.

The algorithms (1–5) of the developed research methodology were used in a kinematic experiment conducted in the city of Olsztyn, Poland. The car journey is planned to take place at Olsztyn-Dajtki Airport. A map showing the Subaru's route is shown in Figure 2. Figures 3 and 4 show the horizontal trajectory of the car's movement and the vertical change in altitude, respectively. The geodetic width coordinates B varied during the drive from 53.773493° to 53.774528° . In turn, the geodetic length L changed during the drive from 20.407058° to 20.419899° . In addition,

the ellipsoidal height h changed during the drive from 163.196 m to 164.920 m. The data sampling period ranges from 29042 s to 32425 s according to GPS Time. Thus, the experiment lasted less than an hour, from 08:04:02 to 09:00:25 GPS Time.

A simplified diagram of the developed research methodology is shown in Figure 5. The most important element in the proposed methodology is the continuity of GPS+EGNOS data collection by satellite receivers No. 1–4. This makes it possible to develop mathematical Equations 1–3. Furthermore, knowing the reference coordinates of the car's movement from the DD solution, it is possible to implement mathematical Equations 4–5. The calculation process for Equations 1–5 was performed in the Scilab v. 6.1.1 environment [61]. The script written in Scilab defined the mathematical formulas and produced the necessary graphical drawings, which will be presented in chapters 6 and 7.

RESEARCH RESULTS

The characteristics of the obtained test results began with showing the number of GPS satellites tracked with EGNOS corrections by GNSS receivers No. 1–4 (see Figure 6). Thus, in the initial phase of the car journey, it can be seen that the number of GPS satellites with EGNOS corrections varies between 9 and 10. Of course, there are noticeable individual jumps



Figure 2. The map with Subaru route [60]

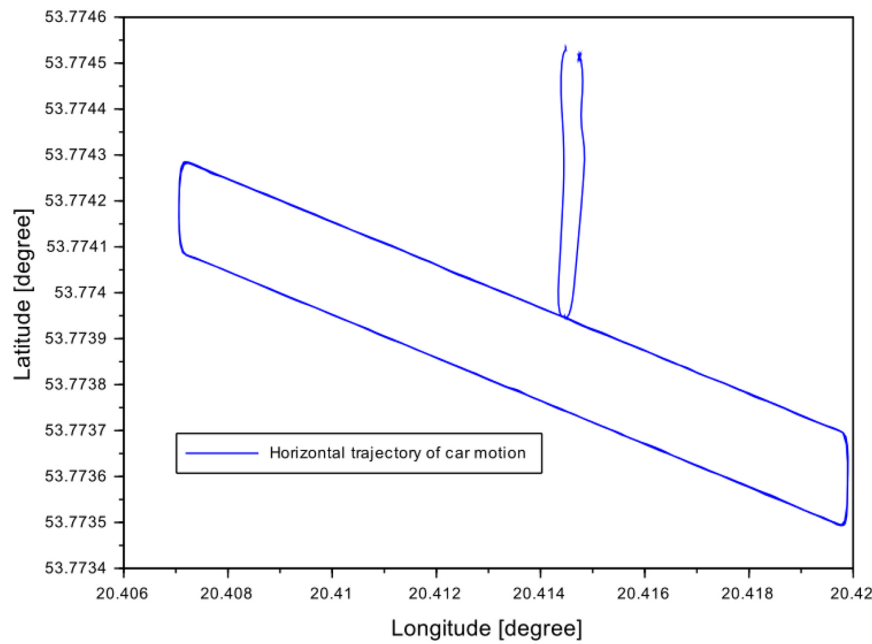


Figure 3. The horizontal trajectory of car motion

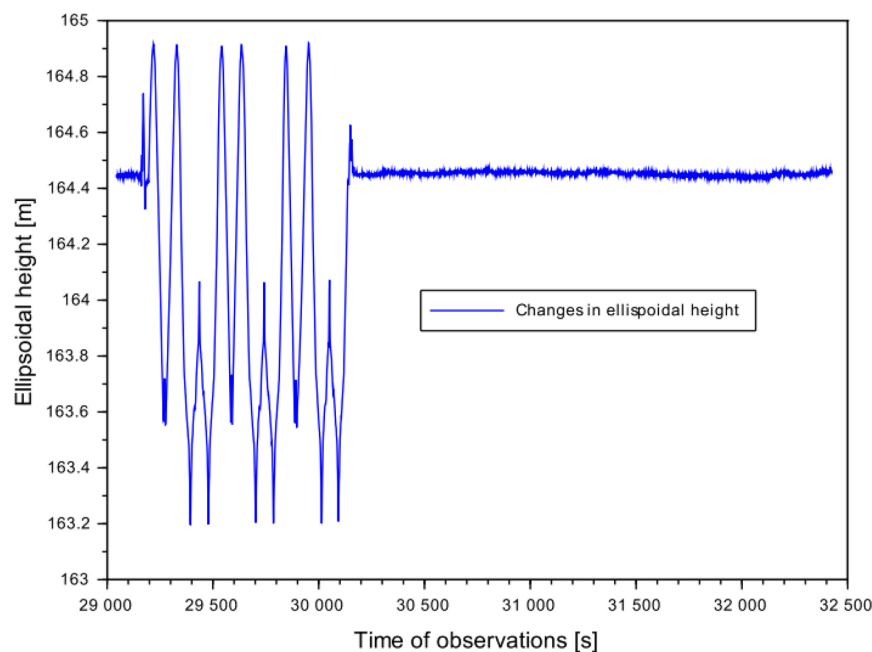


Figure 4. The changes in ellipsoidal height

where the number of satellites changes from 9 to 10 and vice versa. From the middle phase of the experiment to the very end, it can be seen that the number of GPS satellites drops from 10 to 5. This decrease in the number of GPS satellites is visible for all receivers in the same measurement epochs.

The number of GPS satellites tracked with EGNOS corrections directly affects the determination of the geometric PDOP (Position DOP)

coefficient [62]. Figure 7 shows the PDOP coefficients for each GNSS receiver No. 1–4. It can be seen that the quality of the satellite observations was good [63], and the DOP was in the range of 1.8–2.8 for most of the experiment. Only at the end of the experiment, due to a decrease in the number of GPS satellites tracked, did the PDOP values jump to 9.4. Next, Figures 8–10 show the positioning errors (ΔB , ΔL , Δh) determined from mathematical equation (4) for each pair of GNSS

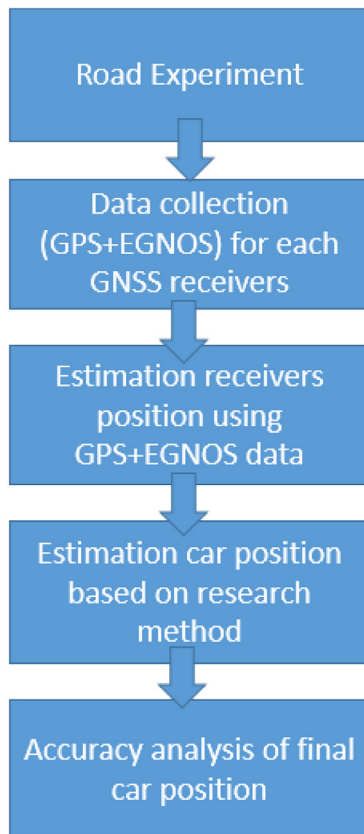


Figure 5. The flowchart of research method

receivers [64, 65]. The positioning errors (ΔB , ΔL , Δh) were as follows:

- from -0.02 m to +1.50 m for the B component for pair no. 1–2,

- from -0.67 m to +1.22 m for component L for pair no. 1–2,
- from -1.28 m to +1.80 m for component h for pair no. 1–2,
- from +0.10 m to +1.57 m for component B for pair no. 3–4,
- from -0.25 m to +0.95 m for the L component for pair no. 3–4,
- from -1.02 m to +2.28 m for the h component for pair no. 3–4.

In addition, the arithmetic mean values for the parameters (ΔB , ΔL , Δh) are as follows:

- +0.72 m for component B for pair no. 1–2,
- +0.40 m for component L for pair no. 1–2,
- +0.65 m for component h for pair no. 1–2,
- +0.75 m for component B for pair no. 3–4,
- +0.40 m for component L for pair no. 3–4,
- +0.65 m for component h for pair no. 3–4.

On this basis, it can be seen that the horizontal coordinates (B and L) and the vertical coordinate h from the proposed methodology match very well with central receiver No. 0. This proves that the proposed algorithm (1–5) is effective for determining the accuracy of horizontal coordinates and ellipsoidal height. For all three BLh components, the average accuracy value is higher than 0.80 m, and this accuracy refers to the kinematic mode of the experiment. For the calculated positioning errors (ΔB , ΔL ,

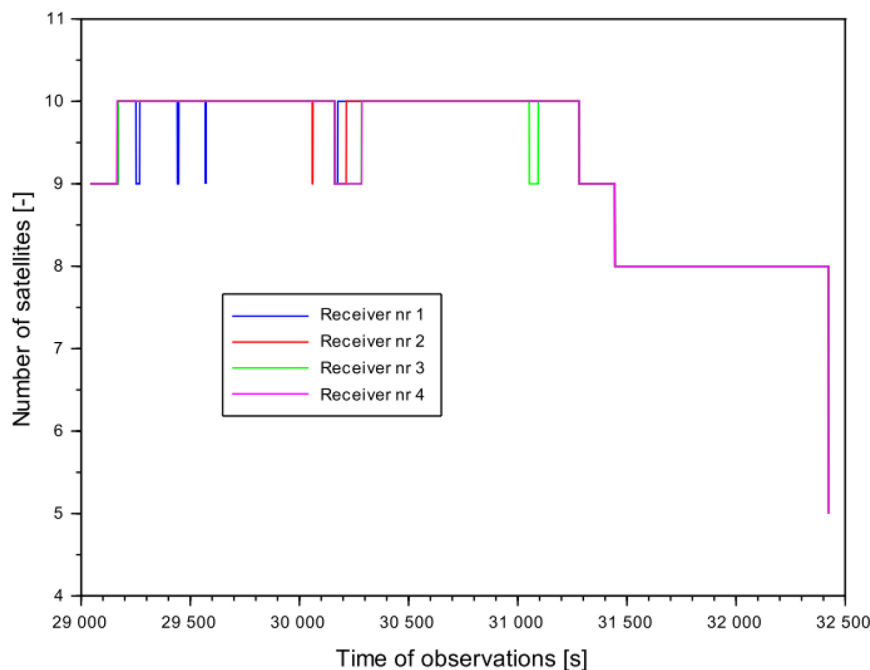


Figure 6. Number of GPS satellites tracked with EGNOS corrections for each GNSS receivers

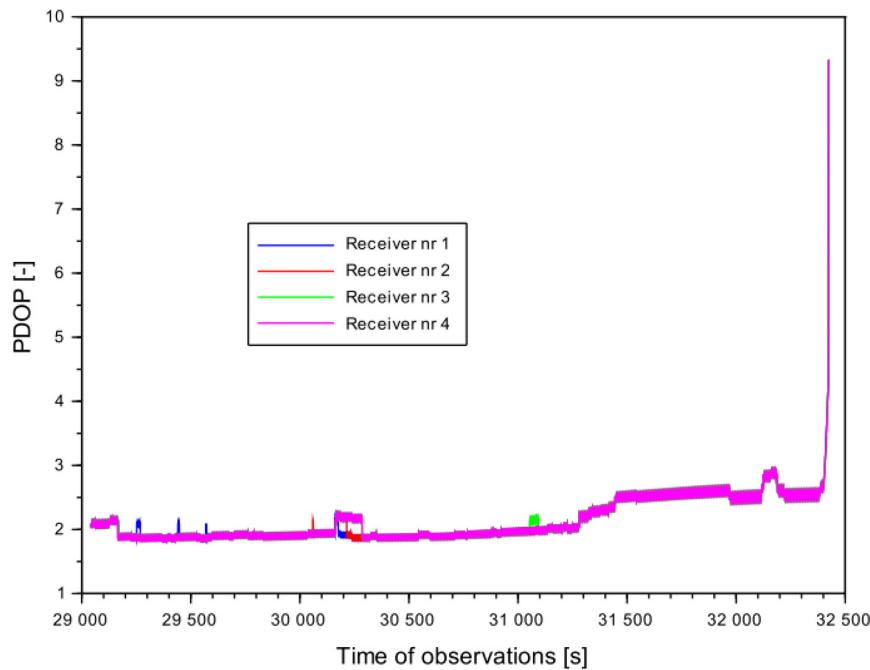


Figure 7. PDOP values based on GPS/EGNOS solution for each GNSS receivers

Δh), RMS errors were also determined according to Equation 5. Thus, it can be said that:

- the RMS errors along the B axis are 0.75 m for receiver pair No. 1–2 and 0.78 m for receiver pair No. 3–4, respectively,
- the RMS errors along the L axis are 0.44 m for the pair of receivers No. 1–2 and 0.43 m for the pair of receivers No. 3–4, respectively,

- RMS errors along the h axis are 0.83 m for receiver pair No. 1–2 and 0.81 m for receiver pair No. 3–4, respectively.

The calculation of the statistical measure of accuracy in the form of RMS errors only confirms that the spread of position error results for horizontal coordinates is smaller than that for the

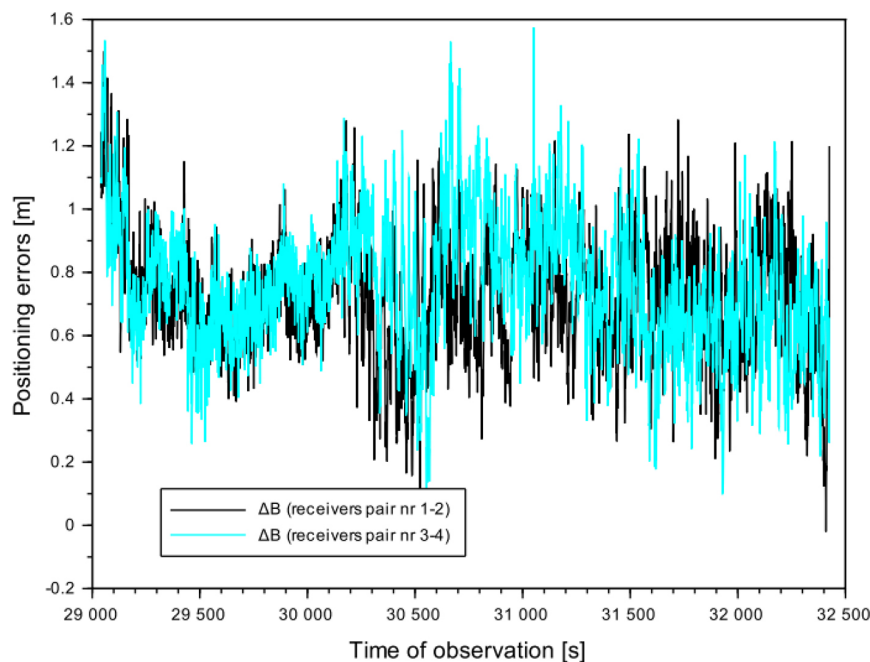


Figure 8. Positioning errors for the latitude

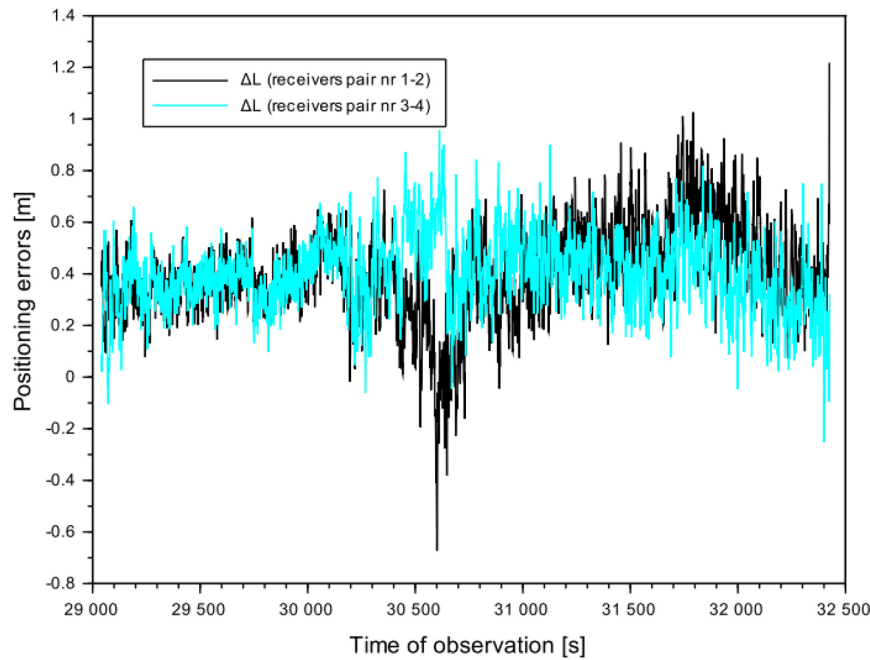


Figure 9. Positioning errors for the longitude

vertical component h . This is important for car navigation in the horizontal plane.

DISCUSSION

The chapter discussing the results takes into account three elements, i.e., determining additional accuracy characteristics of GPS/EGNOS

positioning, developing a new methodology for determining the position of a car based on navigation data readings from four GNSS receivers, and comparing the obtained research results with the analysis of the current state of knowledge. As part of determining the accuracy characteristics of GPS/EGNOS positioning, the following parameters were determined: DRMS (Distance RMS) errors for the horizontal plane and MRSE

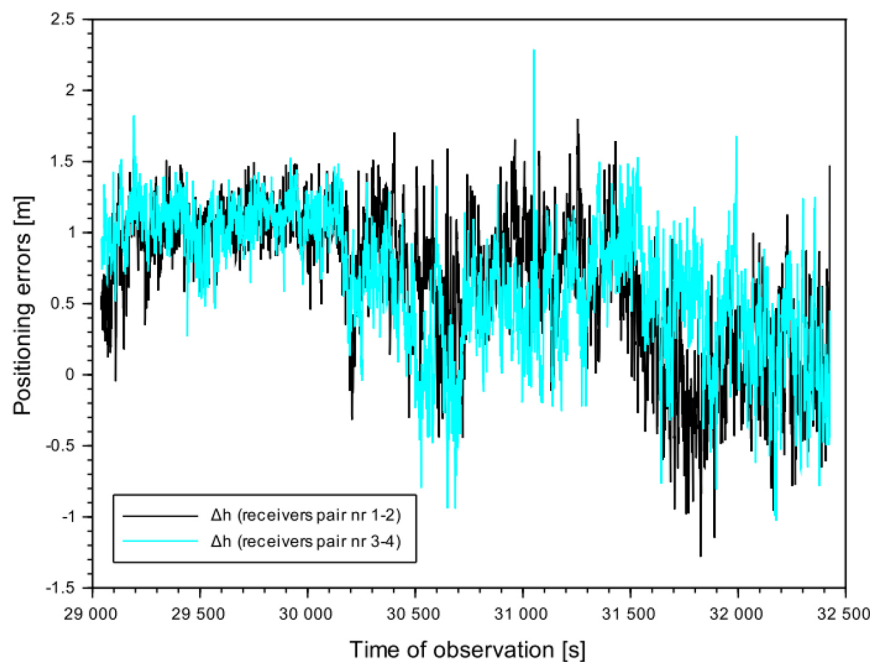


Figure 10. Positioning errors for the ellipsoidal height

(mean radial spherical error) errors for 3D space [66]. Figure 11 shows the results of DRMS positioning errors determined separately for receiver pairs 1–2 and 3–4. The DRMS error values for receiver pair 1–2 ranged from 0.20 m to 1.70 m. For the pair No. 3–4, they ranged from 0.26 m to 1.63 m. It is worth noting that approximately 80% of the DRMS results for the pair No. 1–2 are less than 1 m, and in the case of the pair No. 3–4, we have approximately 76%.

Next, Figure 12 shows the results of MRSE positioning errors determined separately for receiver pairs No. 1–2 and No. 3–4. The MRSE error values for receiver pair No. 1–2 ranged from 0.32 m to 2.26 m. For receiver pair No. 3–4, they ranged from 0.30 m to 2.78 m. It is worth noting that approximately 26% of the MRSE results for pair no. 1–2 are less than 1 m and almost 99% are less than 2 m. For pair no. 3–4, the results are 29% and 99%, respectively. Both the DRMS and MRSE error results are affected by the values of individual positioning errors (ΔB , ΔL , Δh), which are shown in Figures 8–10. The height error Δh has a particularly strong influence on the determination of MRSE errors, as can be seen in Figure 12. On the other hand, horizontal coordinate errors (ΔB , ΔL) influence the calculation of the DRMS parameter.

In the second part of the discussion, a modification of the mathematical algorithm (1–3) was proposed to the following form:

$$B_m = \frac{B_{nr1} + B_{nr2} + B_{nr3} + B_{nr4}}{r} \quad (6)$$

$$L_m = \frac{L_{nr1} + L_{nr2} + L_{nr3} + L_{nr4}}{r} \quad (7)$$

$$h_m = \frac{h_{nr1} + h_{nr2} + h_{nr3} + h_{nr4}}{r} \quad (8)$$

where: r – number of GNSS receivers, $r = 4$.

The proposed algorithm (6–8) refers to determining the resultant position of a car based on coordinate readings from four GNSS receivers numbered 1–4. The algorithm thus implements the arithmetic mean model for a system of four GNSS receivers. Next, positioning errors (ΔB , ΔL , Δh) were determined based on Equation 4. At this stage, Kalman filtering [58, 64, 67] was additionally used to increase positioning accuracy and eliminate any outliers from the time series of the data used. Figure 13 shows the results of positioning errors ΔB , ΔL , Δh for the averaged coordinates (B_m , L_m , h_m) from Equations 6–8 after Kalman filtering. In the case of BLh coordinates, the positioning errors after Kalman filtering are:

- from +0.36 m to +1.33 m for the B component,
- from +0.09 m to +0.74 m for the L component,
- from -0.31 m to +1.37 m for the h component.

In addition, the average values of the parameters (ΔB , ΔL , Δh) were 0.73 m, 0.40 m, and 0.66 m, respectively. In turn, the RMS errors after

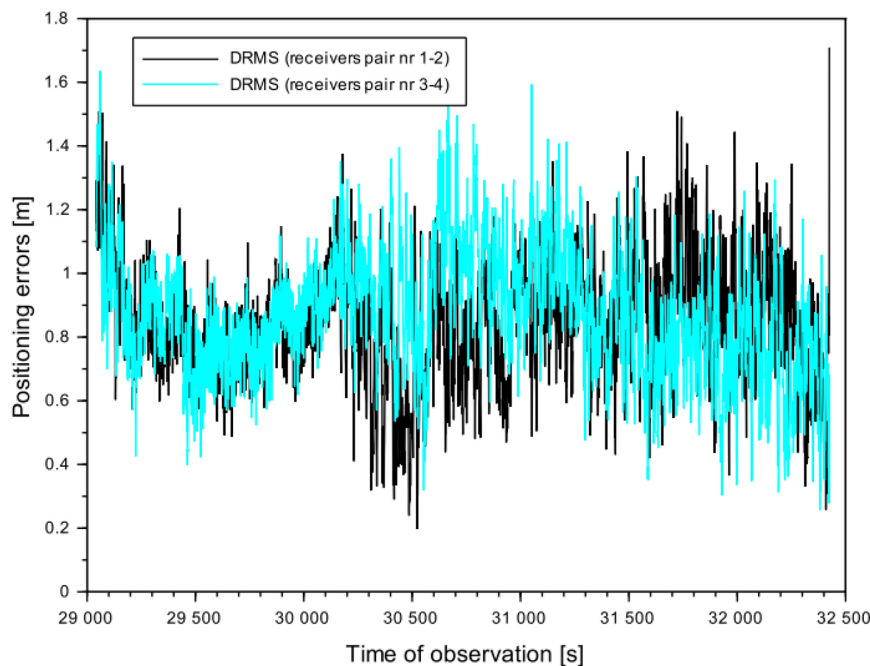


Figure 11. Positioning errors of DRMS term

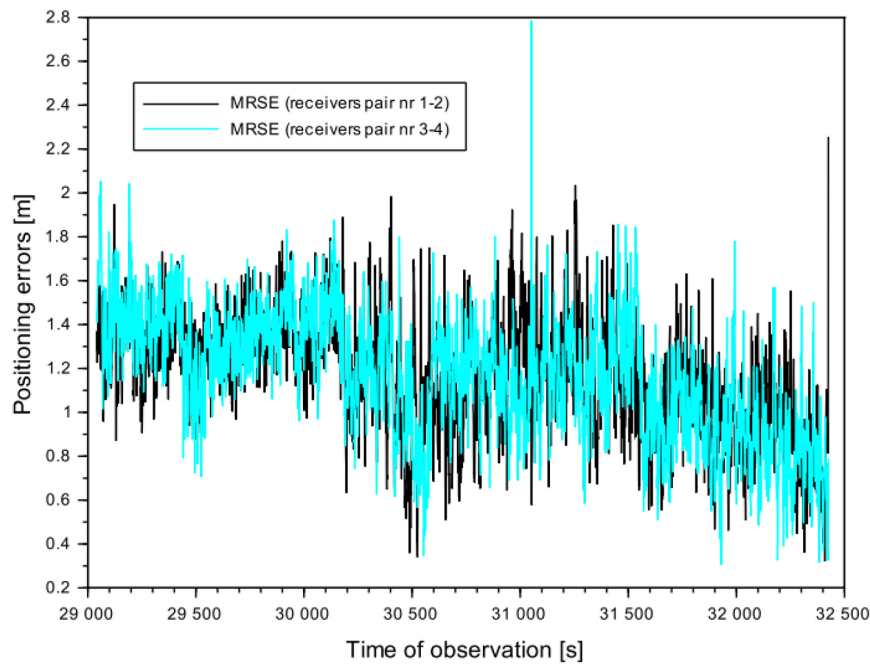


Figure 12. Positioning errors of MRSE term

Kalman filtering are as follows for the BLh components: 0.75 m, 0.41 m, and 0.76 m, respectively. Thanks to Kalman filtering, it was possible to reduce positioning errors:

- for the B component by 11% and 16% compared to the results shown in Figure 8,
- for the L component by 23% and 40% compared to the results shown in Figure 9,

- for the h component by 24% and 40% compared to the results shown in Figure 10.

The third and final part of the discussion compares the research results with an analysis of the current state of knowledge. In the context of comparison with the literature on the subject, it can be concluded that:

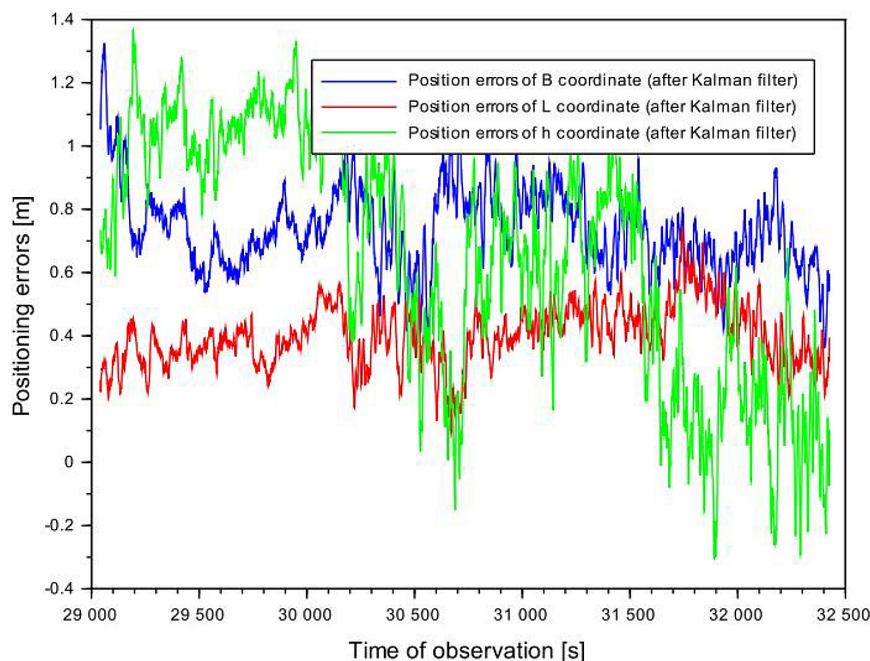


Figure 13. Positioning errors after Kalman filter operation

- the article determines the position of a car using GNSS satellite technology, similarly to the works [13–23, 35, 36, 44–47],
- the article examines the accuracy parameter, as in works [13–23, 35, 36, 44–47],
- the article uses the GPS positioning method with EGNOS corrections, as in works [14–16, 18–21, 23–28, 30–37, 39, 48],
- the article shows the application of the EGNOS assistance system in navigation, as in works [23, 28, 30, 33, 34].
- the GPS/EGNOS positioning accuracy obtained during the research meets the requirements for GNSS systems for road transport applications, as directly stated in work [6].
- the GPS/EGNOS positioning accuracy obtained during the research is higher than the results described in [13–17, 22, 44],
- the presented results and analyses are part of a series of studies on the use of the EGNOS assistance system in road transport in Europe, as in [5–48],
- in paper the car mobile vehicle was used in experiment, as in works [68–70].
- At the end of this chapter, the advantages of our own research in relation to the literature are highlighted:
- this article uses a multi-receiver GPS/EGNOS solution, which is a different approach than in [13–23, 35, 36, 44–47], where single GNSS receivers were used,
- this article conducts a more in-depth statistical analysis of errors by determining DRMS and MRSE parameters, which is not seen in the works [13–23, 35, 36, 44–47],
- in this article, the research test was conducted in north-eastern Poland, i.e., a different location in Poland was chosen than in the studies published in [13–16, 44],
- this article shows that the advantage of the conducted research is the improved positioning accuracy compared to the articles [13–17, 22, 44].
- In addition, it is worth highlighting the advantages of the developed algorithm in comparison with the literature:
- the algorithm is based on a weighted average model within the framework of multi-receiver GPS/EGNOS positioning, which is important from a navigation point of view as it ensures control and redundancy of the determined position. The literature on the subject mainly uses single GNSS receivers, which unfortunately does not guarantee a reliable position. It

is always possible that a single GNSS receiver will provide coordinates with random errors. This means that the determined coordinates may deviate from the reference position.

- thanks to the algorithm, it is possible to determine the user's position and estimate the accuracy of the coordinates. The algorithm scheme proposes the determination of not only single position errors, but also DRMS horizontal errors and 3D resultant errors - the MRSE parameter. Typically, the literature on the subject has been limited to calculating single position errors for three-dimensional coordinates. Hence, the proposed solution provides further improved information on the distribution of measurement errors in statistical terms.
- the use of the Kalman filter in the algorithm shows that the computational strategy is focused on improving positioning and achieving high accuracy of the user's coordinates. In addition, the Kalman filter will ensure the elimination of outliers and the reduction of measurement noise in the time series of position error parameters. Compared to the literature on the subject, the Kalman filter is primarily designed to increase the accuracy of GPS/EGNOS positioning for a multi-receiver GNSS solution in car navigation.

CONCLUSIONS

The main objective of this article was to develop an algorithm and computational strategy for determining the accuracy of a car's position based on a multi-receiver GPS solution with EGNOS corrections in navigation. To this end, a research methodology was developed using the GPS/EGNOS positioning method in kinematic mode for on-board GNSS receivers mounted on a car. Two pairs of GNSS receivers were placed on the roof of the car, forming a geometric section and intersecting at a single central point on the vehicle, where an additional reference receiver was placed. For each pair of GNSS receivers, the central point of the vehicle was determined and its geodetic coordinates BLh were specified. Then, the accuracy of these coordinates was determined in the form of positioning errors and RMS errors. The research showed that the average positioning accuracy for BLh components is higher than 0.80 m. Next, as part of the accuracy characteristics, DRMS and MRSE errors were also calculated, with values not exceeding 1.70 m and 2.78 m,

respectively. Finally, the coordinates of the vehicle's center point were determined from the arithmetic mean model for navigation readings from four GNSS receivers. In this case, the obtained coordinate accuracy results were subjected to Kalman filtering, and the average positioning accuracy for BLh components does not exceed 0.73 m. The research conducted demonstrated the effectiveness of the developed methodology for the GPS/EGNOS positioning method. The presented algorithms can be used, for example, in air navigation for multi-receiver aircraft positioning. In addition, other GNSS positioning methods can be adapted to the proposed research methodology.

REFERENCES

- Grzegorzewski M., Świątek A., Ciećko A., Oszczak S., Ćwiklak J. Study of EGNOS safety of life service during the period of solar maximum activity. *Artificial Satellites*. 2012, 47, 137–145, <https://doi.org/10.2478/v10018-012-0019-5>
- Januszewski J. Satellite navigation systems in therransport, today and in the future. *The Archives of Transport*. 2010, 22(2), 175–187, <https://doi.org/10.2478/v10174-010-0011-4>
- EGNOS and Galileo for ITS and road transport. Available from: https://egnos.gsc-europa.eu/sites/default/files/documents/EGNOS_GALILEO_Road.pdf, (Accessed: 01.06.2025).
- EGNOS Road. Available from: <https://egnos.gsc-europa.eu/segments/road>, (Accessed: 01.06.2025).
- EGNOS High Flexibility, Low Investment. Available from: https://www.euspa.europa.eu/sites/default/files/EGNOS-road_brochure_V_0.pdf, (Accessed: 01.06.2025).
- Fellner R. Analysis of the EGNOS/GNSS parameters in selected aspects of Polish transport. *Transport Problems*. 2014, 4(9), 27–37.
- Galileo and EGNOS for the road transport. Available from: <https://slideplayer.com/slide/739512/>, (Accessed: 01.06.2025).
- New Funding Opportunities to Support Safety of Navigation: EGNOS and Galileo. Available from: <https://www.iala.int/content/uploads/2016/09/1600-Jose-Manuel-Alvarez-e-Navigation-Underway-New-Funding-Opportunities-to-Support-Safety-of-Navigation-EGNOS-and-Galileo.pdf>, (Accessed: 01.06.2025).
- Povero G. SBAS/EGNOS introduction, In: *Proceedings of the GNSS Training Course*, 14–18 January 2019, Bangkok, Thailand, number of slides: 1–29.
- Tyszkowski A., Temlin T., Oszczak S. GNSS Systems in vehicle fleet management. *Transport Problems*. 2007, 2(2), 11–16.
- Ciećko A. Analysis of the EGNOS quality parameters during high ionosphere activity. *IET Radar Sonar Navigation*. 2019, 13, 1131–1139, <https://doi.org/10.1049/iet-rsn.2018.5571>
- Mrozik M. Application of the SBAS positioning method in the aircraft approach procedure, PhD thesis. Silesian University of Technology, Gliwice, 2023, 1–147. (in Polish)
- Jaworski L., Świątek A., Zdunek R. Egnos Limitations over Central and Eastern Poland – results of preliminary tests of egnos-eupos integration project. *Artificial Satellites*. 2013, 48(3), 93–102, <https://doi.org/10.2478/arsa-2013-0008>
- Banachowicz A., Bober R., Szewczuk T., Wolski A. Monitoring of cargo transport in urban areas using GPS/EGNOS technologies as part of the safety system. *Procedia - Social and Behavioral Sciences*. 2014, 151, 371–380, <https://doi.org/10.1016/j.sbspro.2015.05.100>
- Banachowicz A., Bober R., Szewczuk T., Wolski A. Applicability of GPS/EGNOS in city transport. *Logistyka*. 2014, 4, 2639–2648. (in Polish).
- Bober R., Szewczuk T., Wolski A. An effect of urban development on the accuracy of the GPS/EGNOS system. *TransNav*. 2008, 2(3), 235–238.
- Vassilev P. Simulation of EGNOS navigation data for vehicles. *Cybernetics and Information Technologies*. 2008, 8(4), 77–83.
- Santa J., Ubeda B., Toledo R., A. F. G. Skarmeta A. F. G. Monitoring the Position Integrity in Road Transport Localization Based Services, In *Proceedings of IEEE Vehicular Technology Conference*, Montreal, QC, Canada, 2006, 1–5, <https://doi.org/10.1109/VTTCF.2006.575>
- Di Fazio A. Transport of dangerous dangerous goods. In *Proceedings of 7th ETSI Security workshop*, January 2012, Sophia Antipolis, France, number of slides: 1–22.
- Di Fazio A. EGNOS Use in Road Applications. In *Proceedings of Workshop on Science Applications of GNSS in Developing Countries*, 11 April–1 May, 2012, Trieste, Italy, number of slides: 1–39.
- EGNOS: an emerging technology to support urban transport. Available from: <https://www.polisnetwork.eu/wp-content/uploads/2019/06/the-benefits-of-egnos-in-urban-transport.pdf>, (Accessed: 01.06.2025).
- Sheridan K., Wells D., Botteron C., Leclère J., Dominici F., Defina A. An Assisted-GNSS Solution for Demanding Road Applications using the EGNOS Data Access System (EDAS). In *Proceedings of Toulouse Space Show'10*, Toulouse, France, 1–9.
- Di Fazio A., Bettinelli D., Louette E., Mechin J. P., Zazza M., Vecchiarelli P., Domanico L. European pathways to introduce EGNOS and Galileo for dangerous goods transport. *Transportation*

- Research Procedia. 2016, 14, 1482–1491, <https://doi.org/10.1016/j.trpro.2016.05.222>
24. Diani F., Hamet P., Calini G.G., Quantifying and Demonstrating GNSS and EGNOS Benefits in Regulated Road Pricing Applications. In Proceedings of the 23rd International Technical Meeting of the Satellite Division of The Institute of Navigation (ION GNSS 2010), Portland, OR, September 2010, 1124–1129.
25. Ali K., Pini M., Dovis F. Measured performance of the application of EGNOS in the road traffic sector. *GPS Solutions*, 2012, 16, 135–145, <https://doi.org/10.1007/s10291-011-0253-5>
26. Lanza S. G., Gutiérrez C. B., Schortmann J. C. GINA-GNSS for innovative road applications-: EGNOS / Galileo for road user charging and Value Added Services, In Proceedings of 9th International Conference on Intelligent Transport Systems Telecommunications, (ITST), Lille, France, 2009, 348–352, <https://doi.org/10.1109/ITST.2009.5399331>
27. Boniface K., Gioia C., Susi M., Fortuny-Guasch J., Sbardellati F. Galileo and EGNOS Adoption in Automotive Emergency Call System (eCall), In Proceedings of the 32nd International Technical Meeting of the Satellite Division of The Institute of Navigation (ION GNSS+ 2019), Miami, Florida, September 2019, 1284–1294, <https://doi.org/10.33012/2019.17130>
28. Obst M., Mattern N., Schubert R., Wanielik G. Car-to-Car communication for accurate vehicle localization — The CoVeL approach. In Proceedings of International Multi-Conference on Systems, Signals & Devices, Chemnitz, Germany, 2012, 1–6, <https://doi.org/10.1109/SSD.2012.6198050>
29. Gómez Skarmeta A.F., Barberá H.M., Izquierdo M.Z., Miñarro B.Ú., Quiñero J.P.C., Cáceres D.A. Evaluation of EGNOS (GNSS-1) for Application in a Vehicle Driving Support System. In: Rycroft, M. (eds) *Satellite Navigation Systems*. Space Studies, 2003, 8, 175–182, https://doi.org/10.1007/978-94-017-0401-4_19
30. EGNOS System Test Bed. Available from: <https://www.fdc.fr/oregin/fifthmeeting/EGNOSsystem-TestBed2.pdf>, (Accessed: 01.06.2025).
31. Santa J., Úbeda B., Toledo R., Sotomayor C. A facility for GPS/EGNOS signal monitoring. In Proceedings of the “Workshop on EGNOS performance and applications”, October 27–28, 2005, Gdynia, Poland, 183–188.
32. Gómez Skarmeta A., Martínez H., Zamora Izquierdo M., Cánovas Quiñero J., Tomás Balibrea L. Evaluating EGNOS technology in an ITS driving assistance application, In Proceedings of the V GNSS International Symposium (GNSS’2001), Sevilla, Spain, April 2001, 1–5.
33. Schlingelhof M., Betaille D., Bonnifait P., Demasure K. Advanced positioning technologies for co-operative systems. *Intelligent Transport Systems, IET*, 2008, 2(2), 81–91, <https://doi.org/10.1049/iet-its:20070046>
34. Schlingelhof M., Betaille D., Bonnifait P., Poir P., Demasure K. Advanced positioning technology approach for co-operative vehicle infrastructure systems (CVIS). In Proceedings of the 6th European ITC Congress, June 18–20, 2007, Aalborg, Denmark, 1–9.
35. Ali Z., Ahmed Soomro R., Ahmed Dahri F., Mujtaba Shaikh M. Acquisition of positional accuracy with comparative analysis of GPS and EGNOS in urban constituency. *International Journal of Advanced Computer Science and Applications*. 2021, 12(2), 570–574.
36. Lanza S. G., Salmón P. R., Martín Bobis J. M. GINA-GNSS for innovative road applications-: EGNOS / GALILEO for road user charging and value added services. *Revista del Instituto de Navegación de España*. 2009, 36, 24–33.
37. Dodo J. D., Idowu T. O. A proposal for the extension and institutional framework of European Geostationary Navigation Overlay Service (EGNOS) in the implementation of Global Navigation Satellite System (GNSS) in Africa. *FUTY Journal of the Environment*, 2011, 6(1), 79–90.
38. Felski A., Nowak A. On EGNOS monitoring in local conditions. *Artificial Satellites*. 2013, 48(2), 85–92, <https://doi.org/10.2478/arsa-2013-0007>
39. EGNOS and Galileo for the Multimodal Freight Transport. Available from: <https://www.ttsitalia.it/file/papers%20e%20documenti/M-TRADE-Leaflet.pdf>, (Accessed: 01.06.2025).
40. Toledo R., Zamora M. A., Ubeda B., Martínez H., A. Gómez Skarmeta A. A precision positioning system based on EGNOS-SISNET/INS navigation for an autonomous vehicle. In Proceedings of the European Congress on Intelligent Transportation Systems and Services, 4th, 2004, Budapest, Hungary, 1–6.
41. Martinez-Barbed H., Zamora-Izquierdo M.A., Toledo-Mom R., Ubeda B. Gómez-Skarmeta A.F. The MIM-ICS project: An application for intelligent transport systems. In Proceedings of the IEEE Intelligent Vehicles Symposium, Ohio, USA, June 9–11 2003, 633–638.
42. Celestino U. EGNOS status, roadmap, extension to Euromed. In Proceedings of the Joint ACAC/ICAO Workshop on GNSS, Rabat, Morocco, 7–8 November 2017, number of slides: 1–29.
43. EGNOS and Galileo services and applications. Available from: <https://gsv.co.at/wp-content/uploads/2019%2010%2008%20GSA.pdf>, (Accessed: 01.06.2025).
44. Demkowicz J., Bikonis K. EGNOS-Based GNSS Receiver for Precise Positioning in Restricted Areas. *Hydroacoustics*. 2016, 19, 65–73.
45. Peyret F., Vigneau W., Betaille D. EGNOS-ON-THE-ROAD”: what can be expected from EGNOS compared to GPS for road traffic management services.

- In Proceedings of the 17th ITS World Congress, ITS, October 2010, Busan (Corée), South Korea, 1–12.
46. Peyret F., Bétaille D., Laneurit J., Toledo-Moreo R. Lane-level positioning for cooperative systems using EGNOS and enhanced digital maps. In Proceedings of the ENC GNSS Congress "Toulouse Space Show", April 2008, Toulouse, France, 1–9.
47. Celestino U., Di Fazio A., Bettinelli D., Giner Herrera V. J., Vanni P., De Blas F. J. Expanding EGNOS horizons. *InsideGNSS*. 2015, March/April, 42–52.
48. Lanza S. G., Diamandouros K. Meet GINA. *InsideGNSS*. 2011, March/April, 26–38.
49. Ćwiklak J., Jaferník H. The monitoring system for aircraft and vehicles of public order services based on GNSS. *Annual of Navigation*. 2010, 16, 15–24.
50. Ciećko A., Grunwald G. Examination of Autonomous GPS and GPS/EGNOS Integrity and Accuracy for Aeronautical Applications. *Periodica Polytechnica Civil Engineering*. 2017, 61(4), 920–928, <https://doi.org/10.3311/PPci.10022>
51. Auto-info website. Available from: <https://auto-info.gratis/raport/2701968c444b98165e3e38e2d8f5bef4/>, (Accessed: 24.12.2025).
52. Autocentrum website. Available from: <https://www.autocentrum.pl/dane-techniczne/subaru/forester/ii/silnik-benzynowy-2.0-xt-177km-2002-2008/>, (Accessed: 24.12.2025).
53. Grzegorzewski M. Navigating an aircraft by means of a position potential in three dimensional space, *Annual of Navigation*, 2005, 9, 1–111.
54. RTKLIB website. Available from: <http://rtklib.com/>, (Accessed: 01.06.2025).
55. Emlid Studio website. Available from: <https://docs.emlid.com/emlid-studio/>, (Accessed: 01.06.2025).
56. Elkhalea M.F., Hendy H., Kamel A., Abosekeen A., Noureldin A. Centralized measurement level fusion of GNSS and inertial sensors for robust positioning and navigation. *Sensors*. 2025, 25, 2804, <https://doi.org/10.3390/s25092804>
57. Antenna. Available from: <https://igs.org/wg/antenna/#files>, (Accessed: 01.06.2025).
58. Krasuski K., Bakula M., Ciećko A., Grunwald G., Gołda P., Mroziński M., Kozuba J. Algorithms to improve unmanned aerial vehicle positioning accuracy using European Geostationary Navigation Overlay Service and System for Differential Corrections and Monitoring ionospheric corrections. *Advances in Science and Technology Research Journal*. 2025, 19(1), 284–300, <https://doi.org/10.12913/22998624/195195>
59. Specht C., Pawelski J., Smolarek L., Specht M., Dabrowski P. Assessment of the positioning accuracy of DGPS and EGNOS systems in the bay of Gdansk using maritime dynamic measurements, *Journal of Navigation*. 2019, 72(3), 575–587, <https://doi.org/10.1017/s0373463318000838>
60. Google Earth website. Available from: <https://earth.google.com/web/@53.77374482,20.41563652,131.7754312a,2218.05620784d,30.00013219y,0h,0t,0r/data=CgRCaggBOgMKATBCaggASggIeTFoAQQAA>, (Accessed: 24.12.2025).
61. Scilab website. Available from: <https://www.scilab.org/>, (Accessed: 01.06.2025).
62. Specht C., Mania M., Skóra M., Specht M. Accuracy of the GPS positioning system in the context of increasing the number of satellites in the constellation. *Polish Maritime Research*. 2015, 22, 9–14, <https://doi.org/10.1515/pomr-2015-0012>
63. Specht M. Experimental studies on the relationship between HDOP and position error in the GPS system. *Metrology and Measurement Systems*. 2022, 29(1), 17–36, <https://doi.org/10.24425/mms.2022.138549>
64. Ciećko A., Bakula M., Grunwald G., Ćwiklak J. Examination of multi-receiver GPS/EGNOS positioning with kalman filtering and validation based on CORS stations. *Sensors*. 2020, 20, 2732, <https://doi.org/10.3390/s20092732>
65. Beldjilali B., Kahlouche S., Tabti L. Assessment of EGNOS performance for civil aviation flight phase in the edge coverage area. *Int. J. Aviat. Aeronaut. Aerosp.* 2020, 7, 1–25, <https://doi.org/10.15394/ijaaa.2020.1479>
66. GPS Position Accuracy Measures. Available from: https://www.gnss.ca/app_notes/APN-029_GPS_Position_Accuracy_Measures_Application_Note.html, (Accessed: 01.06.2025).
67. Kaniewski P. Extended kalman filter with reduced computational demands for systems with non-linear measurement models. *Sensors*. 2020, 20, 1584, <https://doi.org/10.3390/s20061584>
68. Buckus R., Chlebnikovas A., Strukcinskiene B., Stukas R., Austys D., Caban J., Bogucki M., Sidlauskiene A., Seleviciene V., Kilikevičius A., Matijošius J., Kilikevičienė K., Vainorius D. Simulating the dispersion of the energy flux density of the electromagnetic field generated by antennas for mobile communications. *Electronics* 2022, 11, 2431, <https://doi.org/10.3390/electronics11152431>
69. Ondruš J., Jančár A., Gogola M., Varga P., Šarić Ž., Caban J. Smartphone sensors in motion: Advancing traffic safety with mobile technology. *Appl. Sci.* 2024, 14, 5404, <https://doi.org/10.3390/app14135404>
70. Schlingelhof M., Baille D., Bonnifait P., Poir P., and Demasure K. Advanced positioning technology approach for cooperative vehicle infrastructure systems (cvis). 2007