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Practical implementation of self-balancing method for a washing machine

Tadeusz Majewski[1,2*], Edyta Ładyżyńska-Kozdraś[2], Jerzy Bajkowski[2], Marco Meraz Melo[1], Gale A. Ahearn[3]

[1] Instituto Tecnológico de Puebla, Mexico
[2] Warsaw University of Technology, plac Politechniki 1, 00-661 Warszawa, Poland
[3] Universidad de las Americas-Puebla, Mexico
* Corresponding author's e-mail: t24majewski@gmail.com

ABSTRACT

This article presented the properties of a front-load washing machine equipped with a self-balancing system and its possibility of industrial application in a series of productions. The dynamic model described the behavior of the unbalanced drum and the free elements during balancing. The article defineed the vibratory forces responsible for changing the position of the free elements concerning the drum which leads to the self-balancing phenomena. The kinetic-static model with the vibratory forces was compared with the solution of a set of differential equations. Because the difference between them is small, such a model was used for further analysis. The parameters of a real washing machine have some deviations that influence its work. Various analyses present their impact on decreasing the efficiency of the method. Each of them was studied to establish the residual unbalance generated. Finally, the authors proposed a procedure to establish acceptable maximum deviations for the parameters of the washing machine to obtain the required level of balancing.

Keywords: washing machine, self-balancing, vibrations, vibratory forces, balancing efficiency.

INTRODUCTION

Nowadays, almost every house has a washing machine to help in our daily lives. They come in various forms: a simple top load with a vertical axis of rotation, or a front load with a horizontal axis of rotation, the latter usually with more washing programs. There are two main working stages of washing: the first drum rotates slowly back and forth, sloshing the clothes against the water with detergent, followed by the second stage, the spinning cycle, during which the drum spins at high velocity to eliminate water. During the spinning cycle, the laundry is not symmetrically distributed and generates vibrations. Sometimes, the unbalance is so large that it makes the washing machine move, and some modern washing machines have vibration sensors, to stop it.

The self-balancing method would be the ideal method to eliminate the vibrations during the spinning cycle as the drum unbalance changes for each restart. Since Thearle [1] proposed self-balancing with the balls, some articles analyzed its dynamics [2–10]. Most recently, some articles presented the analytical approach for synchronous and nonsynchronous solutions and their stability due to the balancing effect [11–13]. However, the conclusion from the ideal model cannot be directly applied to the real system as the balls occupy a position that does not entirely compensate for the unbalance. More parameters influence the dynamic of a real washing machine and the final unbalance.

The author of the article proposed his explanation of auto-balancing phenomena because of the vibratory forces that exist in the system. In this way, it was possible to explain why the rotor with one degree of freedom also may eliminate its vibration from the unbalance. If there are more degrees of freedom, the vibratory force results from the vibrations of all component [14–16]. This method was used in the next part of the article. In the case of the ideal model taken in more publications, the balls fully eliminate unbalance and no vibrations.

In practice, some reasons decrease the efficiency of the method resulting in vibrations, noise, and mechanical degradation for the washing machine. Some articles considered only a singular reason for decreasing the efficiency of balancing or proposed unrealistic modifications. The first time the influence of rolling resistance on the final positioning of the free balls was analyzed in the author's article [17]. Recently, this problem was studied again [18–23]. In their model, the component of precession centrifugal force is used to overcome rolling resistance. This is true when the properties of the rotor suspension are the same in both directions. Practically, it is impossible to align the circular path of the ball with the rotor axis and the next error in the ball positioning. In the case of a washing machine, misalignment may cause considerable error in the final ball positions and the drum vibrations. Some additional causes can appear in real systems that decrease the efficiency of the method, e.g., an external vibration or variable speed of the washing machine [24–27]. This is the first article revealing a holistic and systematic approach to evaluating the efficiency of the self-balancing method for a washing machine.

Spinning the laundry

To establish the rate of removing water from clothes, a static experiment with a sloshy towel was performed. The dry towel of mass m_t= 0.44 kg and waterlogged of 1.7 kg, was subjected to a pressure of 4.5 kPa in a container with the same holes as the drum. The diagram in Figure 1 presents the removal of water Δ in time; with increasing dynamic pressure the water extraction is faster. Dynamic pressure during spinning is defined as:

$$p_d = a_n d_c = \left(\frac{\pi n}{30}\right)^2 R d_c \tag{1}$$

where: a_n – is normal acceleration, n [rpm], R – speed and radius of the drum, d_c [kg/m^2] – density of the wet towel.

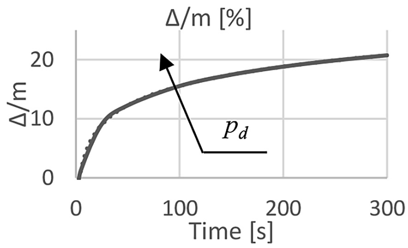


Fig. 1. Extracting water from the towel: Δ – extracting water, *m* – mass of the sloshy towel

The investigation of the behavior of the drum with the self-balancer is limited to one plane without the vibration of the housing. The conclusions from the investigation will be true for a washing machine with more degrees of freedom, but more parameters affect the balancing process. In the article, the parameters of the washing machine Samsung (WD15F5K5ASG/Ax) were taken for further analysis. It is a typical front-loaded washing machine used in most households.

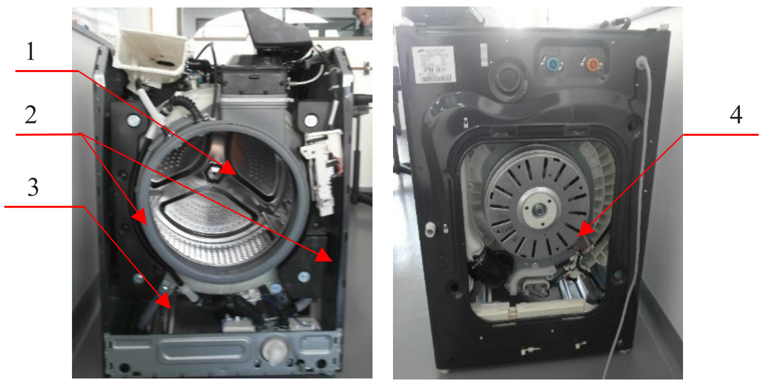


Fig. 2. Front and rear view of the basic elements of Samsung one: 1 – drum; 2 – tub with counterweights m_1, m_2; 3 – dampers; 4 – motor

The tub vibrations measured with two accelerometers B&K 4507 of the sensibility C_s= 0.24 mV/ms^{-2} are shown in Figure 3 at the centrifugal velocity of n=930 rpm and the load of 7.5 kg.

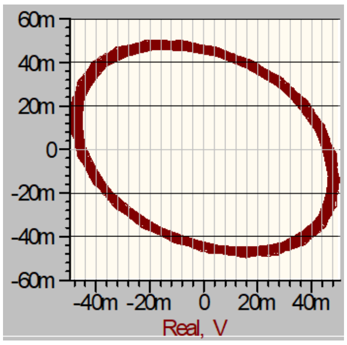


Fig. 3. Vertical and horizontal accelerations of the tub

The amplitudes of acceleration are

$$a_x \cong a_y \cong 47\ mV * 0.24\ \frac{ms^{-2}}{mV} = 11.3\ ms^2$$

The amplitude of vibration $\quad A_x \cong A_y \cong a_x/\omega^2 = 11.28/(\frac{\pi n}{30})^2 \cong 1.2\ mm.$

At the spinning velocity of 680 rpm, the acceleration amplitude is 16 m s^{-2}, and the displacement amplitude is 3.3 mm.

Principle of self-balancing

The inertial forces exist in every vibratory system. When the system is not linear the inertial forces can change its properties. These forces can move free balls continuously or move them to new positions in which they increase or decrease the vibrations. In addition, the vibrational forces can change the stable position into an unstable one and vice versa.

Figure 4 shows the suspension system of the tub-drum assembly, which consists of two springs and four dampers on the bottom. The drum spins inside the tub with the angular velocity ω. Two extra masses M_1 and M_2 are fixed to the tub to lower its natural frequencies. A ring with free elements is attached to the drum. The free elements can be either balls or rollers, in the article they are called balls. The balls have the same mass *m* and can move freely inside the ring of radius *R*. The position of the ball to the static unbalance Me is defined by the angle α_i.

The coordinates *x* and y define the position of the drum axis O. The coordinate system Ox_1y_1 turns with the drum. The rotation of the unbalanced drum causes vibrations, *x*(t) and *y*(t), which generate an inertial force on each ball. The mass center C is defined by OC=e.

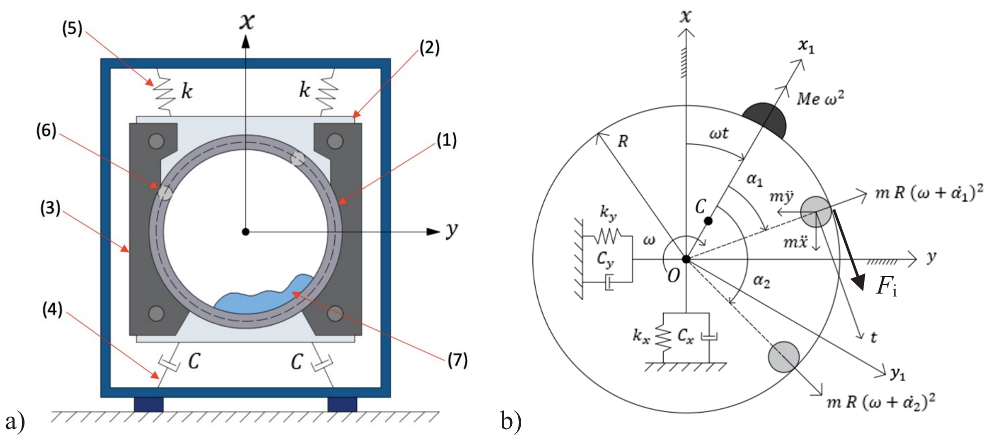


Fig. 4. Front view of washing machine (a), drum with free elements (b):
1 – drum, 2 – tub, 3 – counterweights, 4 – dampers, 5 – springs, 6 – balls, 7vlaundry

The principle parameters of the system: $M = 37$ kg the mass of the tub with two extra masses, the drum, and the motor; $R = 0.28$ m the radius of the circular path of the balls in the ring; the elastic properties in *x* and *y* directions are k_x≅20 N/mm, k_y≅6 N/mm, and the damping of the system in the *x* and *y* directions are c_x≅200 kg/s, c_y≅110 kg/s.

The equations of motion for the drum are as follows:

$$M\ddot{x} + c_x\dot{x} + k_x x = Me\,\omega^2 \cos(\omega t) + m\,R\,\sum_{i=1}^{N}[(\omega + \dot{\alpha}_i)^2 \cos(\omega t + \alpha_i) + \ddot{\alpha}_i \sin(\omega t + \alpha_i)], \quad (2)$$

$$M\ddot{y} + c_y\dot{y} + k_y y = Me\,\omega^2 \sin(\omega t) + m\,R\sum_{i=1}^{N}[(\omega + \dot{\alpha}_i)^2 \sin(\omega t + \alpha_i) - \ddot{\alpha}_i \cos(\omega t + \alpha_i)]\,, \quad (3)$$

The equation of motion for the *ith* ball has a form:

$$m_z\,R\,\ddot{\alpha}_i = m[\ddot{x}\,\sin(\omega t + \alpha_i) - \ddot{y}\,\cos(\omega t + \alpha_i)] - c_i\,R\,m\,\dot{\alpha}_i, \quad i = 1,2,\ldots,N, \quad (4)$$

where: $m_x = m + I_r/r^2$ is the equivalent mass of the rolling ball or roller, c_i is the viscous damping coefficient of the ball in its movement to the drum, and N is the number of balls. The behavior of the system is defined by nonlinear differential equations the solution of which can be obtained only by numerical integration.

The ball changes its position concerning the drum and when this happens, the total unbalance of the system changes. At the final position α_{fi}, the balls may compensate for the unbalance, the resulting force is zero, the tube with the drum does not vibrate, and no inertial forces pushing the balls. The diagrams in Fig. 5 present a numerical solution of the behavior of the washing machine and the balls during spinning at 1000 rpm when $mR=Me$.

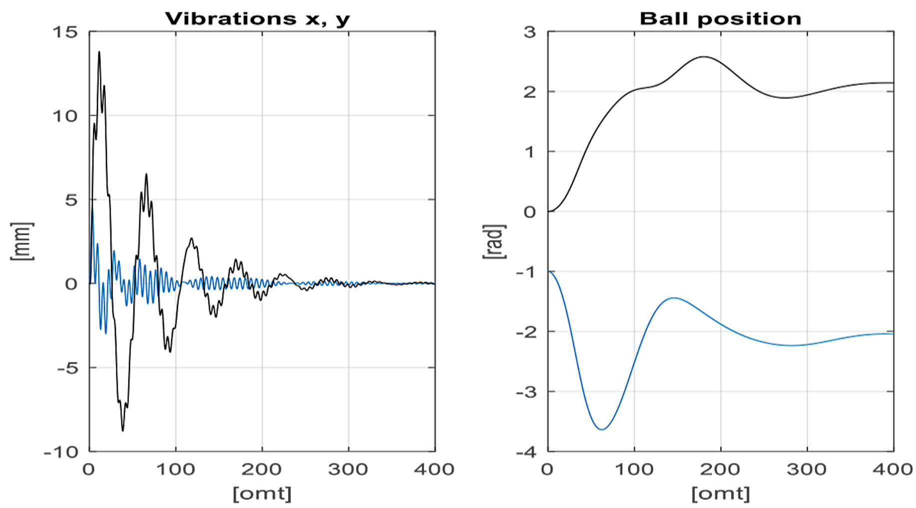


Fig. 5. Behavior of the drum and two balls at 1000 rpm, *Me*=*mR* [ι=om*t=*ωt*]

As a result of the initial unbalance, there are vibrations, the balls move to their equilibrium position and the vibrations vanish. It turns out that the two balls can compensate for the rotor unbalance very quickly, in just 1.5 seconds ($t < 250/\omega$) and there are no vibrations. The balls change their position under the action of the vibratory force that is tangent to the ball trajectory – Eq. (4), (Fig. 4b).

$$F_i^* = m\,[\ddot{x}\,\sin(\omega\,t + \alpha_i) - \ddot{y}\,\cos(\omega\,t + \alpha_i)]. \quad (5)$$

The balls move slowly, so the drum and its vibrations can be approximated as follows:

$$x(t) \cong a_{ox}\cos(\omega t - \varphi_x) + \sum_{i=1}^{N} a_{ix}\cos(\omega t + \alpha_i - \varphi_x), \quad (6)$$

$$y(t) \cong a_{oy}\sin(\omega t - \varphi_y) + \sum_{i=1}^{N} a_{iy}\sin(\omega t + \alpha_i - \varphi_y). \quad (7)$$

The behavior of the balls depends on the average magnitude of the force F_i^*, [14, 15].

$$F_i = \frac{1}{T}\int_0^T F_i^*\,dt \quad (8)$$

$$F_i = -0.5\,\omega^2 m[\,a_{0x}\sin(\alpha_i + \varphi_x) + a_{0y}\sin(\alpha_i + \varphi_y) + \sum_{j=1}^{N} a_{xj}\sin(\alpha_i - \alpha_j + \varphi_x) + \sum_{j=1}^{N} a_{yj}\sin(\alpha_i - \alpha_j + \varphi_y)], \quad (8)$$

where a_{oi} and a_i are amplitudes from the drum unbalance and the ball, respectively. The vibratory forces F_i are responsible for the behavior of the balls and their final position. Each of the components of the drum vibration generates its vibrational force:

$$F_i = F_{ix}(\alpha_1,..,\alpha_N) + F_{iy}(\alpha_1,..,\alpha_N). \quad (9)$$

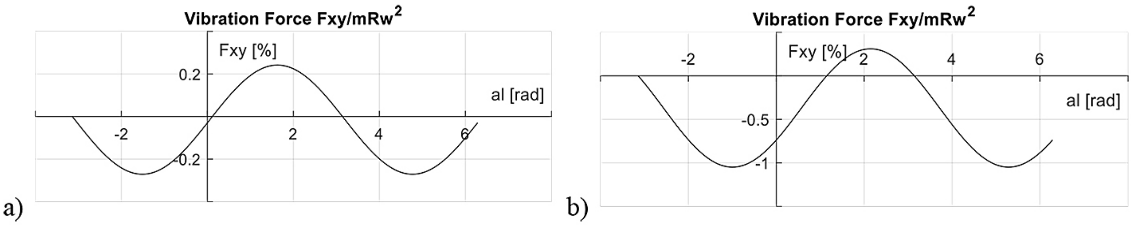
The vibratory force can also be shown as a sum of the forces from each element of the system, i.e. from the static unbalance and each ball

$$F_i = (F_{ix0} + \sum_{i=1}^{N} F_{ijx}) + (F_{iy0} + \sum_{i=1}^{N} F_{ijy}). \quad (10)$$

If only one ball is used with the static moment $mR = Me$

$$F_1 = -0.5\, \omega^2 m[\, a_{0x}\, sin(\alpha + \varphi_x) + \, a_x\, sin(\varphi_x) + \, a_{0y}\, sin(\alpha + \varphi_y) \, + \, a_y\, sin(\varphi_y)]. \tag{11}$$

The diagram below (Fig 6) shows the change in vibratory force F_i with the ball position if only one ball is used. The force F_1 takes the value of zero at two positions: near the unbalance and the opposing the unbalance $\alpha_f = \pi$. By increasing the damping of the drum, the diagram $F_1(\alpha)$ moves downwards and the negative force overrides the positive force. For the spin velocity between two resonances $\omega_y < \omega < \omega_x$, the two components of vibratory force have opposite signs, the resultant force is very small and can be positive or negative - Fig. 6 b. If force F_y is smaller than force F_x, then the balls cannot compensate for the unbalance.



a) b)

Fig. 6. Vibratory force: a) $(F_x+F_y)/(mR\omega^2)$ for $\omega > \omega_x$, ω_y and b) for $\omega_y < \omega < \omega_x$

For the first diagram, both extremes of the vibratory force are almost the same and for the second one, the negative extreme is 2.5 times higher than the positive. In this case, the probability that the ball will move in the direction opposite the drum rotation is much higher than the positive (Fig. 6b).

For higher damping, the diagram of vibratory force is more asymmetrical. For both diagrams, the stable equilibrium position of the ball is $\alpha_f = \pi$ but the margin of stability in Figure 6b is small, a slight impulse can change the ball position into unstable.

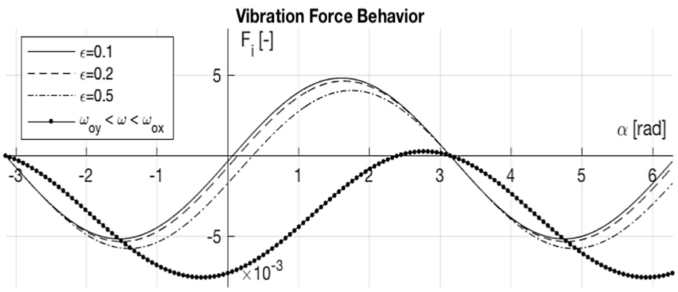


Fig. 7. Vibratory forces $\bar{F_i} = F_i/mR\omega^2$ for $\omega > \omega_{ox}$, ω_{ox} with different damping coefficients ε and for $\omega_{oy} < \omega < \omega_{ox}$

At the spin velocity $\omega_y < \omega < \omega_x$ the vibrational force F_y tries to move the ball to the position in which the ball compensates for the unbalance, whereas F_x would like to increase it (Fig. 7). For low angular velocities $\omega/(\omega_{ox}, \omega_{oy}) < 1$, the balls occupy the positions near the static unbalance, lack of self-balancing. There are two positions of equilibrium, only one of them is dynamically stable. The position of the ball $\alpha_f = \pi$ is stable if the derivate of the vibratory force $F(\alpha)$ with respect to the angle of the ball position is negative – Lagrange-Dirichlet theorem.

$$\left.\frac{\partial F}{\partial \alpha}\right|_{\alpha_f} = -0.5m\omega^2[a_{ox}\, cos(\alpha_f + \varphi_x) + a_{oy}\, cos(\alpha_f + \varphi_y)] < 0. \tag{12}$$

The ball position $\alpha_f = \pi$ is stable when $\omega > \omega_x$, ω_y –the ball can compensate for the unbalance. For more balls, the stability is given in [16]. The balls can compensate for the asymmetrical distribution of laundry at a spin velocity higher than the natural frequencies and the laundry unbalance should be smaller than the static moment of all balls $Me < mR*N$.

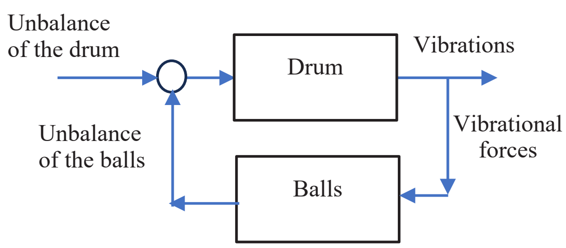


Fig. 8. Principle of self-balancing – an internal feedback loop

It was shown that the balls move under the action of the vibratory force and the behavior of the drum depends on the ball position. Thus, the kinetic-static model can be used for further analyses, i.e., the behavior of the drum is defined by Eqs. (2, 3) and the behavior of the balls is approximated by the following equations:

$$m_z\, R\, \ddot{\alpha}_i = F_i - F_r, \qquad i = 1,2,\dots,N, \tag{13}$$

where: F_i is defined by Eq. (10) and F_r is the resistance force.

There are two vibratory processes with a significant difference in frequency and T. Majewski proposed to separate them and investigate the influence of fast motion on slow motion (vibratory mechanics), and some applications give very good results. The behavior of mechanical systems depends on the ordinary and vibratory forces acting on them.

The diagrams in Figure 9 show the difference between two solutions: $q(t)$ the complete Eqs. (2-4) and the second one $q_k(t)$ for the kinetic-static model for the balls defined by Eq. (14) with the drum Eqs. (2, 3). The solutions are very close to each other, there are only slight differences between them; $D_x = x(t) - x_k(t), D_y = y(t) - y_k(t), D\alpha = \alpha(t) - \alpha_k(t)$. At the beginning of the simulation, $D_x(t)$ is almost 10 µm, for $D_y(t)$ is 25 µm and for the ball positions $D_\alpha(t)$ is 0.005 rad. In a matter of seconds, the difference between the solutions vanishes. Similar results would be for more balls. This confirms that the kinetic-static model can be used in the following sections.

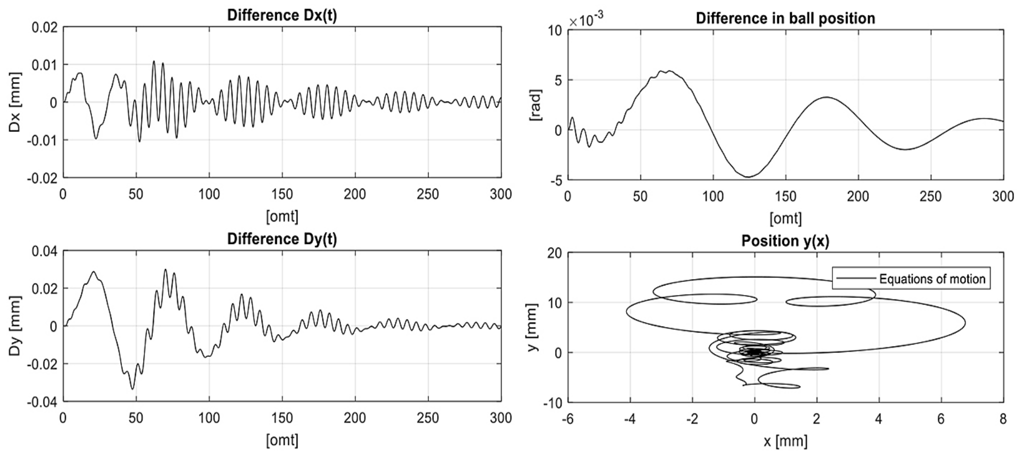


Fig. 9. Difference between the solution of complete equations and kinetic-static model

As it was proven above, as well as in articles [14-17], the vibrational forces are responsible for the self-organizing of the balls and in this application, they lead to self-balancing.

The vibrational forces are very small concerning the centrifugal forces acting on the balls (Fig. 4) and any extra disturbance may seriously affect the balancing process. The balls go to new positions that are different from an ideal balancing model and their dynamic stability also changes. The article showed some disturbances that exist in each real system, analyzed their influence on the residual unbalance, and defined the practicable efficiency of the method.

Transition from washing cycle to spinning cycle

When the washing cycle is finished the washer moves to the spinning cycle to remove the water. The spin velocity increases to 600, 800, or 1000 rpm, and the tub-drum overcomes two resonances. During the increasing velocity, there are large vibrations and angular acceleration that depend on the torque T_M of the motor. Now, the angular velocity varies, and the differential equations should be modified. The kinetic energy of the ball:

$$T_i = \frac{1}{2}m\{[\dot{x} - R(\dot{\varphi} + \dot{\alpha}_i)\sin(\varphi + \alpha_i)]^2 + [\dot{x} + R(\dot{\varphi} + \dot{\alpha}_i)\cos(\varphi + \alpha_i)]^2\} + \frac{1}{2}I\frac{R}{r}(\dot{\varphi} + \dot{\alpha}_i)^2, \tag{14}$$

where $\varphi(t)$ is the angle of the drum rotation.

The rotation and vibration of the drum and the balls are governed by Eqs. (15-18).

$$B_z\,\ddot{\varphi} + c_\varphi\,\dot{\varphi} \cong T_m + Me[\ddot{x}\sin\varphi - \ddot{y}\cos\varphi] + mR\sum_{i=1}^{N}[\ddot{x}\sin(\varphi + \alpha_i) - \ddot{y}\cos(\varphi + \alpha_i) - R\ddot{\alpha}_i], \tag{15}$$

$$M\ddot{x} + c_x\dot{x} + k_x x \cong Me\,[\dot{\varphi}^2\cos(\omega t) + \ddot{\varphi}\sin\omega t] + m\,R\sum[(\omega + \dot{\alpha}_i)^2\cos(\omega t + \alpha_i) + (\ddot{\varphi} + \ddot{\alpha}_i)\sin(\omega t + \alpha_i)], \tag{16}$$

$$M\ddot{y} + c_y\dot{y} + k_y y \cong Me\,[\dot{\varphi}^2\sin(\omega t) - \ddot{\varphi}\cos\omega t] + m\,R\sum[(\omega + \dot{\alpha}_i)^2\sin(\omega t + \alpha_i) - (\ddot{\varphi} + \ddot{\alpha}_i)\cos(\omega t + \alpha_i)], \tag{17}$$

where B_z is a mass moment inertia of the drum. The equation of motion for ith ball has a form:

$$m_z\, R^2\, \ddot{\alpha}_i \cong mR[\ddot{x}\, sin(\varphi + \alpha_i) - \ddot{y}\, cos(\varphi + \alpha_i) - R\ddot{\varphi}] - c_i mR^2 \dot{\alpha}_i, \qquad i = 1,2, \dots, N. \tag{18}$$

Analysis of the differential Eqs. (15-18) should explain if the balls may or not compensate for the unbalance and establish the maximum amplitude of vibration during the increasing spin velocity. The torque of the motor can be constant or variable during this period. In addition, the programmed time for obtaining maximum spin velocity can be different and consequently different maximum amplitudes of vibrations. When the drum spins with acceleration then the tangent inertial forces push the balls in the opposite direction of the drum rotation. Tables 1 and 2 present the time t_f in which the drum reaches its final speed of 1000 rpm (n_f), the maximum amplitudes of the drum, and the number of revolutions that the balls made, before they obtain their final position.

Table 1. T_o = 25 Nm, n_f = 1000 rpm, $Me = mR$, Me = 0.025 kg m

Torque	T_m	t_f	x_{max}	y_{max}	No. of revolutions
–	–	s	mm	mm	–
Constant	T_0	2.23	4.6	4.1	8
Linear	$T_m = T_0\left(1 - {}^{\omega}/_{\omega_f}\right)$	8.2	4.9	4.9	7.6
Harmonic	$T_m = T_0\left(1 - sin\left({}^{\pi\,\omega}/_{2\omega_f}\right)\right)$	8.6	5.1	5.2	6

Table 2. T_o = 25 Nm, n_f = 1000 rpm, $Me = mR$, Me = 0.05 kgm

Torque	T_m	t_f	x_{max}	y_{max}	No. of revolutions
–	–	s	mm	mm	–
Constant	T_0	1.7	8.5	8.7	7.0
Linear	$T_m = T_0\left(1 - {}^{\omega}/_{\omega_f}\right)$	7.2	9.2	8.6	8.0
Harmonic	$T_m = T_0\left(1 - sin\left({}^{\pi\,\omega}/_{2\omega_f}\right)\right)$	8.3	9.9	9.7	5.5

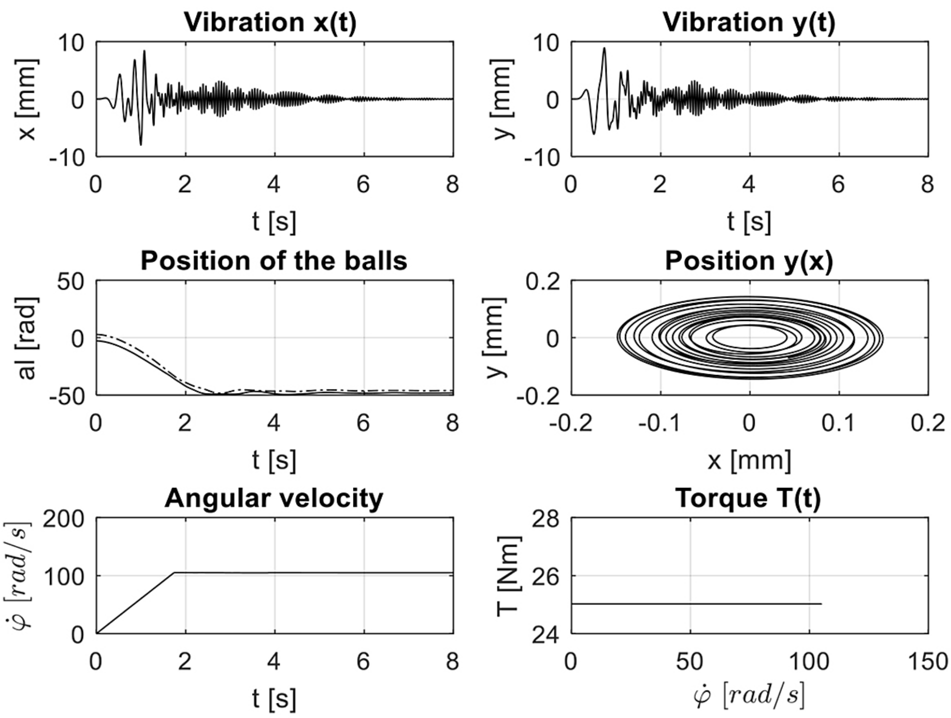


Fig. 10. Behavior of the system with constant torque

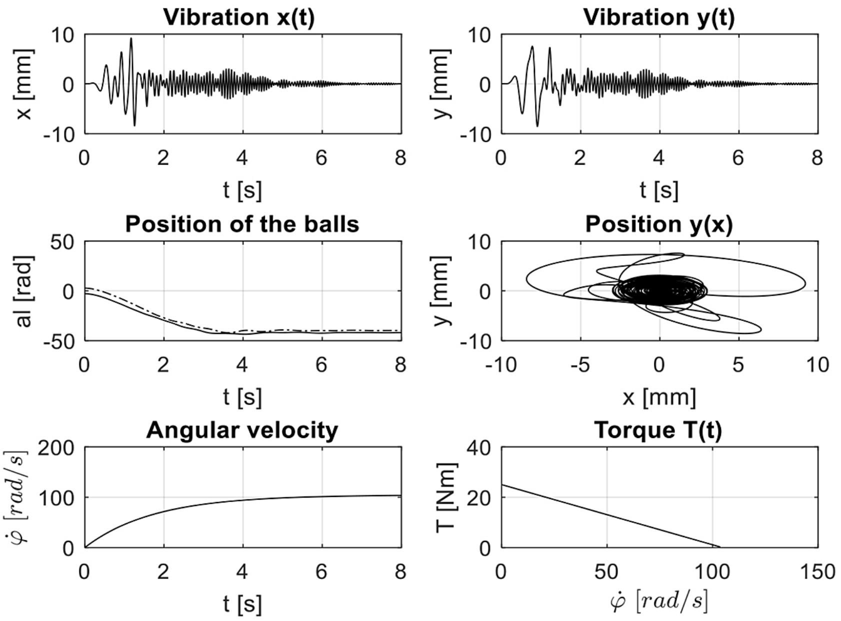


Fig. 11. Behavior of the system with variable torque

The behavior of the system with linear changing torque is shown in Figure 11. The balls turn with the drum about 8 times and when the drum spin velocity stabilizes, the balls go to the position to compensate for the unbalance. During the increasing angular velocity, the drum goes through two resonances, and the amplitudes of vibration increase but in a very short time. There is a very small difference between the maximum amplitude of vibrations with and without the balls.

The process of increasing the spin velocity of the drum does not change the properties of the balls in compensating for the unbalance; it increases the time to obtain the self-balancing effect.

Factors that decrease the efficiency of the method

The principal factors which may influence the efficiency of self-balancing:

- Increased vibrations while changing from washing to spinning cycle (resonances).
- Variable unbalance of laundry during the spinning.
  – extracting water is slow so the balls follow the unbalance change.
- Eccentricity of the ring in which the balls are located.
- Rolling resistance of the balls.
- Friction of the tub suspension.
- Impact between the balls.
- Gravity force.
- External vibrations.

Variable spin velocity

It was shown (Figs.10 and 11) that the balls move inside the drum in the opposite direction as the drum accelerates and when it achieves the working velocity, the balls move to their position of equilibrium to eliminate the vibrations. The time in which the system stabilizes is extended by the time of the variable spin velocity. At the resonances, the amplitudes of vibrations are almost the same for the washing machine with and without the balls.

Rolling resistance of free elements

The centrifugal forces are several dozen times higher than the vibrational forces, so friction forces would give enormous errors in the positioning of the free elements, therefore rolling elements as balls or rollers are used. Their rolling resistance works in position errors Δ_i, residual unbalance, and residual vibrations. The equation of the ball with a rolling resistance has the following form:

$$m_z\, R\, \ddot{\alpha}_i \cong m[\ddot{x}\, sin(\omega t+\alpha_i)-\ddot{y}\, cos(\omega t+\alpha_i)]-c_i\, R\, m\, \dot{\alpha}_i-F_{ir}, \tag{19}$$

where the rolling resistance is equal to:

$$F_{ir}=m\, R\, \omega^2 \frac{f}{r}\, sign(\dot{\alpha}_i). \tag{20}$$

In the equation above, r is the radius of the free element and f is the coefficient of rolling resistance. The ball cannot be moved if the vibrational force is smaller than the rolling resistance:

$$abs(F_i(\alpha_{if} + \Delta_{ir}) - abs(F_{ir}) \leq 0, \qquad i = 1, 2, \ldots, N. \tag{21}$$

From the equations above, the maximum deviations $\Delta\alpha_{1rmax}, \ldots, \Delta\alpha_{Nrmax}$ can be calculated. In the next step, the components of the residual unbalance and the total residual unbalance ΔMe_r as shown.

$$\Delta Me_{rx} = Me + mR\sum_{i=1}^{N} cos\,(\alpha_{if} + \Delta\alpha_{irmax}), \qquad \Delta Me_{ry} = mR\sum_{i=1}^{N} sin\,(\alpha_{if} + \Delta\alpha_{irmax})\,. \tag{22}$$

When the residual unbalance is defined, then the amplitudes of residual vibrations a_{xr}, a_{yr} are obtained from Eqs. (2, 3). The shifted position of the free element can be any from the range $-\Delta\alpha_{irmax} < \Delta\alpha_{ir} < \Delta\alpha_{irmax}$. In the case of one ball with a static moment $mR = Me$, the ball can stop at $\alpha_f = \pi + \Delta\alpha_r$ and Eq. (22) takes a form:

$$abs\{-0.5\,\omega^2 m[\,a_{0x}\,sin(\pi + \Delta\alpha_r + \varphi_x) + \,a_{0y}\,sin(\pi + \Delta\alpha_r + \varphi_y) + \,a_x\,sin(\varphi_x) \,+ a_y\,sin(\varphi_y)\,]\} - m\,R\,\omega^2\frac{f}{r} \leq 0 \tag{23}$$

For a small deviation $\Delta\alpha_r$ the above equation gives a result:

$$abs(\Delta\alpha_r) \leq \frac{2 \cdot R \cdot f}{r(a_x cos\varphi_x + a_y cos\varphi_y)}. \tag{24}$$

Figure 12 shows the maximum error of the position of one ball for different spin velocities and two different coefficients of the rolling resistance.

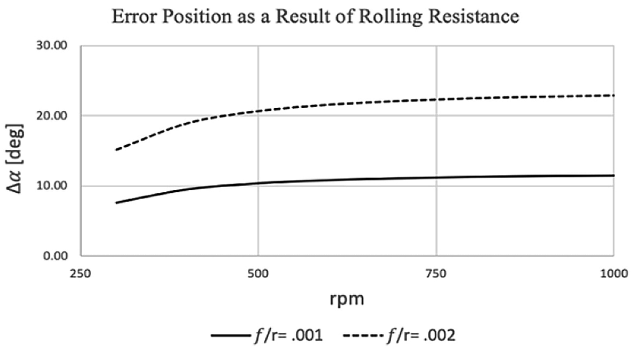


Fig. 12. Maximum deviation $\Delta\alpha_{max}$ vs spin velocity for f/r=0.001 and f/r=0.002

The residual vibration:

$$x_{re} = a_{0x}\,cos(\omega t - \varphi_x) + a_x\,cos(\omega t + \pi + \Delta\alpha_r - \varphi_x) \cong -a_x\Delta\alpha_r\,sin(\omega t - \varphi_x) = -a_{xre} sin\,(\omega t - \varphi_x). \tag{25}$$

The amplitude of residual vibration:

$$a_{xre} \cong a_x\Delta\alpha_r = \frac{2Rf}{r(a_{0x}cos\varphi_x + a_{0y}cos\varphi_{xy})}a_x. \tag{26}$$

And similar to the amplitude a_{yre}. If $\omega >> \omega_x, \omega_y$ then $a_{xr} \cong a_{yr} \approx Rf/re$. The unbalance is unknown and a minimum of two balls must be used. At their final position $\alpha_{1f} + \Delta\alpha_{1r}, \alpha_{2f} + \Delta\alpha_{2r}$ the vibratory forces cannot move the balls until they are smaller than the rolling resistance. For the first ball

$$abs\{F_1(\alpha_{1f} + \Delta\alpha_{1r}, \alpha_{2f} + \Delta\alpha_{2r})\} - m\,R\,\omega^2\frac{f}{r} \leq 0. \tag{27}$$

The same goes for the second ball. If each ball has a static moment mR=Me (than α_{1f}=-α_{2f}= α_f=120 deg) and ω>>ω_x, ω_y, then a_x=a_y≈e, φ_x=φ_y≈π then the balls cannot move if:

$$abs\{-[cos\alpha_f + cos\,(2\alpha_f)]\Delta\alpha_{1r} + cos\,(2\alpha_f)\Delta\alpha_{2r}\}e - m\,R\,\omega^2\frac{f}{r} \leq 0. \tag{28}$$

$$abs\{cos(2\alpha_f)\,\Delta\alpha_{1r} - [cos\alpha_f + cos\,(2\alpha_f)]\Delta\alpha_{2r}\}e - m\,R\,\omega^2\frac{f}{r} \leq 0. \tag{29}$$

As α_f=120 deg, then:

$$|\Delta\alpha_{1r} - 0.5\Delta\alpha_{2r}| - \frac{R\,f}{e\,r} < 0, \tag{30}$$

$$|-0.5\Delta\alpha_{1r} + \Delta\alpha_{2r}| - \frac{R\,f}{e\,r} < 0. \tag{31}$$

There are many possibilities of the ball deviations $\Delta\alpha$. For symmetrical deviations Eqs. (30, 31) give the solutions:

$$\Delta\alpha_{1r} = \Delta\alpha_{2r} = \pm 2\frac{R}{e}\cdot\frac{f}{r} \tag{32}$$

For the following parameters Me=0.05 kgm, R=0.28 m, $mR=Me$, $f/r=10^{-4}$ the maximum deviation is $\Delta\alpha \cong$ 2.4 deg and with the same parameter and the rolling coefficient $f/r = 10^{-3}$ the maximum deviation increases about ten times.

The diagrams in Figure 13 present the behavior of the drum balanced with two balls that move with rolling resistance $f/r=10^{-3}$. Figure 13a shows the vibration $x(\omega t)$, $y(\omega t)$, and Figure 13b the position of the balls in time.

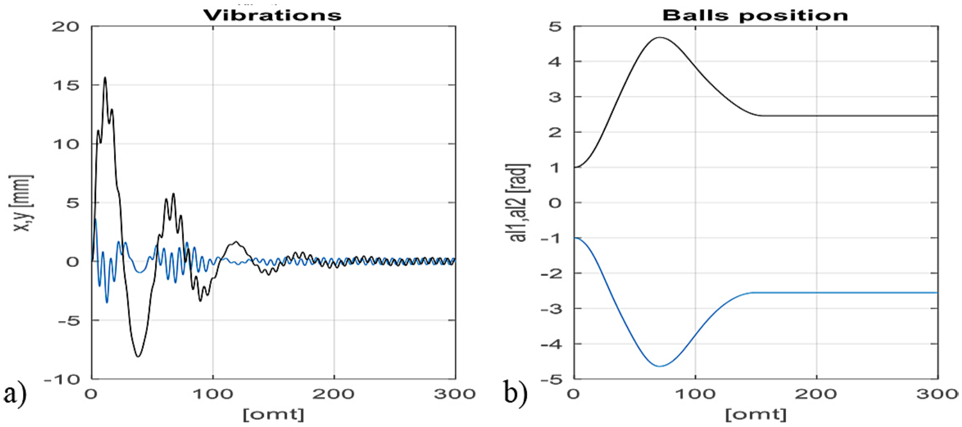


Fig. 13. Vibrations and the position of the balls if Me=0.05 kg m, α_{1f}=- α_{2f}=120 deg, n=1000 rpm, $f/r=10^{-3}$

It can be observed that the first ball comes to a stop with a deviation of 25 deg, and the second ball stops with a difference of 21 deg to the theoretical position $\alpha_f = \pm 120$ deg, just at $t_{imp} \approx 35/\omega$ the balls impact each other. It is impossible to eliminate the rolling resistance; consequently, a residual unbalance causes vibrations.

Eccentricity of the ring

The ring with the balls is fixed to the drum in a free space between the drum and the tub. As the free space between these two elements is small and the unbalance can be significant, it would be better to use the rollers instead of the balls. The drum is an element that is not exactly manufactured, there is an eccentricity between the axis of rotation of the motor and the drum, and between the drum and the ring. It leads to the deviation in the ball position which gives the next resultant unbalance. The eccentricity of the ring center concerning the motor axis is defined by the distance $\rho=OO_1$ and the angle β (Fig 14). It results in a change in the ball velocity and consequently transforms Eqs. (2-4).

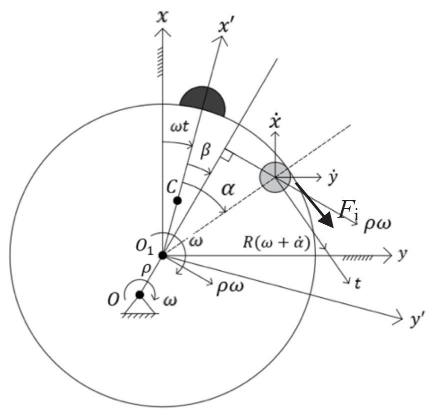


Fig. 14. Position of the ring and the ball concerning the drum

The kinetic energy of the ball will have a new velocity component:

$$T_i = 0.5m\{[\dot{x} - R(\omega + \dot{\alpha}_i)sin(\omega t + \alpha_i) - \rho\omega\, sin(\omega t + \beta)]^2 + [\dot{y} + R(\omega + \dot{\alpha}_i)cos(\omega t + \alpha_i) + \rho\omega\, cos(\omega t + \beta)]^2\} + 0.5I(\omega + \frac{R}{r}\dot{\alpha}_i)^2. \tag{33}$$

The Lagrange´s equations give some extra terms $F_{x\rho}$, $F_{y\rho}$, $F_{i\rho}$ in the differential equations:

$$M\ddot{x} + C_x\dot{x} + k_x x \cong Me\,\omega^2 \cos(\omega t) + m\,R\sum[(\omega + \dot{\alpha}_i)^2 \cos(\omega t + \alpha_i) + F_{x\rho})], \quad (34)$$

$$M\ddot{y} + C_x\dot{y} + k_x y \cong Me\,\omega^2 \sin(\omega t) + m\,R\sum[(\omega + \dot{\alpha}_i)^2 \sin(\omega t + \alpha_i) + F_{y\rho}], \quad (35)$$

$$m_z\,R\,\ddot{\alpha}_i \cong m[\ddot{x}\sin(\omega t + \alpha_i) - \ddot{y}\cos(\omega t + \alpha_i)] - c_i\,R\,m\,\dot{\alpha}_i + F_{i\rho}, \qquad i = 1,2,\ldots,N, \quad (36)$$

where: $F_{x\rho} = \frac{\rho}{R}\omega^2\cos(\omega t + \beta)$, $F_{y\rho} = \frac{\rho}{R}\omega^2\sin(\omega t + \beta)$, $F_{i\rho} = -m\rho\,\omega^2 \sin(\alpha_i - \beta)$ are new terms due to eccentricity. The vibrations of the drum can be approximated as:

$$x(t) \cong a_{0x}\cos(\omega t - \varphi_x) + \sum_{i=1}^{N}[a_{ix}\cos(\omega t + \alpha_i - \varphi_x) + a_{\rho x}\cos(\omega t + \beta - \varphi_x)], \quad (37)$$

$$y(t) \cong a_{0y}\sin(\omega t - \varphi_y) + \sum_{i=1}^{N}[a_{ix}\sin(\omega t + \alpha_i - \varphi_y) + a_{\rho y}\sin(\omega t + \beta - \varphi_y)]. \quad (38)$$

The vibratory force:

$$F_i = \frac{1}{T}\int_0^T m[\ddot{x}\sin(\omega t + \alpha_i) - \ddot{y}\cos(\omega t + \alpha_i) - \rho\omega^2 \sin(\alpha_i - \beta)]dt. \quad (39)$$

It consists of three vibrational forces:

$$F_i = F_{ix} + F_{iy} + F_{i\rho}. \quad (40)$$

The vibrational force F_{ix} from the vibration $x(t)$ has a form:

$$F_{ix} = -0.5m\omega^2[a_{0x}\sin(\alpha_i + \varphi_x) + \sum_{j=1}^{N}(a_{xj}\sin(\alpha_i - \alpha_j + \varphi_x) + a_{x\rho}\sin(\alpha_i - \beta + \varphi_x))]. \quad (41)$$

and similar for F_{iy}. The component $F_{i\rho}$ from the eccentricity:

$$F_{i\rho} = -m\rho\omega^2 \sin(\alpha_i - \beta). \quad (42)$$

For one ball with the static moment $mR=Me$ (then $a_0=a$) the vibrational force F_1 has a form:

$$F_1 = -0.5m\omega^2[a_x\sin(\alpha + \varphi_x) + a_x\sin(\varphi_x) + a_{x\rho}\sin(\alpha - \beta + \varphi_x) + a_y\sin(\alpha + \varphi_y) + a_y\sin(\varphi_y) + a_{y\rho}\sin(\alpha - \beta + \varphi_y)] - m\rho\omega^2 \sin(\alpha - \beta), \quad (43)$$

where $a_{x\rho}/a_x=\rho/R<<1$ is very small and some terms can be neglected.

Without the eccentricity, the ball should take the position $a_f=\pi$ to compensate for the drum unbalance. With the eccentricity, the ball is shifted by $\Delta\alpha_\rho$ with respect α_f and at this position, the vibrational force is zero.

$$F_1 \cong -0.5m\omega^2[-a_x\sin(\Delta\alpha_\rho + \varphi_x) + a_x\sin(\varphi_x) - a_y\sin(\Delta\alpha_\rho + \varphi_y) + a_y\sin(\varphi_y)] + m\rho\omega^2 \sin(\Delta\alpha_\rho - \beta) = 0. \quad (44)$$

For $\omega \gg \omega_x, \omega_y$ it can be taken $a_x = a_y \cong e, \varphi_x = \varphi_y \cong \pi$. The small deviation of the ball concerning the position $\alpha_f=\pi$ is as follows:

$$\Delta\alpha_\rho \cong -\frac{\rho}{e+\rho\cos\beta}\sin\beta. \quad (45)$$

Figure 15 shows in what way the deviation $\Delta\alpha_\rho$ changes with the position of the eccentricity defined by angle β.

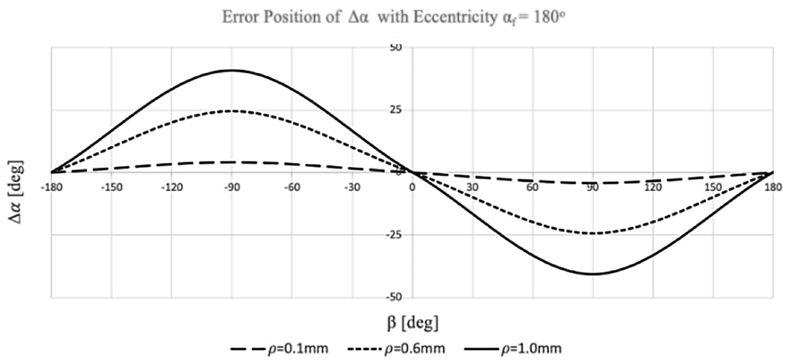


Fig. 15. Deviation of the ball position caused by eccentricity when $\alpha_f=\pi$

The order of magnitude of the deviation $\Delta\alpha_\rho$ can be estimated from the graph in Figure 15. In the case of one roller, its maximum deviation occurs at the angle $\beta = \pi/2$. In practice, a minimum of two rollers should be used to compensate for any drum unbalance. Their deviations $\Delta\alpha_{1\rho}$, $\Delta\alpha_{2\rho}$ can be found from Eq. (46).

$$F_1(\alpha_{1f}+\Delta\alpha_{1\rho},\alpha_{2f}+\Delta\alpha_{2\rho})=0, \qquad F_2(\alpha_{1f}+\Delta\alpha_{1\rho},\alpha_{2f}+\Delta\alpha_{2\rho})=0\,, \tag{46}$$

where the vibratory forces are defined by Eq. (42). Then, the residual unbalance is determined as follows:

$$\Delta Me_{x\rho}\cong -mR\sum_{i=1}^{N} sin(\alpha_{if})\,\Delta\alpha_{i\rho} \qquad \Delta Me_{y\rho}\cong mR\sum_{i=1}^{N} cos\,(\alpha_{if})\Delta\alpha_{i\rho} \tag{47}$$

Figure 16 shows the computer simulation of the drum vibrations and the behavior of the balls when the eccentricity of the ring is equal to $\rho=\ 1\,mm$.

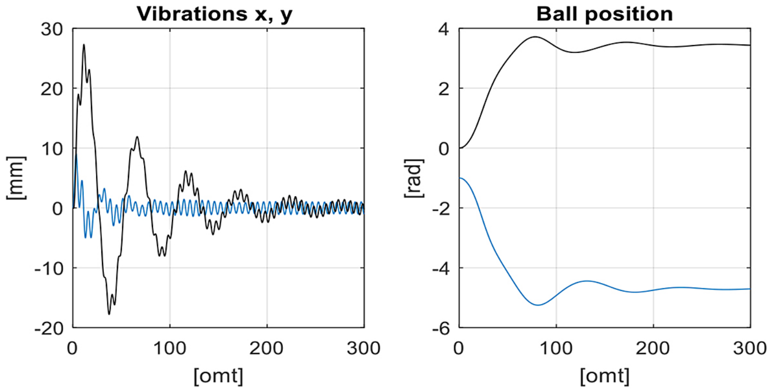


Fig. 16. Behavior of the system with the eccentricity ρ=1 mm at 1000 rpm, *Me*=0.05 kgm, $\beta=\pi/2$.

As the eccentricity is great ρ=1 mm, and β=π/2 the deviations are large: $\Delta\alpha_{1\rho}$=45 deg, $\Delta\alpha_{2\rho}$=34 deg, different for each ball. The drum cannot be balanced and there are residual vibrations with the amplitude of $a_{\rho e}\approx 1.2$ mm.

Variable unbalance

With increasing the drum velocity the distribution of wet laundry changes and the balls roll in the opposite direction of the drum. When the velocity stabilizes, the balls move to the position to compensate for the laundry unbalance (about a second). The next changes in laundry distribution are quickly compensated by the balls. A special mechanism to block the ball in the drum is not necessary.

Damping of the tub

Thus far, the dissipation of energy was approximated by the viscous damping and any unbalance generates vibrations and vibratory forces. The suspension system of the tub consists of two springs and four dampers: the front two with a friction force of up to 80 N, and the rear two 60 N[27]– Fig.17. When the balls are close to their final positions the small unbalance cannot overcome the friction of dampers, no vibrations, no vibratory forces, which leads to the error in the ball position and next residual unbalance.

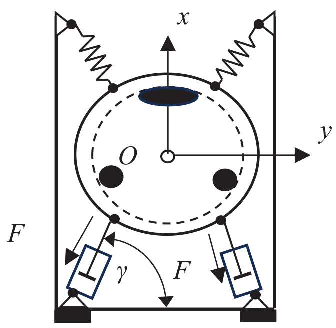


Fig. 17. Sketch of the tub suspension

For instance, when the dynamic force of the unbalance in the vertical direction is smaller than the friction force, the tub cannot move in that direction.

$$\omega^2[Me+mR\sum_{i=1}^{N} cos\,(\alpha_{if}+\Delta\alpha_{if})]\le F_x=2F\cdot\cos(\gamma). \tag{48}$$

where: 2*F*=140 N and γ=61 deg.

For the drum speed *n*=1000 rpm, with two balls *mR*=*Me*=0.05 kg m that are close to their final positions $\alpha_f=\pm 2\pi/3$, the maximum ball error position is

$$\Delta\alpha_f < \frac{F\cdot cos(\gamma)}{Me\cdot sin(2\pi/3)\omega^2} = 0.071\ rad \quad (49)$$

For the lower speed of the drum, the error would be higher. An example of the numerical simulation of the tub vibrations and the ball motion with the friction of the suspension is shown in Figure 18.

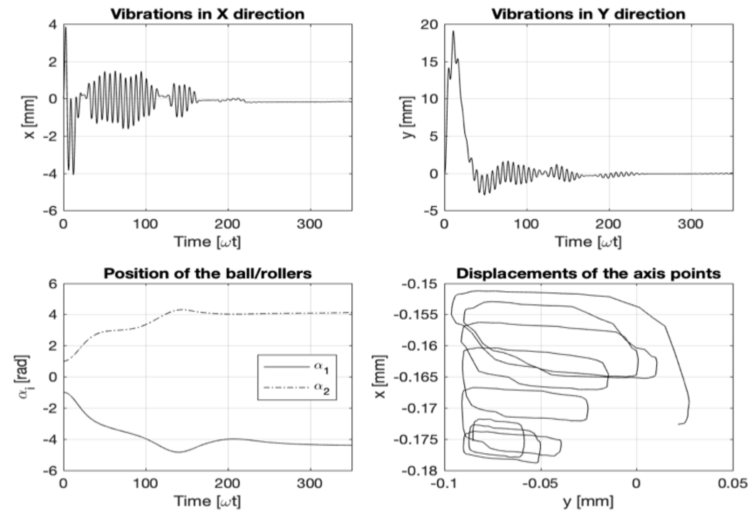


Fig. 18. Behavior of the system with friction forces at spin velocity n=1000 rpm

The theoretical positions of the balls are $\alpha_{1f} = -\alpha_{2f} = -4\pi/3$ rad, for this simulation, the errors of the ball position are $\Delta\alpha_{1f}$=0.22 rad and $\Delta\alpha_{2f}$=0.12 rad. The probability of this error is low as the tub is connected to the metallic casing which can vibrate, the washing machine has anti-vibration pads, and the vibrations of the floor can also get in.

External vibrations

The cabinet has elastic or rigid support and external vibrations can also influence the behavior of the balls and the result of balancing. In this case, the balls want to compensate for the drum unbalance and the external excitation (Fig. 8) – the idea of synchronous eliminator of vibration was given in [28]. It was shown that the vibrations with the same frequency or very close to the drum spin can generate the vibratory forces that change the final position of the freely moving balls to compensate the drum unbalance and the vibration of the floor.

Effect of gravity forces

This type of washing machine has a horizontal axis of rotation. During the washing cycle, the clothes move up and drop down to the water with detergent, while the balls in the drum keep their lower position. During the spinning cycle, large centrifugal forces push the cloths against the drum, keeping them in their position and removing water, the same with the balls. The gravity force of the ball gives a component tangential to its trajectory $F_{ig} = mgsin(\omega t + \alpha_i)$. Hence, the Eq. (52) of the ball shows an additional force.

$$m_z\ R\ \ddot{\alpha}_i = m[\ddot{x}\ sin(\omega t + \alpha_i) - \ddot{y}\ cos(\omega t + \alpha_i) + gsin(\omega t + \alpha_i)] - F_{ir}, \quad i = 1,2,\dots,N. \quad (50)$$

The vibrations of the drum can be approximated as earlier by Eqs. (7, 8) and the vibratory force is defined as:

$$F_i^* = \frac{m}{T}\int_0^T [\ddot{x}\ sin(\omega\ t + \alpha_i) - \ddot{y}\ cos(\omega\ t + \alpha_i) - gsin(\omega t + \alpha_i)]\ dt. \quad (51)$$

It gives the same result as Eq. (10) because the average gravity force in Eq. (51) is equal to zero. The gravity force does not influence the final position of the ball. It generates small oscillations of the ball to its position $\alpha_i(t)$. At the final position of the ball α_{if}, the gravity force with its frequency ω does not change the ball position, because its frequency is much higher than the natural frequency of the ball.

$$mR\Delta\ddot{\alpha}_i = F_i(\alpha_1, \dots, \alpha_N) \cong F_i(\alpha_{1f}, \dots, \alpha_{Nf}) + \sum_{j=1}^{N} \frac{\partial F_i}{\partial \alpha_j}\Delta\alpha_j. \quad (52)$$

where F_i is the vibratory force of the ball i.
For one ballα_f=π then

$$mR\Delta\ddot{\alpha} - 0.5m\omega^2\left[a_{0x}cos\varphi_x + a_{0y}cos\varphi_y\right]\Delta\alpha = 0. \quad (53)$$

The natural frequency:

$$\omega_\alpha = \omega\sqrt{\frac{1}{2R}\left[-a_{0x}cos\varphi_x - a_{0y}cos\varphi_y\right]}. \quad (54)$$

And for $\omega >> \omega_x$, ω_y:

$$\omega_\alpha \approx \omega\sqrt{\frac{e}{R}} \ll \omega. \quad (55)$$

As $\omega/\omega_\alpha >> 1$ the amplitude of oscillation of the ball is very small, and it is not observed in the diagrams from the numerical simulation with the ball gravity.

When the spinning starts then the centrifugal force, and the friction forces, drive the balls to move with the drum. Sometimes the balls move in opposite directions and impact each other. They have the same mass, so after the impact, they move with the same velocities in opposite directions. There can be one or two impacts, and it does not affect the vibrations of the drum and the final position of the balls. During the washing cycle, the balls stay at the lowest position in the drum. The system can be equipped with a mechanism that blocks the balls for the washing cycle and unblocks them for the laundry spinning, but it makes the system more complex.

Evaluation of the final effect of auto-balancing

One by one the article presents some reasons affecting the process of auto-balancing, the importance of each of them is different for the resulting unbalance, and all of them exist simultaneously. Several of them have a significant influence on the deviations of the balls $\Delta\alpha_1, \ldots, \Delta\alpha_N$, some are small, and others are very small and can be ignored. The resultant effect is not a simple sum of them all. To estimate their influence on the efficiency of the method, the probability of each of them should be established.

The eccentricity ρ between ring with the balls and the motor always give the deviation defined by Eqs. 46, 47). On the contrary, the deviations given by the rolling resistance can be any between zero and maximum value given by Eqs. (28, 29).

To initiate a new project for a washing machine with the ball or roller balancer, a designer must define acceptable residual unbalance and distribute it between different reasons of unbalance – the probability of each of them. In this way, the maximum eccentricity ρ_{max}, the coefficient of rolling resistance f_{max}, dry friction force, etc. can be defined. The distribution depends on the designer's experience and the manufacturer's constraints. The probability occurrence of each of them is given by the coefficient p_j

$$\sum_{j=1}^{P} p_j = 1. \quad (56)$$

For instance, the residual unbalance ΔMe_F should be lower than 10% of the initial unbalance *Me* and its distribution can be taken as follows; 50% from the eccentricity, 35% from rolling resistance, friction force 10%, and 5% from undefined reasons, respectively.

$$\Delta Me_F \cong \Delta Me_\rho + \Delta Me_r + \Delta Me_f + \Delta Me_o \leq 0.1Me. \quad (57)$$

The residual unbalance from the eccentricity should be lower than:

$$\Delta Me_\rho \leq 0.5 \cdot 0.1Me = 0.05\, Me. \quad (58)$$

If one ball is used and its final position is $\pi+\Delta\alpha_\rho$ then the unbalance is $\Delta Me_\rho \cong mR\Delta\alpha_\rho$. For $\omega >> \omega_x, \omega_y$ and $\rho/R << 1$ the $\Delta\alpha_\rho$ is given by Eq. (45). Then, the Eq. (58) takes a form:

$$mR\frac{\rho sin\beta}{e+\rho cos\beta} \leq 0.05Me. \quad (59)$$

where: ρ is the eccentricity, r is the radius of the ball, and R is the radius of the path.

The position of the eccentricity β and its magnitude ρ are unknown so the adverse situation arises when $\beta = \pi/2$ and Eq. (59) has a form:

$$mR\frac{\rho}{e} \leq 0.05Me \quad \text{and the maximum eccentricity} \quad \rho_{max} \cong 0.05 \cdot \frac{Me}{M} \quad (60)$$

If Me=0.025 kgm and M= 37 kg the permissible eccentricity of the torus is 34 µm, for $Me = 0.05$ kgm the eccentricity should be smaller of 67 µm. It should be revised if it is technically feasible. If not, then the distribution of the resultant unbalance should be changed or allow a higher residual unbalance. The maximum rolling resistance can be obtained similarly. For one ball that can compensate for the unbalance $Me = mR$ and drum spin velocity $\omega >> \omega_x, \omega_y$, eq. (26) gives.

$$\Delta\alpha_{rmax} = \frac{2fR}{r*abs(a_x cos\varphi_x + a_y cos\varphi_y)} \cong \frac{f}{r}\frac{R}{e}. \quad (61)$$

The ball's deviation can be from the range $-\Delta\alpha_{max} < \Delta\alpha < \Delta\alpha_{max}$ with a small probability that the ball is at one of the maximums $\Delta\alpha_{max}$. Thus, the acceptable unbalance from the rolling resistance can be taken twice higher.

$$mR\Delta\alpha_r = mR\frac{fR}{er} = 2 * \Delta Me_r = 0.7\Delta Me_F = 0.7 * 0.1Me = 0.07Me. \quad (62)$$

The coefficient of the rolling resistance should be:

$$\frac{f}{r} \leq 0.07\frac{e}{R} = 0.07\frac{Me}{MR}. \quad (63)$$

For the unbalance $Me = 0.025$ kgm the rolling resistance should be $f/r \cong 5 \cdot 10^{-5}$. Again, there is the question of whether it is possible in practice to reduce the rolling resistance to this level or not. The suspension friction should give an unbalance lower than 10%, but its probability is small due to vibrations of the housing and its impact can be reduced by half. If there are two balls, then their errors of position are defined by Eq. (51).

$$\Delta\alpha_f < \frac{Fcos(\gamma)}{Me \cdot sin(2\pi/3) \cdot \omega^2}. \quad (64)$$

And its unbalance

$$2mR\Delta\alpha_f = 2mR\frac{Fcos(\gamma)}{Me \cdot sin\,(2\pi/3 \cdot)\omega^2} \leq \Delta Me_f = 0.05\Delta Me_F = 0.005Me, \quad (65)$$

The maximum friction force of the suspension can be defined from the equation above.

$$F < 0.005Me \cdot \omega^2\frac{\sin\left(\frac{2\pi}{3}\right)}{\cos(\gamma)}. \quad (66)$$

For the drum unbalance and the spin velocity 1000 rpm, the friction force of suspension F should be lower than 57 N, the friction of this washing machine is close to that magnitude.

Some results of the numerical simulation with several reasons mentioned above (ball gravity, eccentricity, and rolling resistance are presented in Figure 19. The motion of the balls stops rapidly and vibrations become small. It is seen that the balls cannot completely compensate for the drum unbalance. Tables 3 and 4 present the final amplitude of the drum vibrations a, the position error of

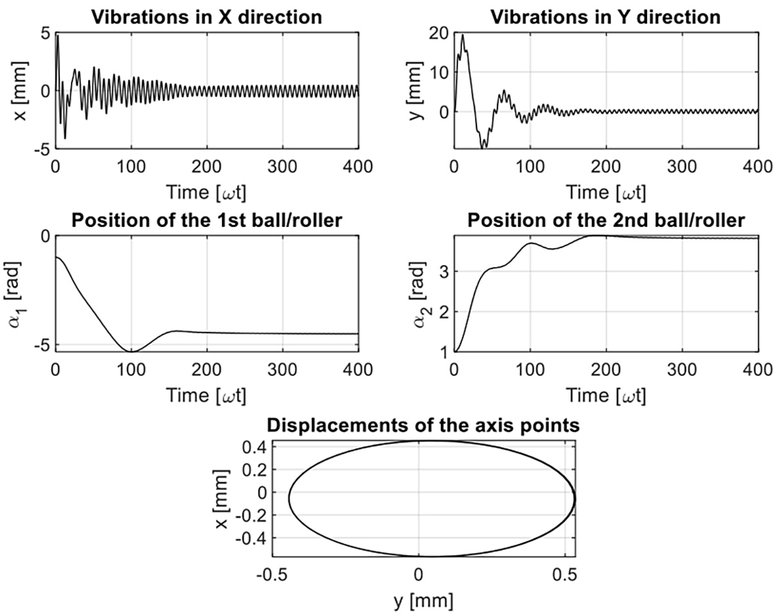


Figure 19. Behavior of the system if $Me = mR = 0.05$ kg m, $\omega = 100$ rad/s, $\rho = 0.5$ mm, $\beta = \pi/2$, $f = 10^{-4}r$

Table 3. $Me = mR = 0.05$ km, $\beta = \pi/2$, $f = 10^{-3}r$

ρ [mm]	A [mm]	$\Delta\alpha_1$ [deg]	$\Delta\alpha_2$ [deg]	RMe [%]
0	0.35	3.5	12.3	74.5
0.05	0.34	4.7	10.9	75.2
0.1	0.334	6.0	9.4	75.7
0.5	0.34	17.3	0.3	68.9
1	0.38	37.5	7.8	44.4

Table 4. $Me = mR = 0.05$ km, $\beta = \pi/2$, $f = 10^{-4}r$

ρ [mm]	A [mm]	$\Delta\alpha_1$ [deg]	$\Delta\alpha_2$ [deg]	RMe [%]
0	0.05	0.1	0.1	99.8
0.05	0.05	2.0	1.5	96.4
0.1	0.1	4.1	3.9	93
0.5	0.5	18.4	21.1	64.7
1	1	29.9	42.9	29.3

the balls $\Delta\alpha_1$ and $\Delta\alpha_2$, the efficiency of unbalance removing R*Me* = (*Me*–Δ*Me*)/Me if the eccentricity ρ changes from 0 to 1 mm, its position is defined by the angle $\beta = \pi/2$, and two coefficients of rolling resistance $10^{-4}r$ or $10^{-3}r$.

For the coefficient of the rolling resistance $f = 10^{-3}\,r$, the effect of the eccentricity is much smaller (Table 3) than the resistance, for a change the greater influence of eccentricity and smaller resistance as in Table 4. The acceleration measured during the centrifugal force of the wet towel at 720 rpm was $a_x \cong a_y = 30$ mV, which gives an amplitude of vibration of 1.3 mm. Comparing this result with the vibration of a washing machine equipped with a self-balancing system, as shown in Table 3, reveals a 70% decrease in vibration. However, comparing it with the results in Table 4 shows a reduction of only 25% when the eccentricity was 1 mm. To achieve optimal self-balancing, the eccentricity should be less than 0.1 mm and the rolling resistance coefficient *f/r* should be below 0.001. In this case, the vibrations of the washing machine will be much smaller than those shown in Figure 3.

CONCLUSIONS

Most articles on the self-balancing of rotating systems lead to the conclusion that free elements can compensate for the initial unbalance in 100% and eliminate vibrations. The article showed that some extra parameters should be taken for a model of the washing machine if its real possibility is to be determined and decision on using this method to eliminate vibrations is to be made. The eccentricity between the ring and the motor, resistance of the balls, variable speed of the washing machine, properties of the drum suspension system, variable unbalance, gravity forces, and external vibrations are important reasons that decrease the efficiency of the washing machine.

The article showed in what way each of them influences the ball distribution concerning the unbalance, the errors in their positioning, how large they are, and what residual unbalance they introduce. Some simulations demonstrated in what way the vibrations of the drum change during the spinning of the laundry and the behavior of the balls during this process. The balls reach their final position very quickly $t < 150/\omega$ sec, and one or two impacts can happen between them. It has been proven that ball resistance and especially eccentricity have the greatest impact on balancing efficiency. Finally, the article proposed a method of distribution of the partial unbalances to achieve the required residual unbalance. In this way, the deviations of the most important parameters can be established, whether self-balancing happens or not, and how large the amplitudes of residual vibrations can be.

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