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Beyond the Dirac delta: Experimental challenges in modelling random impulse responses of discrete systems

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ABSTRACT

There are many studies that present mathematical models of a system's response to excitation by a random sequence of impulses, but none of these models has been experimentally verified. This article presents a study investigating the discrepancies between a mathematical model, which includes the Dirac delta, and the actual dynamic response of an RLC system. The model describes vibrations in a discrete system subjected to a stochastic sequence of impulses. The system parameters were identified using an algorithm based on statistical analyses such as interval and point estimation, nonlinear curve fitting to experimental data, and analysis of fitting errors. The research showed that, system's response to individual impulses depends on both the value of the impulse and its distribution, resulting in dynamic variations of system parameters. Furthermore, a signal flattening effect is observed, which significantly influences the vibrational output characteristics. The results indicate that identifying the underlying impulse distribution will require the application of artificial intelligence algorithms based on image analysis. This approach gives hope for high effectiveness, particularly in the case of low-intensity impulses occurring in a random temporal sequence.

Keywords: stochastic mechanic, Dirac delta in experiment, random series of impulses, random vibrations, system identification.

INTRODUCTION

A common denominator in the work of many mathematicians and physicists was the pursuit of a rigorous, mathematical formalization of the laws of nature. Augustin-Louis Cauchy's [1] contributions to mathematical analysis and the theory of differential equations laid the foundations for the development of modern analysis. Siméon Denis Poisson [2] made significant contributions to the advancement of theoretical mechanics. Lord Kelvin (William Thomson) [3, 4] sought to unify the description of natural phenomena through mathematical analysis and physical analogies.

Scientific concepts [5-9] whose mathematical models include the Dirac delta described in 1930 in "The Principles of Quantum Mechanics" [10] also belong to the research combining mathematics with classical mechanics. Roberts developed a theory for predicting the response of a linear vibratory system to impulses that occur at random times and have random strength [11]. This theory was used to calculate the mean square response of a single degree-of-freedom system to two simple types of random impulse excitation [12, 13], and introduced the concept of "mean square resonance". The dynamic response of non-linear systems to external excitations in the form of a Poisson distributed train of random impulses is considered by Iwankiewicz [14–16]. However these scientific concepts whose mathematical models include the Dirac delta that have never been verified through experiments [17–19].

One of the key issues in signal and systems theory is the impulse response of linear systems. Such analysis allows not only the description of the dynamic properties of electrical and electronic systems, but also the formulation of analogical models for mechanical and vibroacoustic systems [20–22]. Classical models based on RLC elements are still widely used however, they are increasingly complemented by generalized approaches, such as fractional-order models or strongly nonlinear systems. Recent research pays particular attention to fractional-order circuits. Studies on the impulse response of such systems indicate diverse types of transient behaviors [23]. Similar results were reported in the study [24], which emphasized the influence of the modified transfer function on the nature of the impulse response.

Currently, RLC circuits find numerous applications, including energy harvesting systems. In [25], the concept of an electromagnetic energy harvester was presented, in which the impulse response of the resonant RLC circuit coupled with a magnetorheological damper plays a key role. Another application of RLC circuits is their use in active and passive vibration damping. The article [26] presented the concept of coupling a mechanical system with a shunted RLC circuit. It has been demonstrated that appropriately selected values of resistance, inductance, and capacitance allow for the regulation of the damping characteristics of the impulse response of the structure. An important aspect of contemporary research is also the use of the impulse response for the identification of dynamic systems. The study [27] focuses on the analysis of the dynamic response of high-damped test systems, based on impulsive excitations, for the purpose of parameter estimation.

The team consisting of the authors of the present paper is also conducting studies of systems [28, 29] whose vibrations are forced by a random series of impulses [30].The authors have prepared an experiment for the RLC system, and the results of the study are presented in the current paper. Excitations with the help of the Dirac delta and the response of the system was executed using the NI USB-6251 card by National Instruments. The goal of this study is to assess to what extent the model reflects the phenomena occurring in the real RLC system. The differences between the model and the experiment discussed in this study are significant. This requires a redefinition of the mathematical models.

MATHEMATICAL MODEL

This study deals with the motion of a oscillator $x(t)$ with damping b and frequency of damped vibrations ω. The force $f(t)$ exciting the vibrations of the system is defined as a series of random impulses with values A_i occurring at random instants of time t_i.

$$f(t) = \sum_{t_i<t} A_i\, \delta(t - t_i) \tag{1}$$

Dirac delta δt_i at t_i, which are the basic tool in mathematical physics and the theory [31–33] of signals [34–36], were used to model a series of impulses. For zero initial conditions until the moment when the first impulse occurs, $x(0) = 0$, $\dot{x}(0) = 0$. The first impulse of random value A_1 will occur at random time t_1, and causes the oscillator to vibrate. At the moment of hit, the impulse of the action time close to zero causes a rapid increase in the velocity up to the value issued by the law of momentum variation. The initial conditions from which the constants occur in the equation of motion $x(t)$ of the oscillator are the form: $x(0) = 0$, $\dot{x}(0) = A_1$. Applying the rule of superposition, the characteristics of the solutions of differential equations manifests itself in the fact that the sum of the solutions is also a solution to the equation of the system, and the mathematical model for a random series of impulses is determined in the same way for each of the impulses occurring in the random series. The response of the system [37, 38] to forcing assumes the form (2).

$$x(t) = \begin{cases} 0 \text{ for } t \le t_1 \\ \frac{1}{\omega} A_1 e^{-b(t-t_1)} \sin(\omega(t-t_1)) \text{ for } t_1 < t \le t_2 \\ \frac{1}{\omega} A_1 e^{-b(t-t_1)} \sin(\omega(t-t_1)) + \frac{1}{\omega} A_2 e^{-b(t-t_2)} \sin(\omega(t-t_2)) \\ \qquad \text{for } t_2 < t \le t_3 \\ \qquad \cdots \\ \frac{1}{\omega} A_1 e^{-b(t-t_1)} \sin(\omega(t-t_1)) + \frac{1}{\omega} A_2 e^{-b(t-t_2)} \sin(\omega(t-t_2)) \\ + \cdots + \frac{1}{\omega} A_i e^{-b(t-t_i)} \sin(\omega(t-t_i)) \text{ for } t_i < t \le t_{i+1} \end{cases} \tag{2}$$

Simplification of (2) requires indicating of interdependencies between particular elements of the equation. Between the subsequent impulses, the solutions can be presented as follows (3).

$$x(t) = \frac{1}{\omega} e^{-b(t - t_i)} \left(C_{1i} \sin(\omega t) - C_{2i} \sin(\omega t) \right) \quad (3)$$

The coefficients C_{1i} and C_{2i} change after the subsequent impulses, in accordance with (4) and (5).

$$C_{1i} = C_{1(i-1)} e^{-b(t_i - t_{i-1})} + A_i \cos(\omega t_i) \quad (4)$$

$$C_{2i} = C_{2(i-1)} e^{-b(t_i - t_{i-1})} + A_i \cos(\omega t_i) \quad (5)$$

The coefficients C_{1i} and C_{2i}, which change after each hit, can be presented using sum (6):

$$x(t) = \frac{1}{\omega} \sin(\omega t) \sum_{0 \le t_i < t} A_i e^{-b(t-t_i)} \cos(\omega t_i) - \\ - \frac{1}{\omega} \cos(\omega t) \sum_{0 \le t_i < t} A_i e^{-b(t-t_i)} \sin(\omega t_i) \quad (6)$$

This, in turn, simplifies the mathematical description of the continuous random variable *x(t)* by inserting the factors sin(ωt) and cos(ωt) into the sum (7).

$$x(t) = \frac{1}{\omega} \sum_{0 \le t_i < t} A_i e^{-b(t-t_i)} \sin(\omega(t - t_i)) \quad (7)$$

Hitherto, several simulations have been carried out and in this study, experimental research has been conducted. Earlier simulation research [39] has shown that in order to develop an analytical model to recognize the distribution of stochastic sizes of impulses, oscillators with strong damping and high frequencies should be studied. The high frequencies of the vibrations that are required can only be obtained using electronic systems. Simulation studies show that for appropriately selected oscillators, the time required to identify the parameters of the estimators of the distributions of impulse values is limited to a few or several minutes. This paper discusses investigations aimed at checking the differences between the model (7) and the actual response of the RLC system. The identification of the response parameters of the RLC system to such forcing is becoming the key element that answers the research question stated in this article. As shown below, parameter identification required writing an algorithm based on coarse identification and fine identification to estimate the response parameters of the RLC system.

EXPERIMENTAL INVESTIGATIONS

An RLC system with the parameters L = 22.7 µH and C = 940 nF was investigated. In our research, the circuit resistance R was determined based on the impulse response. The investigations are executed with the help of the NI USB-6251 card by National Instruments, whose sampling rate equals 1 µs. The Figure 1 shows a simplified schematic of the experiment aimed at analyzing the response of an RLC system to a random series of impulses. An application developed in the LabVIEW environment generates a random sequence of impulses. The randomly selected impulse value and the random times of their occurrence were transmitted to the data acquisition (DAQ) card via the USB interface. The DAQ card performs two functions in the setup. First, it implements the excitation generated by the computer in the form of a voltage impulse, which was sent to the tested RLC system through a BNC connector. Second, it records the response of the RLC system to the applied excitation and transferred the recorded signal to the computer via the USB interface.

In the first stage, the voltage was varied from 100 mV to 1 V in steps of 100 mV. The probability of the occurrence of each impulse was 0.1. Impulses occur with intensity $\lambda = 10$, at random times. The distance between impulses is executed with the help of a continuous random variable whose cumulative distribution function is represented by (8).

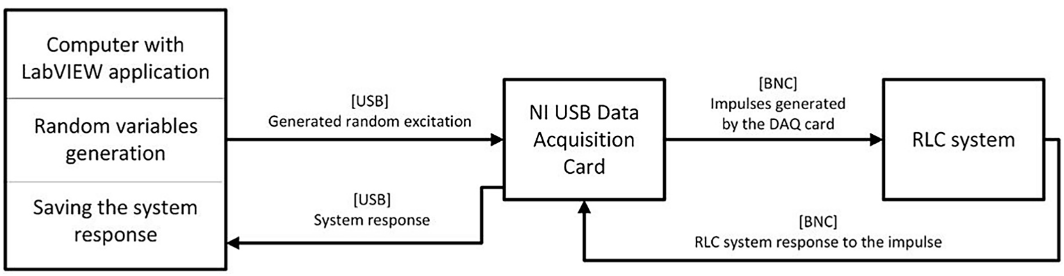

Figure 1. The simplified schematic of the experiment

$$F(\tau) = \begin{cases} 1 - e^{-\lambda\tau} \text{ for } \tau \geq 0 \\ 0 \text{ for } \tau < 0 \end{cases} \tag{8}$$

For λ = 10, impulses occur on the average every 0.1 s, with standard deviation also equal to 0.1 s. The parameters were selected so that a number of impulses sufficient to execute an analysis could be acquired within a few minutes (Figure 2). As mentioned before, the identification of parameters of the RLC system response is the key element that allows to answer the research question asked in this paper. Unfortunately, RLC circuit is also the greatest challenge. Since Dirac Delta is merely a mathematical tool and its experimental execution is impossible, therefore the authors are considering forcing with a random series of hits in the shape of a triangle. The NI USB card executing a single hit generates a signal in the form shown in Figure 3.

This signal assumed the form of an impulse. In the diagram, one should focus on the form of the function responsible for increasing to 100 mV and decreasing of the signal. After achieving the top values, characteristic exponential fading was observed. The parabolic increase and exponential decrease of the signal were not symmetric. It is worth noting that the background noise, which has a changeable course both before and after the impulse. In the mathematical model, the response of the system to a single impulse is equivalent to exciting free vibrations by a rapid increase in the initial velocity of the system at zero initial displacement. However, the increase and decrease of the impulse at the time 2 μs generated by the card (Figure 3) changes the response of the system by introducing of the deviations from the mathematical model. For all impulse forcings with values ranging from 100 mV to 1 V, only the value of the impulse changes, whereas the process of signal rise and decay remains the same. There were unexpected differences between the model and the experiment. Exploratory studies show that when forcing assumes values greater than 400 mV (Figures 4,5), the effect of flattening the systems response in the initial phase of the interval 8 μs to 20 μs is noticeable. Thus, the experiment did not describe model (2) for this values of forcings.

Another problem that can be noticed involves changing the parameters of the response, namely, the frequency of damped vibrations ω and damping b. In this manner we obtain a few vibrating systems with different parameters. In the analysis of vibrations, there is sense of applying superposition of the sum of linear solutions of differential equations with different damping, provided that cases in which the interval between impulses is less than 200 μs are omitted (Figure 4). Probability (8) that the distance between impulses would be less than the duration of the vibrations evoked by the impulse is approximately 0.01998.

Exploratory research indicates that the differences between damping coefficients are smallest for excitations of 100 mV and 200 mV; therefore, in the next phase of the study, only these two cases will be considered. It is possible that the mathematical model (7) will provide an approximate description of the experiment, even in those

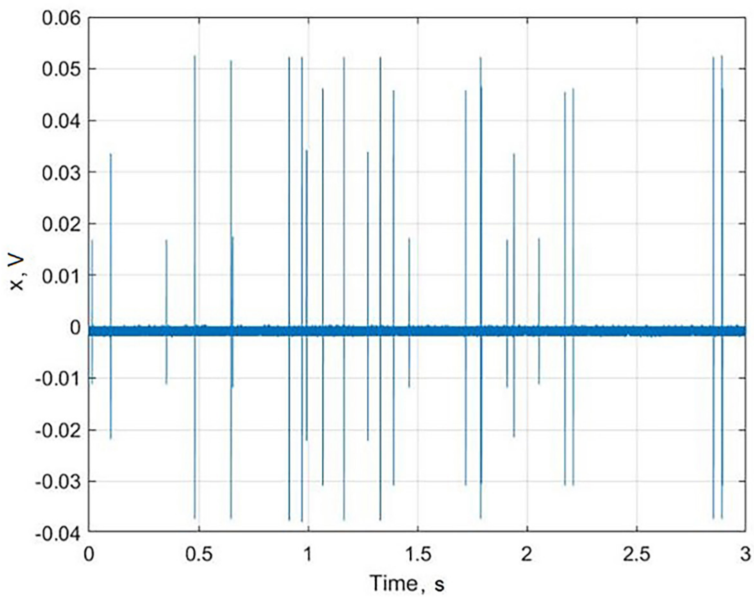

Figure 2. The response of an RLC system to forcing with a random series of impulses

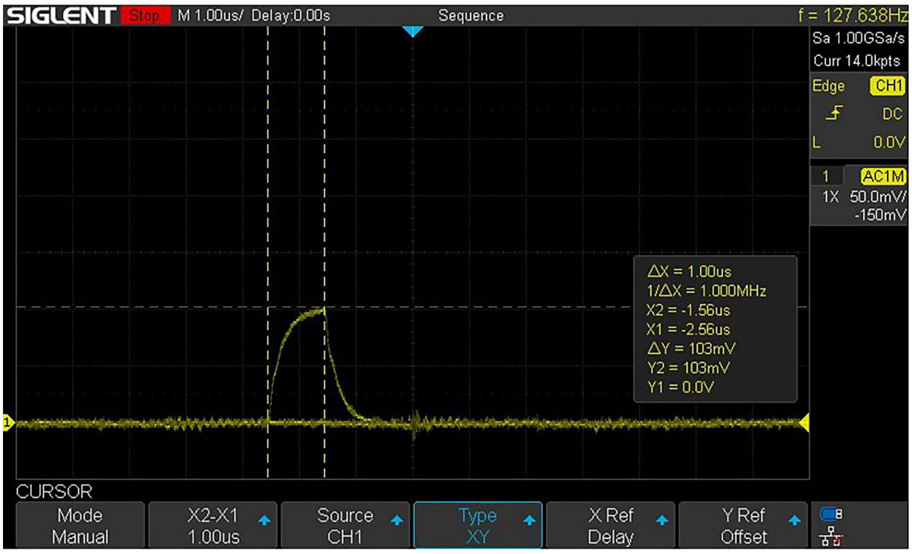

Figure 3. NI USB-6251 card response to single impulse forcing

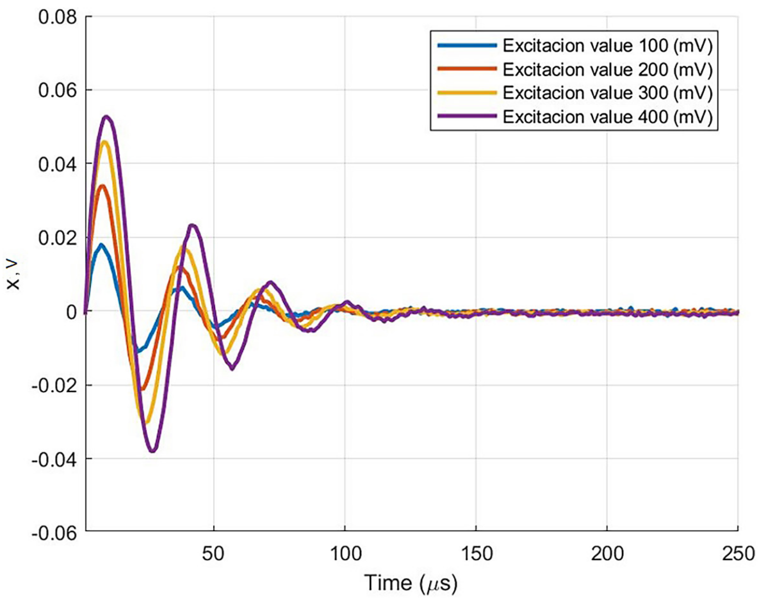

Figure 4. Responses of the RLC system to excitations value ranging from 100 mV to 400 mV

extremely rare instances when the next impulse occurs before vibrations induced by the previous impulse have fully decayed.

DETERMINING A RELIABLE PROPAGATION TIME OF THE SYSTEM RESPONSE

The algorithm that records the response of the RLC system registers a random time interval t_i during which the impulse occurred, as well as its random value A_i. Thus, it is possible to distinguish random events forced by the impulse of a definite amplitude. It is possible to compute the mean value of the number of trials for the recorded responses of the system for each amplitude of excitation. All responses of the RLC system, selected to determine the mean value, start at the same point. In the analysis it is also possible to neglect those cases in which a subsequent impulse occurred before the vibrations forced by the previous impulse has disappeared.

The study comprised 242 trials for the responses of the RLC system to forcings with an amplitude of 100 mV (Figure 6), and 269 trials for the responses of the RLC system to forcings with an amplitude of 200 mV (Figure 7). To determine the duration of signal propagation, the

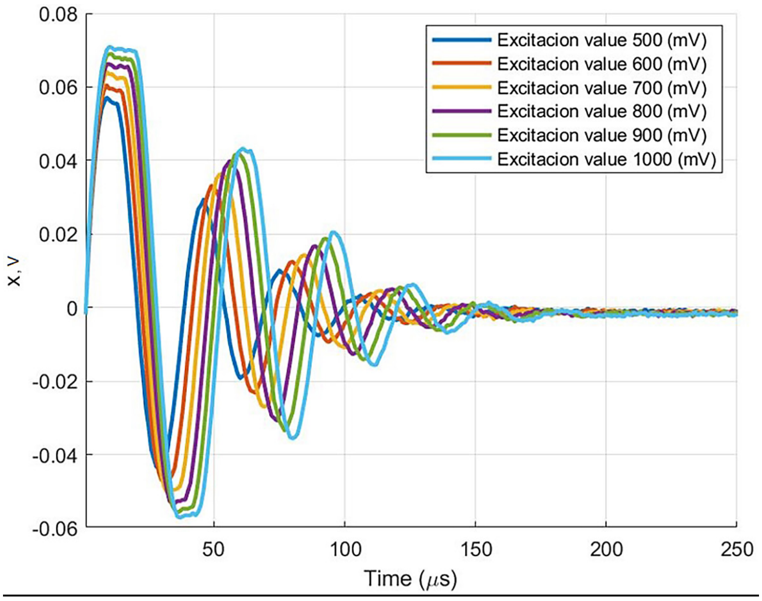

Figure 5. Responses of the RLC system to excitations ranging from 500 mV to 1 V

response of the system to forcing with a random series of impulses shown in Figure 2 point estimation and interval estimation were executed for each time unit *dt* up 0 to 250 *μs* with sampling at 1 MHz. Subsequently, 251 estimators of medium values were prepared for this study. It is necessary to check the reliability of each estimator because the data is charged with errors related to signal discretization and measurement uncertainties.

To evaluate the estimation, the relative error of the point estimator (10) and the relative precision of the interval estimator (11) were used. The estimator was assumed to be unreliable when the relative error value was greater than 15% or when the relative precision value of the estimation was greater than 10%. In Figure 6, 7 unreliable estimators are indicated by red lines.

$$V = \frac{\hat{s} \cdot \bar{m}}{\sqrt{n}} \cdot 100\% \tag{9}$$

where: $\hat{s}$ – the standard deviation, $\bar{m}$ – the mean value of the samples, n – is the number of elements in the sample.

$$B = \frac{u_{\left(1-\frac{\alpha}{2}\right)} \cdot \hat{s}}{\bar{m} \cdot \sqrt{n}} \cdot 100\% \tag{10}$$

where: u – is the inverse cumulative distribution function for $1 - \alpha/2$ and the significance level was $\alpha = 0.05$.

The duration of the signal propagation determined on the basis of the executed estimation, which can be used for identification equals 59 μs for the impulse of 100 mV, and for the impulse with an amplitude of 200 mV it equals 89 μs.

ESTIMATION OF THE PARAMETERS OF AN RLC SYSTEM FOR A FORCING WITH AN IMPULSE

Both before and after single impulses, the noise of the background, whose average value of oscillations was other than zero, was recorded. Unfortunately, background noise has a significant influence on the estimation of damping values in the RLC system. Discrete values of the noise signal cause the impulse to start with an initial value that is different from zero. Additionally, the errors that occur at low deviations in the final stage of the vibration change *x(t)* significantly. The influence of the noise of a recorded present signal, transfer of the start of oscillation and the occurrence of the constant component C_n required the transformation of (1) into (12).

$$x_n(t_i) = a_n e^{-b_n t_i} \sin(\omega_n t_i + \phi_n) + C_n \tag{11}$$

where: n – assumes the values 1 or 2, signifying the response to forcing with impulses of 100 mV and 200 mV, respectively, C_n – the constant around which vibrations occur, a_n – is equal to $\frac{A_n}{\omega_n}$, ϕ_n appears as a result to start of the impulse with an initial value that is different from zero.

$\hat{s}$

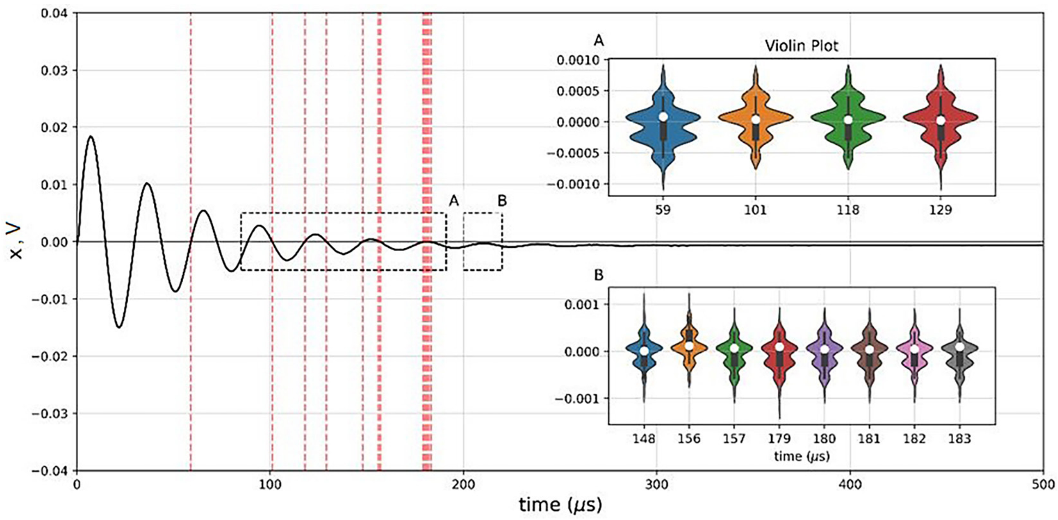

Figure 6. Estimates of the mean value determined from 242 trials for the response of the RLC system to an excitation of amplitude 100 mV. The red line indicates unreliable estimators

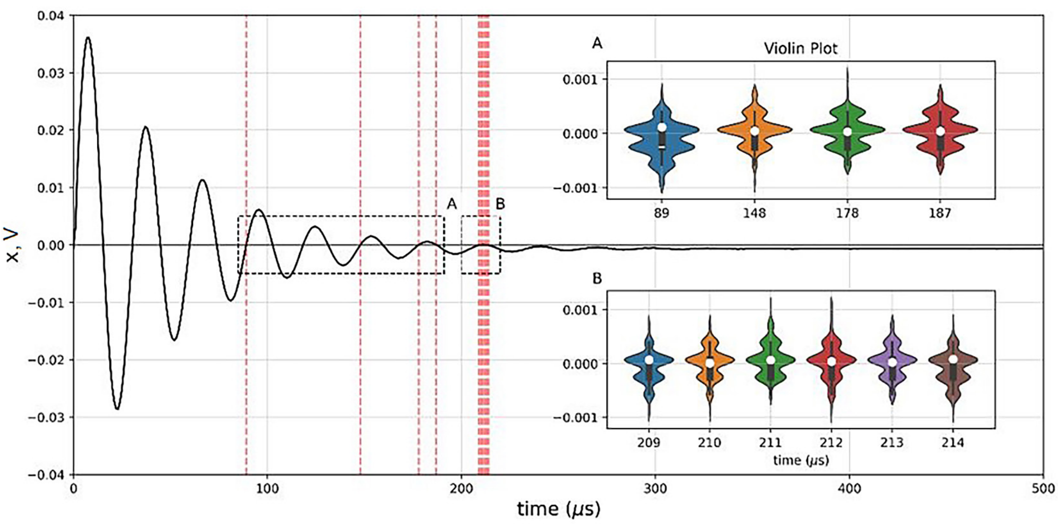

Figure 7. Estimates of the mean value determined from 269 trials for an excitation of 200 mV. The red line indicates unreliable estimators

The identification process was conducted in a MATLAB environment, and the function *lscurvefit()* was used in the study. While identifying the parameters of (12), it is difficult to determine the value of damping b. Using the principle of electric modelling of linear vibrations of mechanical systems with a finite number of degrees of freedom [40], the frequency of the damped vibrations of the RLC system is derived from (12).

$$b = \sqrt{\frac{1}{LC} - \omega^2} \tag{12}$$

The applied function *lscurvefit()* negated (13) hence, the algorithm had to be modified for an iterative search. In each iteration, the value of the frequency of damped vibrations, as well as the value of damping, change in accordance with Equation 13.

The parameter identification process runs using the matching function (12). The parameters a_n, ϕ_n and C_n are identified at the ω parameter indicated at the iteration and the b parameter is computed from (13). Another important step is the assessment of the quality of matching, which is designed to check how well the model reproduces experimental data. To this end, four error measures for each iteration of the data-matching process were determined. The relative error, absolute error, mean squared error, and value of the maximum error were computed.

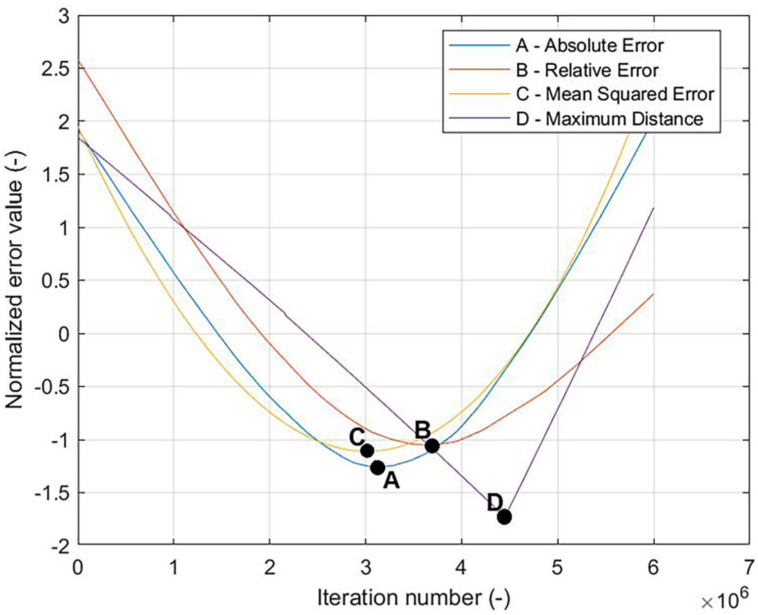

Figure 8. Representation of normalized error measures

Relative error is a measure that expresses the quotient of the absolute and actual measured values. The absolute error describes the difference between the mean value and the identified absolute value. The mean square error, which is the most popular measure for assessing the quality of data matching, is a measure that estimates the mean value of the square of differences. The final measure used in the assessment of matching was the value of the error of the maximum deviation of the prognosis from the actual value in the data set. These measures and their changes for each iteration of the matching process are presented in Figure 8.

On the horizontal axis we can see the number of iterations while the vertical axis (Y) shows normalized values of errors. The curves illustrate in what way the particular measures change with subsequent iterations of the matching process. The points indicate the local minima of each of the presented error measures. On the basis of the diagram it can be inferred that particular error measures may reach minima at different iteration numbers, which emphasizes that the choice of the error measure influences the obtained solution and leads to different results. Therefore, it is hard to speak about the identification of parameters, but it is possible to speak about their assessment. The parameters of the system for forcing at 100 mV and 200 mV depending on the adopter error criteria can be found in Table 1 and Table 2 respectively.

Exploratory analyses indicate that the larger the impulse values, the smaller ω becomes, which in turn implies stronger damping (13). Comparing the data in Tables 1–2, this criterion is satisfied by two cases: B – relative error and C – MSE. However, the difference between the damping values for the 100 mV and 200 mV excitations in the case of relative error is significantly larger than in the other cases. Based on the results obtained, the best fit was selected by applying the mean squared error (MSE) metric. Furthermore, in Tables 1 and 2, the estimated parameters yielding the best fit for the excitation values of 100 mV and 200 mV are highlighted as underlined values.

In classical (linear) vibration mechanics, the damping coefficient of a system is a property of the system and does not "increase" simply because we increase the amplitude of the driving

Table 1. Analysis of parameters for 100 mV

Error type	b_1	ω_1	a_1	Φ_1	C_1
A - Absolute error	35 895.516	213 485.97	$2.16\cdot10^{-2}$	$2.87\cdot10^{-1}$	$-1.70\cdot10^{-4}$
B - Relative error	34 750.185	213 675.39	$2.12\cdot10^{-2}$	$2.88\cdot10^{-1}$	$-1.32\cdot10^{-4}$
C - MSE	35 426.193	213 564.35	$2.14\cdot10^{-2}$	$2.88\cdot10^{-1}$	$-1.55\cdot10^{-4}$
D - Maximum distance	35 426.193	213 675.39	$2.12\cdot10^{-2}$	$2.88\cdot10^{-1}$	$-1.32\cdot10^{-4}$

Table 2. Analysis of parameters for 200 mV

Error type	b_2	ω_2	a_2	Φ_2	C_2
A - Absolute error	35 665.853	213 524.461	$4.29 \cdot 10^{-2}$	$2.17 \cdot 10^{-1}$	$-6.39 \cdot 10^{-4}$
B - Relative error	35 322.111	213 581.593	$4.27 \cdot 10^{-2}$	$2.17 \cdot 10^{-1}$	$-2.01 \cdot 10^{-4}$
C - MSE	35 730.713	213 513.617	$4.30 \cdot 10^{-2}$	$2.17 \cdot 10^{-1}$	$-3.82 \cdot 10^{-4}$
D - Maximum distance	34 861.371	213 657.280	$4.24 \cdot 10^{-2}$	$2.17 \cdot 10^{-1}$	$-3.86 \cdot 10^{-4}$

force. Observed changes in damping at larger impulses usually result from nonlinear losses or heating of components, which increases damping.

CONCLUSIONS

This article discusses the studies that were conducted to determine the differences between the mathematical model and the actual response of an RLC system. These changes result from the experimental realization of a mathematical concept, namely Dirac Delta, and the properties of an RLC system. Experiments are characterized by obvious phenomena such as noise and the occurrence of a constant component in the response of the system, which transforms the mathematical model. In an ideal RLC system, there is no tension threshold in which the condenser stops absorbing the energy and flattens out at the initial phase of the response. If it were strictly linear (no saturations, protection diodes, amplifier limitations), the amplitude of the oscillation would grow at a stronger forcing. In the actual RLC system discussed in this article, the flattening out of the signal excluded six out of ten impulse values that could be used in the model. Additional impulses were excluded because the stronger impulse, the lower frequency of damped vibrations therefore, stronger damping occurs. By executing these experiments, we realized that the mathematical model did not reflect the experiment. The application of the presented algorithm to identify the oscillator parameters indicates that the damping also changes in a system where smaller or larger impulses occur with higher or lower probability. The differences are significant and require the creation of new algorithms of artificial intelligence based on image recognition and not on the analysis of statistical time series.

The logical sense of application of such algorithms AI exists only when the probability that a subsequent impulse occurs before the vibrations forced by a previous one expire is as small as in this case. Then incidents of overlaying vibrations can be neglected in recognition because they are extremely rare. Because of the random time at which an impulse may occur, it is impossible to attempt to recognize two or more impulses occurring close to one another, there are infinitely many cases to recognize.

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