Advances in Science and Technology Research Journal

Advances in Science and Technology Research Journal, 2025, 19(12), 300–306
https://doi.org/10.12913/22998624/209630
ISSN 2299-8624, License CC-BY 4.0

Received: 2025.08.11
Accepted: 2025.10.01
Published: 2025.11.01

Analysis of the influence of non-minimal phase and time delay on the dynamic behavior of systems

Ľubica Miková[1], Dominik Novotný[2*], Peter Frankovský[3]

[1] Department of Industrial Automation and Mechatronics, Faculty of Mechanical Engineering, Technical University of Košice, Letná 1/9, Košice-Sever, Slovak Republic
[2] Department of Production Systems and Robotics, Faculty of Mechanical Engineering, Technical University of Košice, Letná 1/9, Košice-Sever, Slovak Republic
[3] Department of Applied Mechanics and Mechanical Engineering, Faculty of Mechanical Engineering, Technical University of Košice, Letná 1/9, Košice-Sever, Slovak Republic
* Corresponding author's e-mail: dominik.novotny@tuke.sk

ABSTRACT

This article deals with non-minimal phase systems, which are characterized by unstable zeros in the transfer function. Such systems are more difficult to control, because they can cause opposite transient phenomena than expected. In addition, time delays occur in many technical applications, which complicates the design of the controller. To model this, the paper used Padé approximation, which allows the delay to be replaced by a rational transfer function. The aim of the article was to analyze the influence of non-minimum phase characteristics and delay on system behavior.

Keywords: non-minimal phase, unstable zeros, Padé approximantion.

INTRODUCTION

The issue of dynamic system control can be characterized as a synergistic combination of rigorous theoretical foundations with advanced design strategies that reflect practical limitations and model uncertainties.

Chen and Francis provided a comprehensive theory of optimal control of sampled systems based on continuous models, formally analyzing the transformation to the discrete domain as well as formulating LQR and H∞ controller designs with an emphasis on the accuracy of the mathematical formulation and implementation in both the time and frequency domains [1]. He et al. (2019) focused on the issue of delay in non-minimum phase systems, where classical compensation approaches fail due to inherent instabilities. They proposed an adaptive method based on online delay estimation and dynamic control law adjustment, thereby achieving improved robustness and performance in real-time industrial applications [2]. Skogestad and Postlethwaite elaborated on the methodology for the design and analysis of multiple-input multiple-output (MIMO) feedback systems with a focus on robustness to model uncertainties, sensitivity to disturbances, and advanced design approaches, including H∞ and μ-synthesis. Their work provided a bridge between formal theory and engineering practice through practical design guidelines and heuristics [3].

Zhou et al. (2020) presented a modern theory of robust and optimal control with an emphasis on H∞ optimization, μ-analysis, as well as the design of controllers with guaranteed stability and performance under structural and parametric uncertainties [1, 4]. From an integration framework perspective, the works of Chen and Francis and Zhou et al. represent a fundamental theoretical basis, while Skogestad and Postlethwaite and He et al. provide application-oriented methodologies enabling the implementation of robust and adaptive strategies into real technological processes, taking into account system constraints, delays, and uncertainties [1–5]. In control theory, systems with non-minimal phase play an important role, mainly due to their specific dynamic properties, which significantly influence the design of control algorithms. Non-minimal phase refers to the systems the transfer function of which contains zeros in the right half of the complex plane (for continuous systems) or zeros outside the unit circle (for discrete systems). These zeros cause transient phenomena that are counterintuitive - for example, the output first changes sign in the opposite direction to that implied by the input signal. Such behavior has a fundamental impact on feedback stability, control performance limitations, and, in particular, the feasibility of inverse control. For many technical applications, such as aircraft control, robotic arms, or process control, it is necessary to take this property into account when designing controllers [6, 7].

The theoretical foundations of non-minimal phase systems were intensively studied as early as the 1970s, with significant contributions such as [8], which pointed out the limits of feedback performance in such systems. In modern times, research focuses mainly on optimization methods, robust control, and model predictive control (MPC) applied to non-minimum phase systems [9, 10]. As it was mentioned above, the systems with non-minimal phase are the systems the transfer function of which has at least one zero in the right half of the complex variable s. Their presence is reflected in the frequency characteristics as well as in the time responses. In the frequency characteristics, the relationship between the slope of the amplitude frequency characteristic and the phase ceases to apply because the phase is no longer minimal. In the time domain, the presence of an unstable zero manifests itself as an initial undershoot of the transition characteristic to negative values. An example of such a system is a coal dust boiler. Pouring coal dust into the boiler initially causes a kind of extinguishing, which manifests itself in an initial drop in temperature. After a while, however, the fuel ignites and the temperature rises.

CHARACTERIZATION OF SYSTEM POLES AND ZEROS IN STATE-SPACE REPRESENTATION

A continuous system with one input and one output is defined Equation 1:

$$\begin{aligned} \dot{X}(t) &= AX(t) + Bu(t) \\ y(t) &= CX(t) + Du(t) \end{aligned} \tag{1}$$

where: $u(t)$ – represents the input, $y(t)$ – represents the output, the state vector $X(t)$ is a column vector that includes n elements for the nth order of the system, and its components represent state variables Equation 2.

$$X(t) = \left(x_1(t), x_2(t) \dots x_n(t)\right)^T \tag{2}$$

The matrix of system A consists of $n \ x \ n$ elements and represents the matrix of internal connections (system or feedback matrix). Vector B contains $n \ x \ 1$ columns and represents the effect of the action elements. Row vector C has a dimension of $1 \ x \ n$ and represents the links between the output and the state. D represents the direct links between the output and the input in Equation 3. If $D=0$, then the input $u(t)$ has no direct influence on the output $y(t)$.

$$G(s) = \frac{Y(s)}{U(s)} = C(sI - A)^{-1}B \tag{3}$$

The non-minimal phase is not directly "visible" from the state description - matrix A may be stable, but the system may have unstable zeros that cause a non-minimal phase. These zeros do not depend only on A, but on all matrices A, B, C, D. Therefore, the transfer function or the so-called invariant zeros of the system (system zeros), which are formally defined as values s for which the system (5) is singular, must be analyzed. The poles and zeros can be determined by writing the transfer G(s) Equation 4 as:

$$G(s) = \frac{N(s)}{D(s)} \tag{4}$$

where: the numerator polynomial is Equation 5:

$$N(s) \triangleq det \begin{pmatrix} sI & -A & -B \\ C & & 0 \end{pmatrix} \tag{5}$$

and the polynomial of the denominator Equation 6:

$$D(s) \triangleq det\,(sI - A) \tag{6}$$

The functions $N(s)$ and $D(s)$ represent the numerator and denominator of the transfer function $G(s)$, and their roots correspond to the zeros and poles of the system, respectively. This relationship holds only if $N(s)$ and $D(s)$ do not share any common roots. The poles of the transfer function $G(s)$ are fundamental in determining the system's natural frequency and damping ratio. Additionally, they play a crucial role in defining the stability of the system.

The system with non-minimal phase

Let us consider the standard form of a transfer function that contains a single zero and a pair of complex-conjugate poles, which is common in many control system applications, such as modeling second-order systems with added zeros in Eq. (7):

$$F(s) = \frac{\frac{s}{a\zeta\omega_n}+1}{\frac{s^2}{\omega_n^2}+2\frac{\zeta s}{\omega_n}+1} \tag{7}$$

In this expression, the numerator includes a zero at $s = -a\zeta\omega_n$, while the denominator represents a standard second-order system with natural frequency ω_n. Substitution ω_n and damping ratio ζ. The presence of the zero alters the system dynamics by modifying the frequency response and transient behavior. To facilitate analysis, a common step is to perform frequency normalization by $s = \frac{s}{\omega_n}$. This substitution not only normalizes the frequency variable but also effectively corresponds to normalizing the time scale of the system, since ω_n relates to the system's natural oscillation frequency. The normalized transfer function is then given by Equation 8:

$$F(s) = \frac{\frac{s}{a\zeta}+1}{s^2+2\zeta s+1} \tag{8}$$

This normalized form is advantageous because it simplifies the analysis by removing explicit dependence on ω_n, allowing the focus to be on the effects of the zero and the damping ratio. This standard transfer function $F(s)$ can be decomposed into the sum of two separate functions Equation 9:

$$F_f(s) = F_1(s) + F_2(s) = \frac{1}{s^2+2\zeta s+1} + + \frac{1}{a\zeta}\frac{s}{s^2+2\zeta s+1} \tag{9}$$

Here, $F_1(s) = \frac{1}{s^2+2\zeta s+1}$ represents the original second-order system without any zeros. On the other hand, $F_2(s) = \frac{1}{a\zeta}\frac{s}{s^2+2\zeta s+1}$ accounts for the effect of the zero introduced in the numerator. By expressing the transfer function as a sum of these two parts, the system's behavior can be interpreted as a combination of the base second-order response plus an additional component influenced by the zero.

In the time domain, considering the Laplace transform properties, the derivative of the output *y(t)* corresponds to multiplication by sss in the Laplace domain, i.e., $L\left\{\frac{dy}{dt}\right\} = sY(s)$.

Using this, the system's response to a step input can be expressed as Equation 10:

$$y_n(t) = y_1(t) + y_2(t) = y_1(t) + \frac{1}{a\zeta}\dot{y}_1(t) \tag{10}$$

where: $y_1(t)$ and $y_2(t)$ correspond to the system responses associated with $F_1(s)$ and $F_2(s)$, respectively. This means that the overall system response $y_n(t)$ can be seen as the original system response plus a scaled time derivative of this response, reflecting the influence of the zero.

The system response for the case where $a > 0$, corresponding to the introduction of a zero in the

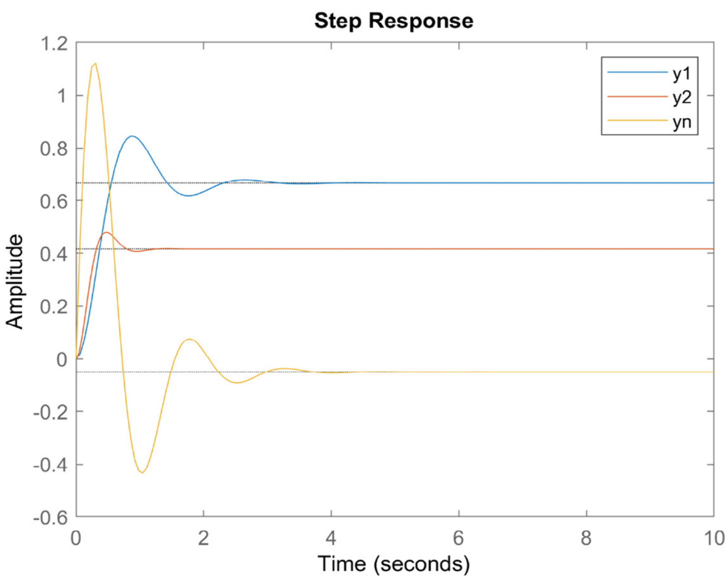

Figure 1. Influence of the position of zero in the left half-plane of the root plane s

left half of the s-plane, is illustrated in Figure 1. The derivative term in the output expression y_2 which introduces the zero, amplifies the overall response of the system $F_f(s)$ and leads to a larger overshoot. This effect is characteristic of systems with left-half-plane zeros, where the initial energy injection due to the zero can temporarily boost the response before settling. The response of the system for the case when $a < 0$ is shown in Figure 2. If the transfer zero is located in the right half-plane of the root plane s, then such a system is called a system with non-minimal phase. This zero causes negative overshoot. The transfer of a dynamic system can generally be written in the form Equation 11:

$$G(s) = \frac{N(s)}{D(s)} = \frac{b_m s^m + b_{m-1}s^{m-1} + \cdots + b_1 s + b_0}{a_n s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0} \quad (11)$$

The system is strictly pure if the condition $n \geq m$ is satisfied. If the transfer function G(s) is asymptotically stable, i.e. if the roots of the denominator D(s) are in the left half of the root plane s, then each zero has a specific effect on the system for specific input variables. The roots of the numerator N(s) can be real or complex. If the zeros are located near the poles, they reduce the influence of the system response on the input variable. Assuming that the poles of the transfer function pi are real or complex but conjugate, the transfer function G(s) can be written in the form Equation 12:

$$G(s) = \frac{C_1}{s-p_1} + \frac{C_2}{s-p_2} + \cdots + \frac{C_n}{s-p_n} \quad (12)$$

The equation for coefficient C_1 can be written as Equation 13:

$$C_1 = (s - p_1)G(s)|_{s=p_1} \quad (13)$$

As it can be seen in this case, if the transfer function G(s) has a root in the left half-plane with zero near the pole at $s = p_1$, the value of coefficient C_1 will be reduced. which determines the contribution of the specific term in the response will be small. In general, it can be noted that each zero in the left half-plane of the root plane limits the specific input signal. However, the question is what happens if the zero is located in the right half-plane. This situation can be illustrated by applying an unbounded signal such as $u(t) = e^t$ to the system input. Figure 3 and Figure 4 show the response of two transfer functions, namely Equation 14, Equation 15:

$$G_1(s) = \frac{2(s+b_1)}{(s+a_1)(s+a_2)} \quad (14)$$

$$G_2(s) = \frac{2(s-b_1)}{(s+a_1)(s+a_2)} \quad (15)$$

A system with a non-minimal phase can be defined as a system that has a zero or pole in the right half-plane of the s-plane. It can also be defined as a system whose transfer function contains a zero in the right half-plane of the s-plane, or has a time delay, or both. In this section, emphasis will be placed on systems with non-minimal phase, where the output is either an inverse

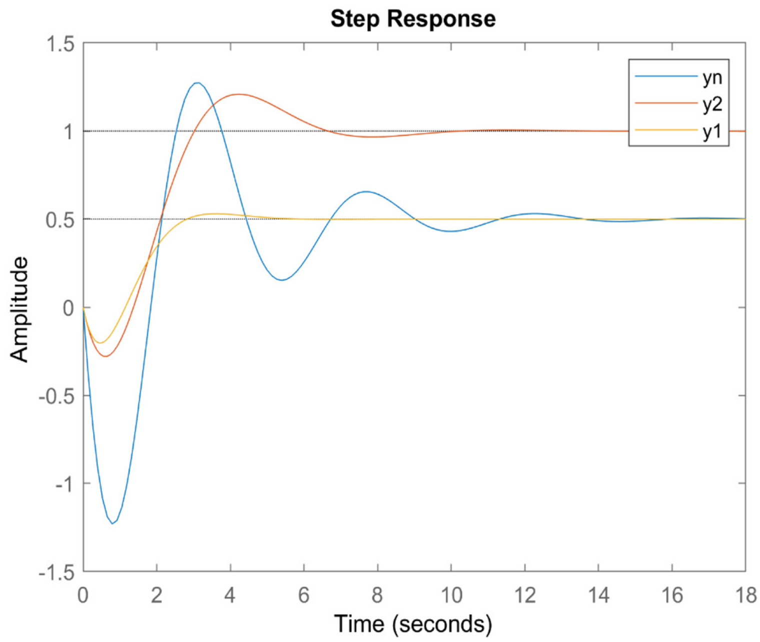

Figure 2. Influence of the position of zero in the right half-plane of the root plane s

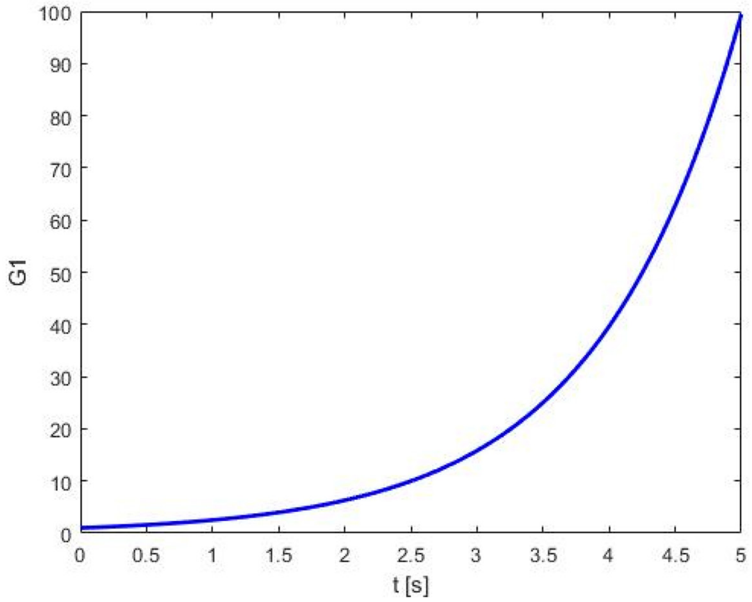

Figure 3. Unbounded response of transfer $G_1(s)$ to unbounded input

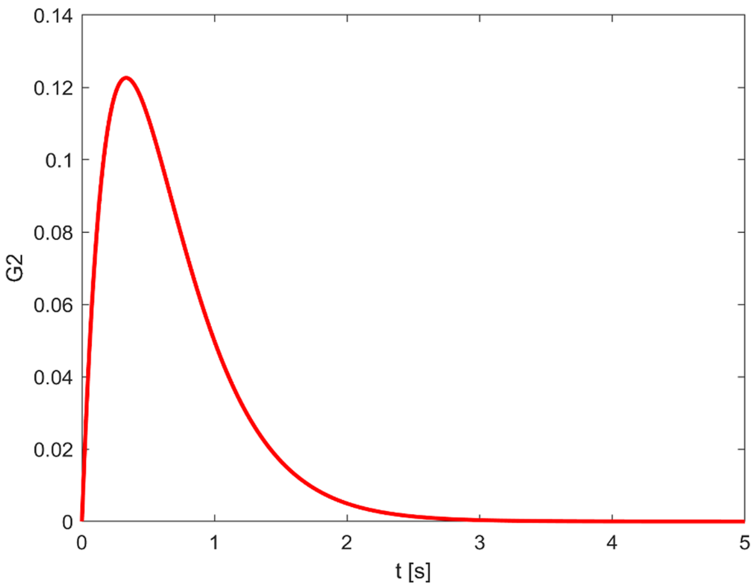

Figure 4. Bounded response of transfer $G_2(s)$ to unbounded input

response or a time delay. The undershoot refers to the initial response of the system, which is in the opposite direction to the steady state. Continuous systems that have an odd number of real zeros in the right half-plane are characterized by an inverse response to a step change.

THE PADÉ APPROXIMATION

A system with time delay is a special case of a system with non-minimum phase. To express a transfer function that also includes time delay, the so-called Padé approximation is often used Equation 16:

$$G(s) = \frac{K}{1+s\tau} e^{-sT} \tag{16}$$

Padé approximation is a method for approximating transcendental functions using rational fractions, i.e., the ratio of two polynomials. In the field of automatic control, it is often used to approximate time delay, which in the Laplace domain has the form of an exponential function e^{-sT}, where T is the delay length. Since this function is not rational, it is not directly suitable for analysis using classical tools such as Bode diagrams or PID controller design. Therefore, the delay is replaced by a rational approximation using a Padé expansion [11]. Padé approximation is used in the design of classical controllers (e.g., PID), model predictive control (MPC), and frequency analysis (e.g., Bode diagrams), where it is advantageous to work with rational functions [13, 14]. However, it should be noted that Padé approximation introduces artificial poles and zeros into the system, which can adversely affect the stability or robustness of the control, especially if the delay is large or if a higher order of approximation is chosen [6, 15].

The first-order Padé aproximation

In Equation 17, k represents the amplification constant, τ is the time constant, and T denotes the time delay (also known as time delay) of the system. To handle the exponential delay term e^{-sT}, a Padé approximation is used, which provides a rational function approximation of the time delay. The first-order Padé approximation of e^{-sT} is given by:

$$e^{-sT} \cong \frac{N_r(sT)}{D_r(sT)} \tag{17}$$

Where Equation 18 and Equation 19:

$$N_r(sT) = \sum_{k=0}^{r} \frac{(2r-k)!}{k!(r-k)!}(-sT)^k \tag{18}$$

$$D_r(sT) = \sum_{k=0}^{r} \frac{(2r-k)!}{k!(r-k)!}(sT)^k \tag{19}$$

where: r is the degree of approximation.

In general, Padé approximation of type n/n uses the same degree for both the numerator and denominator, with the aim of making the Taylor expansion of the resulting fraction coincide with the expansion of the exponential function of the highest possible order around the point $s=0$ [12].

The second-order Padé aproximation

The second order Padého approximation provides a more accurate approximation of the exponential term of the time delay e^{-sT} compared to the first order. Mathematically, it is expressed as Equation 20:

$$e^{-sT} \cong \frac{1-\frac{sT}{2}+\frac{(sT)^2}{12}}{1+\frac{sT}{2}+\frac{(sT)^2}{12}} \tag{20}$$

This approximation is more accurate than the first order and better captures the dynamics of the time delay, but at the same time increases the order of the transfer function. The second order of Padé approximation provides a more accurate capture of the dynamics of time delay compared to lower order approximations, thereby reducing the error between the actual delay and its model. At the same time, however, it increases the order of the transfer function, which leads to more complex dynamic behavior of the system. As a result, see Figure 5, more poles and zeros appear in the model, the position of which in the complex plane can significantly affect the stability of the system. Therefore, it is necessary to thoroughly analyze

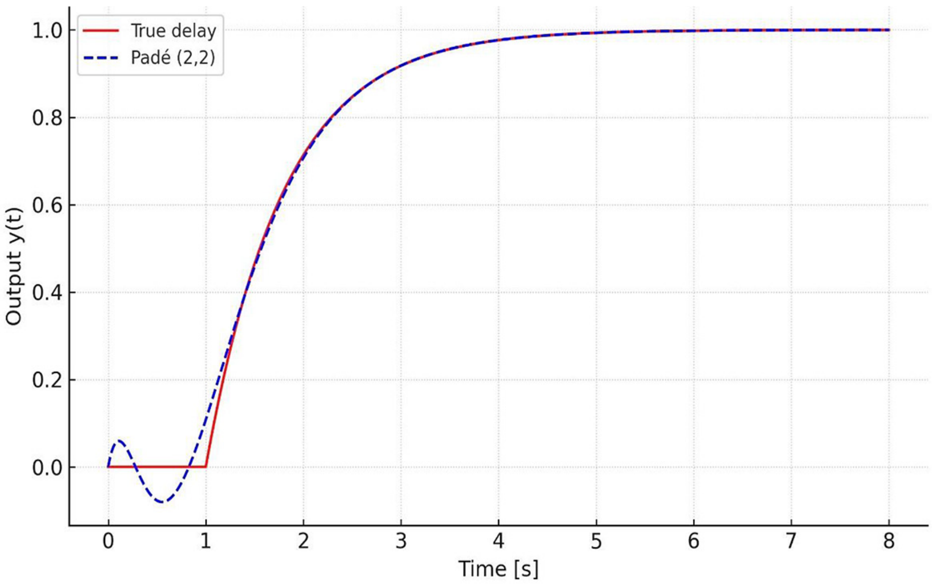

Figure 5. Comparison of step response with Padé approximation of time delay

these new poles and zeros to avoid possible instability. In addition, as with other Padé approximations, the second order can cause non-minimum-phase behavior of the system, where the output after a sudden change in input may initially respond in the opposite direction than expected, which can affect the design and performance of the control system [16, 17].

CONCLUSIONS

This article analyzed the properties of systems with non-minimal phase, which occur in many technical applications and pose a significant challenge in control design. Their specific behavior, caused by unstable zeros, significantly affects transient phenomena, stability, and the overall performance of the control system. An important part of the analysis was also the presence of time delay, which was approximated using Padé approximation. This made it possible to convert the irrational term into a rational form and thus simplify the design of controllers using classical methods.

The analysis shows that the use of Padé approximation is particularly practical in frequency analysis and the design of feedback systems, but caution must be exercised when selecting its order. Too high a degree can lead to a deterioration in stability and introduce unrealistic dynamic effects into the system. For the design of non-minimum phase systems with delay, it is therefore advisable to combine the knowledge from classical and modern control theory, including robust and optimization methods that can compensate for these negative effects. The simulations and examples presented in the paper confirmed the theoretical findings and highlighted the importance of correct model approximation and an appropriate approach to control design.

Acknowledgements

The authors would like to thank to project KEGA 016TUKE-4/2025, KEGA 040TUKE-4/2025, VEGA 1/0152/24, project ITMS 2014+ project 313011AVF5 INTELTEX – Centre for the Development of Textile Intelligence and Antimicrobial Technologies and project APVV-23-0364 – Identification and Quantification of Key Parameters of Core Drilling of Rocks by New Diagnostic Methods.

REFERENCES

1. Chen, T., Francis, B.A. Optimal Sampled-Data Control Systems, New York: Springer, 1995.
2. He, W., Zhang, Y., Li, Z. Delay compensation in non-minimum phase systems using adaptive control, IEEE Transactions on Industrial Electronics, 66(5), 3872–3881, 2019.
3. Skogestad, S., Postlethwaite, I.: Multivariable Feedback Control, Analysis and Design. 2nd ed. Chichester: Wiley, 2005.
4. Zhou, K., Doyle, J.C., Glover, K. Robust and Optimal Control, Upper Saddle River: Prentice Hall, 1996.
5. Fridman, E., Xia, Y., Liu K. Introduction to Time-Delay Systems a Networked Control Under Communication Constraints, Springer, 2020.
6. Franklin, G.F., Powell, J.D., Emami-Naeini, A. Feedback Control of Dynamic Systems, 7th ed. Pearson, 2015.
7. Morari, M., Zafiriou, E. Robust Process Control. Prentice-Hall, 1989.
8. Doyle, J. C., Francis, B. A., Tannenbaum, A. R.: Feedback Control Theory. Macmillan Publishing Co., 1992.
9. Zhu, Q., Lin, W. Model predictive control for non-minimum phase systems using output reparameterization, IEEE Transactions on Control Systems Technology, 2020; 28(1): 168–176.
10. Wang, Y., Boyd, S. Fast model predictive control using online optimization, IEEE Transactions on Control Systems Technology, 2010; 18(2): 267–278.
11. Ogata, K.: Modern Control Engineering, 5th ed., Prentice Hall, 2010.
12. Houpis, C.H., Sheldon, S.N. Linear Control System Analysis and Design with MATLAB, 6th ed., CRC Press, 2013.
13. Middleton, R.H., Goodwin, G.C. Digital Control and Estimation: A Unified Approach, Prentice Hall, 1990.
14. Visioli, A. Practical PID Control, Springer, 2006.
15. Uren, K., Schoor, G.: Predictive PID Control of Non-Minimum Phase Systems, Advances in PID Control, 2011.
16. Estrada, M. Toward the Control of Non-Linear, Non-Minimum Phase Systems via Feedback Linearization and Reinforcement Learning. University of California, Berkeley, 2021.
17. Miková, Ľ., Prada, E., Virgala, I., Hroncová, D. Modelling of dynamic systems in state space, Acta Mechatronica, 2021; 6(1): 7–10.
18. Xing, H., Ploeg, J., Nijmeijer, H. Padé approximation of delays in cooperative ACC based on string stability requirements. IEEE Transactions on Intelligent Vehicles, 2016; 1(3): 277–286.