

# Development and evaluation of a vibratory book-block cutting method using eccentric disk knives

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## ABSTRACT

The kinematics and cutting forces that arise when book blocks are trimmed with one or two eccentrically mounted vibrating disk knives were investigated. The eccentricity of disk knife induces a reciprocating motion relative to the material, resulting in two distinct cutting modes: continuous and intermittent. The research includes theoretical calculations of cutting force distribution for both single and dual vibrating disk knife configurations, which were subsequently validated experimentally. The study culminated in design recommendations for advanced cutting machines that employ a pair of vibrating disk knives.

**Keywords:** book-edge trimming, disk knife, eccentricity, two-knife cutting.

## INTRODUCTION

In the final stage of book production, three-side trimming is performed by guillotine-type flat knives the low speed of which now represents the main bottleneck in high-throughput book-binding lines. Although three-knife trimmers are mechanically mature, the throughput of modern book-binding lines has risen sharply. Disk knives are widely used in the printing industry for cutting cardboard sheets and slim brochures. This method offers significant technological advantages, such as high processing speed and cutting quality. However, when cutting thicker books (over 5–8 mm), the cutting force increases significantly. Additionally, friction between the knife and the paper leads to a substantial rise in temperature, potentially causing the cutting surface to burn. Cutting thick book blocks with conventional disk knives is currently impossible. Consequently, new high-efficiency trimming methods are needed. In recent years, there has been growing interest in the use of vibrations in the processing of various materials [1–4].

Research has also focused on the application of vibratory cutting to paper-based materials [5–7]. Previous studies [8,9] have shown that applying longitudinal vibrations to flat knives during paper cutting demonstrated that cutting forces could be reduced by up to 1.8 times compared to conventional cutting [8]. The most effective results were achieved with vibration amplitudes of 3–5 mm and frequencies in the range of 30–50 Hz.

These findings suggest that vibratory knives operating at low frequencies (up to 50 Hz) can be effectively applied for book trimming. One promising solution to the challenges of high-speed book cutting is the use of vibrating disk knives in automated bookbinding lines.

## PROPOSED METHOD OF VIBRATORY BOOK CUTTING

To lower the cutting force, a disk knife was mounted so that its axis of rotation was offset (eccentric) by a small amount (0.5–1 mm) from the

geometric center of the blade. Eccentricity refers to the offset between the geometric center of the blade and axis of rotation.

Figure 1a depicts vibratory cutting of booklets, while Figure 1b extends the principle to full book blocks in automated lines. The book block (booklet) 3 is clamped in the conveyor 4 of the automatic line and fed toward the vibrating disk knife 1 for cutting. Figure 1b shows the method proposed for cutting thicker book blocks using two eccentrically mounted vibrating disk knives. In this configuration, the first disk knife 1 cuts through half the thickness of the book block on one side, while the second disk knife 2, positioned sequentially, cuts through the remaining thickness of the book block 3. By setting the rotating disk knives with an eccentricity of value  $e$ , vibratory cutting is achieved. This significantly reduces the cutting force and enables high-performance, continuous (streamlined) processing of book blocks in automatic bookbinding lines.

The previously conducted comparative studies on the kinematics of the cutting process using a traditional single disk knife (without vibrations) [10] and using a single vibrating disk knife with eccentricity [11–13], have made it possible to identify the advantages and potential of the proposed method of vibratory book block cutting.

Mathematical modeling of the eccentric blade trajectory during cutting revealed that the process can occur in two distinct modes: continuous cutting and intermittent cutting (impulse) cutting [14]. Both modes are possible in direct and counter-cutting configurations. In continuous cutting, the knife blade remains in constant contact with the paper throughout the process. In contrast, in intermittent cutting, the blade during a specific

phase of rotation, the knife blade temporarily disengages from the paper. This interruption allows the knife edge to cool more effectively, as it does not touch the material for a portion of the cycle [15]. In both cutting modes, a significant reduction in the cutting force of book blocks is expected.

To verify the scientific hypothesis regarding the existence of two vibratory cutting modes – continuous and intermittent – and to validate the theoretical predictions on the influence of kinematic parameters, a series of experimental tests were conducted using a laboratory setup. The results of preliminary tests confirmed the hypothesis [16].

The aim of this paper was to further investigate the theoretical and practical aspects of vibratory cutting using one and two eccentric disk knives, and to propose new design concepts for advanced book trimming machines suitable for integration into high-speed bookbinding lines.

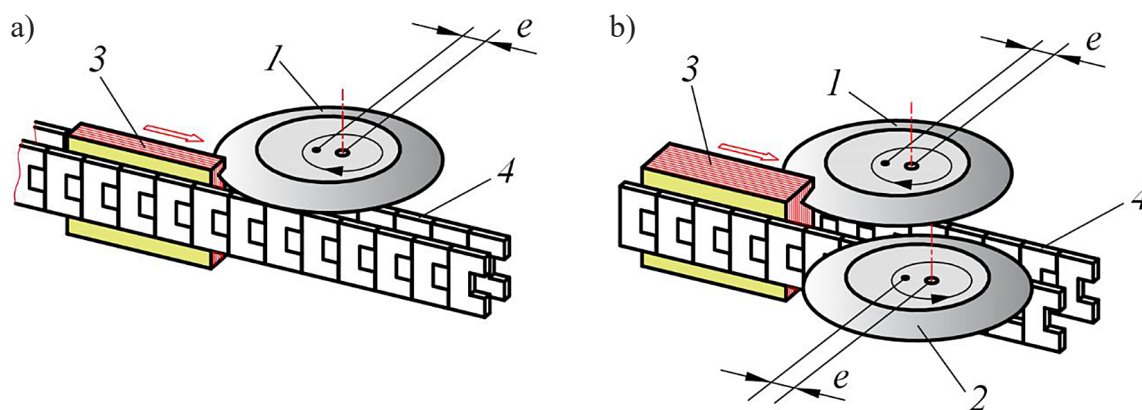
## KINEMATICS OF CUTTING WITH AN ECCENTRIC DISK KNIFE

To investigate the distribution of point velocities on the eccentric knife blade during cutting, the velocities of points at any position, at a distance  $y$  from the knife rotation axis, were analyzed. The cutting speed is determined from the linear speed  $v_R$  and the feeding rate  $v_0$  (Figure 2).

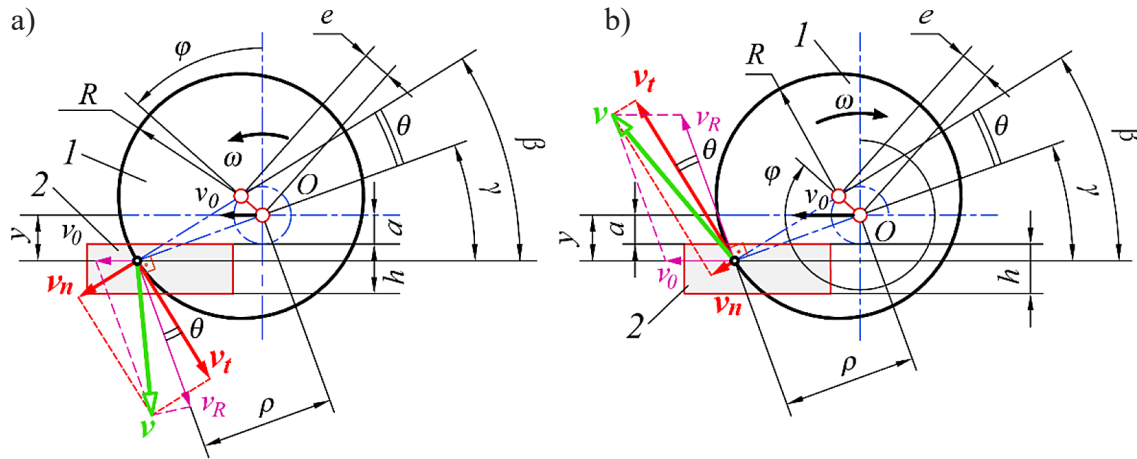
The value of the speed rate  $v_0$  is known and the other components of the full velocity could be calculated using the following equations:

$$\varphi = \omega \cdot t; v_R = \omega \cdot \rho \quad (1)$$

The distance  $\rho$  is determined by solving the triangles formed by the straight segments  $a, y, e$ ,



**Figure 1.** The principle of vibratory cutting of book block by vibrating knives. 1, 2 – disk knives, 3 – book block, 4 – conveyor,  $e$  – eccentricity of the disk knife



**Figure 2.** Schemes for determining the total cutting speed of any point of the knife blade: 1 – disk knife, 2 – book block, (a) direct cutting; (b) counter-cutting

and  $R$ , as shown in Figure 2. This distance varies depending on the position  $y$  of the cutting point relative to the knife center of rotation, the orientation of the knife center relative to its axis of rotation, and the magnitude of the eccentricity  $e$ :

$$\rho(y) = \sqrt{\left\{ R \cdot \cos \left[ \arcsin \left( \frac{y + e \cdot \cos(\omega t)}{R} \right) \right] \pm e \cdot \sin(\omega t) \right\}^2 + y^2} \quad (2)$$

From this, the following was derived:

$$v_R = \omega \cdot \sqrt{\left\{ R \cdot \cos \left[ \arcsin \left( \frac{y + e \cdot \cos(\omega t)}{R} \right) \right] \pm e \cdot \sin(\omega t) \right\}^2 + y^2} \quad (3)$$

It is assumed that at the initial moment ( $t = 0$ ), the geometric center of the knife is positioned vertically above its center of rotation, with the distance between them equal to the eccentricity value  $e$ . In formulas (2)–(4), the upper signs correspond to direct cutting, while the lower signs apply to counter-cutting. The tangential  $v_t$  and normal  $v_n$  components of the total cutting speed are obtained by decomposing the velocity vectors  $v_R$  and  $v_0$  into their radial and tangential directions (see Figure 2).

$$\begin{aligned} v_n &= v_0 \cos \beta \pm v_R \sin \theta; \\ v_t &= \mp v_R \cos \theta - v_0 \sin \beta \end{aligned} \quad (4)$$

where:

$$\begin{aligned} \gamma &= \arcsin \left( \frac{y}{\rho} \right); \\ \beta &= \arcsin \left[ \frac{y + e \cdot \cos(\omega t)}{R} \right]; \\ \theta &= \beta - \gamma \end{aligned} \quad (5)$$

The distance  $\rho$  for a point on the blade depends on the knife's rotational position, and the magnitude as well as direction of the cutting velocity components change depending on the current time and the position of the point on the knife blade.

In deriving the formulas, the following assumptions were made:

- The book block remains unmovable, while the knife rotates and advances toward it at a specified feed rate.
- The velocity of each point on the blade in contact with the book block during cutting consists of two components:
  - the normal component  $v_n$ , aligned along the radius of the knife, embodies the plunging velocity of the blade as it drives into the book block. Due to the blade eccentricity, the normal velocity periodically reverses, generating an oscillating motion. This dynamic motion defines the plunging speed, more precisely known as the normal cutting speed.
  - the tangential component  $v_t$ , which is oriented tangentially to the blade and perpendicular to the radius at a given point on the blade, represents the sliding velocity of the blade over the book surface.

To conceptualize this, the blade of the disk knife can be imagined as a polygonal prism composed of elementary straight knives of infinitesimally small length. Each elementary knife segment follows its own trajectory and has its own varying normal as well as tangential cutting velocities. The amplitude of vibration in normal direction equals the eccentricity value. Figure 3 illustrates the typical distribution of normal speed

across the lower left quarter of the disk blade during direct cutting, derived from calculations using Equations 1 through 5.

As shown by the distribution, the magnitude of the normal component  $v_n$  decreases from the center of the disk downward. Overall, the magnitude of  $v_n$  increases along with feed rate  $v_0$ . A change in the direction of  $v_n$  means that in this zone, the knife blade moves away from the book block, leading to intermittent cutting. When the block is placed closer to the center of the disk knife, the cut will be continuous; when the block is moved farther from the center, a pulsed cut may occur (Figure 3b). Additionally, increasing eccentricity amplifies the variation in the magnitude of the normal speed (see Figure 4).

At any point on the knife edge, the magnitude of the tangential component  $v_t$  is approximately equal to the linear velocity and is independent of the knife rotation angle. The transformed knife cutting edge angle decreases with distance from the knife center. Therefore, the book block during cutting should be positioned as far from the knife center as possible, in a direction perpendicular to the feed direction. This applies to both counter-cutting and direct cutting.

### THEORETICAL CALCULATION OF CUTTING FORCE WITH AN ECCENTRICALLY INSTALLED DISK KNIFE

The cutting force can be accurately approximated as the sum of two separate components: the penetrating force, directed perpendicular to the

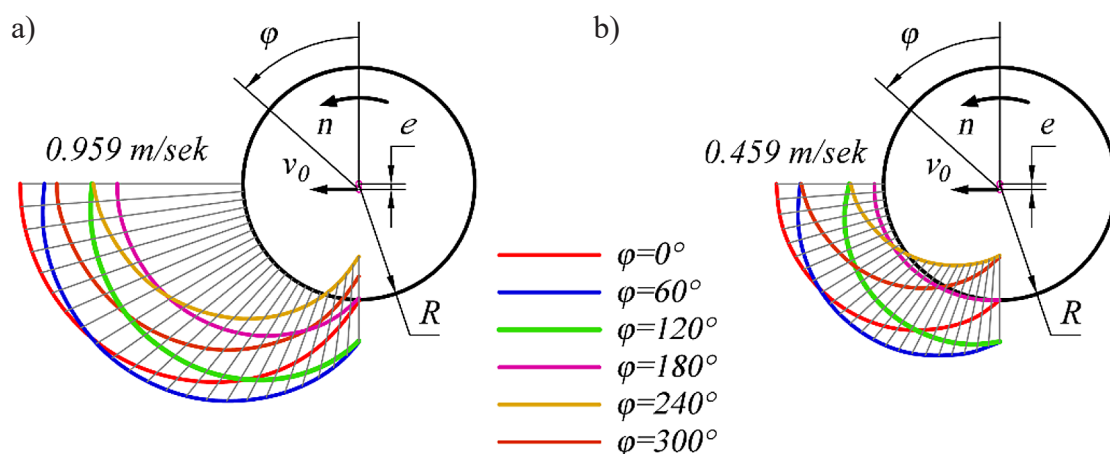
blade, and the sawing force, directed along the blade or tangentially to it. Accordingly, the total cutting force is a vector sum of two components:

The normal component, directed along the radius, which is caused by the destruction of paper fibers when the blade penetrates the paper sheets at a speed  $v_n$ .

The tangential component, directed tangentially at the cutting edge to the knife radius. It is the sum of the force of sawing through the layers of paper and the friction force. The sawing occurs at a speed of  $v_t$ .

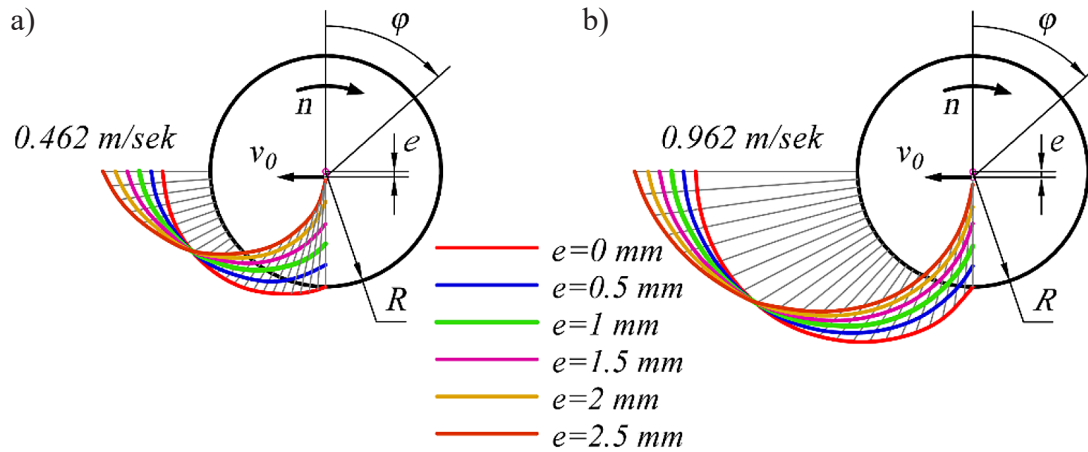
The circular knife blade can be conceptualized as being composed of an infinite number of small straight prismatic blades, each of infinitely small length  $dl$ . The transformed cutting edge angle of each prismatic blade depends on its position on the disk and the angle of rotation of the knife. The transformation of the cutting edge angle for a circular knife without eccentricity has been previously studied [10]. For a disk knife with eccentricity, this transformation becomes more complex. Each individual small prismatic blades follows its unique path, cutting through the book block with a corresponding elemental force,  $dF$  (see Figure 5).

The elemental cutting force,  $dF$ , can be presented as the vector sum of its components,  $dF_n$  and  $dF_t$ . In the experimental studies [16, 17], three components of the total cutting force were recorded: in the feed direction ( $F_x$ ), in the transverse direction ( $F_z$ ) and along the axis of cutter rotation ( $F_y$ ). The value of the component,  $F_y$ , turned out to be insignificant and was not taken into account in this analysis.

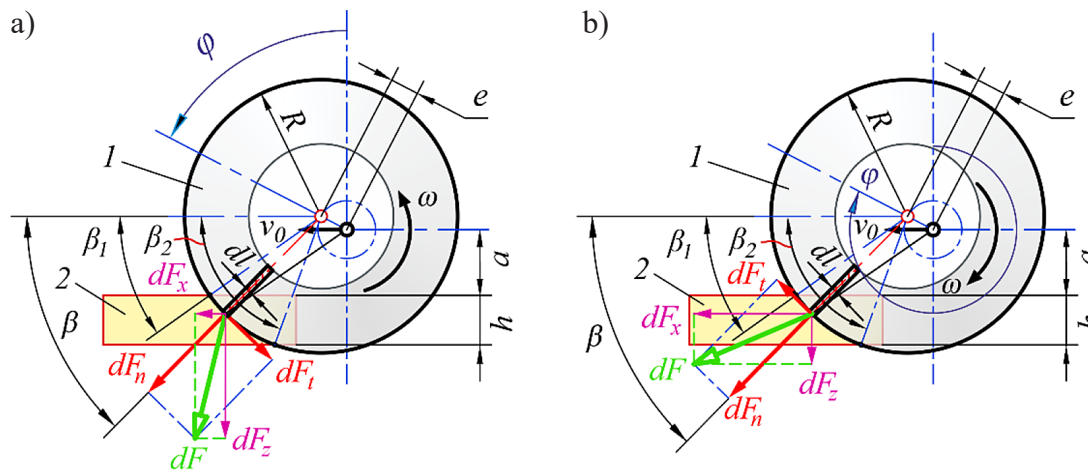


**Figure 3.** The normal speed distribution across the blade during rotation. Parameters: the knife rotation angle  $\phi$ , disk radius  $R = 100$  mm, eccentricity  $e = 1$  mm, disk rotational speed  $n = 2000$  rpm, direct cutting, a)  $v_0 = 0.75$  m/sec; b)  $v_0 = 0.25$  m/sec





**Figure 4.** Diagrams of normal speed distribution on the blade relative to the knife eccentricity. Parameters: disk radius  $R = 100$  mm, disk rotation angle  $\varphi = 240^\circ$ , disk rotational speed  $n = 2000$  rpm, counter-cutting; (a)  $v_0 = 0.2$  m/sec, (b)  $v_0 = 0.7$  m/sec



**Figure 5.** Distribution of the elementary cutting force components in a random point on the blade: 1 – disk knife, 2 – book block; (a) direct cutting; (b) counter-cutting

The vector of the elementary cutting force  $dF$  can be decomposed into two components: one in the feed direction,  $dF_x$ , and the other in the transverse direction,  $dF_z$ . The magnitude of the elementary cutting force varies due to differences in the direction of speed and position of each elementary knife. As established in the research [8], the main factors influencing the unit normal force when cutting a stack of paper with a prismatic knife are the type of paper, the condition of the knife blade and the kinematics of the cutting process (transformed blade cutting edge angle). According to [18] the transformed angle for the prismatic knife is:

$$\alpha_T = \arctg \left[ \tan(\alpha_0) \frac{v_n}{\sqrt{v_n^2 + v_t^2}} \right] \quad (6)$$

where:  $v_n$  – the normal component of the total cutting speed;  $v_t$  – the tangential component of the total cutting speed;  $\alpha_0$  – blade cutting edge angle.

Each individual elementary blade on the circular knife can be treated as prismatic, so formula (6) can be applied to each of them, calculating the velocity components using formulas (1–5). According to studies [8,18], the normal cutting force of a flat knife can be approximated with sufficient accuracy by the following relationship:

$$F_n = K_0 \cdot \alpha_T^\gamma \cdot L \quad (7)$$

where:  $\alpha_T$  – transformed cutting edge angle,  $L$  – cutting line length,  $K_0$  and  $\gamma$  – empirical factors taking into account the condition of the knife blade, the type of paper and

other processing parameters. According to our research [8], the factors for offset paper with a grammage of 70–80 g/m<sup>2</sup> can take the following values:  $K_0 = 19.3\text{--}22.6$ ,  $\gamma = 1.6\text{--}2.2$ .

This approach is applied to approximate the normal cutting force for a disk knife. Unlike a flat knife, cutting with a circular knife creates an arc-shaped cut, and the actual angle of cutting varies along this arc. In the tests [16] conducted on cutting book blocks with a disk knife without eccentricity, we used the unit cutting force, that is, the cutting force per unit length of the knife blade. The unit force for a disk knife is just the elementary cutting force shown in Figure 5. To approximate the unit cutting force, we proposed using the same power dependence, as for a flat knife.

The ratio of the unit tangential force to the unit normal force remains nearly constant value and depends only slightly on the block thickness [16]. The cutting factor  $f$  defined as the ratio of the tangential component to the normal component of the cutting force was introduced. The elementary tangential force was expressed by the cutting factor  $f$ , which was assumed to be constant:

$$dF_n = K_0 \cdot \alpha_T^\gamma; dF_t = dF_n \cdot f \quad (8)$$

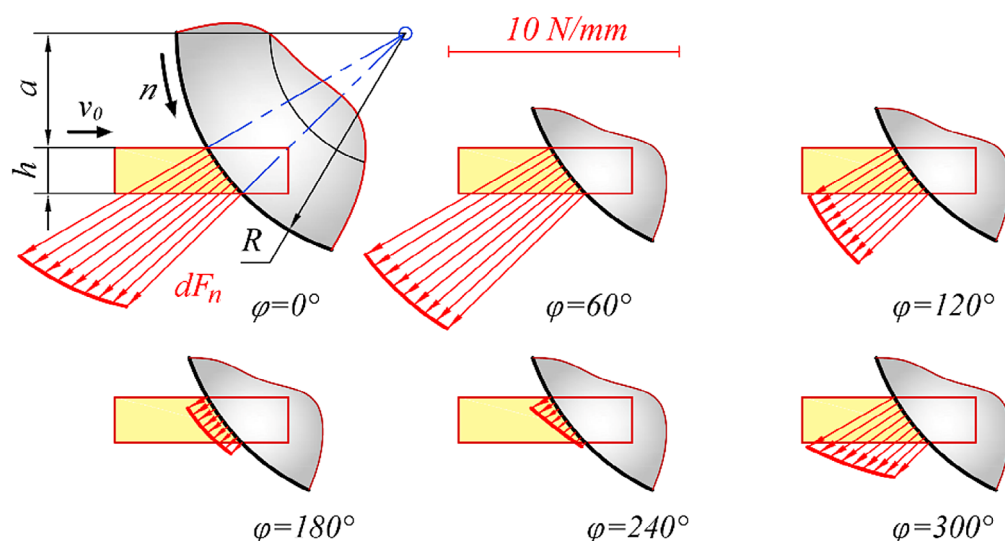
Figures 6 and 7 illustrate typical theoretical distributions of the normal cutting force along the cut path of a book block under different kinematic parameters, calculated using formulas

1–8. On the basis of on authors' estimations, for offset paper with a grammage of 70–80 g/m<sup>2</sup>, block thickness  $h=10\text{--}15$  mm, and knife rotational speed of 1000–2000 rpm, the cutting factor  $f$  ranges between 0.5 and 0.62 [19].

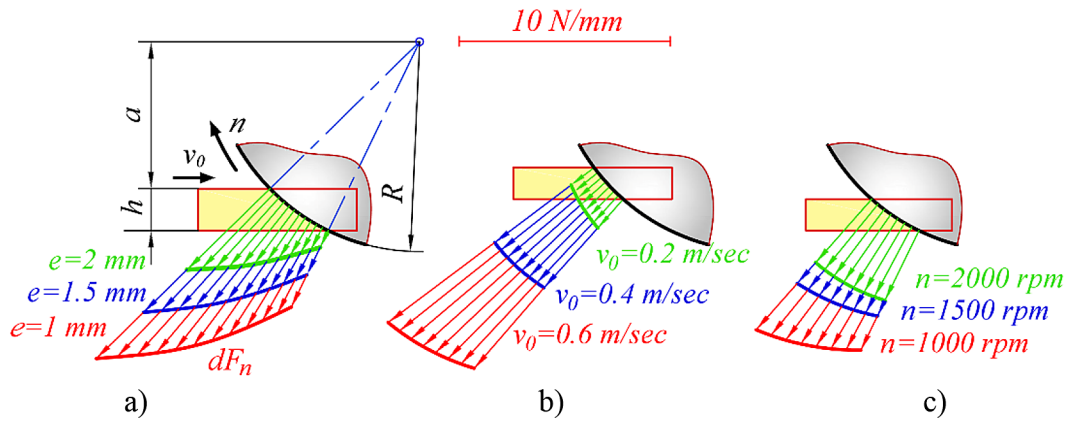
From the analysis of the normal force distribution during cutting with an eccentric knife, the following conclusions can be drawn: the normal force decreases as the rotational speed of the knife increases and increases with higher feed rates. The overall pattern of normal force distribution remains similar for both counter-cutting and direct cutting, although the magnitude of the force is slightly lower in counter-cutting under identical conditions.

Additionally, its magnitude will be lower with an increase in the distance of the block from the knife rotation center. The presence of eccentricity leads to significant fluctuations in normal force, and the range of fluctuations increases with the increase in eccentricity. At certain ratios of kinematic parameters, the normal force drops to zero, and cutting becomes intermittent.

To derive values of the longitudinal component of the total cutting force  $F_x$  and its transverse counterpart  $F_z$  theoretically, the vector of the normal force,  $dN$ , must be projected onto the  $x$  and  $z$  directions, with the resulting expressions integrated along the cutting arc over the angle  $\beta$ , spanning from  $\beta_1$  to  $\beta_2$  (refer to Figures 3 and 5). Taking into account expressions (1) to (7), the value of the normal force is calculated using formula 8.



**Figure 6.** Diagrams of normal cutting force distribution along the cut path by different rotation angle  $\varphi$  of the knife. Parameters: disk radius  $R = 100$  mm, disk rotational speed  $n = 1800$  rpm, feed rate  $v_0 = 0.3$  m/sec, block thickness  $h = 20$  mm, distance of the block from the knife rotation center,  $a = 50$  mm, direct cutting



**Figure 7.** Diagrams of normal cutting force distribution by counter-cutting, disk radius  $R = 100$  mm; (a) disk rotation angle  $\varphi = 330^\circ$ , disk rotational speed  $n = 1500$  rpm, feed rate  $v_0 = 0.8$  m/sec, block thickness  $h = 20$  mm, distance of the block from the knife rotation center,  $a = 70$  mm, (b)  $\varphi = 40^\circ$ ,  $n = 1500$  rpm,  $h = 15$  mm,  $a = 60$  mm, eccentricity  $e = 1.5$  mm, (c)  $\varphi = 75^\circ$ ,  $v_0 = 0.25$  m/sec,  $h = 15$  mm,  $a = 75$  mm,  $e = 1$  mm

$$F_x(t) = \int_{\arcsin\left(\frac{a+e\cos(\omega t)}{R}\right)}^{\arcsin\left(\frac{a+h+e\cos(\omega t)}{R}\right)} K_0 \left[ \arctan\left( \tan(\alpha_0) \frac{v_n}{\sqrt{v_n^2 + v_t^2}} \right) \right]^\gamma R [\cos\beta \pm f \sin\beta] d\beta \quad (8)$$

$$F_z(t) = \int_{\arcsin\left(\frac{a+e\cos(\omega t)}{R}\right)}^{\arcsin\left(\frac{a+h+e\cos(\omega t)}{R}\right)} K_0 \left[ \arctan\left( \tan(\alpha_0) \frac{v_n}{\sqrt{v_n^2 + v_t^2}} \right) \right]^\gamma R [\sin\beta \mp f \cos\beta] d\beta \quad (9)$$

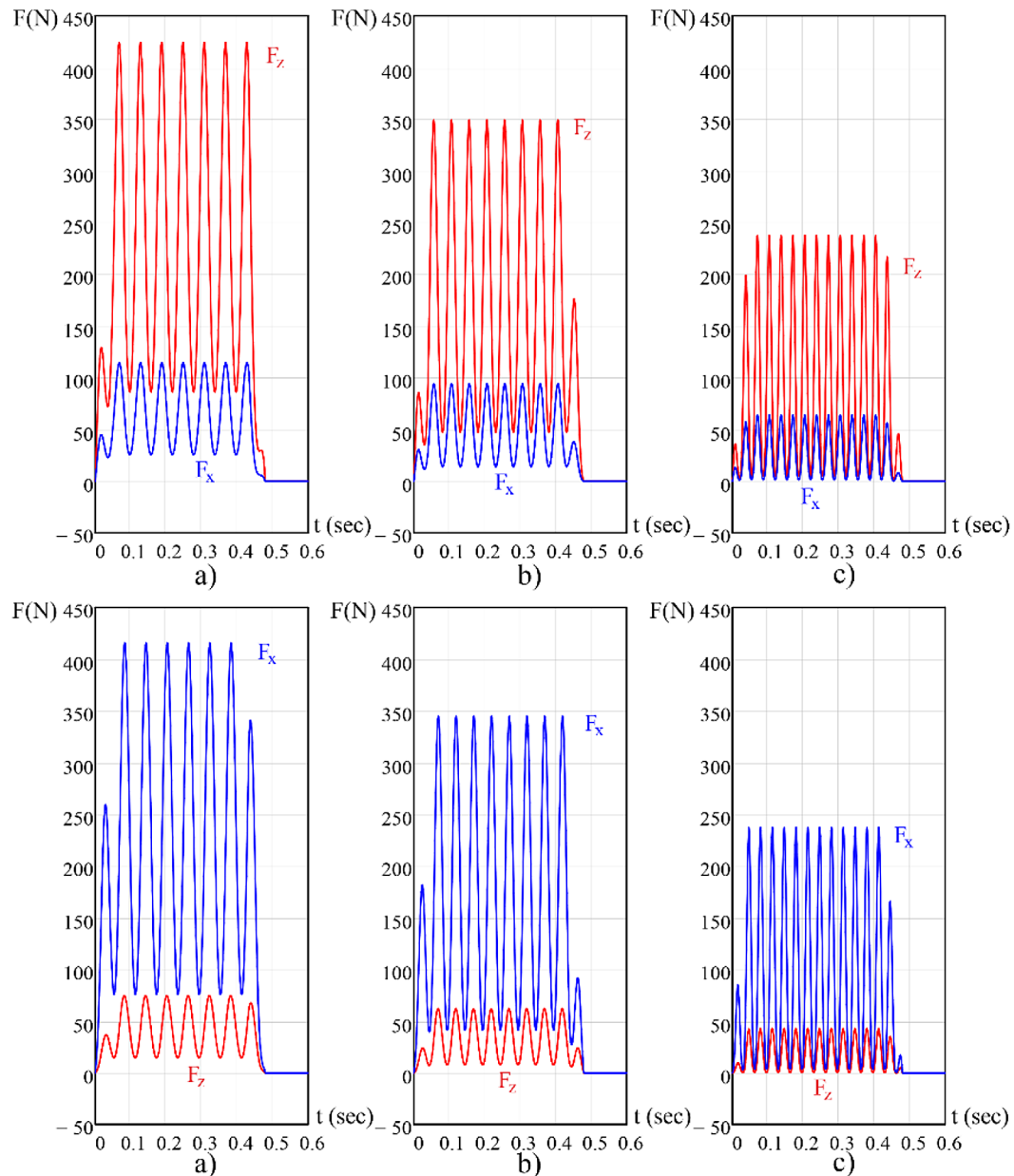
In formulas 9–10 the upper characters refer to the counter-cutting, while the lower characters refer to the direct cutting. Figure 8 shows typical results of theoretical calculations according to formulas (9) and (10) for different parameters of book block cutting. Calculations of the cutting force components across various parameters revealed that the total cutting force is reduced in counter-cutting. Moreover, the longitudinal component  $F_x$  is of higher value in counter-cutting, whereas the transversal component  $F_z$  is greater in direct cutting. In Figure 8, the absolute magnitudes of the force components are shown, although their actual values may be negative depending on the direction of the knife's rotation and the position of the block relative to the rotation axis. This behavior was also described in [19] in studies of cutting with a disk knife without eccentricity.

As it can be seen in Figure 8, both the magnitude and amplitude of the cutting force components decrease as the rotational speed of the knife increases. Figures 8a and 8b depict continuous cutting, while Figure 8c illustrates intermittent cutting, characterized by the cutting force dropping to zero when the knife blade loses contact

with the paper. On the basis of theoretical calculations using formulas (9) and (10), further studies were conducted to examine the influence of additional cutting process parameters, including the knife eccentricity, the distance of the block from the knife rotation center, the feed speed, and the thickness of the book block.

## EXPERIMENTAL VALIDATION OF THEORETICAL MODEL OF CUTTING

To validate the theoretical model, a specialized laboratory stand [16] was constructed, enabling testing of the vibratory cutting process of book blocks under conditions similar to those in production bookbinding lines. The laboratory stand was equipped with two mounted disk knives driven by electric motors controlled via frequency converters (Figure 9). Book blocks were clamped in a special carriage and fed at regulated speeds to the vibrating disk knives. The laboratory stand allows testing of the cutting process on clamped books with variable parameters: cutting lengths up to 240 mm, book thicknesses up to 30 mm,



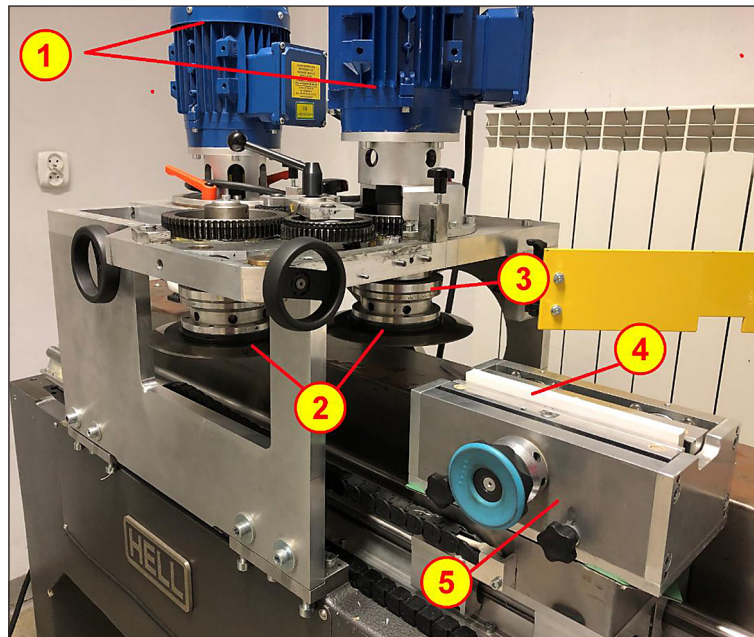
**Figure 8.** Theoretical calculations of cutting force components at different disk knife rotational speeds: (a)  $n = 1000$  rpm, (b)  $n = 1200$  rpm, (c)  $n = 1800$  rpm. Top row – direct cutting, bottom row – counter-cutting, disk radius  $R = 100$  mm, feed rate  $v_0 = 0.3$  m/sec, block thickness  $h = 15$  mm, distance of the block from the knife rotation center  $a = 60$  mm, block length  $L = 130$  mm, eccentricity  $e = 1$  mm

book feed rate up to 3 m/s, knife rotational speeds from 750 to 2000 rpm (12.5 to 33.3 Hz), and eccentricities up to 1 mm. Disk knives with a diameter of 200 mm and a cutting edge angle of  $26^\circ$  were used for cutting the book blocks.

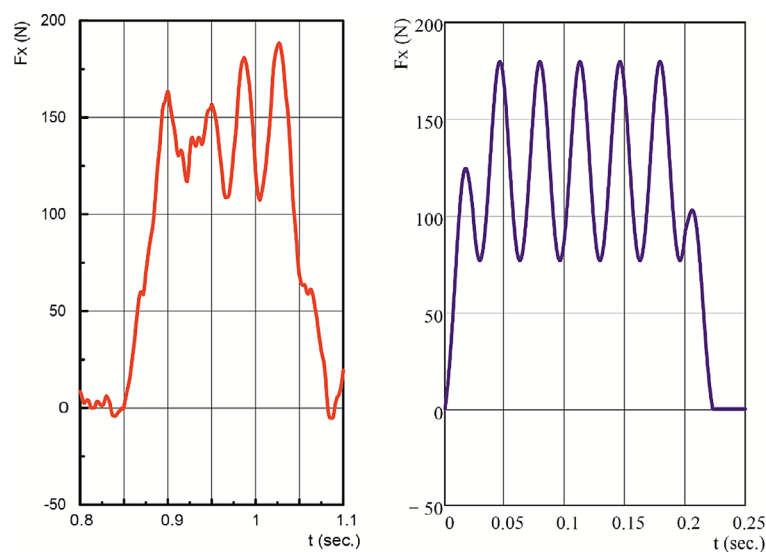
A special made tensometric dynamometer was installed in the carriage carrying the book blocks, enabling simultaneous measurement of three components of the cutting force. A data acquisition and analysis system from National Instruments (USA) was used to record and analyze rapidly changing signals during the cutting process, using specialized

LabVIEW SignalExpress and DIAdem programs. Figure 10a shows an experimentally recorded oscillogram of changes of longitudinal component  $F_x$  when cutting a 10 mm thick book block. Figure 10b shows a graph of the  $F_x$  force, which was calculated based on computer simulations of the process of cutting a book block of the same thickness. Figure 10 shows that the cutting force falls to zero, marking the intermittent phase, the recorded force  $F_x$ , at the moment of separation of the knife blade from the paper, drops practically to zero and increases again from the moment the cutting edge





**Figure 9.** Special laboratory stand for investigating book trimming with two eccentrically installed disk knives. 1 – electrical drive motors; 2 – disk knives; 3 – eccentricity adjustment mechanism; 4 – book block; 5 – movable carriage (conveyor) with sensors for measuring cutting forces in three directions



**Figure 10.** Depicts the recorded (a) and theoretical (b) oscillograms of the cutting force component  $F_x$  in intermittent mode of book block vibratory cutting; disk radius  $R = 100$  mm, rotational speed  $n = 1800$  rpm, book block thickness  $h = 10$  mm, distance of the block from the rotation center  $a = 70$  mm, book block length  $L = 100$  mm, eccentricity  $e = 1$  mm, feed rate  $v_0 = 0.5$  m/s

reenters contact with the block. The dynamic impact of the blade enhances processing efficiency, reduces the cutting force and contributes to lowering the temperature in the cutting zone. The conducted experimental research demonstrated the practical feasibility of using vibratory cutting technology with an eccentrically positioned disk knife, enabling the development of new cutting sections for high-speed automatic bookbinding lines.

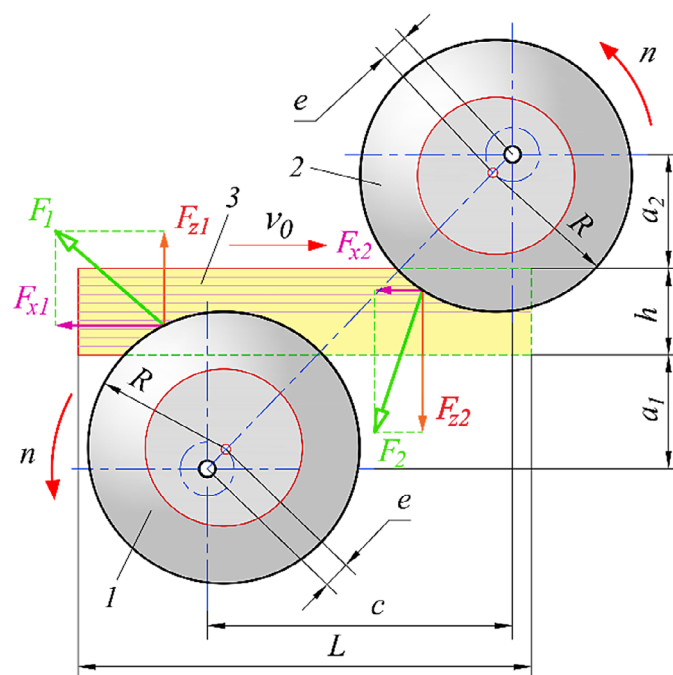
## CUTTING FORCE ANALYSES IN BOOK BLOCK VIBRATORY TRIMMING WITH TWO KNIVES

During vibratory cutting with a single eccentrically installed disk knife, significant vibrations caused by unbalanced inertia forces can be transmitted to the book conveyor. In order to eliminate the negative impact of vibrations on

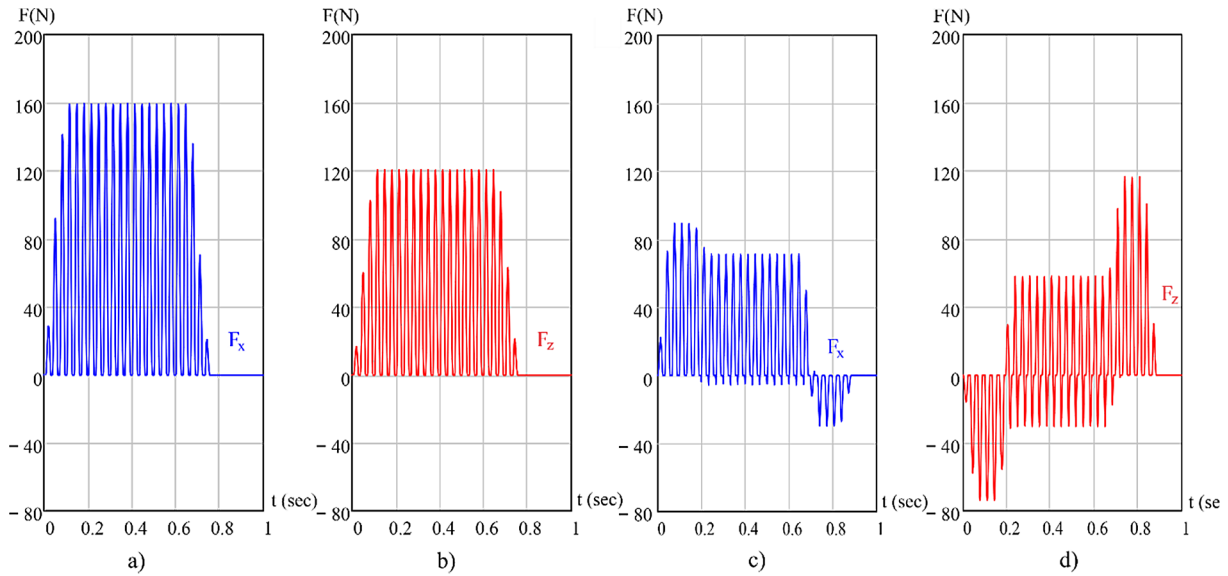
the book conveyor it was proposed to replace the single vibrating disk knife with two disk knives with identical eccentricity rotating synchronously. This configuration helps to compensate for unbalanced inertial forces and reduces vibrations. In the proposed concept of the two-knife setting, the first knife performs a counter-cutting, while the second performs direct cutting (Figure 11). In this configuration of the two-knife setting, the rotational speed of the knives can be increased due to partial system balancing. The height of the trimmed material is reduced by half, the distance between the book block and the axis of rotation of the knife is maximized on both sides, and the transverse forces  $F_{z1}$  and  $F_{z2}$  are also partially balanced. Figure 12 presents the results of computational research based on theoretical calculations according to the formulas (9), (10) for two possible cutting processes applied to the same book block: counter-cutting with a single disk knife (Figure 5b) and cutting with two disk knives (Figure 11). All diagrams illustrate intermittent cutting. The oscillograms in Figures 12c and 12d represent counter-cutting with the first knife during the 0–0.2 s period, cutting with two knives from 0.2–0.7 s, and direct cutting with the second knife from 0.7–0.9 s. The longitudinal force decreases by more than 60%, while the reduction in the transversal force is less significant.

Figure 13 compares four different cutting processes for the same book block under varying kinematic parameters, all ensuring continuous cutting. Figure 13a illustrates direct cutting with one knife, while Figure 13b shows counter-cutting with the same knife. Figure 13c depicts two-knife trimming with equal cutting depths on both knives. Since the cutting force in counter-cutting mode is lower than in direct cutting mode, it is recommended to position the book block asymmetrically so that the cutting depth on the counter-cutting side is greater. Figure 13d demonstrates this approach, which further reduces the cutting force. Vibratory cutting using two eccentrically mounted disk knives effectively reduces the cutting force magnitude, supporting the rationale for designing the machines based on this concept.

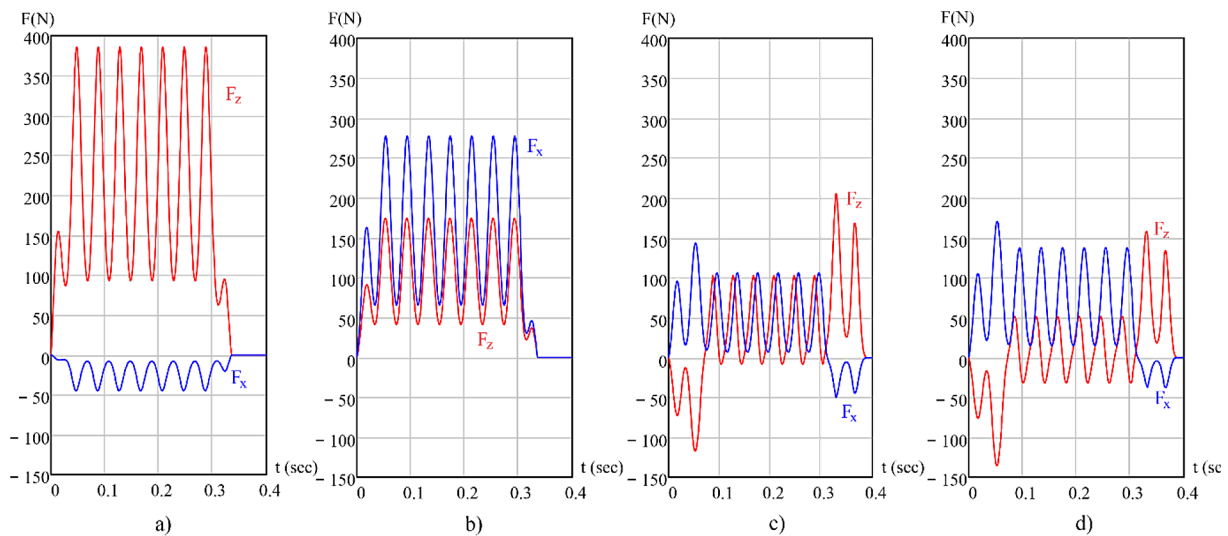
Computer simulations of the vibratory cutting process using two eccentrically mounted disk knives – where the first knife performs counter-rotating cutting and the second performs direct cutting – demonstrated a significant reduction in the cutting force applied to book blocks. As a result of the experimental studies, it was established that the book block feed speed, the rotational speed of the disk knives, value of the eccentric, and thickness of the book have the greatest influence on the cutting force. When cutting in counter-rotary



**Figure 11.** Shows the diagram of the forces components distribution by trimming of a book block with two eccentrically installed knives: 1 – first disk knife, counter-cutting; 2 – second knife, direct cutting; 3 – book block



**Figure 12.** Computational comparison of the cutting force components by cutting book-block with one (a, b) and two (c, d) knives, disk radius  $R = 100$  mm, feed rate  $v_0 = 0.3$  m/s, disk rotational speed  $n = 1800$  rpm, block thickness  $h = 10$  mm, distance of the block bottom from the knife rotation center  $a+h = 98$  mm, block length  $L = 200$  mm, eccentricity  $e = 1$  mm, distance between knives centers  $c = 50$  mm, (a, b) counter-cutting with one knife (c, d) counter-cutting (1<sup>st</sup> knife) plus direct cutting (2<sup>nd</sup> knife)



**Figure 13.** Computational comparison of the cutting force components by cutting; book-block with one (a, b) and two (c,d) knives, disk radius  $R = 100$  mm, feed rate  $v_0 = 0.66$  m/s, block thickness  $h = 10$  mm, block length  $L = 200$  mm, eccentricity  $e = 0.8$  mm, disk rotational speed  $n = 1500$  rpm, distance between knives centers  $c = 40$  mm, a) direct cutting with one knife,  $a = 85$  mm, b) counter-cutting with one knife,  $a = 85$  mm, c) counter-cutting (1<sup>st</sup> knife) plus direct cutting (2<sup>nd</sup> knife),  $a_1 = a_2 = 94.5$  mm (d) counter-cutting (1<sup>st</sup> knife) plus direct cutting (2<sup>nd</sup> knife),  $a_1 = 93.5$  mm,  $a_2 = 95.5$  mm

mode and setting the eccentricity in the range of  $e = 0.5$ – $1$  mm, it is possible to reduce the cutting force of book blocks by 1.6–1.9 times (depending on the type of paper) compared to cutting with a circular knife without setting the eccentricity. The tests also confirmed the feasibility of cutting book blocks up to 30 mm thick. Comparison between

calculated and experimental cutting force values showed good agreement within certain parameter ranges, leading to practical recommendations for reducing cutting force: favor counter-cutting over direct cutting, increase the rotational speed of the disk knives, and maximize the distance between the book block and the rotation axes of knives.

## CONCLUSIONS

Two eccentric knives ( $e = 0.5\text{--}1\text{ mm}$ ) lower the cutting force by 1.6–1.9 times, paving the way for high-speed trimming systems. While eccentric installation of disk knives can induce undesirable vibrations, employing two knives with identical eccentricities rotating synchronously in the same direction (clockwise or counterclockwise) effectively compensates for unbalanced inertial forces and reduces vibration transmission to the machinery. It is well-known that a single disk knife cannot efficiently trim blocks thicker than 5–8 mm due to blade overheating. Using two vibrating disk knives on opposite sides of the book block significantly lowers the cutting force and allows cutting blocks up to 30 mm thick without overheating the blades. The developed analytical formulas and computer simulations provide a reliable theoretical basis for calculating cutting forces in machines equipped with two eccentrically vibrating knives, facilitating the design of new high-speed vibrating cutting sections in automatic bookbinding lines.

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