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Simulation of influence of diameter and other circle parameters on results of incomplete round profile testing

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ABSTRACT

Coordinate measuring technique offers a wide range of possibilities for verifying the quality of a product. Each measurement method has different properties, which predispose it to a certain group of measurements depending on the required measurement uncertainty, dimensions and type of recorded data. However, it should be remembered that the measurement strategy has an equally significant impact on the obtained results. Depending on the size of the objects being tested, research can be conducted on different scales: macro, meso, micro or nanoscale. Each scale requires a different approach and measurement strategy. The article concerns research conducted on the macro-scale, which involves the measurement of geometric features. An example of such a measurement would be a circle and its angular segment, i.e., an arc of varying length expressed in angular measure. Determining the diameter or radius of such a curve is not an easy task in practice and requires collecting a sufficient number of information contained by coordinates of measuring points. The article presents the results of simulated measurements of a 10 mm diameter circle with an ovality deviation. In each case, the full outline (360°) and arc segments of different lengths were evaluated. For a circle with an ovality deviation, three characteristic cases of the location of the extreme points of the circle outline can be distinguished. In the first case, the arc position corresponds to the outline peak, in the second to the valley, and, in the third, it lies between the two previous cases, the most characteristic being the intermediate position. The simulation results are presented for four reference circles (LSCI, MZCI, MICI, and MCCI).

Keywords: diameter, form deviation, roundness, coordinate measurement technique.

INTRODUCTION

One of the most important challenges of any study is to measure related phenomena with sufficient precision and, in many cases, to prove theoretical and simulation results with appropriate experimental tests [1]. The evaluated parameters are used, for example, to determine the geometric characteristics of the element (dimension, position, form) and/or the accordance of the element with the imposed tolerances [2]. As a wide-universal means of determining geometrical characteristics, measurement devices have become widespread, in which a coordinate measuring technique is used, the essence of which is collecting of the coordinates of a series of points in space and then calculation on it a various parameter. A special feature of coordinate measurements is the direct measurement of individual points on the surface of the part and the calculation of the geometric parameters from the results obtained [3]. Coordinate measurements can be made for products composed of free-form surfaces, as well as objects with regular geometric shapes [4]. A precise analysis of measuring errors, taking into account their origin, is essential to obtain reliable and repeatable results, as inaccuracies in the measurement process can lead to distortion of the actual values of the measured parameters [5]. Unfortunately, despite the indisputable benefits of the coordinate measuring technique (CMT), it also has one fault of inaccuracy of results despite using CMMs with similar technical parameters and comparable conditions of measurement [6]. Therefore, measurement simulations are carried out, which allow the identification and modelling of potential sources of error and the consideration of various conditions influencing the measurement, which allows for a better understanding of the influence of individual factors. As a result, simulations can help develop more uniform measurement procedures, increasing the consistency and comparability of results. The user should be aware of this important fact and develop task-related measuring strategies for each measured that will provide the appropriate level of confidence in the final result [7].

One of the actual measurement challenges is the inspection of various holes and shafts [8]. The problem of determining the coordinates of a circle and its radius from a set of measured points of interest is of practical interest [9]. Round holes in industrial parts are a very important feature that is often used as a reference for assembly of other parts, so accurate manufacturing and measurements are required [10]. Estimating the center and radius of a circle from a set of coordinates of points is important in practical applications. However, if measurements are made from only a small part of the circle (small arc), the estimation may have large variations in the results [11].

The degree of approximation depends on the accuracy of the applied measurement method. It should be based on the test objective, which leads to functionality-oriented evaluation processes and the application of correct sampling strategy, which determines the degree to which the function-relevant form deviations are represented in the data sets of measured points collected during measurements. The functionality-oriented evaluation procedure should lead to an optimal sampling strategy, a minimum number and optimal distribution of data points, as well as correct metrological determination of parameters [12]. Before performing measurements, it is necessary to properly define the locations of measuring points and determine their parameters [13]. The sampling strategy must distribute these points across the surface in such a way that the feature is effectively characterized [14], which is valid also for optical CMS [15]. The common practice is to distribute the sample points in a uniform pattern. This does not take into account the surface complexity that may vary across the surface regions. Though the method is very simple, it may often result in inadequate sampling when there are sharp changes in curvatures and unnecessarily more sampling at relatively flat regions, both of which are undesirable in the measurement process. As a result, two cases may arise, i.e. sampling unnecessary data points in flat regions, or ignoring complex regions of the surface in a similar way [16]. Cho and Kim [17] proposed a strategy for selecting the distribution of discrete data points based on the surface curvature. This strategy consists of two stages of surface subdivision. The first stage uniformly divides the surface into several regions. The second stage uniformly subdivides each region into smaller subregions, calculates the surface mean curvature at a uniform grid of points calculated on the subregion, and ranks the subregions according to their average curvature values. Another example could be the new methods for arranging measuring points, which were also proposed by Rajamohan et al. [14]. Similarly to the simulation studies conducted, these methods use the lengths of the curves being studied.

The accuracy, efficiency, and robustness of the localization process are also influenced by many other factors e g. the number of points [4]. The measurement of diameter and roundness deviation for an incomplete contour is ambiguous. The problem of non-measured points is one of the most important issues in measurements. The presence of even a small number of measured points can cause false estimation of surface parameters which can substantially affect the quality assessment of machined elements [18]. These issues can cause potential changes in the results obtained during verification [19]. The position of measurement length about the extrema of the contour has also an appreciable influence on the value of roundness deviation. The higher the deviation value for the full contour, the higher the variation of the deviation for reduced measurement sector length. Variation in the length of the measurement sector and its position also have an impact on the measured value of diameter [20].

One of the key factors that should be properly defined is the measurement strategy, i.e. the number and location of measurement points, evaluation criteria, and filters [21]. An important issue is an assessment of the fidelity of representation regarding an actual shape and the one calculated from measurement data, using calculation algorithms included in CMM software. In many cases, users use algorithms based on the least squares method, which can lead to erroneous shape analysis [22]. When the diameter is being measured, the fitting element plays an important role. The standard ISO 6318 gives four fitting elements for a circle: least square circle (LSCI), minimal circumscribed circle (MCCI), maximal inscribed circle (MICI), and the minimal zone circle (MZCI) [23]. The conducted research shows differences between the radii calculated for the same measurement points but using different fitting methods. A question arises about which method to choose and what criteria should be then applied [24].

SCOPE OF RESEARCH

One of the most common geometric elements in mechanical engineering is the circle. Due to the prevalence of curvatures, measuring them may seem like a trivial task. However, when a precise definition is required, important metrological aspects come into play, such as the type of measurement device, the measurement strategy, and the surface irregularities parameters. Coordinate metrology is one of the most suitable measurement techniques for checking the requirements of the products.

The article presents the results of simulated measurements of a circle with different diameters: 10 mm, 50 mm, 100 mm, 150 mm, and 200 mm. Research includes an analytical part related to simulating individual factors that influence the diameter measurements obtained. The simulation studies assessed the impact of parameters such as diameter size, angular length of the arc, form deviations, position of the circle contour's vertex, and measurement errors on the obtained results. To simulate these factors, an original algorithm was used, which calculated the coordinates of points in the Cartesian system according to the proposed methodology. Then the calculation on simulated points was done by Inspect V8 software.

SELECTION OF PARAMETERS FOR RESEARCH

In the case of a circle with an ovality deviation, three characteristic cases of the location of the vertices of the circle contour can be distinguished (Fig. 1). First, the arc position according to the coordinate system corresponds to the contour vertex, second, the valley, and in the third, it is between the two previous cases, the most characteristic is the intermediate position.

The results were affected by a roundness deviation (RONt) of 0.010 mm, 0.050 mm, and 0.100 mm, respectively. The simulations were conducted without considering the inaccuracy of the measuring device. In each case, the full contour (360°) and arc segments of lengths 270°, 180°, 120°, 105°, 90°, 75º, 60°, 45°, 30°, and 15° were evaluated (Fig. 2).

Random detection of point coordinates with form deviation (ovality) causes significant discrepancies in the results. Therefore, in the study, 1441 points were simulated, evenly spaced every 0.25 degrees along the entire circumference of the circle. As the length of the measurement section decreases, the number of points decreases, while the sampling density between points remains unchanged.

OBTAINED SIMULATION RESULTS

Different reference elements can yield varying results depending on the type of form deviation, accuracy of the measuring device, and distribution of points along the circumference. Therefore, the results are presented for four different circle fitting elements: LSCI, MCCI, MICI, and MZCI. Each method of calculating the reference circle can react differently to errors and inaccuracies in the measurement data. By comparing these methods, it is possible to identify which are more

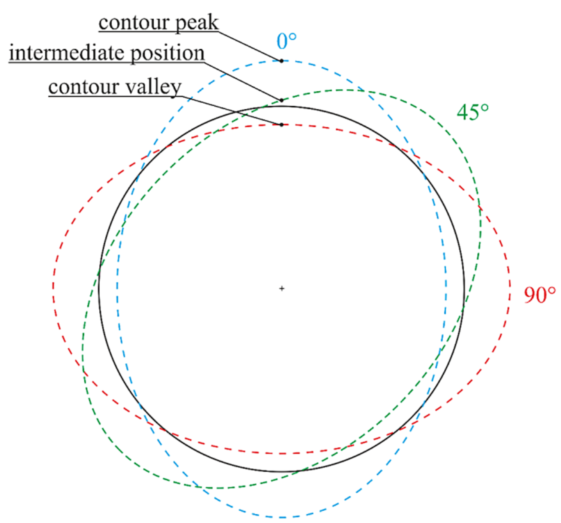

Figure 1. Location of the circle contour vertices

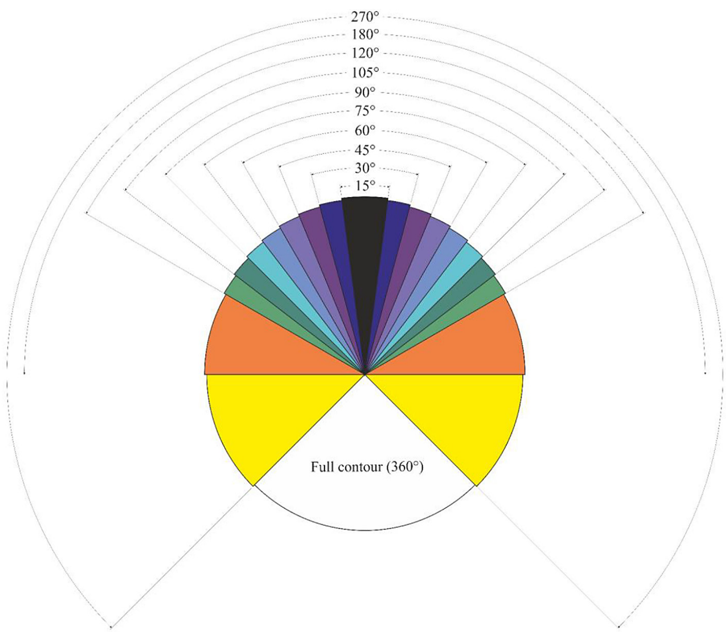

Figure 2. The outline of the measurement segments for the full contour and arc segments

robust to specific types of errors, which allows for a better interpretation of the measurement results.

The simulation results for a diameter of 10mm for different reference elements are shown above. Analyzing the variations in diameter values presented in the graphs, it can be observed that these changes depend on the selected variables. The results are influenced by the shape and magnitude of the form deviation, as well as the length of the measured arc and the contour of the element remaining within the measurement area. This leads to variations in the obtained diameter values and form deviations. For the MZCI (Fig. 3) and LSCI (Fig. 4) elements, the relationship between diameter variations and the length of the measured arc segment is similar. As the angular segment shortens and the RONt deviation increases, the diameter value rises when associated with a contour valley. Conversely, when the angular segment is linked to a contour peak, the diameter value decreases. The changes in diameter for form deviations located between the peak and valley follow a similar trend; however, the diameter varies between these two reference values—it increases when the analyzed region is closer to the valley and decreases when it is closer to the peak of the oval contour. For the other two reference elements, MICI (Fig. 5) and MCCI (Fig. 6), a different pattern of diameter variation can be observed. In particular, the changes are not dependent on the position of contour extremes or the magnitude of the deviation. In both cases, the diameter value remains practically constant until reaching half of the contour (180°). Subsequently, it decreases with different trends for MICI and MCCI, ultimately approaching a value close to zero at an angular segment length of 180°.

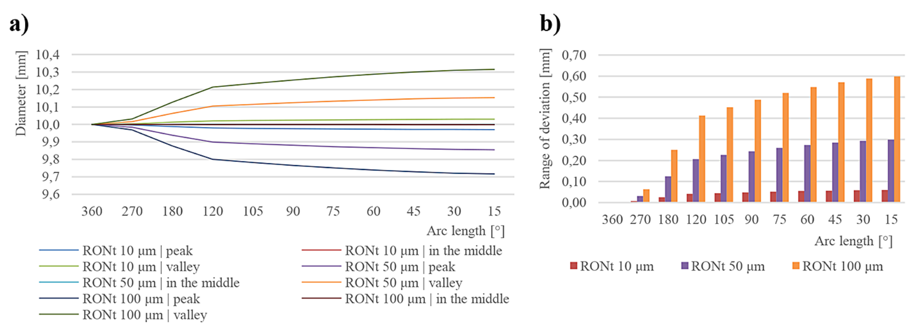

Figure 3. Minimum zone reference circle (MZCI) - Measurement results of a diameter of 10 mm with different deviation values: a) diameter, b) range of deviation

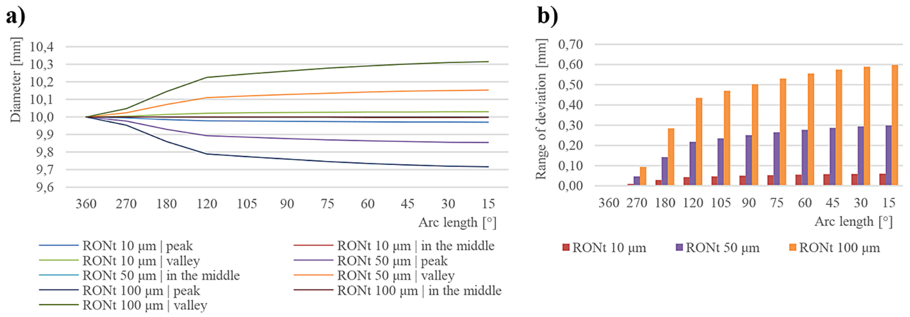

Figure 4. Least squares reference circle (LSCI) - Measurement results of a diameter of 10 mm with different deviation values: a) diameter, b) range of deviation

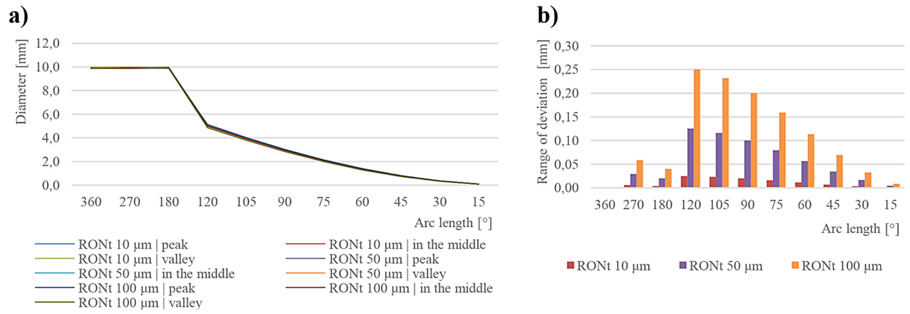

Figure 5. Maximum inscribed circle (MICI) – measurement results of a diameter of 10 mm with different deviation values: a) diameter, b) range of deviation

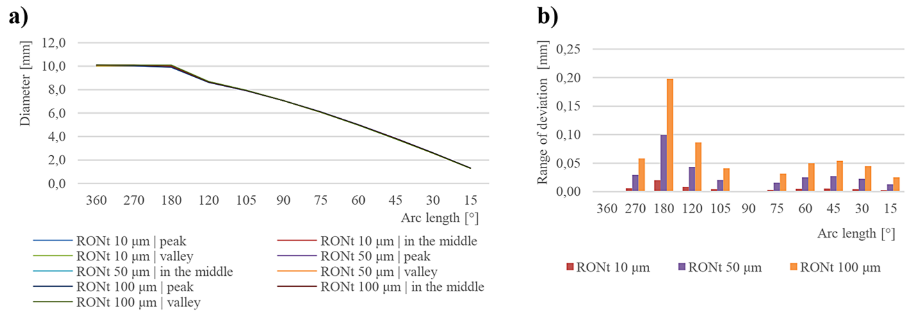

Figure 6. Minimum circumscribed circle (MCCI) – measurement results of a diameter of 10 mm with different deviation values: a) diameter, b) range of deviation

When comparing the values obtained from simulations for circles with diameters of 10 mm and 50 mm, it can be observed that the pattern of diameter variations for the corresponding surrogate elements remains the same. Similarly, when analyzing the variability of deviation values (figures marked with letter b), a consistency in the obtained results can also be observed. The highest variability is noted for circles with a deviation of RONt = 100 µm, while the lowest occurs for RONt = 10 µm. Once again, for the MZCI (Fig.7) and LSCI (Fig. 8) circles, the trend of changes is very similar. In the case of a full circle (360°), the variation in diameter values is close to zero. It then gradually increases, reaching 0.6 mm for an angular segment of 15° and a deviation of RONt = 100 µm. When assessed using the MICI (Fig. 9) circle, a variability pattern can be observed that is not visible in the diameter variation graph (figure marked a). In this case, for an arc with an angular length of up to 180°, the variability remains below 0.06 mm. Then, for a contour of 120°, it rises sharply, reaching up to 0.25 mm for RONt = 100 µm, with proportionally lower values for smaller RONt values. Subsequently, the values decrease as the analyzed arc segment is reduced, approaching zero at a segment of 15°. A different variability pattern is

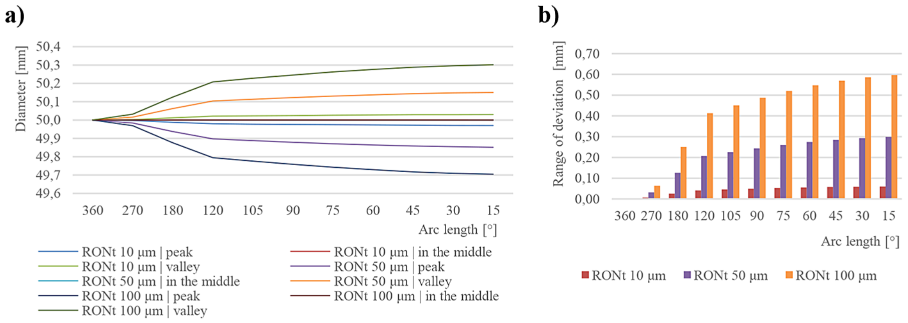

Figure 7. Minimum zone reference circles (MZCI) – measurement results of a diameter of 50 mm with different deviation values: a) diameter, b) range of deviation

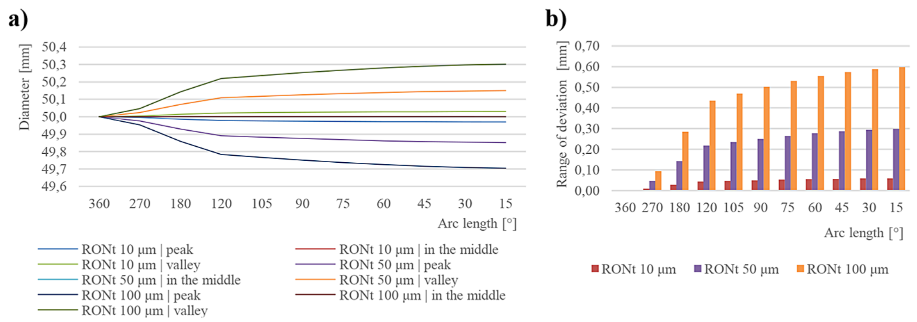

Figure 8. Least squares reference circle (LSCI) – measurement results of a diameter of 50 mm with different deviation values: a) diameter, b) range of deviation

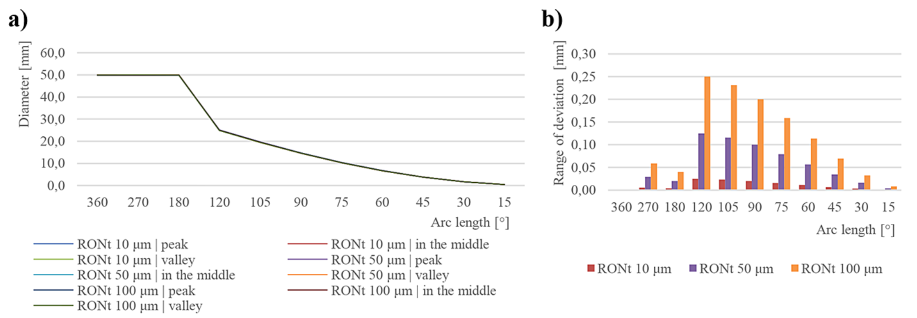

Figure 9. Maximum inscribed circle (MICI) – measurement results of a diameter of 50 mm with different deviation values: a) diameter, b) range of deviation

observed in measurements using the MCCI (Fig. 10) circle. In this case, the maximum value is reached at 180°, followed by a decrease to zero at 90°. It then increases to 0.055 mm at a 45° segment and RONt = 100 µm, before decreasing again to approximately 0.025mm at a 15° segment and RONt = 100 µm.

Analyzing the data presented in the graphs corresponding to the simulation of the measurement of a 100 mm diameter circle, it can be concluded that the pattern of variations is identical to the results obtained in the simulations of 10 mm and 50 mm diameter circles. The only difference concerns the nominal diameter value. Consequently, the variability in diameter values for the MZCI (Fig. 11) and LSCI (Fig. 12) circles oscillates around 100 mm, with fluctuations dependent on the form deviation and the angular length of the analyzed segment. In contrast, for the MICI (Fig. 13) and MCCI (Fig. 14) circles, the diameter remains at approximately 100 mm for segments up to 180°, then decreases almost to zero for a 15° segment in the case of the MICI circle and to approximately 13 mm for a 15° segment in the case of the MCCI circle.

The results of the simulation of the measurement of a 150 mm diameter circle (Fig. 15–18) confirm the consistent trend in the variability of

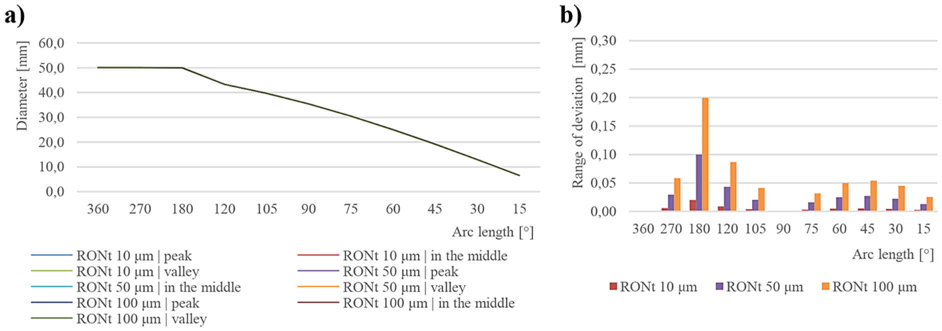

Figure 10. Minimum circumscribed circle (MCCI) – measurement results of a diameter of 50 mm with different deviation values: a) diameter, b) range of deviation

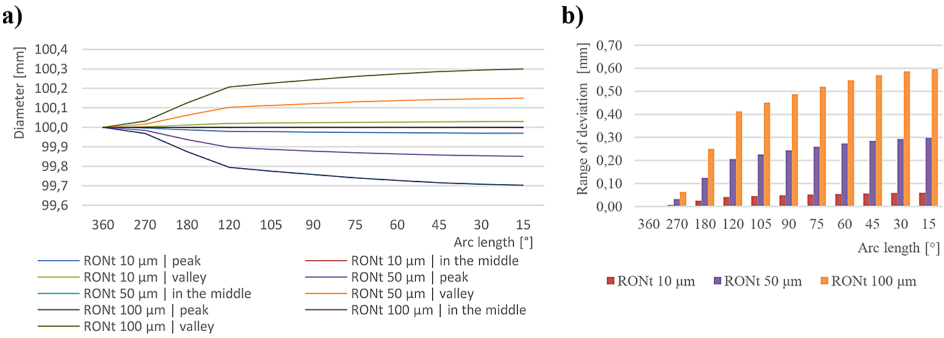

Figure 11. Minimum zone reference circles (MZCI) – measurement results of a diameter of 100 mm with different deviation values: a) diameter, b) range of deviation

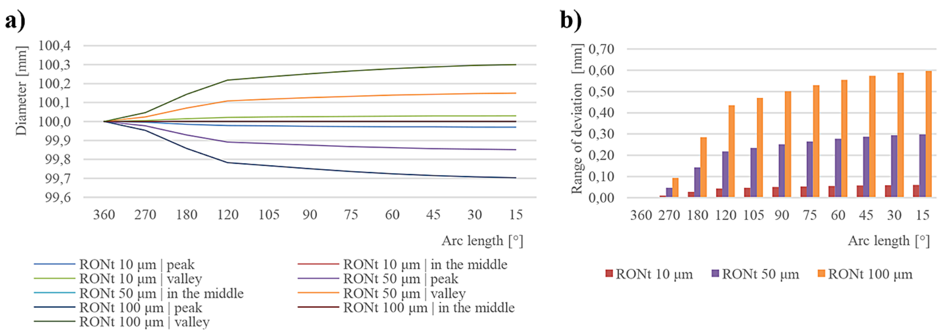

Figure 12. Least squares reference circle (LSCI) – measurement results of a diameter of 100 mm with different deviation values: a) diameter, b) range of deviation

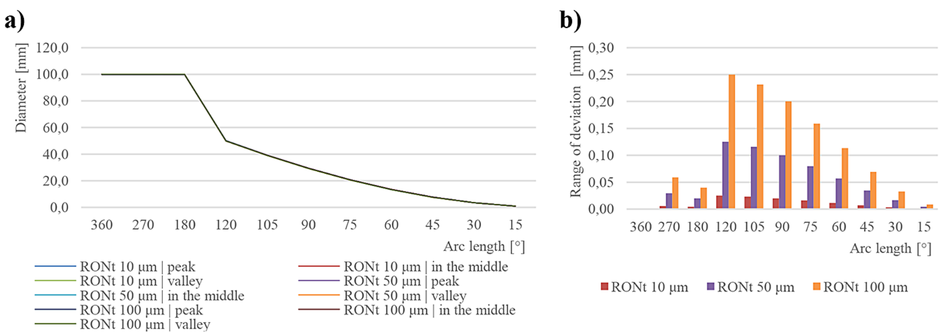

Figure 13. Maximum inscribed circle (MICI) – measurement results of a diameter of 100 mm with different deviation values: a) diameter, b) range of deviation

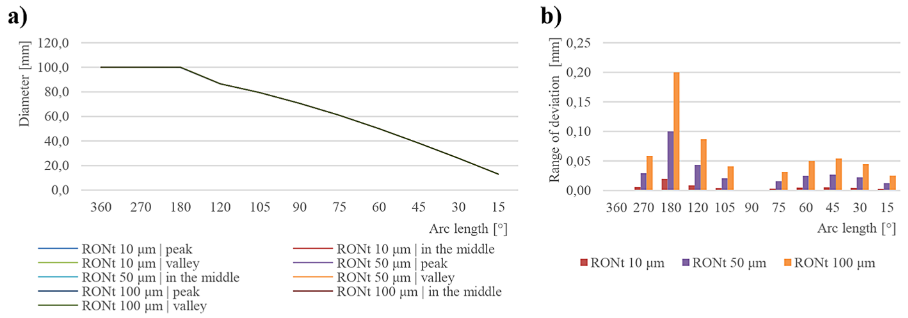

Figure 14. Minimum circumscribed circle (MCCI) – measurement results of a diameter of 100 mm with different deviation values: a) diameter, b) range of deviation

the obtained results, which is influenced by the length of the angular segment, the magnitude of the ovality deviation, and the position of its extrema (peak/valley) relative to the analyzed angular segment.

The simulation results for a diameter of 200 mm for different reference elements are shown above: minimum zone reference circle (Fig. 19), least squares reference circle (Fig. 20), maximum inscribed circle (Fig. 21), and minimum circumscribed circle (Fig. 22). When the results in the figures are analyzed, it can be seen that the diameter values change depending on the angular length of the arc and the type of evaluation element. This phenomenon applies to all analyzed cases, i.e., diameters of 10 mm, 50 mm, 100 mm, 150 mm, and 200 mm. The article presents the results obtained through simulation for all examined diameters. Although some of the obtained values overlap to a certain extent, only the presentation of complete data allows for a comprehensive understanding of the extent to which the values considered in the simulation influence the results. A key observation is that for the LSCI and MZCI circles, the values are consistent, and, most notably, the diameter decreases when a peak

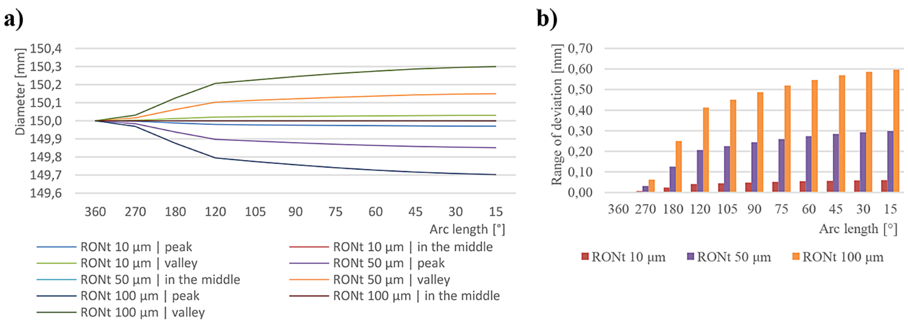

Figure 15. Minimum zone reference circles (MZCI) – measurement results of a diameter of 150 mm with different deviation values: a) diameter, b) range of deviation

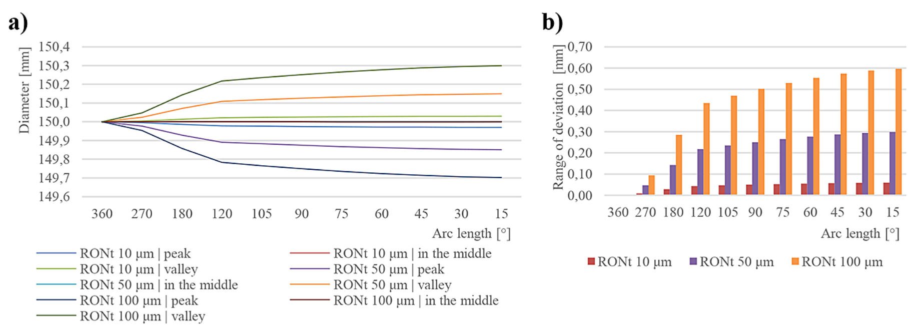

Figure 16. Least squares reference circle (LSCI) – measurement results of a diameter of 150 mm with different deviation values: a) diameter, b) range of deviation

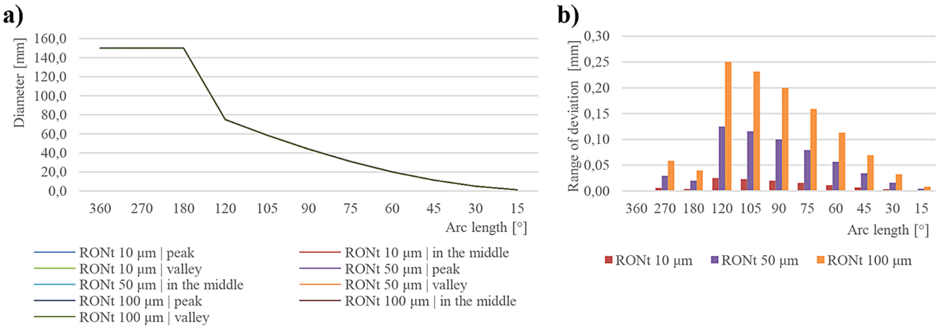

Figure 17. Maximum inscribed circle (MICI) – measurement results of a diameter of 150 mm with different deviation values: a) diameter, b) range of deviation

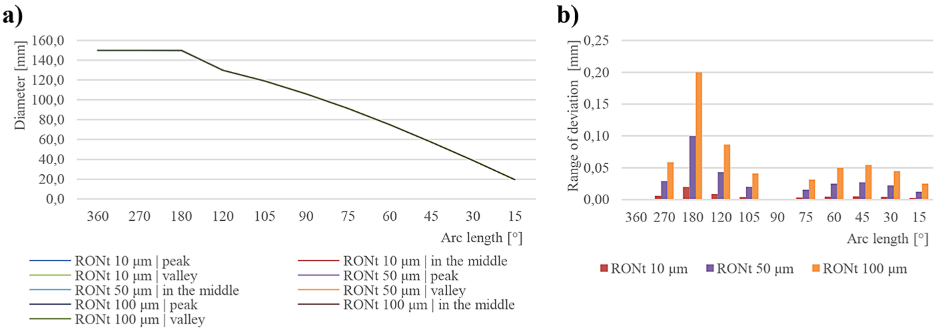

Figure 18. Minimum circumscribed circle (MCCI) – measurement results of a diameter of 150 mm with different deviation values: a) diameter, b) range of deviation

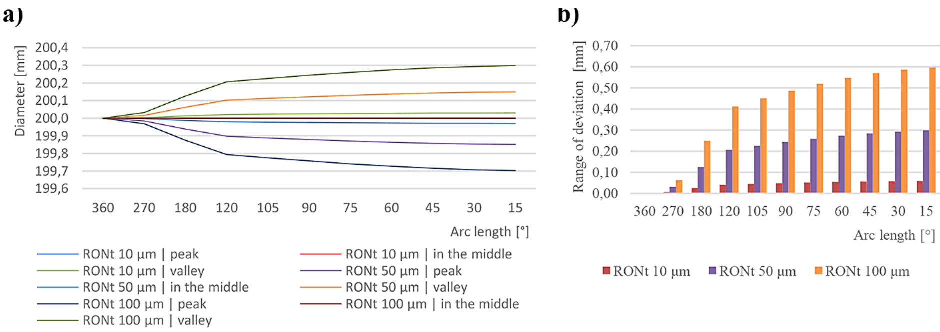

Figure 19. Minimum zone reference circles (MZCI) – measurement results of a diameter of 200 mm with different deviation values: a) diameter, b) range of deviation

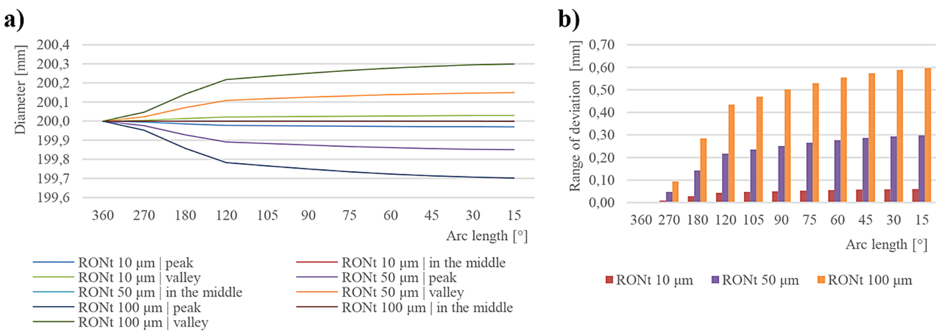

Figure 20. Least squares reference circle (LSCI) – measurement results of a diameter of 200 mm with different deviation values: a) diameter, b) range of deviation

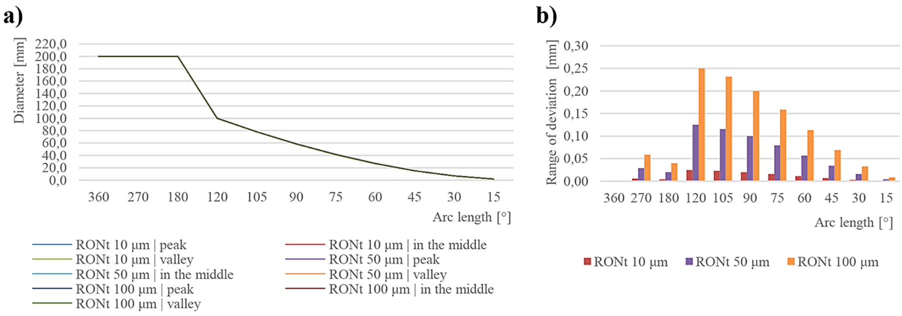

Figure 21. Maximum inscribed circle (MICI) – measurement results of a diameter of 200 mm with different deviation values: a) diameter, b) range of deviation

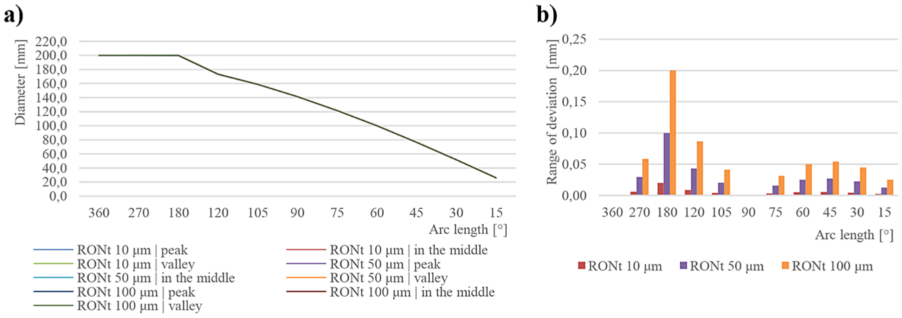

Figure 22. Minimum circumscribed circle (MCCI) – measurement results of a diameter of 200 mm with different deviation values: a) diameter, b) range of deviation

is present in the analyzed segment and increases when a valley is present. The variations in diameter values are directly dependent on the ovality deviation, which is the most common form of roundness deviation in industrial practice. For the MICI and MCCI circles, the diameter value oscillates around the nominal value for a measurement segment within the range of 360° to 180°, after which it decreases almost to zero for a 15° segment.

CONCLUSIONS

This article focuses on simulating and analyzing the results of measuring a circle and its parts (arcs) with the most typical form deviation, which is ovality. This study emphasized the importance of properly selecting measurement and analysis methods, depending on the specificity of the objects studied. The study considered the changes in the roundness value for five different diameters across various angular lengths of the arc – from the full diameter at 360° down to 15°. Different positions of the oval apex were also considered: peak, middle, and valley. A series of comparisons were performed, and then the relationship between different reference elements was analyzed. MZCI, LSCI length of the arc and the type of reference element. The nature of the changes in the results is similar for wall diameters (10 mm, 50 mm, 100 mm, 150 mm and 200 mm). The results of the minimum zone reference circle and least squares reference circle elements follow a similar pattern. It can be seen that for the arc located at the peak, there is a narrowing of the diameter, and for the arc located in the valley, on the contrary, there is a widening of the diameter. The largest increase of the value of roundness deviation occurs in the range of 120° to 270°. In the intermediate position, the diameter remains very stable and almost constant throughout the entire range, indicating that no significant deviations were observed. Analysis of the MZCI and LSCI diameters shows that in ovality measurements, the differences in the RONt deviations have a significant effect on the accuracy of the diameter measurements. As the RONt value increases (from 10 µm to 100 µm), clear changes in the diameter of the peaks and valleys of the oval can be seen. Therefore, at higher deviations (RONt), special attention should be paid to the location of the peaks and valleys, because these places are most susceptible to deviation. At roundness deviations RONt = 10 μm, the diameter changes are small and uniform, whereas at larger deviation values (RONt 50 μm and 100 μm) significant differences in the results can be observed, especially at the peaks and valleys where the diameter decreases and increases, respectively. This translates into a scatter of values that gradually increases with the shortening of the arc. The highest scatter value of 0.6 mm was recorded for the angular sector of 15° with a RONt roundness deviation of 100 μm.

The situation is different for MICI and MCCI. The value varies significantly and drops almost to 0 for the 15° arc. Changes occur in the same way regardless of the value of the deviation of the form and position of the arc relative to the oval vertices. For the maximum inscribed circle, the largest decrease in the diameter can be seen for angular segments between 120–180°. On the other hand, for the minimum circumscribed circle, the diameter gradually decreases from the arc with an angular length of 180°. Differences in this area can be seen in the scatter plots of values. The largest scatter of results for MICI occurs for the angular segment with a length of 120° and is 0.25 mm, and then with the decrease of the segment length the scatter value gradually decreases. The scatter values for the MCCI are irregular. For an angular section of 180° length, the scatter value is the largest for RONt 100 μm and is 0.20 mm. For an angular section of 90 ° length, regardless of the value of the oval form deviation, the scatter of the results is 0 mm.

Each method has its advantages and disadvantages, and its application depends on the purpose of the analysis. In the context of measurements, it is important to understand that different methods may lead to various interpretations of the results, depending on the degree of deformation and the selected angular segment. The least squares reference circle method aims to minimize the sum of the squares of the deviations between the measured points and the theoretical circle. This means that the LSCI graph illustrates the average deformation trend, considering the entire area. The deviations for different RONt values (e.g. 10 μm, 50 μm, 100 μm) are uniform but do not show extremes. This method is better at stably reproducing a circle, especially in shorter angular segments. The MCCI method better reflects extreme deformations, whereas LSCI provides more averaged values, especially over longer angular sections. The MICI method aims to include as many measured points as possible, resulting in greater variability in peak and valley deviations. The minimum zone reference circle includes as much deformation as possible, leading to larger differences between peaks and valleys. On the other hand, MICI minimizes these differences, making the results smoother. The graphs for MZCI have more pronounced differences in diameter size for the RONt values (50 μm, 100 μm), while MICI minimizes these differences.

When analyzing arc fragments, methods that consider both local deformations and the overall shape should be used to make the results more representative of the actual object. Using maximum inscribed circle and minimum circumscribed circle for arc fragments can lead to serious distortions of the diameter results. The MICI consists of finding the largest circle that can be inscribed in a set of points that represent the outline or the arc fragment. For a full circle, this method gives reasonable results because it contains the entire curve, and any form deviations (e.g. peaks, valleys) have an even effect on the results. However, for small arc fragments, this method tends to underestimate the value of form deviations. In practice, using MICI for arc fragments leads to results that underestimate the diameter value. Minimum circular circle finds the smallest circular circle that contains all the points on the outline. For a full circle, this method works as intended because it includes the largest deviations around the entire circumference. However, for arc segments, this method tends to overestimate the diameter. This method only includes those points that are on the outline, ignoring the rest of the circumference. As a result, the results for arc segments may show much larger diameter values, which falsifies the true deformation measurement, especially in cases where the angular segments are very small.

In summary, measuring the circle for small arc segments is not an obvious task. The curvature, which in this case is ovality, has a significant impact on the obtained results. To perform a detailed analysis of the results presented, it is necessary to pay attention to several key elements related to the position of the circle diameter (peak, intermediate position, and valley), the angular length of the arc, and the type of the evaluation element in the context of different deviations of ovality RONt. The authors plan to expand on this topic in future publications. To ensure a precise analysis, preliminary research has been conducted, including computer simulations of various measurement scenarios, which will be compared with real data obtained from contact and non-contact measurements. During laboratory tests, the influence of parameters such as diameter, angular length of the arc, measurement errors, and the number of measurement points on the obtained results will be analyzed. For each diameter, a series of measurements will be carried out under controlled and repeatable laboratory conditions, allowing for the acquisition of reliable data and a detailed analysis of the impact of various parameters on measurement results. Additionally, the authors plan to extend the study to include various forms of roundness deviation. So far, only ovality has been investigated, but future research will consider other geometric deviations, such as lobing. The objective of future studies is to compare the results obtained using different contact and optical measuring devices, with various measurement strategies, considering their influence on the results, and to identify best practices for selecting a metrologically correct measurement strategy.

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