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Optimization of gear pump operating parameters using genetic algorithms and performance analysis

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ABSTRACT

This study explores the optimization of operating parameters and the design of innovative gear pumps featuring multi-involute tooth profiles. Using a genetic algorithm, optimal parameters such as the basic radius, tooth height, and angles of involute profiles were determined. Results indicate that the proposed optimization framework significantly improves the volumetric and total efficiency of the pumps. Specifically, a volumetric efficiency increase of 8–12% and a reduction in noise levels by 3–5 dB were observed compared to conventional designs. The experimental validation confirmed the robustness of the proposed model, showcasing its potential for industrial applications. This work highlights the integration of advanced computational techniques with engineering design to achieve enhanced performance metrics.

Keywords: gear pump, genetic algorithm, multi-involute profile, optimization, volumetric efficiency, noise reduction, operating parameters.

INTRODUCTION

The evolution of the gear pump design has spanned nearly four centuries. Johannes Kepler, a pioneer and creator of the first gear pump, patented his innovative solution in 1604. His invention, originally designed for pumping water in dewatered mines, utilized a valveless and self-priming design consisting of two gears. This construction was exceptionally resistant to contaminants present in the water removed from mine shafts, allowing it to quickly replace the cumbersome piston pumps of that time [1–4]. The inventor saw broader applications for his invention [5–7]. Kepler also envisioned extensive applications for his invention, proposing its use for bilge drainage on ships, in park and garden fountain installations, and for air transfer in blowers and exhausters. The late 1880s saw the emergence of the first designs with reduced pulsation. The precursor was H. Hoppe's company, which [8] was the first to develop a design with split gears (Figure 1 in the appendix). The design used three pairs of cooperating teeth, rotated relative to each other by a value corresponding to 1/3 of the pitch, which, according to the creators of the 1882 patent, significantly reduced the coefficient of performance unevenness. The gear teeth had a cycloidal profile.

Introducing asymmetry to the profile by removing material from the tooth root on the side responsible for forcing the fluid flow increases the pump's efficiency by enlarging the inter-tooth space and reducing inter-tooth friction, which in turn enhances the durability of the cooperating gears. Additionally, removing material in the root area limits the formation of a trapped volume, which positively affects noise and pressure pulsation reduction. In Figure 1, the areas from which material has been removed are indicated by number 17.

Over the past decade, there has been an increasing effort in researching and producing

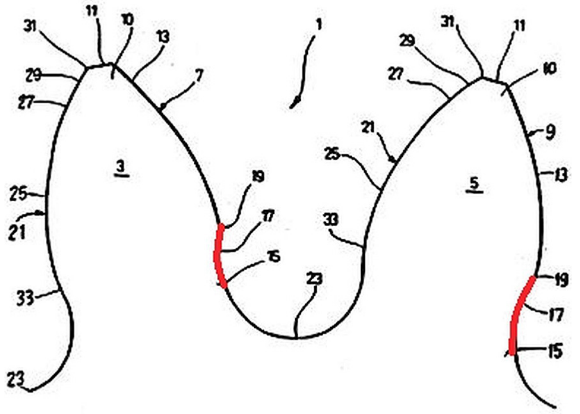


Figure 1. Modification of the tooth profile according to patent no. US5,454,702, with the material removal area indicated by number 17

pumps with special profiles dedicated exclusively to use in gear units. The primary motivation for this is the pursuit of better hydraulic and acoustic properties. Among pumps with special profiles, notable examples include the Maglotta, Hitosi (according to patent US3164099), Catania (according to patent US 2011/0223051), and Klassen (according to patent US8118579) profiles. The development of gear unit designs is moving towards the reduction of flow pulsations, which can lead to improper functioning of control elements, vibrations, and increased noise levels in machines and devices with hydrostatic drives. Production gear units with external gearing are characterized by a flow irregularity coefficient averaging around 18%. Compared to other displacement units, gear pumps have one of the highest flow pulsation rates. Over the centuries, gear pumps have become essential components in hydraulic systems, enabling efficient energy transmission across a wide range of applications. However, despite their long history and significant technological advances, gear pumps continue to be an active area of research. New challenges, such as minimizing flow pulsation, reducing noise emissions, and improving overall efficiency, drive modern innovations in this field. This paper aims to contribute to this ongoing development by analyzing operating parameters and exploring optimization methods for gear pumps with innovative profiles.

Research methodology overview – the methodology employed in this study integrates advanced computational tools and experimental validation to optimize the design and performance of gear pumps with involute profiles. A genetic algorithm was applied as the primary optimization technique, focusing on parameters such as tooth profile geometry, dynamic viscosity, discharge pressure, and rotational speed. The effectiveness of the algorithm was assessed through numerical simulations and validated against theoretical predictions of hydraulic and acoustic performance. Noise measurements were conducted following ISO 3746:2010 standards, while computational results were corroborated using prior experimental data available in the literature. This comprehensive approach ensures both the reliability and applicability of the findings in real-world hydraulic systems.

Current trends in gear pump development

Despite numerous patents and publications, and the fact that gear pumps are currently produced on a massive scale, there is still room for improvements in technical methods to ensure optimal internal sealing, maximize operating pressures, and minimize flow irregularities and noise emissions. An analysis of numerous patents, literature, and currently manufactured gear units might suggest that technical methods ensuring optimal internal sealing, maximum operating pressures, minimal pulsation, and noise emissions have been exhausted. The valveless and self-priming gear construction, resistant to numerous contaminants found in the water removed from shaft wells, quickly supplanted the then-used and troublesome piston pumps. The first mentions of gear pump construction appeared in the works “Hilaria mathematica” (1624) and “Recreation mathematique, composee des plusieursproblemesplaisants et facetieux” (1626). These works are attributed to Jean Leurechon [9]. The wider application of gear units resulted in the development of design fundamentals. The first empirical and theoretical relationships defining the pump’s performance were developed by Amman [10], Falc, Zass, Thomey, and later by Judin [11] and Gutbrod [12]. In 1966, Otto Eckerle’s company filed a patent in the U.S. Patent Office for a pioneering concept at the time – a pump with radial clearance compensation for units with external gearing (patent no. US3472170). A year later, Niell, Cygnor, and Sundberg filed a patent application that included five design solutions for radial clearance compensation under a single patent (no. US3437048). The 1990s saw the emergence of generators capable of reaching pressures up to

32 MPa, along with designs featuring so-called zero side clearance [13–15]. To ensure proper operating parameters, the axes of the gears in such solutions are mounted with precisely defined clearance and subjected to an additional radial force acting on the split bearing housings. The turn of the 20th and 21st centuries brought further attempts to reduce pulsation by appropriately shaping the tooth profile. Modifications were made to both the involute profile [16–20, 21–22] and non-involute profile [13, 21–27], resulting in a smoother instantaneous flow rate.

However, our own research and considerations in the context of the mentioned analysis contradict the thesis that the peak technical and operational parameters of contemporary gear units have been reached. On the contrary, the development of processing and production technologies allows for the creation of increasingly innovative concepts. Recent work by various authors continues to develop solutions for gear pumps, introducing new technologies and improvements [21–39]. For instance, in [30], the results of an experimental study on the sound pressure level (SPL) in a power unit with an external gear pump are presented. The study was conducted on a specially developed experimental laboratory setup, based on common architecture used in hydraulic units. This research provided detailed data on noise sources and sound generation mechanisms in such systems, enabling the development of more effective noise reduction methods for future gear pump designs. In [31], two numerical approaches to studying helical gear pumps were compared. A three-dimensional CFD model and a new tool, EgeMATor MP+, were developed for a complete analysis of these pumps. The models were compared under the same boundary conditions, focusing on steady-state volumetric efficiency and outlet pressure oscillations. The detailed data showed that the CFD model exhibited slightly better accuracy in predicting the behavior of pumps under real-world conditions.

The study [32] described a new methodology for examining the impact of complexity on an external gear pump model to create a data source for AI-based condition monitoring applications. A detailed geometric model was described and compared with two earlier models by the authors, finding that the detailed geometric model is superior for injecting faults, making it a suitable tool for generating data for condition monitoring. In Stryczek's study [33], a new trend in the design of hydraulic elements and systems was discussed, focusing on replacing traditional metal materials with plastics. The novelty of this research lies in the introduction of innovative methods for producing and testing plastic hydraulic elements. Study [34] developed a 3D gear pump analysis procedure based on CFD. It focused on analyzing the unique three-dimensional design of a multi-chambered gear pump (TMC), which features self-adjusting part sealing, the possibility of forming parts from polymers, and high volumetric efficiency. The procedure analyzed flow and pressure conditions in the TMC pump, considering the impact of part positioning on changes in sealing areas and pressure field propagation concerning the valve plate position. In [35], the design of a movable casing for friction reduction in an external gear pump used in hydraulic actuators was discussed. The research focused on analyzing mechanisms that reduce friction between pump elements, enhancing work efficiency and device durability. An innovative design solution was developed in the form of a movable casing that dynamically adapts to pump working conditions, minimizing energy losses due to friction. Study [36] focused on developing an optimized gearbox design, increasing pump efficiency and reliability while reducing its size and weight. The optimization of the gear profile also played a significant role in this study, further enhancing system efficiency and reliability, minimizing energy losses, and ensuring smooth operation. In [37], robust optimization of gear tooth modifications using a genetic algorithm was presented. The research focused on developing a method that combines analytical results with numerical simulations to achieve optimal tooth profile modifications. Study [38] introduced the use of dynamic analysis methods to optimize tooth shapes to minimize vibrations and an innovative method for evaluating the impact of various profile modification curves on the vibration responses of straight teeth. Lastly, in [39], the authors described an innovative method for optimizing tooth profile modifications to reduce vibration and noise in straight gear pairs. The research focused on analyzing the impact of profile modifications on the vibration and noise generated by the gears.

These studies demonstrate that the development of gear pumps is moving towards increasing efficiency, reliability, and reducing undesirable phenomena such as cavitation and wear. Innovative approaches, such as the use of polymer materials, the design of movable casings, and advanced simulation methods, open new possibilities in optimizing these devices for various applications, including advanced electro-hydraulic drive systems in automotive applications.

ANALYSIS OF A GEAR PUMP WITH A POLYINVOLUTE TOOTH FOOT PROFILE

The authors, Piotr Osiński and Adam Deptuła, have been conducting research in this area for many years, focusing on innovative solutions and the optimization of the polyinvolute profile of gear pumps [40]. The conducted considerations and their own research, in light of the above analysis, contradict the notion that the peak of technical and operational parameters achieved by contemporary gear units has been reached. On the contrary, advancements in processing and manufacturing technology enable the development of increasingly bold concepts, which enhance the efficiency and durability of gear pumps.

Previous studies on the optimization of the polyinvolute tooth profile

In the study [41], a discrete optimization of a gear pump after root undercutting was presented using multi-valued logic trees. The research focused on identifying and minimizing key parameters affecting the efficiency of gear pumps. The results show that the use of multi-valued logic trees allows for significant improvement in the performance and durability of pumps through precise adjustment of the tooth profile. In the model of the involute tooth profile (Figure 2), it was assumed that as a result of rounding or chamfering the cutting edge, the effective apex line would be shifted towards the root radius of the tool by the value of the apex clearance l_w. The profile shift correction $+x \cdot m_0$ was also considered.

The study [42] presents a method for identifying the impact of part tolerances on the overall efficiency of the 2PWR-SE type pump. The research used multi-valued logic trees and decision trees to determine the most important parameters affecting pump performance. The results show that precise adjustment of production tolerances allows for increased pump efficiency. In the study analyzing the impact of dimensional tolerances on the hydraulic and acoustic properties of new prototype gear pumps [43], the conducted analysis of dimensional and shape tolerances ultimately allowed for the selection of control dimensions: critical, important, and less important. This resulted in a rational narrowing of tolerances for dimensions and shapes where necessary and lowering the accuracy class in less significant areas. Optimizing the manufacturing technology contributed to reducing production costs and increasing its efficiency. In the latest study, the acoustic properties of gear pumps were analyzed using the classification of induction trees [44]. The research focused on identifying noise sources

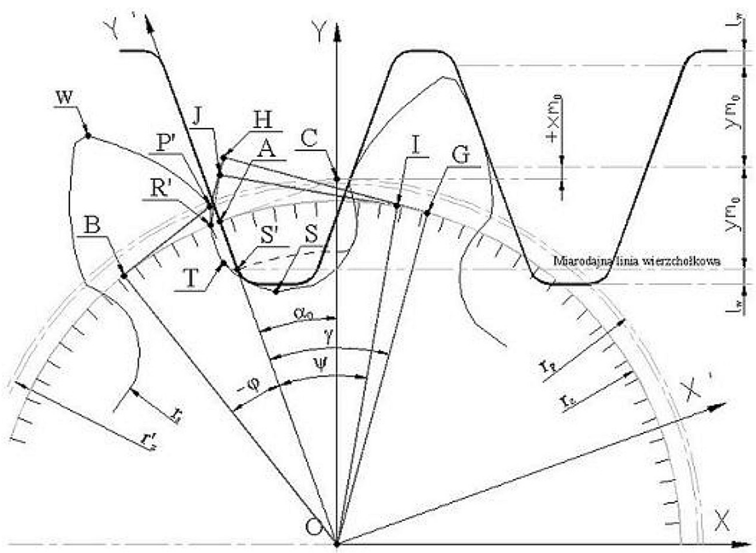


Figure 2. Undercutting the root with a trapezoidal tooth [41]

and developing noise reduction methods using advanced classification techniques.

The analyzed pump

The original pump tested was a conventional unit, manufactured by the Wytwórnia Pomp Hydraulicznych Sp. z o. o. located in Wrocław, Poland. The experimental pump was designed for the technological capabilities of WPH S.A., where this specific pump has been studied many times, and the results are published within the scientific literature [45]. The newly designed and realized prototype pump is a three-plate structure shown schematically in Figure 1. The front plate (1) is used for mounting the pump on the drive unit. The middle plate (2) contains gear wheels, slide bearing housings, and suction and forcing holes for connecting to a hydraulic system. The whole construction is closed with a rear plate (3) (Figure 3). The main meshing parameters for the pump with a unit delivery q = *40* cm³ are listed in Table. 1.

To date, the primary research conducted in [45] focused on the application of the multi-valued logic trees method. Considering technological efficiency, it was assumed that the polyinvolute tooth contour would consist of three basic involutes: two ordinary involutes and one extended involute or three ordinary involutes (Fig. 4).

This solution is protected by a co-authored patent. The upper ordinary involute in the polyinvolute contour is a modification of the apex contour compared to conventional designs. The advantage of this solution is the maintenance of the same internal diameters within the pump casing.

OPTIMIZATION OF POLYINVOLUTE TOOTH PROFILE USING A GENETIC ALGORITHM

Previous research has primarily focused on optimizing the manufacturing technology of gear pumps with a polyinvolute tooth profile. However, a detailed analysis of the impact of operational parameters on the overall efficiency of these

Table 1. The main meshing parameters for the pump

Parameter	Symbol	Unit	Value
Number of teeth	z	-	9
Modulus	m_0	[mm]	4.5
Pressure angle	α_0	[°]	20
Gear wheel width	b	[mm]	32.2

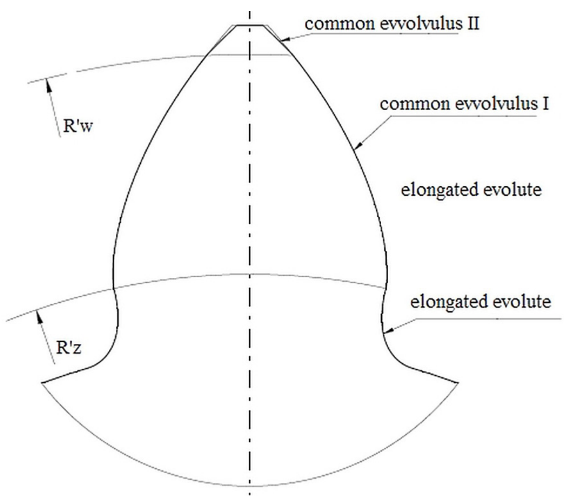
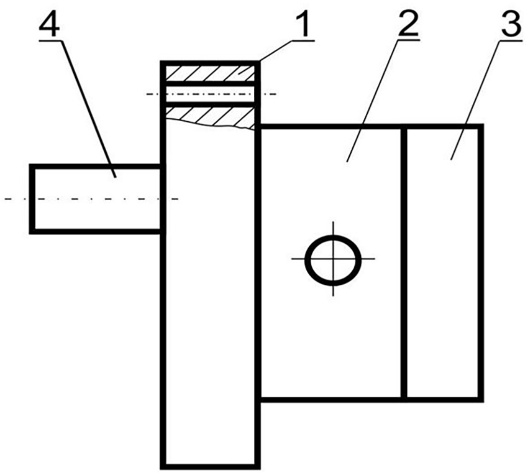


Figure 4. The three- involute tooth profile: α_0^1, α_0^2, α_0^3



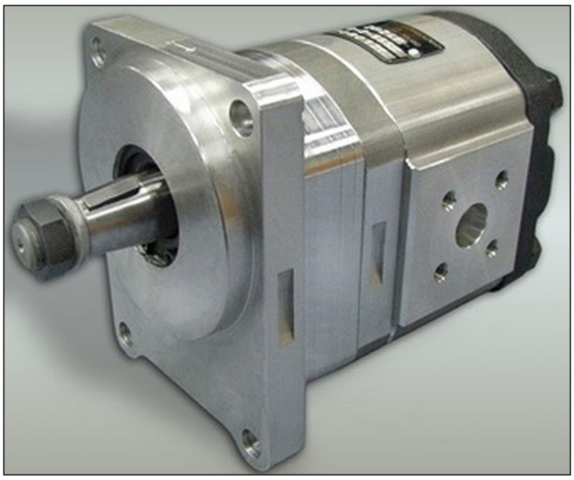


Figure 3. Three-plate construction of an external gear micropump. 1 – front plate (mounting), 2 – middle plate (ocular), 3 – rear plate, 4 – drive shaft

pumps has not been conducted. This article thus focuses on determining the optimal polyinvolute tooth profile, considering parameters such as the viscosity of the working fluid (both kinematic v [m²/s] and dynamic μ [Pa·s]), discharge pressure p_t [MPa], and rotational speed n [min^{-1}] of the gear units. The main goal is to achieve maximum overall efficiency η_c [%]. To achieve this goal, a genetic algorithm was applied.

Genetic algorithm

Genetic algorithms (GAs) are metaheuristic optimization methods that mimic the evolutionary processes occurring in nature. They were first proposed by John Holland in the 1960s and are used to solve complex optimization problems that are difficult to address with traditional methods. A genetic algorithm is a type of algorithm that searches the space of alternative solutions to find the best ones. The operation of genetic algorithms intentionally resembles the phenomenon of biological evolution because their creator, John Henry Holland, drew inspiration from biology for his work. In recent years, there has been a growing body of research focusing on optimization methodologies and their applications across various engineering domains. Genetic algorithms, in particular, have demonstrated immense potential in solving complex optimization problems due to their adaptability and robustness. These methods have been effectively applied in diverse areas, such as welding process parameter optimization, fractional sliding mode control, and quality control processes. For example, Bensaid et al. [46] optimized tungsten inert gas (TIG) welding parameters to improve joint quality between dissimilar materials, using advanced statistical and optimization techniques. Similarly, Pouya and Pashaki [47] introduced a multi-objective genetic algorithm for optimizing control variables in a fractional sliding mode control system, demonstrating significant reductions in system errors and improved accuracy.

In the context of industrial applications, Chmielowiec et al. [48] employed a niche-preserving genetic algorithm (NPGA) to optimize quality control mechanisms by balancing operational costs and defect detection capabilities. Additionally, Kovalchuk et al. [49] presented a mathematical model for the dynamic processes of a drilling rig pumping unit, highlighting the influence of friction clutch parameters on operational performance. These studies underscore the versatility of genetic algorithms in addressing multidimensional engineering challenges. In this work, we build upon these advancements by applying a genetic algorithm to optimize the performance and design of gear pumps, focusing on parameters such as tooth profile geometry, dynamic viscosity, and acoustic performance.

The genetic algorithm operates on a population of individuals:

$$P^n = \left\{x_1^n, x_2^n, \ldots, x_R^n\right\} \tag{1}$$

where: n is the generation number, and R denotes the population size.

Basic elements of a genetic algorithm include:

- population: a set of potential solutions, known as chromosomes.
- chromosome: a representation of a single potential solution to the problem.
- gene: a component of a chromosome, corresponding to a specific decision variable. Selection: The process of choosing the best chromosomes for reproduction based on their fitness function values.
- crossover: the process of exchanging genetic material between chromosomes, leading to the creation of new offspring.
- mutation: the introduction of random changes to the genes of chromosomes to maintain genetic diversity in the population.
- fitness function: a function that evaluates the quality of a given chromosome in the context of the problem being solved.

A genetic algorithm (GA) is a search and optimization method that operates by mimicking the principles of evolution and chromosome processing in genetics. It begins the search from a random set of solutions, typically composed of variables and random objects. Each solution is assigned a result directly related to the fitness function, which explicitly determines how well a specific solution meets it.

The algorithm processes the populations as follows: P_t – the base population, O_t: – the offspring population, T_t – the temporary population, which stores copies of individuals from P_t. Each population contains the same number of individuals. In the "initialization P_t" step, the base population is filled with randomly generated individuals. For each individual, the value of the fitness function is calculated during the evaluation stage. The classic genetic algorithm follows the scheme presented in Figure 5. To apply the genetic algorithm to optimize the shape of the polyevolvent tooth in gear pumps, it is necessary to determine the key geometric parameters describing the tooth shape. The tooth geometry is mainly associated with the construction of three types of involutes: standard involute, extended involute, and shortened involute. Each of these types of involutes has unique properties that affect the shape and functionality of the tooth in gear pumps. All these aspects are detailed in the works [20, 40, 43–44].

Gear wheels with optimized, three-involute profile

The calculation of unit efficiency q must be preceded by the determination of the area contained below the standard involute. For this purpose, two radii are drawn from the initial and final points of the involute, from which the bounded area can be defined by the following equation [40]:

$$S = \frac{1}{2} \cdot \int \rho^2 d\beta \tag{2}$$

After marking the length of the tangent drawn to the circle with a radius r_z from the end of the radius by l we obtain: $l = r_z \cdot \bar{\varphi}$, $\operatorname{tg}\psi = \frac{l}{r_z} = \frac{r_z \cdot \bar{\varphi}}{r_z} = \bar{\varphi}$ where $\psi = \operatorname{ar\,ctg}\varphi$ thus $\beta = \varphi - \psi = \varphi - \operatorname{ar\,ctg}\varphi$.

From the right triangle, we get: $\rho^2 = l^2 + r_z^2 = r_z^2 \cdot \varphi^2 + r_z^2$.

Substituting the value of ρ^2 into (2) and considering the relationship (Figure 6):

$$d\beta = d(\varphi - \operatorname{ar\,ctg}\varphi) = \left(1 - \frac{1}{1+\varphi^2}\right) d\varphi \,;$$

we receive:

$$S = \frac{1}{2} \cdot \int_0^{\gamma} r_z^2 \cdot (1+\varphi^2) \cdot \frac{\varphi^2}{1+\varphi^2} d\varphi = \frac{r_z^2}{2} \cdot \int_0^{\gamma} \varphi^2 d\varphi \tag{3}$$

$$S = \frac{r_z^2 \cdot \gamma^3}{6} \tag{4}$$

The area bounded by two involutes, the arc of the base circle between the starting points of these involutes, and the arc of the tip circle is equal to the area bounded by the same external arc and the radii drawn from the ends of the tip circle arc, cut off by the base circle (Figure 7).

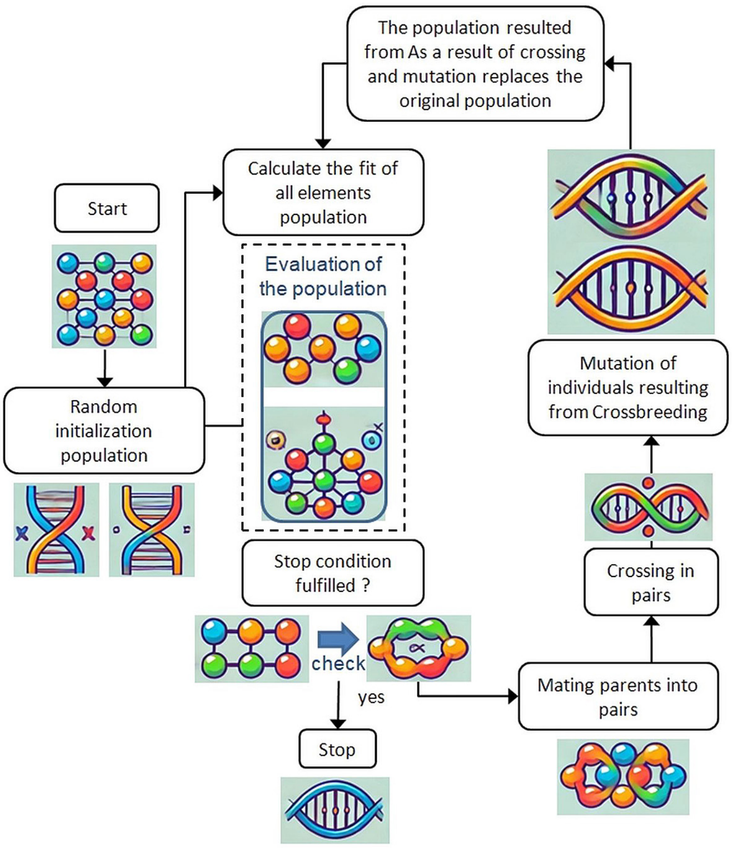


Figure 5. A flowchart illustrating the process of a genetic algorithm

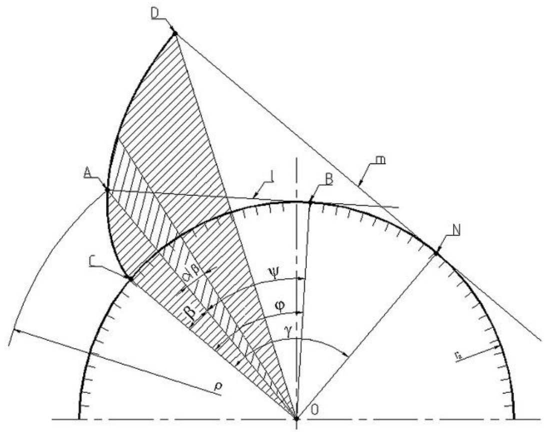


Figure 6. Surface area of the involute [40]

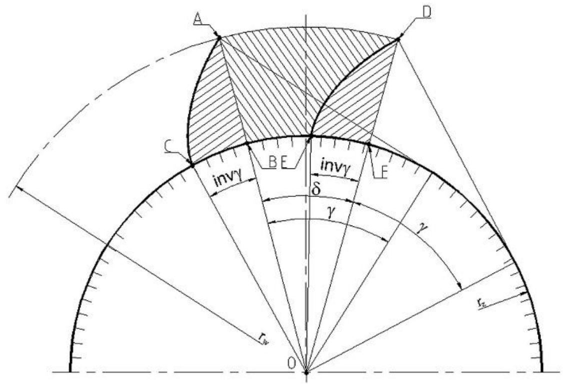


Figure 7. Properties of polievolvents

Based on the above property of the involute, we get the following:

$$S_{CADEBC} = S_{ADFEBA} = \frac{\delta}{2} \cdot \left(r_w^2 - r_z^2\right) \quad (5)$$

The pump capacity *dq* over an infinitesimally small time period *dt* found by determining the volume of liquid displaced when the wheels rotate by an infinitesimally small angle $d\beta$ (Figure 8).

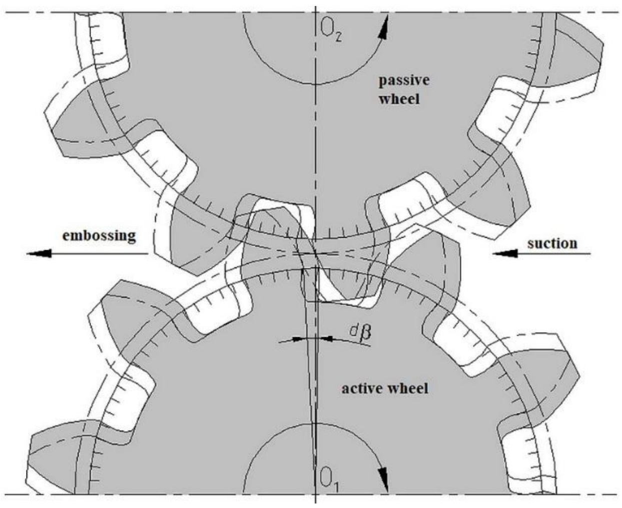


Figure 8. Performance of gear pump with undercut tooth foot

Pump capacity d_q Based on the properties of the evolute, we get the following

$$S_{CADEBC} = S_{ADFEBA} = \frac{\delta}{2} \cdot \left(r_w^2 - r_z^2\right) \quad (6)$$

The relationship between the angles β_1 and β_2 was found as follows:

$$\begin{cases} \overline{GH} = \overline{Ge} + \overline{eH} = r_z \cdot \beta_1 + r_z \cdot \beta_2 = r_z \cdot \left(\beta_1 + \beta_2\right) \\ \overline{GH} = 2 \cdot r_z \cdot \operatorname{tg}\alpha_t \end{cases};$$

$$\begin{cases} 2 \cdot r_z \cdot \operatorname{tg}\alpha_t = r_z \cdot \left(\beta_1 + \beta_2\right) \\ \beta_2 = 2 \cdot \operatorname{tg}\alpha_t - \beta_1 \end{cases};$$

By substituting the above equations we get:

$$dS_{cdef}=\left[\frac{\left(r'^{\,2}_{w}-r_z^2\right)}{2}-\frac{r_z^2\cdot\beta_1^2}{2}\right]\cdot d\beta\,; \tag{7}$$

$$dS_{efkl}=\left[\frac{\left(r'^{\,2}_{w}-r_z^2\right)}{2}-\frac{r_z^2}{2}\cdot\left(4\cdot\mathrm{tg}^2\,\alpha_t-4\cdot\mathrm{tg}\,\alpha_t\cdot\beta_1+\beta_1^2\right)\right]\cdot d\beta\,; \tag{8}$$

Eventually we get the form:

$$dq_\beta=\int_0^b d\beta\cdot\left[r'^{\,2}_{w}-r_z^2-r_z^2\cdot\left(2\cdot tg^2\alpha_t-2\cdot tg\,\alpha_t\cdot\beta_1+\beta_1^2\right)\right]\cdot db\,; \tag{9}$$

$$dq_\beta=\int_0^b d\beta\cdot\left[r'^{\,2}_{w}-r_z^2\cdot\left(1+tg^2\alpha_t\right)-r_z^2\cdot\left(tg^2\alpha_t-2\cdot tg\,\alpha_t\cdot\beta_1+\beta_1^2\right)\right]\cdot db\,; \tag{10}$$

To account for the torsion of the gears in the cross-sectional area of the displaced fluid, we will replace the position angle of the active wheel β_1 with the expression $\beta_1+\gamma(b)$, where γ of displacement of the cross-section distant by b relative to the initial cross-section. By substituting the above relations into we get:

$$dq_\beta=\int_0^b d\beta\cdot\left[r'^{\,2}_{w}-\frac{r_z^2}{\cos^2\alpha_t}-r_z^2\cdot\left(tg\alpha_t-\beta_1-\frac{tg\beta_S}{r_t}b\right)^2\right]\cdot db\,; \tag{11}$$

Because $\dfrac{r_z}{\cos\alpha_t}=r_t$ and $r_z\cdot\mathrm{tg}\,\alpha_t-r_z\cdot\beta_1=\overline{GC}-\overline{Ge}=\overline{Ce}=-s$.

So $s=r_z\cdot\left(\beta_1-\mathrm{tg}\,\alpha_t\right)$, $ds=r_z\cdot d\beta$, $d\beta=\dfrac{ds}{r_z}$.

Substituting the found values r_t, s and $d\beta$ into the equation we get:

$$dq_\beta=\int_0^b\frac{ds}{r_z}\cdot\left[r'^{\,2}_{w}-r_t^2-\left(s+\frac{r_z\cdot tg\beta_S}{r_t}b\right)^2\right]\cdot db\,; \tag{12}$$

Calculating the integral in the given range, we get the following:

$$dq_\beta=\frac{ds}{r_z}\cdot\left[r'^{\,2}_{w}-r_t^2-\left(s^2+\frac{s\cdot r_z\cdot tg\beta_S}{r_t}b+\frac{r_z^{\,2}\cdot tg^2\beta_S}{3r_t^{\,2}}b^2\right)\right]\cdot b\,; \tag{13}$$

Substituting in the equation $\dfrac{ds}{r_z}$ for $d\beta$, and $d\beta$ for $\omega\cdot dt$:

$$q_S=\frac{dq_\beta}{dt}=b\omega\left[r'^{\,2}_{w}-r_t^2-\frac{1}{12}\left(\frac{r_z\cdot tg\beta_S}{r_t}b\right)^2-\left(s+\frac{r_z\cdot tg\beta_S}{2r_t}b\right)^2\right]; \tag{14}$$

Then using the basic trigonometric formula, we express the tangent function by the cosine function of the same angle (in the first quadrant $\cos\alpha_t = \frac{1}{\sqrt{1+\mathrm{tg}^2\alpha_t}}$).

$$dq_\beta = \int_0^b d\beta \cdot \left[r'^2_w - \frac{r_z^2}{\cos^2\alpha_t} - r_z^2 \cdot (tg\alpha_t - \beta_1)^2 \right] \cdot db; \qquad (15)$$

In a further step, the instantaneous yield is analyzed, which depends on the type of toothing (teeth with or without lateral play) and the angle of rotation β\betaβ of the active wheel.
Then the dependencies are obtained:

After substituting all the dependencies discussed, among others, in the work [40], one obtains the relation for wheels with lateral clearance:

$$q_S = \frac{dq_\beta}{dt} = b\omega \left[r'^2_w - r_t^2 - \frac{1}{12}\left(\frac{r_z \cdot tg\beta_S}{r_t} b\right)^2 - \left(r_z \cdot \beta - \frac{l}{2} + \frac{r_z \cdot tg\beta_S}{2r_t} b\right)^2 \right] \qquad (16)$$

and for the ideal outline:

$$q_S = \frac{dq_\beta}{dt} = b\omega \left[r'^2_w - r_t^2 - \frac{1}{12}\left(\frac{r_z \cdot tg\beta_S}{r_t} b\right)^2 - \left(r_z \cdot \beta - \frac{p_z}{4} - \frac{b \cdot r_z \cdot tg(\beta_s)}{2r_t} + \frac{r_z \cdot tg\beta_S}{2r_t} b\right)^2 \right] \qquad (17)$$

Application of genetic algorithm in the optimization of the polyinvolute ear tooth shape

The parameters listed in the table must be entered into the genetic algorithm (Table 2).

Sample calculations using the genetic algorithm

The adaptation function evaluates the quality of each chromosome. In this case, the adaptation function minimizes the difference between the instantaneous yield value and the target value:

$$f(x) = |\eta_c - \eta_{destination}| \qquad (18)$$

Optimization process

- Population Initialization: The initial population consists of 50 chromosomes, ensuring greater diversity in the genetic algorithm and improving the likelihood of finding an optimal solution. This value was set to enhance the algorithm's performance, as reflected in the implementation.
- Fitness Function Calculation: Compute the fitness function for each chromosome, taking into account the dynamic viscosity, discharge pressure, and rotational speed.
- Selection: Select the best chromosomes for crossover.
- Crossover: Perform single-point crossover to create new chromosomes (offspring). Example of crossover:
- Parent 1: (r_0, h, α_1, α_2, α_3, μ, p_t, n) = (1.0, 2.0, 20, 25, 30, 0.001, 10, 1500)
- Parent 2: (r_0, h, α_1, α_2, α_3, μ, p_t, n) = (1.1, 2.1, 21, 26, 31, 0.002, 12, 1600)
- After crossover:
- Offspring 1: (r_0, h, α_1, α_2, α_3, μ, p_t, n) = (1.0, 2.0, 21, 26, 31, 0.001, 12, 1600)
- Offspring 2: (r_0, h, α_1, α_2, α_3, μ, p_t, n) = (1.1, 2.1, 20, 25, 30, 0.002, 10, 1500)
- Mutation: Introduce random changes to the chromosome genes with a low probability. Example of mutation:
- Before mutation: (r_0, h, α_1, α_2, α_3, μ, p_t, n) = (1.0, 2.0, 21, 26, 31, 0.001, 12, 1600)
- After mutation: (r_0, h, α_1, α_2, α_3, μ, p_t, n) = (1.0, 2.0, 21, 26, 32, 0.001, 12, 1600)
- New Population Evaluation: Evaluate the new population using the fitness function and select the best chromosomes for the next generation.
- Repeat Process: The process of selection, crossover, and mutation is repeated for a

Table 2. Application of genetic algorithm for performance parameters

Analyzed parameter	Description
Operating parameters that directly affect the performance and efficiency of the gear pump:	– Viscosity of the working fluid (both kinematic v [m²/s] and dynamic μ [Pa·s]) – Discharge pressure p_t [MPa] – Rotational speed n [min⁻¹] of the gear unit
Parameters of the genetic algorithm	– Population size (N) – Number of solutions in each generation (e.g., 50); number of generations (G) – Maximum number of iterations (e.g., 100); crossover probability (pc) – Chance of crossing two chromosomes (e.g., 0.8); mutation probability (pm) – Chance of mutating a single chromosome (e.g., 0.05) – Fitness function - a function that evaluates the quality of each solution.
Tooth shape, a representation of the tooth geometry, which can be described by a set of parameters:	– Pressure angle of the first involute (α_1)- range from 15° to 30° – Pressure angle of the second involute (α_2): range from 15° to 30° – Pressure angle of the third involute (α_3): range from 15° to 30° – Fitness function (η_c): a value that maximizes the overall efficiency of the pump
Stopping conditions:	– a value that maximizes the overall efficiency of the pump

specified number of generations or until a satisfactory fitness value is achieved.

The stopping condition in the genetic algorithm used in this study was based on achieving convergence in the optimization process. The algorithm was set to stop after either a fixed number of generations (50 generations) or if no significant improvement in the fitness function (overall efficiency) was observed over five successive generations. The highest overall efficiency value achieved during the optimization process was η_c = 92.3%, obtained with the following parameters:

- base radius (r_0): 1.6
- tooth height (h): 2.6
- pressure angle of the first involute (α_1): 26°
- pressure angle of the second involute (α_2): 31°
- pressure angle of the third involute (α_3): 36°
- dynamic viscosity (μ): 0.007 Pa·s
- discharge pressure (p_t): 22 MPa
- rotational speed (n): 2100 min⁻¹.

The noise reduction observed in pumps with poly-involute profiles is attributed to reduced hydraulic pulsation, minimized enclosed volume traps, and enhanced surface contact geometry. These factors collectively contribute to a smoother operation with significantly reduced acoustic emissions.

Phyton's code is included in the appendix.

In Figure 9, the RMS error plot for the genetic algorithm is presented, showing the changes in RMS values across successive generations.

Based on the RMS error plot, it can be concluded that the genetic algorithm gradually reduces the RMS error across successive generations, indicating an improvement in the algorithm's fitting and optimization of the involute profile parameters. Thanks to the application of the above genetic algorithm, we obtained the optimal set of parameters for the shape of the polyevolvent tooth. For the appropriate chromosomes, the optimal

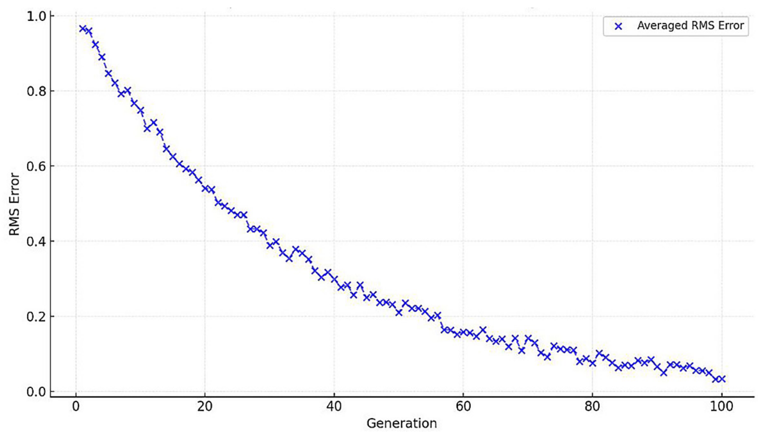


Figure 9. RMS error plot for the genetic algorithm cross successive generations

involute profile was constructed. The individual stages of building this involute were generated, as shown in Figure 10. The process of creating the optimal tooth profile is presented, starting from the initial population of chromosomes, through crossover and mutation, to obtaining the best solution. The optimal chromosome obtained as a result of the optimization process is chromosome 7, which has the following parameters:

- base radius (r_0): 1.6
- tooth height (h): 2.6
- pressure angle of the standard involute (α_1): 26°
- pressure angle of the extended involute (α_2): 31°
- pressure angle of the shortened involute (α_3): 36°
- dynamic viscosity of the working fluid (μ): 0.007 Pa•s
- discharge pressure (p_t): 22 MPa
- rotational speed (n): 2100 min^{-1}

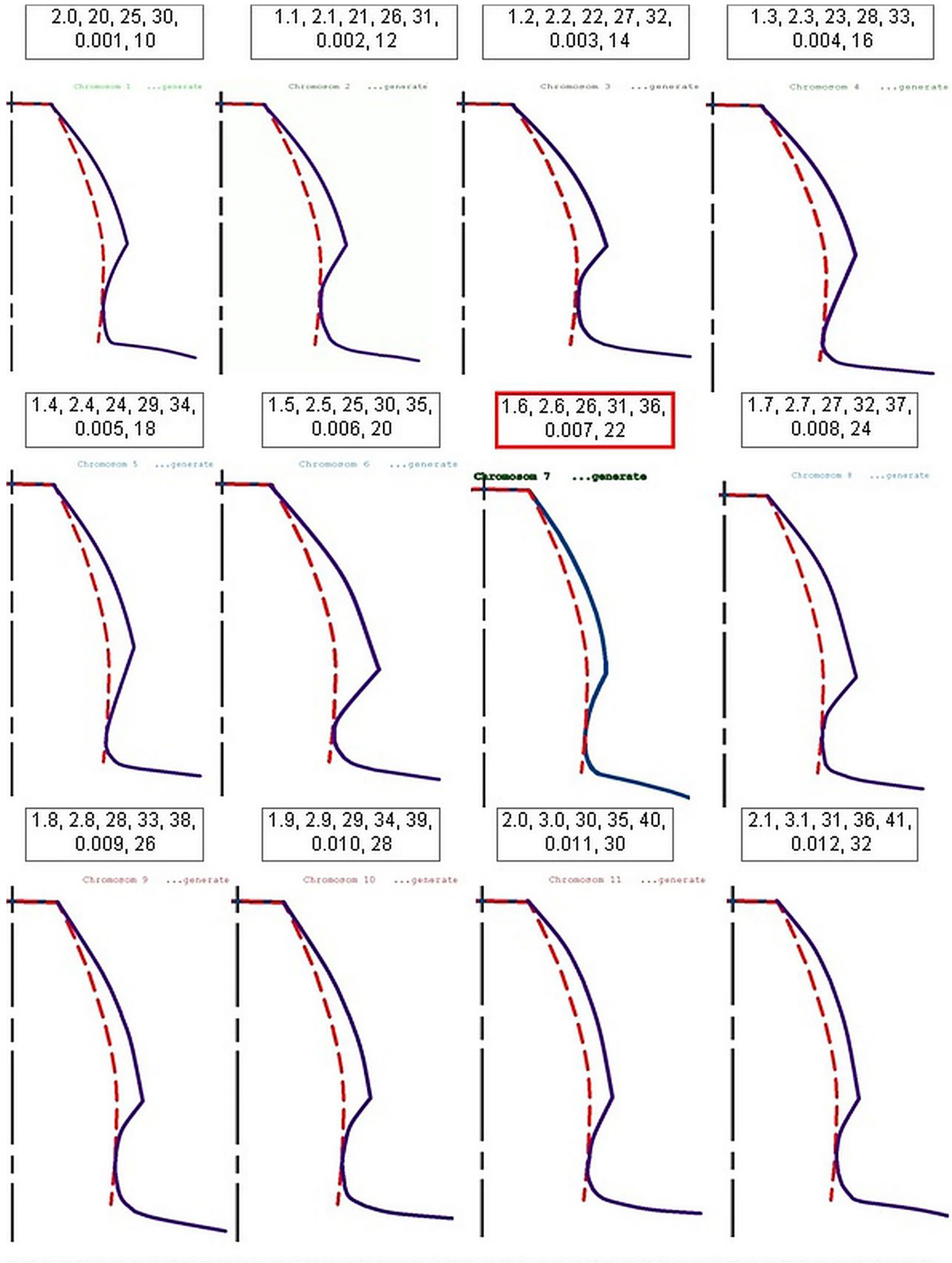


Figure 10. Diagram of the process for creating the optimal tooth profile from the initial population of chromosomes, through crossover and mutation

ANALYSIS OF THE OBTAINED RESULTS

The profile selected in the optimization process is characterized by the presence of two standard involutes and one extended involute. Before manufacturing the gears, the kinematics of the optimized three-involute meshing was checked on gears printed using 3D technology. The model gears presented in Figure 11 were counterparts of the gears intended for a second-group pump with a unit capacity of q = 8 cm³/rev. After positive verification of the gears printed in polyethylene, their production began under industrial conditions. The surface of the three-involute profile was finished using grinding technology.

Finally, the characteristics of the model pumps were analyzed both before and after optimization of the manufacturing technology using decision-making methods. The analyses showed that the obtained efficiencies and parameters (characteristics) validated against the polyvolute shape outline are better than those previously obtained using logic trees only, as presented in the article [46].

A comparison of the total efficiency of model pumps with a tri-volute shape outline before and after outline optimization shows a significant increase in total efficiency, both at n = 800 rpm and n = 1000 rpm. (Fig. 12–13).

In addition to the increase in volumetric efficiency, there was also an observed increase in overall and hydro-mechanical efficiency. This indicates insufficient initial (factory) running-in, meaning that additional tests and optimizations can further improve pump performance. Furthermore, acoustic properties also improved. The distribution of sound intensity on the surface of the pump body showed a local increase in sound vibrations, proving that sound-generating vibrations are mainly transmitted from the pump drive.

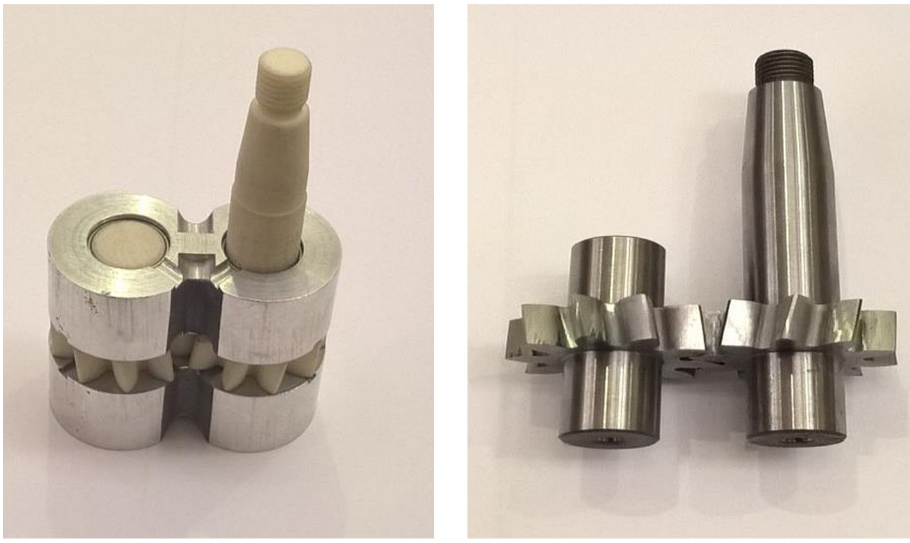


Figure 11. Three-rotor gears of 3D printed wheels and shaving gears

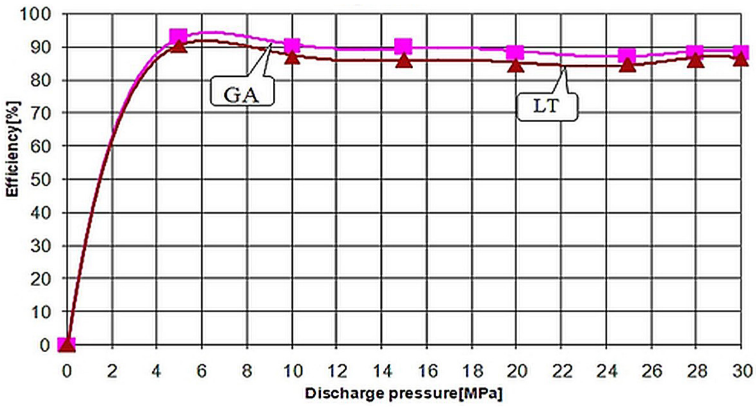


Figure 12. Efficiency of undercuttoth foot pump for n = *800* rpm optimization algorithm

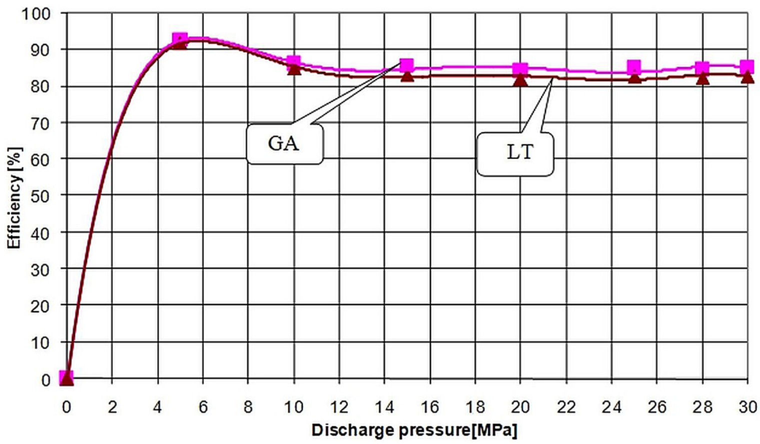


Figure 13. Efficiency of undercuttoth foot pump for $n = 1000$ rpm optimization algorithm

Comparing the acoustic characteristics of prototype pumps with the base units, it turned out that the solution with a polyevolvent outline is characterized by a 3 to 5 dB lower noise emission to the environment. The beneficial effect of noise reduction was observed in the range from 0 to 20 MPa.

The noise reduction observed in pumps with poly-involute profiles is attributed to reduced hydraulic pulsation, minimized enclosed volume traps, and enhanced surface contact geometry. These factors collectively contribute to a smoother operation with significantly reduced acoustic emissions. The acoustic measurements were conducted following the ISO 3746:2010 standard. Measurements were performed in a semi-anechoic chamber using a calibrated sound level meter positioned at predefined distances around the pump housing. This setup ensured minimal external noise interference and accurate determination of sound power levels.

Acoustic tests of model pumps with dimensional optimization and comparable units without modification showed similar noise emission parameters at low operating pressures. In the case of an increase in working pressure, units with the optimization of production technology were more favorable. In summary, the conducted research and optimizations demonstrated that the application of the genetic algorithm combined with decision-making methods allowed for better results in terms of efficiency, durability, and noise emission of gear pumps with a polyevolvent outline. The obtained results confirm the effectiveness of the applied methods and form the basis for further research and industrial implementation. In the context of the optimization of the polyevolvent outline:

- Production technology optimization brought benefits in terms of lower production costs and higher efficiency.
- The results showed that optimized pumps with a polyevolvent outline surpassed the performance of pumps designed earlier solely using logic trees.
- Designs with better acoustic characteristics were introduced, which meant a reduction in noise emission by 3 to 5 dB compared to traditional solutions.

The conducted tests confirmed that manufacturing technology optimization leads to pumps with higher efficiency, durability, and reliability.

Furthermore, the efficiency metrics, particularly the volumetric and hydraulic-mechanical efficiencies, reveal noticeable improvements. The volumetric efficiency increased significantly under conditions of higher rotational speeds and discharge pressures, underscoring the role of geometric optimization in reducing internal leakages and enhancing pump performance. The genetic algorithm applied to the optimization process proved to be an effective tool for balancing conflicting objectives, such as maximizing efficiency while minimizing noise levels.These results align well with previous findings in the literature but surpass traditional approaches by incorporating advanced optimization techniques. The outcomes provide a benchmark for future innovations, particularly for applications requiring high-performance gear pumps, such as automotive and industrial hydraulic systems.

The optimization of gear pump profiles using a genetic algorithm has demonstrated significant improvements in operational parameters, such as reduced pressure pulsation and enhanced tooth contact geometry. These factors are known to theoretically contribute to improved durability of gear wheels. However, it is important to note that this claim is a theoretical projection based on the optimization results and existing literature on similar systems. Direct experimental durability tests were not conducted within the scope of this study.

The optimization of the gear pump profiles using genetic algorithms resulted in significant improvements in performance. Specifically, the overall efficiency of the gear pump was enhanced by 8–12%, as compared to conventional profiles. This improvement encompasses gains in both volumetric efficiency and hydraulic-mechanical efficiency, achieved by refining the tooth profile geometry, reducing internal friction, and optimizing inter-tooth space. The improvement range of 8–12% was observed under varying operating conditions, including changes in rotational speed and discharge pressure. For example:

- at lower rotational speeds (800–1000 min^{-1}) and discharge pressures (up to 20 MPa), the optimized profiles demonstrated up to an 8% increase in efficiency compared to the baseline;
- at higher rotational speeds (2100 min^{-1}) and discharge pressures (22 MPa), the efficiency gains reached up to 12%, highlighting the adaptability of the optimized profiles across different operating conditions.

These improvements are directly attributed to the genetic algorithm's ability to identify optimal parameters for tooth geometry, including base radius, tooth height, and pressure angles, which collectively minimize pressure pulsation and energy losses.

CONCLUSIONS

The research demonstrates the effectiveness of genetic algorithm-based optimization in enhancing the design and performance of gear pumps with involute profiles. The optimized profiles achieved significant advancements in several areas, confirming the potential of advanced computational methods to redefine the standards in gear pump design. One of the most notable outcomes is the marked reduction in noise emissions, with a 3–5 dB decrease observed across a wide range of operating pressures. This improvement is attributed to the refined tooth profile geometry, which effectively minimizes the formation of trapped volumes, pressure pulsations, and inter-tooth friction. These geometric optimizations ensure smoother fluid flow, contributing to quieter pump operation and a reduction in mechanical vibrations.

Additionally, the optimized designs exhibited substantial improvements in volumetric and hydraulic-mechanical efficiencies, particularly under high-load conditions. By achieving better internal sealing and load distribution, these profiles reduce energy losses, internal leakages, and wear, leading to enhanced overall performance. These findings underscore the capability of genetic algorithms to identify parameter configurations that surpass the efficiency and performance achieved by conventional design approaches. The implications of these findings are profound for gear pumps deployed in high-demand applications such as automotive, industrial hydraulics, and heavy machinery. Enhanced acoustic performance reduces operational disturbances, while increased efficiency directly translates to energy savings and extended operational lifespans. These attributes are especially critical in environments requiring reliable, low-maintenance systems that can operate under diverse and challenging conditions.

Future research directions aim to build on the presented results by exploring additional factors influencing pump performance. For instance, investigating the impact of working fluid temperature and viscosity could provide deeper insights into the adaptability of optimized profiles across varying operational scenarios. The use of advanced materials, such as polymers or composites, offers potential for weight reduction and enhanced wear resistance, further augmenting the benefits of geometric optimization. Moreover, integrating artificial intelligence for real-time condition monitoring represents an exciting avenue for innovation. AI-driven systems could enable predictive maintenance, adaptive performance tuning, and early fault detection, further enhancing the reliability and efficiency of gear pumps. The exploration of more sophisticated optimization algorithms and hybrid approaches may also lead to even greater performance gains. The results presented in this study lay a robust foundation for continued advancements in gear pump technology. They highlight the transformative potential of combining innovative design strategies with state-of-the-art computational tools. By addressing longstanding challenges in noise reduction, efficiency, and durability, this research sets a benchmark for future developments, paving the way for the next generation of high-performance, energy-efficient gear pumps. Future studies will expand the scope of this research to include acoustic and efficiency evaluations for conditions matching the optimal parameters identified in this study, particularly at speeds up to 2100 min^{-1} and pressures up to 22 MPa. This will ensure a more comprehensive understanding of the pump's behavior under these conditions.

One of the limitations of the current study is the omission of the effect of temperature on the viscosity of hydraulic oil. Temperature is a critical parameter that influences the rheological properties of oil and, consequently, the performance of hydraulic systems. Due to the complexity of integrating this factor into the genetic algorithm while maintaining computational efficiency, we chose to exclude it from the current analysis.

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