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# On a possibility of complicating the mathematical model of a collision between two motor vehicles

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## ABSTRACT

In this paper, a survey on some selected problems of motor vehicle collisions modeling was undertaken. The aim of this paper was to replace the classic approach based on a planar motion with a more complex resultant motion to examine the potential additional factors affecting the process of a collision. In the case of a collision between two vehicles, especially for a large velocity, the colliding vehicles may break away from the road surface and perform a motion that requires including a more sophisticated approach to understand a road collision process, particularly in view of its short duration. Moreover, the most common frontal or rear collision model was replaced with the side impact collision model in which one vehicle struck the side of the other. Such approach makes the collision model even more complex. It was also necessary to consider other phenomena as well. Apart from adopting a simple momentum – impulse collision model, the authors took into account the friction between the bodies of the vehicles involved in a collision in order to analyze the possibility of the coefficient of restitution in the case of both normal and tangential directions versus the plane of collision common for both vehicles. In the case of a resultant motion during the collision the forces of inertia, transportation and Coriolis were included. Such analysis can be a tool to better understand the crucial parameters of any collision between motor vehicles, instead of performing a forensic expertise. The novelty and one of the most important results of this paper may be creation of the more complex mathematical model of a collision between two vehicles when compared to the regular models. Such model can include such factors as the resultant motion during a collision that enable providing more realistic description of a collision process than in a typical scenario. Of course, the paper deals with the momentary phenomena, which last for a very short period of time (e.g. 0.1 s), hence including the factors which make the collision model more complex may allow understanding its course and, what is more, the potential effect on the post-collision motion of the vehicles involved.

**Keywords:** car accidents, vehicle collision modelling, parameters of collision between motor vehicles, analytical calculations.

## INTRODUCTION

There is a variety of the effects of a road collision, depending on either dynamic or geometric factors, e.g. a place of the road event. Most dangerous accidents in road traffic seem to be those taking place on intersections, hills and turns as well as at high speed. This in connection with, e.g. side impact collisions, which can cause the vehicles to perform not only a planar motion, as assumed in most common approach towards collision modeling. Thus far, analyses of motor vehicles collision have been conducted with the use of both simplified and more complex models, e.g. [1, 2, 3]. Mathematical description of the simplified models can be reduced to the principles of momentum and angular momentum only. Yet, such approach seems good for the cases in which a forensic expert must examine only the most obvious road traffic events without any additional factors determining the propriety of their opinions. In more advanced collision models, the geometrical parameters can also include the resultant motion of the colliding vehicles which enables understanding the physical nature of a collision between two motor vehicles.

There are, of course, some other directions in which research on road traffic events has been developed. One of them is the use of neural networks and fuzzy logic to develop a collision model, as, e.g. in [4] and [5] or the so-called machine learning to analyze the impact on the so-called vulnerable (non-protected) road users [6, 7, 8]. As for the road users, the average male was regarded as a norm to the analyses regarding the effects of road accidents in [9]. Research on biomechanics of a spine damage in a crash with the road barrier was a subject of, e.g. [10].

Another approach was based on the statistical modeling [11] enabling the crash prediction modeling. In [12], separation of the two vehicle collisions from the multi vehicle ones took place. In [13], a review of the techniques and tools used in road accident analysis was performed, while in [14] the factors determining the severity of each accident under analysis were studied. Thus far, some works have been devoted to other types of road accidents than only a frontal impact. In [15], the authors considered the impact into a solid object on a roadside, while in [16] the preliminary conclusions on the use of the restitution coefficient in the side impact collision were presented. The techniques used in mathematical modeling of road accidents were presented in [15], while the special attention to frontal collisions was paid in [16], where various overlaps were considered. Of course, the current research also relates to the autonomous vehicles in terms of road accidents e.g. [17] and more advanced techniques, such as the finite element method [18] and the big data approach to understand a collision process [19].

This paper concentrated on the selected aspects of a road traffic collision modeling with the inclusion of the additional phenomena, such as a resultant motion for the example of a side impact collision. Occurrence of the impulses of the forces acting on the vehicles can be related to the Mandelsztam criterion [3], determining the weak or strong coupling of the dynamic systems such as motor vehicles while in a collision. A mathematical model of a side impact collision seems complicated, including the additional phenomena. Yet, some simplifications are necessary in order to provide the soluble equations without involving the time-consuming mathematical operations. Therefore, the following assumptions have been made, some of which refer to Figure 1:

- the models of the vehicles involved are quasi – stiff and linear. Non – linearity would be advisable if, e.g. vibrations were considered. Here, however, the vibrations during a collision are neglected,
- the bodies of both vehicles are the cuboids with the constant stiffness and mass so their motion can be considered in terms of material points,

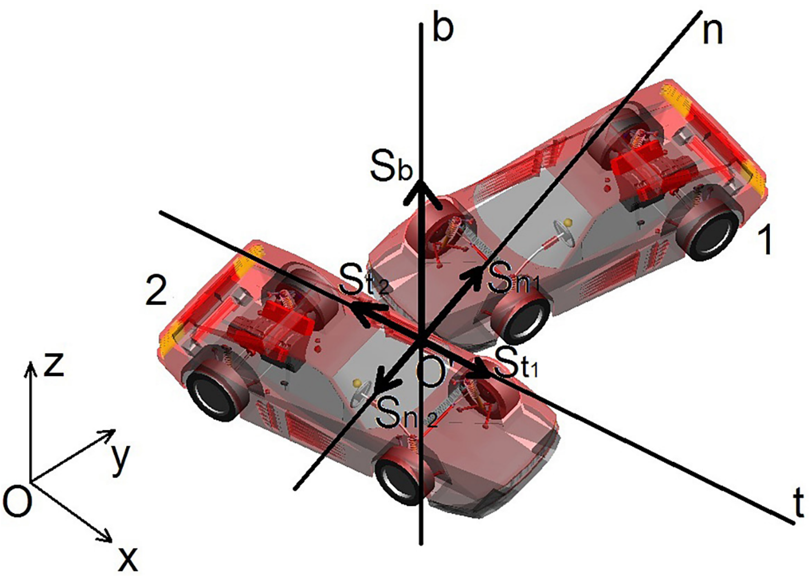


**Figure 1.** The location of both vehicles at the beginning of the collision with the components of the collision force impulse and the numbers of vehicles marked [based on 3]

- the discussed example of a collision took place on a dry road with the coefficient of adhesion 0.8,
- the system $O_{xyz}$ is an inertial system (Fig. 1) and the $O'_{ntb}$ system located at the center of an impact moves with respect to $O_{xyz}$,
- in this case, the models of the vehicles performed a resultant motion, so to describe their motion in relation to the non-inertial $O'_{ntb}$ coordinate system, three coordinates were used: *n* (normal), *t* (tangential) and *b* (binormal). The normal direction is perpendicular to the plane of collision (i.e. the vertical plane of initial contact common for both vehicles), the tangential direction is coherent with the plane of collision and the binormal is vertical and perpendicular to the road plane,
- the geometric center of this collision is primarily the point O', i.e. the origin of the $O'_{ntb}$ system located in the center of the area of initial contact between the vehicles,
- the impulses of all forces taken into account were decomposed into the components along the normal, tangent and binormal direction versus the plane of collision. The external forces and moments resulting from, e.g. varying adhesion between the wheels and the road are not considered;
- the impulses of the forces in resultant motion (the forces of inertia in relative motion, the forces of transportation and the Coriolis forces) were included in the equations specifying the kinematic state of the vehicles and applied at their centers of mass. Although their directions were unknown, their components in the three directions of the $O_{'ntb}$ system could be used;
- both the translational and the angular velocity components after the collision were marked with an apostrophe.

## MATERIALS AND METHODS

### General assumptions

The mutual location of both vehicles (no. 1 and 2) at the beginning of the collision is presented in Figs. 1–3. In Fig.1, the components of the impulse of the collision force was also added to mark what type of a collision will be analyzed in the further part of this paper. The markings in all of the figures presented in this paper should be explained here, as they refer to further analyses in this paper:

- *1, 2* – the numbers of both vehicles involved in a discussed collision,
- *n*, *t*, *b* –the axes of the local coordinate system attached to the center of the collision *O'*,
- $S_{in}$, $S_{it}$, $S_b$ – the components of the collision force impulse for each vehicle, along the axes of the local coordinate system,
- $n_1$, $t_1$, $b_1$, $n_2$, $t_2$, $b_2$ – coordinates of the center of mass of the vehicles no. 1 and 2, respectively, in relation to the origin of the local coordinate system *O'ntb*,
- $C_1$, $C_2$ – the center of mass of the vehicles 1 and 2, respectively,
- $S_{Ini}$, $S_{Iti}$ – the components of the impulse of the inertia force for each vehicle in the normal (*n*) and the tangential (*t*) direction,
- $S_{Tni}$, $S_{Tti}$ – the components of the impulse of the transportation force for each vehicle in the normal and the tangential direction,
- $S_{Cni}$, $S_{Cti}$ – the components of the impulse of the Coriolis force for each vehicle in the normal and the tangential direction,
- $\upsilon_{1t}$, $\upsilon_{2n}$ – the tangential velocities directed parallel to the *t* axis for each vehicle,
- the normal velocities directed parallel to the *n* axis for each vehicle,
- $\upsilon_{1b}$, $\upsilon_{2b}$ – the binormal velocities directed parallel to the b axis for each vehicle,
- $\omega_1$, $\omega_2$ the angular velocities about the vertical axis of each vehicle,
- $I_1$, $I_2$ – the moment of inertia of vehicle no. 1 and 2 respectively, relative to the vertical axis of each vehicle.

In Figs. 2 and 3, the distances between the center of mass of each vehicle and the origin of the *O'ntb* system were presented in all three directions. Let us assume that these three figures will be a basis for further discussion on the resultant motion included in the analyzed example of the side impact collision.

### Description of a collision model while driving along a road curve

In general, the equations regarding a collision modeling for a planar motion can be formulated as below (e.g. based on [3]), regarding Figs. 1–3:

$$m_i(v'_{in} - v_{in}) = S_{ni}, m_i(v'_{it} - v_{it}) = S_{ti} \quad (1)$$

$$I_i(\omega'_i - \omega_i) = S_{ti}n_i + S_{ni}t_i \quad (2)$$

$$i = 1, 2,$$

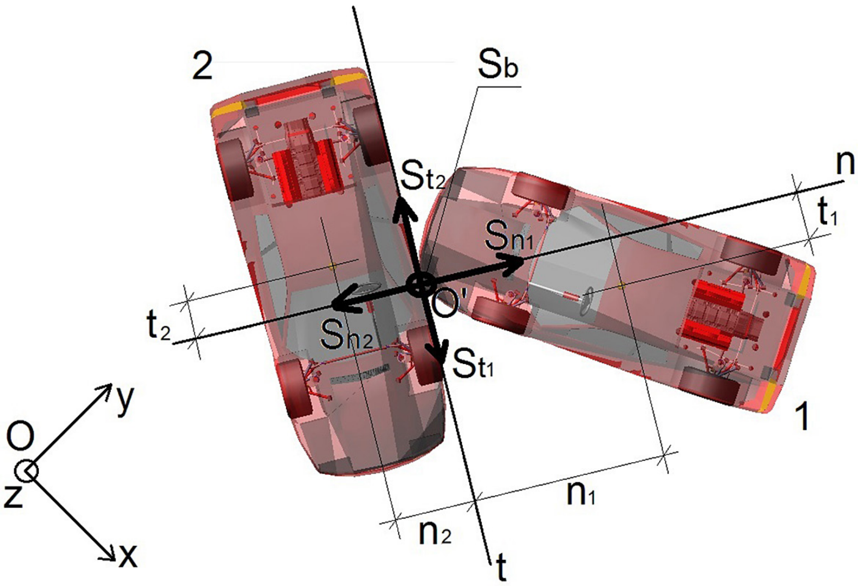


**Figure 2.** The physical model of the front, eccentric and oblique impact with the impulses of the impact force marked in a plan view [based on 3]

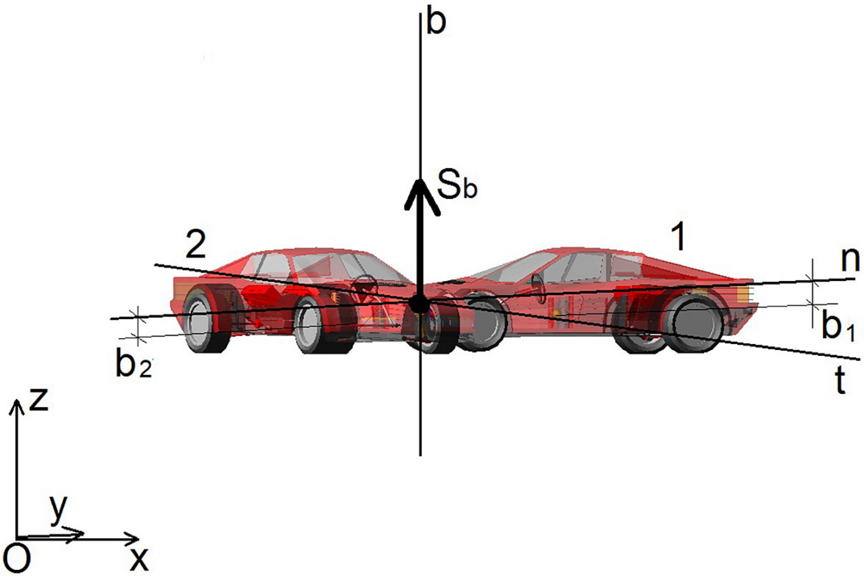


**Figure 3.** The physical model of the front, eccentric and oblique impact with the impulses of the impact force marked in a plan view [based on 3]

where: $m_i$ – the mass of the particular vehicle, $v_{it}$ – the velocity tangential to the plane of collision for each vehicle, $v_{in}$ – the velocity normal to the plane of collision for each vehicle, $\omega_i$ – the angular velocity of each vehicle, $S_n$ – the moment of inertia relative to the vertical axis of each vehicle, $S_{ni}$, $S_{ti}$ – the normal and the tangential components of the collision force impulse for each vehicle.

In more specific examples, such as the one discussed in this paper, more equations related to the resultant motion are needed. In the most general case, they would be as follows:

$$m_i(v'_{in} - v_{in}) = S_{ni} + S_{Ini} + S_{Tni} + S_{Cni} \quad (3)$$

$$m_i(v'_{it} - v_{it}) = S_{ti} + S_{Iti} + S_{Tti} + S_{Cti} \quad (4)$$

$$m_i(v'_{ib} - v_{ib}) = S_b + S_{Ibi} + S_{Tbi} + S_{Cbi} \quad (5)$$

$$I_i(\omega'_{in} - \omega_{in}) = S_{ti}b_i + S_b t_i \quad (6)$$

$$I_i(\omega'_{it} - \omega_{it}) = S_{ni}b_i + S_b n_i \quad (7)$$

$$I_i(\omega'_{ib} - \omega_{ib}) = S_{ti}n_i + S_{ni}t_i, \quad (8)$$

$i = 1, 2,$

where: $S_{in}$, $S_{it}$, $S_b$ – the normal, the tangential and the binormal components of the collision force impulse for each vehicle; – the normal and the tangential components of the inertia force impulse for each vehicle; – the normal and the tangential components of the transportation force impulse for each vehicle; – the normal and the tangential components of the Coriolis force impulse for each vehicle; – coordinates of the center of mass of each vehicle in relation to the center of the collision *O'*.

Analysis of the presented case of an eccentric side impact collision needs to be performed with the use of some proper figures. Therefore, additional impulses of the forces included in the resultant motion were presented in Figs. 4–6. Of course, here it is most important to assume that the forces in the resultant motion are of such great value that they can be included in the equations describing the kinematic state of both vehicles during the collision. In Figs. 4–6, the location of the center of mass for both vehicles is depicted by $C_1$ and $C_2$. In Equations 3 to 8, there seem to be too many insignificant parameters and unknowns, so in order to make these equations more usable, some of them should be removed, having no meaning for the description of the analyzed example.

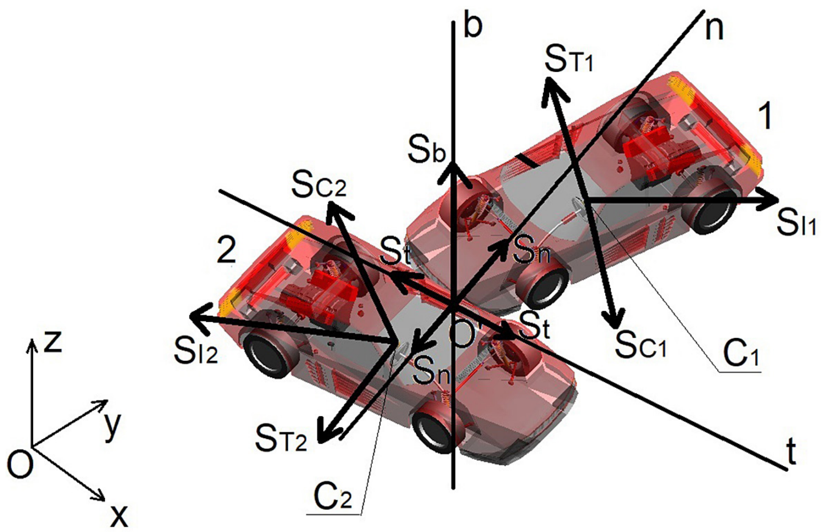


**Figure 4.** The impulses of the forces in resultant motion [based on 3]

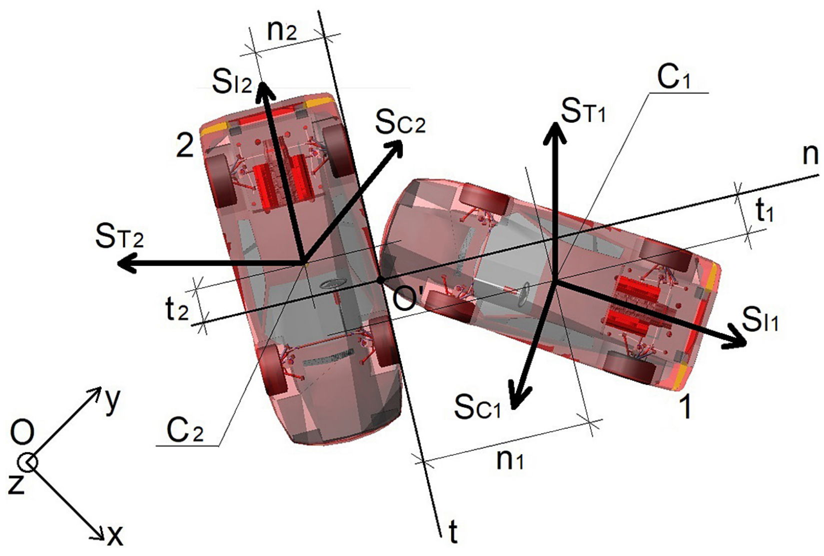


**Figure 5.** The impulses of the forces in resultant motion in a plan view [based on 3]

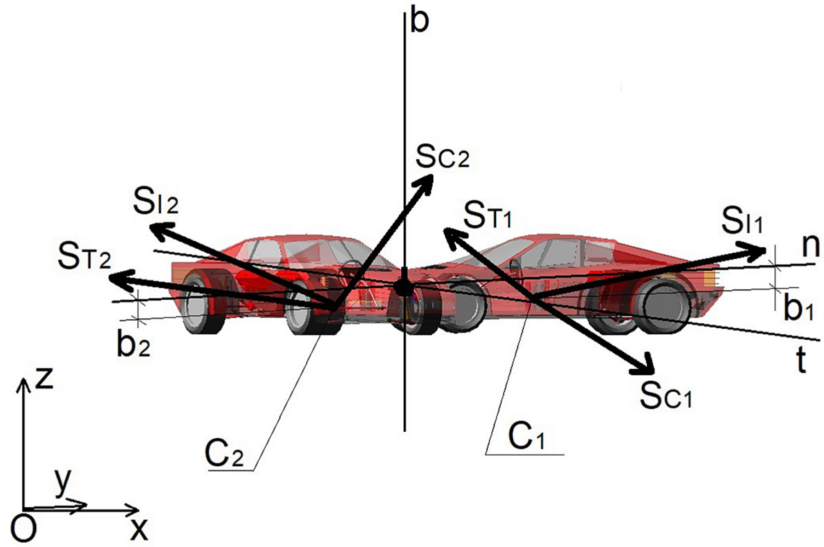


**Figure 6.** The impulses of the forces in resultant motion in a side view [based on 3]

The main simplifications regard the most insignificant phenomena. First, it seems unimportant to take the angular velocity about the longitudinal axis of each vehicle (i.e. the roll velocity). In the case of such a short period phenomenon as a collision, it seems that the rotation about the vertical axis providing the most necessary information about the change of direction of the forward velocity of each vehicle is the most important. $S_o$ the roll angular velocity ($\omega_n$ and $\omega_t$ in Equations 2) can be neglected or at least assumed to remain constant and therefore equal to 0 as a difference between the beginning and the end of the collision). However, it has to be stressed that for vehicle no. 2, $\omega_t$ will be the roll angular velocity and for vehicle no. 1, it will be $\omega_n$.

The same refers to the pitch angular velocity. Of course, the vehicle could move pitch-like with, e.g. the rear wheels above the road surface, but this kind of angular velocity will not affect the change in the forward direction of each vehicle, so let it be assumed that the difference between the post and after the collision, the pitch velocity will be equal. Again, for vehicle no. 2, $\omega_n$ will be the pitch angular velocity and for vehicle no. 1, it will be $\omega_t$. The component of the binormal impulse Sb has a common vector return for both vehicles. Therefore, the equations in this direction for both

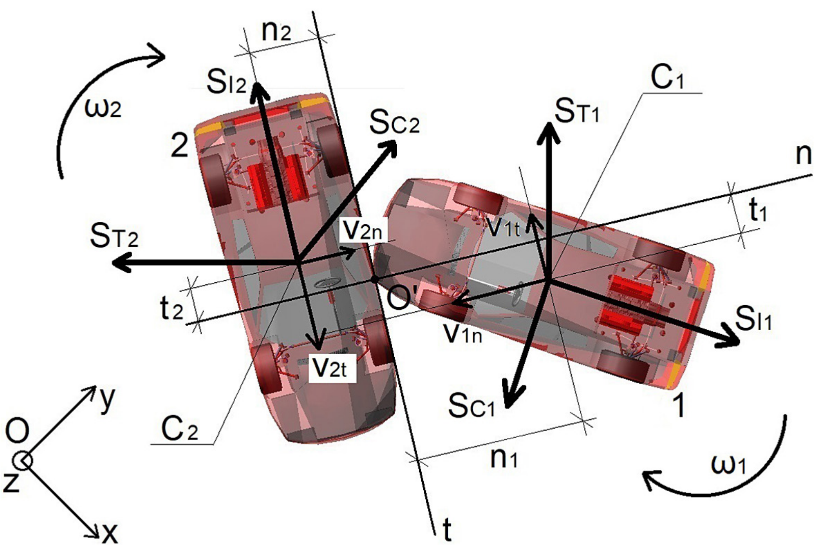


**Figure 7.** The components of the translational and the angular velocities of both vehicles in a plan view [based on 3]

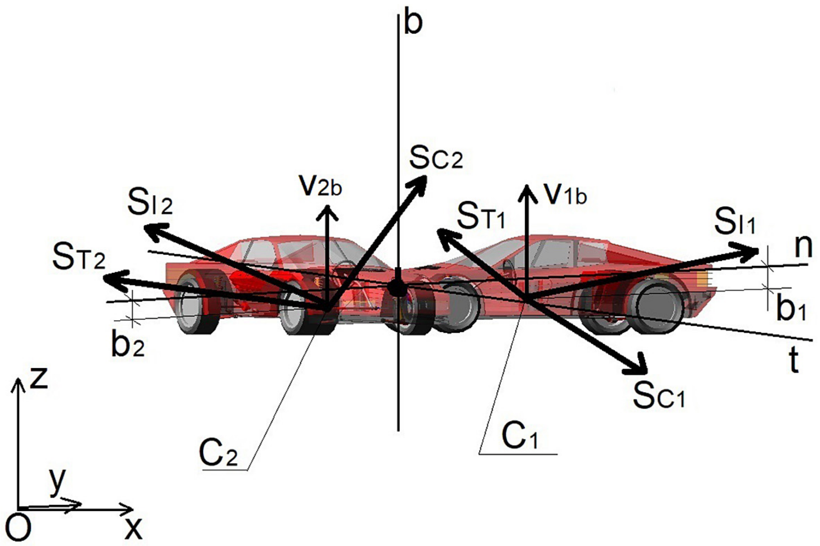


**Figure 8.** The components of the velocities of both vehicles in a side view [based on 3]

vehicles will be similar. With the assumed simplifications which may not affect the whole process of the analyzed collision, some simplified equations of motion can be specified. The components of the angular velocities which do not affect the change in the forward direction of motion before and after the collision were 0, while the impulses remained. In Figures 7 and 8, the components of the translational velocities of each vehicle were presented. Also, in Figure 7, the angular velocities used here were marked. This, in turn, may let us assume that the normal velocity of the vehicle 2 (v2n) is 0 at the beginning of the collision, because the vehicle moves parallel to the adopted $O'_t$ axis. Only after the collision, the velocity $v_{2n}'$ component will differ from 0.

Hence, based on Figs. 2, 3, 5, 6, 7 and 8, the general, yet specified for the given example, equations for vehicle no. 1 would be:

a) along the *O'n* axis:

$$m_1(-v_{1n} + v'_{1n}) = S_{n1} + S_{In1} + S_{Tn1} + S_{Cn1} \tag{9}$$

b) along the *O't* axis:

$$m_1(-v_{1t} + v'_{1t}) = S_{t1} + S_{It1} + S_{Tt1} + S_{Ct1} \tag{10}$$

c) along the *O'b* axis:

$$m_1(v'_{1b} - v_{1b}) = S_b + S_{Ib1} + S_{Tb1} + S_{Cb1} \tag{11}$$

d) around the longitudinal axis of vehicle no. 1:

$$0 = S_{t1}b_1 + S_b t_1 \tag{12}$$

e) around the lateral axis of vehicle no. 1:

$$0 = S_{n1}b_1 + S_b n_1 \tag{13}$$

f) around the vertical axis of vehicle no. 1:

$$I_1(\omega'_{1b} - \omega_{1b}) = S_{t1}n_1 + S_{n1}t_1, \tag{14}$$

and for vehicle no. 2:

a) along the *O'n* axis:

$$m_2(v'_{2n}) = -S_{n2} - S_{In2} - S_{Tn2} - S_{Cn2} \tag{15}$$

b) along the *O't* axis:

$$m_2(v_{2t} - v'_{2t}) = -S_{t2} - S_{It2} - S_{Tt2} - S_{Ct2} \tag{16}$$

c) along the $O'b$ axis:

$$m_2(v'_{2b} - v_{2b}) = S_b + S_{Ib2} + S_{Tb2} + S_{Cb2} \tag{17}$$

d) around the longitudinal axis of vehicle no. 1:

$$0 = -S_{t2}b_2 - S_b t_2 \tag{18}$$

e) around the lateral axis of vehicle no. 1:

$$0 = -S_{n2}b_2 - S_b n_2 \tag{19}$$

f) around the vertical axis of vehicle no 1:

$$I_2(\omega'_{2b} - \omega_{2b}) = -S_{t2}n_2 - S_{n2}t_2, \tag{20}$$

where: $m_1, m_2$ – the mass of vehicle no. 1 and 2 respectively; $v_{1n}, v_{2n}$ – velocity components of both vehicles in the $O'n$ direction; $v_{1t}, v_{2t}$ – velocity components of both vehicles in the $O't$ direction; $v_{1b}, v_{2b}$ – velocity components of both vehicles in the $O'b$ direction; $\omega_1, \omega_2$ – angular velocities of both vehicles involved in the collision; $I_1, I_2$ – the moments of inertia of vehicle no. 1 and 2 relative to the vertical axes passing through the center of mass of each vehicle; $S_n, S_t, S_b$ – the components of the impulse of a collision force in the normal, tangential and binormal direction; $S_{In1}, S_{It1}$ – the normal and the tangential components of the inertia force impulse of vehicle no. 1; $S_{Tn1}, S_{Tt1}$ – the normal and the tangential components of the transportation force impulse of vehicle no. 1; $S_{Cn1}, S_{Ct1}$ – the normal and the tangential components of the Coriolis force impulse of vehicle no. 1; $S_{In2}, S_{It2}$ – the normal and the tangential components of the inertia force impulse of vehicle no. 2; $S_{Tn2}, S_{Tt2}$ – the normal and the tangential components of the transportation force impulse of vehicle no. 2; $S_{Cn2}, S_{Ct2}$ – the normal and the tangential components of the Coriolis force impulse of vehicle no. 2; $n_1, t_1, n_2, t_2, b_1, b_2$ – the coordinates of the center of mass of each vehicle with respect to the $O'$ origin of the local coordinate system; $\omega_{1b}, \omega_{2b}$ – the angular velocity of each vehicle involved in the collision about their vertical axis; $I_1, I_2$ – the moment of inertia of each vehicle relative to the vertical axis passing through their centers of mass.

In the next step, a kinematic state of both vehicles was presented in order to sum up which parameters are necessary to consider such a modified and expanded side impact collision model. Thus, the kinematic state of the vehicles for the post collision moment can be presented with the use of the following formulas:

a) for vehicle no. 1:

$$v'_{1n} = v_{1n} + \frac{S_{n1} + S_{In1} + S_{Tn1} + S_{Cn1}}{m_1}, \tag{21}$$

$$v'_{1t} = v_{1t} + \frac{S_{t1} + S_{It1} + S_{Tt1} + S_{Ct1}}{m_1}, \tag{22}$$

$$v'_{1b} = v_{1b} + \frac{S_b + S_{Ib1} + S_{Tb1} + S_{Cb1}}{m_1}, \tag{23}$$

$$S_{t1}b_1 = -S_b t_1, \tag{24}$$

$$S_{n1}b_1 = -S_b n_1, \tag{25}$$

$$\omega'_{1b} = \omega_{1b} + \frac{S_{t1}n_1 + S_{n1}t_1}{I_1}, \tag{26}$$

b) for vehicle no. 2:

$$v'_{2n} = -\frac{S_{n2} + S_{In2} + S_{Tn2} + S_{Cn2}}{m_2}, \tag{27}$$

$$v'_{2t} = v_{2t} + \frac{S_{t2} + S_{It2} + S_{Tt2} + S_{Ct2}}{m_2}, \tag{28}$$

$$v'_{2b} = v_{2b} + \frac{S_b + S_{Ib2} + S_{Tb2} + S_{Cb2}}{m_2}, \tag{29}$$

$$S_{t2}b_2 = -S_b t_2, \tag{30}$$

$$S_{n2}b_2 = -S_b n_2, \tag{31}$$

$$\omega'_{2b} = \omega_{2b} - \frac{S_{t2}n_2 + S_{n2}t_2}{I_2}. \tag{32}$$

In this case, the obtained 12 Equations 21–32 contain 32 unknowns, because it has been assumed that $S_b$ is common for both vehicles. From these equations, it can also be concluded that the parameters necessary to solve such a problem are mainly the components of the impulses of all forces taken into account in the resultant motion, as well as the impulses of the collision forces. In the typical software, such as V-Sim or PC-Crash the impulse of a collision force is usually calculated, but given a one common magnitude, internal for a period of two vehicles remaining in contact and not decomposed into two separate impulses, each for every vehicle.

Of course, it is difficult to specify both the resultant impulse of a collision force and the resultant impulse of the absolute force stemming from the resultant motion as the directions of both of them are unknown. Some additional simplifications may enable the discussed collision model more easy without removing the resultant motion phenomena. One of them may reflect the impulse of the Coriolis force, as it is responsible for the rotation of each vehicle, mainly about its vertical axis passing through the center of mass. Let us assume that the Coriolis force for each vehicle remains parallel to the road, so no additional binormal components would be taken into account. Then, the Equations 23 and 29 will be simpler:

$$v'_{1b} = v_{1b} + \frac{S_b + S_{Ib1} + S_{Tb1}}{m_1} \tag{33}$$

$$v'_{2b} = v_{2b} + \frac{S_b + S_{Ib2} + S_{Tb2}}{m_2} \tag{34}$$

As for the rest of the simplifications, a certain hypothesis can be formulated to enable determination of some unknowns so that the angles between the resultant impulse and the $O'_n$, $O'_t$ and $O'_b$ directions will not be necessary.

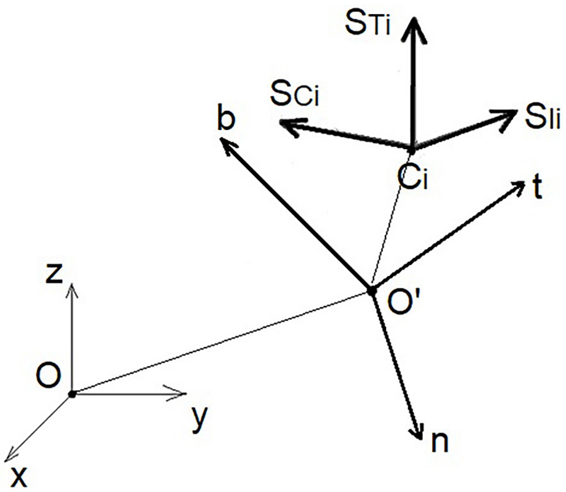


**Figure 9.** Mutual location of the impulses of forces in the resultant motion during the collision

## CONCLUSIONS

Modeling of collisions may be easy or complicated, depending on the needs and tasks ahead of a person doing so. It may also be a method that enables demonstrating the influence of the selected parameters on the motion of the vehicles taking part in the modelled collision. The more complicated model, the more unknowns and the more equations needed to be solved. The degree of complexity of such a model enables a more thorough analysis of the phenomena occurring during a road traffic collision. Of course, the most complex models need a specified software, but the problems presented in this paper may be a background for further development and implementation of the more complex cases regarding the road accidents. Introduction of the potentially less important parameters to the model enables more accurate reflection of reality of accidents that may have taken place in reality.

For example, a hypothesis regarding the coefficient of restitution can be made. Let the coefficient of restitution in the normal direction be marked with $R$ and the coefficient of restitution on the normal and binormal direction be marked with $\theta$. It seems fair to say that the coefficient of restitution in these directions is the same as the axes $O'_t$ and $O'_b$ compose the plane of collision, along which both vehicles can move while in contact. The additional equation can therefore be specified in which the normal coefficient of restitution ($R$) is regarded as an ordinary parameter used, e.g. in planar motion collisions, while the coefficient of restitution for the tangential and the binormal direction ($\theta$) can be regarded as a slip coefficient, similar to the one used in friction analysis (the vehicles in contact rub mutually against each other laterally and vertically). In [16] it was stressed – following the results of previous research by, e.g. Japanese scientists – that the $R$ coefficient takes values between 0 and 1, whereas the $\theta$ coefficient can take values between –1 and 1, depending on the nature of collision.

The coefficient $R$ can be used as in the case of a regular planar motion cases to determine the relative velocity, as, e.g. in R. Gryboś collision theory cited, e.g. in [3].

Therefore, using these coefficients such a hypothesis, making the model of the collision more useful, can be made. Regarding the impulses $S_n$, $S_t$, and $S_b$ as the forces with a very short duration period a general form of the equations with the coefficients of restitution:

$$w_t' = w_t\theta, w_n' = -w_nR, w_b' = w_b\theta \quad (35)$$

where: $w_t$ – the relative tangential velocity of each vehicle (along the $O't$ axis), $w_n$ – the relative normal velocity of each vehicle (along the $O'n$ axis), $w_b$ – the relative binormal velocity of each vehicle (along the $O'b$ axis).

The given relative velocities can be associated to the components of the translational velocity and the angular velocity. The concept of relative velocities regard both the slip (tangential and binormal) and the compression (normal) directions. When solving the Equations 21 to 32, the additional equations can be obtained to supplement them:

$$w_t' = (v_{1t}' + n_1\omega_1') - (-v_{2t}' - n_2\omega_2') = w_t\theta \quad (36)$$

$$w_n' = (v_{1n}' - t_1\omega_1') - (-v_{2n}' + t_2\omega_2') = -w_nR \quad (37)$$

$$w_b' = v_{1b}' - v_{2b}' = w_b\theta \quad (38)$$

In the case of the binormal velocity, it has been assumed that the angular component is equal to 0 because it cannot affect the process of a collision. The signs in Equation 38 result from the same returns of the $v_{1b}$ and $v_{2b}$ vectors in Figure 8.

According to the Coriolis theorem from the classical mechanics, the absolute acceleration in relation to the moving reference coordinate system is the sum of the relative, translation and Coriolis accelerations. $S_o$, if the accelerations were replaced by the impulses of the forces (which seems right because of the short period of duration of these forces), then the three components taken into account in the relative motion can be described as:

$$S_{Ii} = m_i a_{Ii} t, i = 1,2 \quad (39)$$

$$S_{Ti} = m_i a_{Ti} t, i = 1,2 \quad (40)$$

$$S_{Ci} = m_i a_{Ci} = 2m_i\omega_i w_i \sin\phi_i\, t, i = 1,2 \quad (41)$$

where: $m_i$ – the mass of each vehicle, $t$ – duration of the collision, – the relative acceleration of inertia of each vehicle, – the translation acceleration of each vehicle, – the Coriolis acceleration, – the relative velocity of inertia of each vehicle, – the angular velocity of each vehicle, – the angle between the relative velocity of inertia and the angular velocity of each vehicle.

In order to solve the matter of location of these three impulses, it can be assumed that they are mutually perpendicular. The impulse of inertia was assumed to be parallel to the longitudinal plane of symmetry of each vehicle. Thus, the remaining impulses can be located as presented, e.g. in Figure 9.

Of course, there is only one possible approach towards collision modelling. Yet, a possibility to include more complicated or specific phenomena, regarding them as momentary at the same time, seems to be the most important. Due to incorporating the more sophisticated mathematical model into a dedicated software it seems possible to reflect the real road traffic situations in virtual environment.

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