Advances in Science and Technology Research Journal

*Advances in Science and Technology Research Journal*, 18(8), 341–350
https://doi.org/10.12913/22998624/193789
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Received: 2024.07.01
Accepted: 2024.09.26
Published: 2024.11.01

# Multi-Criteria Decision Support in the Evaluation of Hydrodynamic Cavitation Effects – A Case Study

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## ABSTRACT

The work describes a method for searching for the best variant(s) of cavitation of coffee waste, taking into account the defined five criteria: sCOD/COD and DOC/TOC, which were maximised, and the others, i.e. caffeine concentration, phenols concentration and energy consumption, which were minimised. The method used in the first stage determines non-dominated variants, then a compromise variant, assuming that all criteria used are equally important. Further analysis allows determining further compromise options. The method of determining compromise solutions is based on the use of Chebyshev metric, which has been enriched with a mechanism for normalising individual criteria (which allows for their comparison) and the ability to define the importance of individual criteria. For normalisation, the so-called the ideal point, which is an internal property of the analysed variants, is determined each time for a given subset of non-dominated variants. After determining the first compromise solution, the process of generating new ideal points (their number is equal to the size of the criteria space) begins by associating the components of this solution with the components of the ideal point. Using new reference points, the method determines additional compromise solutions. The five-criteria evaluation of 20 variants obtained in the experiment showed that when adopting different values of the importance of individual criteria, some variants are never selected as a compromise. The subset of compromise variants ranged from 3 to 6. The variants that were repeatedly selected as compromise variants, with different values of the importance of individual criteria, were the variants for which the cavitation process time was: 20 or 30 minutes and the cavitation inlet pressure was 5 bar. The multi-criteria assessment showed that these are the best compromise options and can be recommended for the coffee waste cavitation process.

**Keywords:** multi-criteria decision support, hydrodynamic cavitation, coffee waste, Chebyshev metric

## INTRODUCTION

Conducting various types of laboratory experiments involving the study of technological processes usually leads to determining the values of many indicators describing their effects. These indicators are expressed in different units, and the obtained values sometimes differ by many orders of magnitude, depending on the applicable units and the accuracy typical of a given measurement equipment. By leveraging the expertise in the relevant domain, it can be discerned whether the aim is to achieve the smallest or the largest values of individual indicators, thereby enhancing process efficiency. This concept gives rise to criteria that are appropriately minimised or maximised, based on domain knowledge. In this way, a typical situation occurs in which, with many opposing criteria describing the effects of the experiment, it is difficult to choose the best of the obtained variants. Multi-criteria decision support is a discipline that deals with solving such problems. Through the use of various approaches and built-in mechanisms, it allows:

- more or less effective selection of some variants (methods from the ELECTRE family [1–3] e.g. the problem of selecting technology suppliers, cycling path selection for sustainable tourism, PROMETHE [4, 5] e.g. the problem of choosing an organisational system for expansion into new economic markets,
- building rankings from available variants,
  - complete order, e.g. various types of scalarisation methods [6–8], e.g. in economic development policy, agriculture economics, road network design, oil industry regulation, international water systems and flood control, energy policy, traffic assignment, the AHP method – Analytic Hierarchy Process [9–14] in various versions, e.g. in an assessment of the environmental performance of administrative regions,
  - partial pre-order – methods using the "surpassing" category [15, 16], e.g. in designing the life cycle of a production system,
- selecting the "best" variant (compromise variant, e.g. methods using lexicographic order [17, 18] or distance function, e.g. TOPSIS [19, 20]), e.g. for choosing an organisational system for the renovation of expressways,
- determining the best option when making group decisions [21, 22], e.g. in making decisions in emergency situations,
- determining a certain subset of compromise variants [23], e.g. when designing ferromagnetic particle separators,
- determining the Pareto front [24–27], the Multi-Skill Resource-Constrained Project Scheduling Problem.

The developed methods are specific and are never universal. The ELECTRE and PROMETHE family of methods are based on the outranking theory published by Roy in 1985 [28]. Obtaining selected variants requires entering such data as: the value of the equivalence threshold, the outranking threshold, the incomparability threshold and the importance weights of individual criteria or preference functions for comparing criteria, which in many cases leads to the situation that the variants obtain the status of incomparable variants. The rankings built using scalarisation methods depend on the applied normalisation or coding method (normalisation with respect to the extreme value, Neumann-Morgenstern coding, Pattern coding) to obtain dimensionless values that can be aggregated and the applied scalarisation strategy (additive – compensatory scalarisation or multiplicative – non-compensatory scalarisation). The AHP method presented by Saaty in 1980 [29] requires a very laborious evaluation of the compared criteria and variants, and changing these values usually completely changes the obtained ranking. Obtaining a single "best" variant is completely dependent on the hierarchy of criteria introduced in the lexicographic method or the introduced importance weights of individual criteria and the adopted distance function (e.g. Hamming, Euclid or Chebyshev). In existing group decision-making methods, hidden scalarisation is very often used to aggregate the decisions of individual decision-makers. As a result of determining the Pareto front, we usually obtain a subset of non-dominated solutions with a large number from several to several hundred, which can be treated only as a preliminary selection of the analysed variants.

The methods used are more or less complicated, based on various types of defined indicators, with or without physical interpretation, more or less intuitive and to a greater or lesser extent dependent on data entered arbitrarily by the user. Therefore, making decisions in multi-criteria spaces is not an obvious or simple matter.

The work presents an original method enabling the generation of a subset of the "best" variants, using the internal properties of the considered variants, while introducing a mechanism for normalising the values of individual criteria. This ensures the possibility of comparing them and introducing their importance by specifying weights. The presented method does not introduce a scalarisation mechanism at any stage of operation, and the obtained small subset (from 2 to 4 elements) presents the variants that are the least susceptible to the actions of the user (decision maker) in terms of introducing their own preferences, e.g. arbitrarily determining the importance of individual criteria.

The main aim of the work was to present the method used for multi-criteria decision support with regard to the hydrodynamic cavitation of spent coffee grounds. Through this approach, it is possible to indicate for which values of decision variables (cavitation process time and inlet pressure into the cavitation zone) hydrodynamic cavitation is the most effective in terms of energy consumption and the effects of destruction of lignocellulosic structures of coffee waste.

## METHODS AND MATERIALS

The author's method of determining a subset with a small number of variants from the original set of acceptable variants is a multi-stage method. The method analyses variants in a criteria space the size of which is greater than or equal to 2. There is no limit to the size of the criteria space, it can range from several to a dozen or so criteria. However, the general rule is that the criteria space should be as small as possible and should not exceed the value of 10. The method allows for determining compromise variants in a situation where some of the criteria are minimised and the remaining ones are maximised.

### Distance function – Chebyshev standard

The author's method seeks a compromise variant using the Chebyshev norm, which is a special case of calculating the distance between two points in N-dimensional space. For the minimisation task when considering a single variant, the general notation of the distance function is presented in formula (1).

$$\left[\sum_{i=1}^{N}\left(\frac{F_i(\boldsymbol{x}^*)-F_i(\boldsymbol{x})}{F_i(\boldsymbol{x}^*)}\right)^p\right]^{\frac{1}{p}} \rightarrow min \qquad (1)$$

where: $i$ – criterion index, $N$ – size of the criteria set, $p$ – power exponent (for $p$=2 the Euclidean distance is obtained), $x$ – vector of decision variables. The use of the $F_i(x^*)$ component in the denominator makes it possible to switch to dimensionless values, which allows the summation of criteria values expressed by different quantities and units. There are many ways to determine the vector $x$*, e.g. formula (3).

In the Chebyshev norm, the exponent $p$ tends to infinity ($p \rightarrow +\infty$), which implies the following form of the distance function when considering many variants (2).

$$F^*(\boldsymbol{x}) = \min_{k \in M} \max_{i \in N} \left\{ \varpi_i \frac{|F_i^o(\boldsymbol{x}) - F_i^k(\boldsymbol{x})|}{|F_i^o(\boldsymbol{x})|} \right\} \qquad (2)$$

where: k – index of the considered variant, M – number of variants, $F_i^o(x)$ – i-th component of the ideal point [27], $F_i^k(x)$ – $i$-th component of the criteria vector of variant k.

Formula (2) also introduced the $\omega_i$ component – a weight reflecting the importance of a given criterion. Thus, when the exponent $p \rightarrow +\infty$, only its largest component remains from the sum (1), while all the others can be ignored. The components of the ideal point – $F^o_i(x)$ for the minimisation task are determined using the formula

$$F_i^o(\boldsymbol{x}) = \min_{k \in N} F_i^k(\boldsymbol{x}) \qquad (3)$$

### Multi-criteria decision support

The author's method – Multi-criteria Determination of Recommended Variants (MCDRV) is a multi-stage method that can be divided into stages and steps. The block diagram of the method is shown in Figure 1.

*Stage 0*

Preparation of data for the multi-criteria evaluation process. Constructing a minimisation task by replacing the maximised criteria with minimised criteria. Values representing the maximised criteria are converted to negative values by adding a unary "minus", in accordance with the generally applicable principle that maximising a value is minimising its negative value.

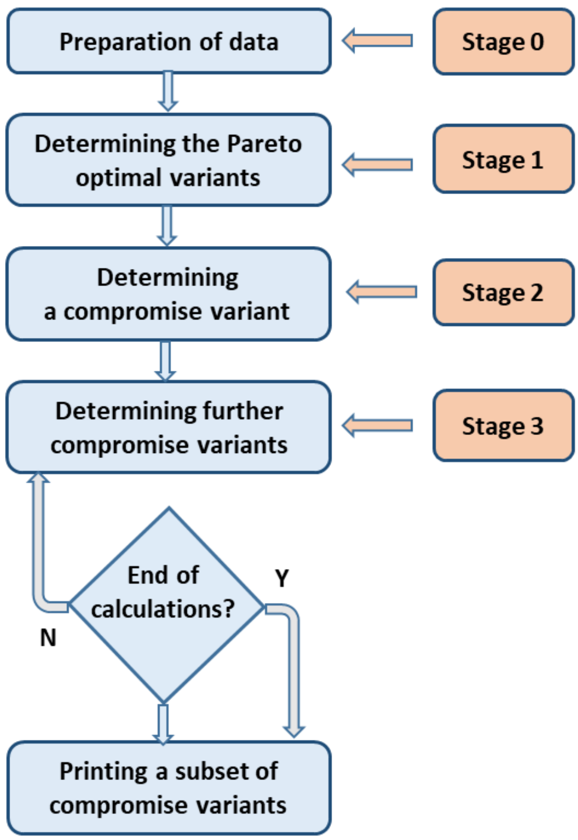


**Fig. 1.** Block diagram of the developed MCDRV method

*Stage 1 – determining the Pareto optimal variants*

Pareto optimal variants are determined (in other words, non-dominated variants, i.e. those that cannot eliminate each other). These variants create the so-called Pareto front [26] and are clearly defined in the mathematical sense. This action usually reduces the size of the set of primary variants, which is beneficial for further actions.

*Stage 2 – determining a compromise option variant*

- Step 2.1 includes determining the ideal point $F^{o}(x)$ (such a point is an internal property of the considered subset of Pareto optimal variants), which is created by combining the smallest values representing all the considered criteria (3).
- *Step 2.2* allows determining one compromise variant $F^{*}(x) = [F^{*}_{1}(x), F^{*}_{2}(x), \ldots, F^{*}_{N}(x)]^{T}$ using the Chebyshev norm (2), when the point the reference point is the ideal point $F^{o}(x) = [F^{o}_{1}(x), F^{o}_{2}(x), \ldots, F^{o}_{N}(x)]^{T}$.
- *Step 2.3* allows generating new compromise variants after introducing different importance weights of individual criteria. The rule adopted is that the sum of the weights is 1. This facilitates the interpretation of the adopted values.
- Stage 3 – determining further compromise variants
- At this stage, many compromise variants can be determined using the same weight values reflecting the importance of the adopted criteria. The process of generating further compromise variants can be repeated at subsequent levels by repeating steps 3.1 to 3.3.
- Step 3.1 is used to determine new ideal points: $F^{o1}(x)$, $F^{o2}(x)$, …, $F^{oN}(x)$ using the initial ideal point and the determined compromise variant, formula (4).

$$
\begin{aligned}
\boldsymbol{F}^{o1}(\boldsymbol{x}) &= [\boldsymbol{F}^{*}_{1}(\boldsymbol{x}), \boldsymbol{F}^{o}_{2}(\boldsymbol{x}), \ldots, \boldsymbol{F}^{o}_{N}(\boldsymbol{x})\\
\boldsymbol{F}^{o2}(\boldsymbol{x}) &= [\boldsymbol{F}^{o}_{1}(\boldsymbol{x}), \boldsymbol{F}^{*}_{2}(\boldsymbol{x}), \ldots, \boldsymbol{F}^{o}_{N}(\boldsymbol{x})\\
\boldsymbol{F}^{oN}(\boldsymbol{x}) &= [\boldsymbol{F}^{o}_{1}(\boldsymbol{x}), \boldsymbol{F}^{o}_{2}(\boldsymbol{x}), \ldots, \boldsymbol{F}^{*}_{N}(\boldsymbol{x})
\end{aligned}
\tag{4}
$$

- Step 3.2 is used to withdraw the compromise variant from the considered set and to assign the remaining variants to the newly generated ideal points according to the inverted formula (3). A given subset includes only those variants for which the values of the individual criteria components will not be better than the values of the new ideal point. The number of new subsets is equal to the number of criteria.
- Step 3.3 involves generating new compromise variants $F^{*1}(x) = [F^{*1}_{1}(x), F^{*1}_{2}(x), \ldots, F^{*1}_{N}(x)]^{T}$, $F^{*2}(x) = [F^{*2}_{1}(x), F^{*2}_{2}(x), \ldots, F^{*2}_{N}(x)]^{T}$ etc. Theoretically, there may be as many variants as the number of criteria. In practice, the user decides on the size of the subset from which a new compromise variant is determined. Thus, the number of designated compromise options usually ranges from 1 to 3.

When a very large set of variants is considered, e.g. several dozen or more, the actions described in stage 3 can be repeated. In this case, in the next step 3.1', new ideal points are determined using formula (5) repeatedly.

$$
\begin{aligned}
\boldsymbol{F}^{oN+1}(\boldsymbol{x}) &= [\boldsymbol{F}^{*1}_{1}(\boldsymbol{x}), \boldsymbol{F}^{o1}_{2}(\boldsymbol{x}), \ldots, \boldsymbol{F}^{o1}_{N}(\boldsymbol{x})]^{T}\\
\boldsymbol{F}^{oN+2}(\boldsymbol{x}) &= [\boldsymbol{F}^{o1}_{1}(\boldsymbol{x}), \boldsymbol{F}^{*1}_{2}(\boldsymbol{x}), \ldots, \boldsymbol{F}^{o1}_{N}(\boldsymbol{x})]^{T}
\end{aligned}
\tag{5}
$$

The final product of the presented method is a generated subset of compromise variants, the elements of which were determined while maintaining the same importance values of individual criteria. It is possible to designate several such subsets with different user preferences in relation to the analysed set of criteria. Therefore, it is possible to analyse which variants constitute a single subset for the same criterion importance weights, and to compare the contents of subsets with different values of the adopted weights (Fig. 2). In practice, this approach provides significantly more information about the impact of the weights used and allows for informed decisions about which variants are the best.

## Hydrodynamic cavitation of coffee waste

Coffee waste, like many other organic wastes, can be efficiently converted into energy products (methane) through anaerobic digestion processes. The specificity of coffee waste rich in lignocellulosic compounds resistant to biochemical degradation requires that it be subjected to pretreatment to ensure the highest possible increase in the degree of biodegradability. The use of hydrodynamic cavitation enables the transformation of the original structure of coffee waste, disintegration of lignocellulosic fibres and solubilisation of organic matter, which consequently leads to an increase biodegradability of such waste and hence improves the efficiency of methane production. The cavitation process with a properly

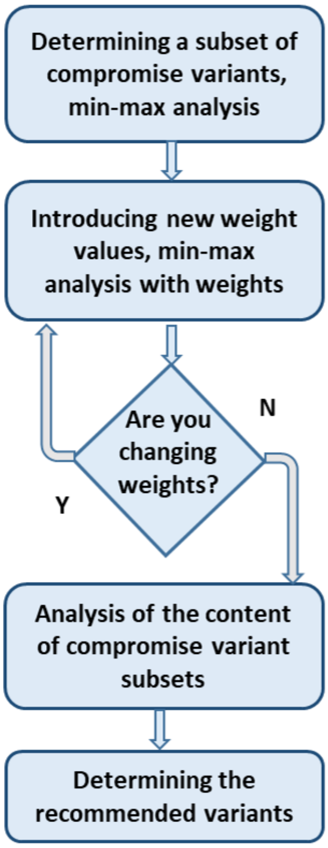


**Fig. 2.** Block diagram of generating multiple subsets of compromise variants

selected inducer is usually described by two decision variables: $x_1$ – inlet pressure to the cavitation zone and $x_2$ – process duration directly related to the number of times the stream passes through the cavitation zone. In turn, cavitation effects are described by many different physical and chemical quantities expressed in different units, and their values sometimes differ by many orders of magnitude (Table 1). These values can be used directly or can be used to construct dimensionless indicators, which will become criteria for assessing the effectiveness of the process. It should be recalled that a criterion is a quantity describing the analysed object for which the need to minimise or maximise it is indicated. In a properly constructed optimisation task, the criteria should be opposite, so some of them should be minimised and the others maximised.

Among the examined quantities describing the cavitation process of coffee waste, 5 criteria were constructed [30]. The following criteria were selected for the multi-criteria assessment: C1 – sCOD/COD (MAX.) – the proportion of dissolved COD/total COD describing the share of the solubilised fraction in the total COD, C2 – DOC/TOC (MAX.) – the proportion of dissolved

**Table 1.** Values of decision variables and criteria for the hydrodynamic cavitation process

| Variant number | Variable | | Criteria | | | | |
|---|---|---|---|---|---|---|---|
| | Time | Pressure | sCOD/COD | DOC/TOC | Concentration of caffeine | Phenols | Energy use |
| | $x_1$ | $x_2$ | C1–MAX. | C2–MAX. | C3–MIN. | C4–MIN. | C5–MIN. |
| | min. | bar | - | - | ppm | mg/l | kWh |
| v1 | 0 | 3 | 0.15 | 0.0006 | nd | 20.8 | - |
| v2 | 5 | 3 | 0.20 | 0.0008 | nd | 22.3 | 0.039 |
| v3 | 10 | 3 | 0.21 | 0.0009 | nd | 21.5 | 0.077 |
| v4 | 20 | 3 | 0.22 | 0.0009 | nd | 22.8 | 0.154 |
| v5 | 30 | 3 | 0.24 | 0.0010 | nd | 24.5 | 0.232 |
| v6 | 45 | 3 | 0.27 | 0.0011 | nd | 25.4 | 0.347 |
| v7 | 0 | 5 | 0.12 | 0.0007 | nd | 18.5 | - |
| v8 | 5 | 5 | 0.20 | 0.0010 | nd | 21.3 | 0.057 |
| v9 | 10 | 5 | 0.22 | 0.0010 | nd | 22.8 | 0.114 |
| v10 | 20 | 5 | 0.25 | 0.0011 | nd | 23.9 | 0.227 |
| v11 | 30 | 5 | 0.28 | 0.0012 | 4.8 | 26.1 | 0.343 |
| v12 | 45 | 5 | 0.32 | 0.0013 | 6.12 | 27.6 | 0.515 |
| v13 | 0 | 7 | 0.14 | 0.0005 | nd | 19.8 | - |
| v14 | 5 | 7 | 0.19 | 0.0009 | 9.64 | 22.5 | 0.077 |
| v15 | 10 | 7 | 0.24 | 0.0009 | 8.73 | 24.3 | 0.154 |
| v16 | 20 | 7 | 0.28 | 0.0010 | 5.50 | 26.9 | 0.308 |
| v17 | 30 | 7 | 0.30 | 0.0011 | 13.81 | 29.7 | 0.463 |
| v18 | 45 | 7 | 0.33 | 0.0013 | 7.55 | 33.8 | 0.694 |

organic carbon/total organic carbon, indicating the degree of biodegradability of the matter, C3 – concentration of caffeine (MIN.), C4 – concentration of phenols (MIN.) and C5 – Energy use (MIN.), i.e. overall energy consumption via hydrodynamic cavitation.

## RESULTS OF MULTI-CRITERIA DECISION SUPPORT

Conducting laboratory experiments leads to many cavitation variants in which individual values of individual criteria are obtained for a pair of decision variables ($x_1$ – inlet pressure and $x_2$ – cavitation time) (Table 1). With a large number of criteria, deciding which variant should be recommended is impossible without multi-criteria decision support.

The multi-criteria assessment of the effectiveness of hydrodynamic cavitation of coffee waste was carried out many times to determine several subsets of compromise variants for different importance weights of individual criteria. It was assumed that when calculating compromise variants, the minimum subset size is 8.

### Stage 0

The task of minimising the multi-criteria assessment was created by introducing an unary minus to the values representing the C1 and C2 criteria. In this way, the maximised criteria were replaced with minimised criteria.

### Stage 1

Pareto optimal variants were determined (non-dominated variants). Of the 18 variants introduced into the analysis, 3 variants were eliminated (v4, v5 and v14, Table 1), which means that the size of the original set (18) was reduced to a subset of 15.

**Analysis 1.** Identical weight values for all criteria.

The results of the min-max analysis for identical importance of all criteria are presented in Table 2.

**Analysis 2.** Weight values different for individual criteria – the highest importance of C2 and C5

The results of the min-max analysis with weights, for the following weight values: $\omega_1$=0.20; $\omega_2$=0.30; $\omega_3$=0.10; $\omega_4$=0.15; $\omega_5$=0.25; are presented in Table 3. It was assumed that the value of the biodegradability index (DOC/TOC) and energy consumption (Table 1) were the most important. The ideal point does not change.

**Analysis 3.** Weight values different for individual criteria – the highest importance of C5

The results of the min-max analysis with weights, for the following weight values: $\omega_1$=0.10; $\omega_2$=0.10; $\omega_3$=0.10; $\omega_4$=0.10; $\omega_5$=0.60 are presented in Table 4. It was assumed that energy consumption is the most important. The ideal point does not change.

**Analysis 4.** Weight values different for individual criteria – C2 is the most important, C3 is the least important

**Table 2.** Compromise variants in min-max analysis

| Stage 2 | Variant number | Min-max analysis<br>$\omega_1$=0.20; $\omega_2$=0.20; $\omega_3$=0.20; $\omega_4$=0.20; $\omega_5$=0.20 |
|---|---|---|
| Step 2.1 | | |
| Ideal point | – | $F^o_1$ = -0.33 $F^o_2$ = -0.13E-02 $F^o_3$ = 0.00 $F^o_4$ = 18.5 $F^o_5$ = 0.00 |
| Step 2.2 | | |
| Compromise variant | v10 | $F^*_1$ = -0.25 $F^*_2$ = -0.11E-02 $F^*_3$ = 0.00 $F^*_4$ = 23.9 $F^*_5$ = 0.227 |
| Stage 3 | | |
| Step 3.1 | | |
| New ideal points | – | $F^{o1}_1$ = -0.25 $F^{o1}_2$ = -0.13E-02 $F^{o1}_3$ = 0.00 $F^{o1}_4$ = 18.5 $F^{o1}_5$ = 0.00<br>$F^{o2}_1$ = -0.33 $F^{o2}_2$ = -0.11E-02 $F^{o2}_3$ = 0.00 $F^{o2}_4$ = 18.5 $F^{o2}_5$ = 0.00<br>$F^{o3}_1$ = -0.33 $F^{o3}_2$ = -0.13E-02 $F^{o3}_3$ = 0.00 $F^{o3}_4$ = 18.5 $F^{o3}_5$ = 0.00<br>$F^{o4}_1$ = -0.33 $F^{o4}_2$ = -0.13E-02 $F^{o4}_3$ = 0.00 $F^{o4}_4$ = 23.9 $F^{o4}_5$ = 0.00<br>$F^{o5}_1$ = -0.33 $F^{o5}_2$ = -0.13E-02 $F^{o5}_3$ = 0.00 $F^{o5}_4$ = 18.5 $F^{o5}_5$ = 0.227 |
| Step 3.3 | | |
| New compromise variants | v8<br>v15 | $F^{*1}_1$=-0.20 $F^{*1}_2$=-0.10E-02 $F^{*1}_3$= 0.00 $F^{*1}_4$= 21.3 $F^{*1}_5$= 0.057<br>$F^{*2}_1$=-0.24 $F^{*1}_2$=-0.09E-02 $F^{*1}_3$= 8.73 $F^{*1}_4$= 24.3 $F^{*1}_5$= 0.154 |

**Table 3.** Compromise variants in min-max analysis with weights – the greatest importance of C2 and C5

| Stage 2 | Variant number | Min-max analysis with weights $\omega_1$=0.20; $\omega_2$=0.30; $\omega_3$=0.10; $\omega_4$=0.15; $\omega_5$=0.25 |
|---|---|---|
| Step 2.2 | | |
| Chosen variant | v10 | $F^*_1$ = -0.25 $F^*_2$ = -0.11E-02 $F^*_3$ = 0.00 $F^*_4$ = 23.9 $F^*_5$ = 0.227 |
| Stage 3 | | |
| Step 3.1 | | |
| New ideal points | --- | *They are identical to the min-max analysis because the ideal point and the compromise option (v10) are the same* |
| Step 3.3 | | |
| New compromise variants | v8<br>v15<br>v6 | $F^{*1}_1$ =-0.20 $F^{*1}_2$ =-0.10E-02 $F^{*1}_3$ = 0.00 $F^{*1}_4$ = 21.3 $F^{*1}_5$ = 0.057<br>$F^{*2}_1$ =-0.24 $F^{*2}_2$ =-0.09E-02 $F^{*2}_3$ = 8.73 $F^{*2}_4$ = 24.3 $F^{*2}_5$ = 0.154<br>$F^{*3}_1$ =-0.27 $F^{*3}_2$ =-0.11E-02 $F^{*3}_3$ = 0.00 $F^{*3}_4$ = 25.4 $F^{*3}_5$ = 0.347 |

**Table 4.** Compromise variants in min-max analysis with weights – the highest importance of C5

| Stage 2 | Variant number | Min-max analysis with weights $\omega_1$=0,10; $\omega_2$=0,10; $\omega_3$=0,10; $\omega_4$=0,10; $\omega_5$=0,60 |
|---|---|---|
| Step 2.2 | | |
| Chosen variant | v10 | $F^*_1$ = -0.25 $F^*_2$ = -0.11E-02 $F^*_3$ = 0.00 $F^*_4$ = 23.9 $F^*_5$ = 0.227 |
| Stage 3 | | |
| Step 3.1 | | |
| New ideal points | – | *They are identical to the min-max analysis because the ideal point and the compromise variant (v10) are the same* |
| Step 3.3 | | |
| New compromise variants | v8<br>v15 | $F^{*1}_1$ =-0.20 $F^{*1}_2$ =-0.10E-02 $F^{*1}_3$ = 0.00 $F^{*1}_4$ = 21.3 $F^{*1}_5$ = 0.057<br>$F^{*2}_1$ =-0.24 $F^{*2}_2$ =-0.09E-03 $F^{*2}_3$ = 8.73 $F^{*2}_4$ = 24.3 $F^{*2}_5$ = 0.154 |

The results of the min-max analysis with weights, for the following weight values: $\omega_1$=0.10; $\omega_2$=0.50; $\omega_3$=0.05; $\omega_4$=0.10; $\omega_5$=0.25 are presented in Table 4. It was assumed that the value of the biodegradability index (DOC/TOC) was the most important, and the concentration of caffeine was the least important (Table 1). The ideal point does not change.

**Analysis 5.** Weight values different for individual criteria – C2 and C3 are the most important

The results of the min-max analysis with weights, for the following weight values: $\omega_1$=0.10; $\omega_2$=0.40; $\omega_3$=0.25; $\omega_4$=0.10; $\omega_5$=0.15 are presented in Table 6. It was assumed that the value of the biodegradability index (DOC/TOC) and the caffeine concentration (Table 1) were the most important. The ideal point does not change.

**Analysis 6.** Weight values different for individual criteria – C3 and C5 are the least important

The results of the min-max analysis with weights, for the following weight values: $\omega_1$=0.20; $\omega_2$=0.50; $\omega_3$=0.05; $\omega_4$=0.20; $\omega_5$=0.05 are presented in Table 7. It was assumed that caffeine concentration and energy consumption were the least important (Table 1). The ideal point does not change. The summary results of the analyses performed are presented in Table 8.

## DISCUSSION

The obtained results (Table 8) indicate that the most frequently represented variants were those for which the inlet pressure to the cavitation zone was $x_2$ = 5 bar (variants: v8. v9. v10. v11). For inlet pressures $x_2$ = 3 and $x_2$ = 7 bar. the system generated only one compromise variant – v6 and v15. respectively. This fact proves the dominance of compromise variants at a pressure of 5 bar. The selected compromise variants in subsequent analyses represented all durations of the hydrodynamic cavitation process – from 5 to 45 min. Content analysis of the subsets of compromise variants shows that only the v10 variant was included in all subsets. This proves that this variant is the least sensitive to the introduced weight values representing the importance of individual criteria and therefore should be recommended as the optimal variant, inference analogous to multi-criteria analysis was used in [23]. Variant v6 (process time 45 min. pressure 3 bar) was chosen as the first compromise variant when a very low value was assumed for the validity of criterion 5 (energy consumption) ($\omega_5$ = 0.05), compare [30].

**Table 5.** Compromise variants in min-max analysis with weights – the highest importance of C2, the least importance of C3

| Stage 2 | Variant number | Min-max analysis with weights<br>$\omega_1$=0.10; $\omega_2$=0.50; $\omega_3$=0.05; $\omega_4$=0.10; $\omega_5$=0.25 |
|---|---|---|
| Step 2.2 | | |
| Chosen variant | v11 | $F^*_1$ = -0.28 $F^*_2$ = -0.12E-02 $F^*_3$ = 4.80 $F^*_4$ = 26.1 $F^*_5$ = 0.343 |
| Stage 3 | | |
| Step 3.1 | | |
| New ideal points | – | $F^{o1}_1$ = -0.28 $F^{o1}_2$ = -0.13E-02 $F^{o1}_3$ = 0.00 $F^{o1}_4$ = 18.5 $F^{o1}_5$ = 0.00<br>$F^{o2}_1$ = -0.33 $F^{o2}_2$ = -0.12E-02 $F^{o2}_3$ = 0.00 $F^{o2}_4$ = 18.5 $F^{o2}_5$ = 0.00<br>$F^{o3}_1$ = -0.33 $F^{o3}_2$ = -0.13E-02 $F^{o3}_3$ = 4.80 $F^{o3}_4$ = 18.5 $F^{o3}_5$ = 0.00<br>$F^{o4}_1$ = -0.33 $F^{o4}_2$ = -0.13E-02 $F^{o4}_3$ = 0.00 $F^{o4}_4$ = 26.1 $F^{o4}_5$ = 0.00<br>$F^{o5}_1$ = -0.33 $F^{o5}_2$ = -0.13E-02 $F^{o5}_3$ = 0.00 $F^{o5}_4$ = 18.5 $F^{o5}_5$ = 0.343 |
| Step 3.3 | | |
| New compromise variants | v6 | $F^{*1}_1$ =-0.27 $F^{*1}_2$ =-0.11E-02 $F^{*1}_3$ = 0.00 $F^{*1}_4$ = 25.4 $F^{*1}_5$ = 0.347 |
| | v10 | $F^{*2}_1$ =-0.25 $F^{*2}_2$ =-0.11E-02 $F^{*2}_3$ = 0.00 $F^{*2}_4$ = 23.9 $F^{*2}_5$ = 0.227 |

**Table 6.** Compromise variants in min-max analysis with weights – the greatest importance of C2 and C3

| Stage 2 | Variant number | Min-max analysis with weights<br>$\omega_1$=0.10; $\omega_2$=0.40; $\omega_3$=0.25; $\omega_4$=0.10; $\omega_5$=0.15 |
|---|---|---|
| Step 2.2 | | |
| Chosen variant | v11 | $F^*_1$ = -0.28 $F^*_2$ = -0.12E-02 $F^*_3$ = 4.80 $F^*_4$ = 26.1 $F^*_5$ = 0.343 |
| Stage 3 | | |
| Step 3.1 | | |
| New ideal points | – | *They are identical to the min-max analysis with weights because the ideal point and the compromise variant (v11) are the same* |
| Step 3.3 | | |
| New compromise variants | v6<br>v10 | $F^{*1}_1$ =-0.27 $F^{*1}_2$ =-0.11E-02 $F^{*1}_3$ = 0.00 $F^{*1}_4$ = 25.4 $F^{*1}_5$ = 0.347<br>$F^{*2}_1$ =-0.25 $F^{*2}_2$ =-0.11E-02 $F^{*2}_3$ = 0.00 $F^{*2}_4$ = 23.9 $F^{*2}_5$ = 0.227 |

**Table 7.** Compromise variants in min-max analysis with weights – least importance C3 and C5

| Stage 2 | Variant number | Min-max analysis with weights<br>$\omega_1$=0.20; $\omega_2$=0.50; $\omega_3$=0.05; $\omega_4$=0.20; $\omega_5$=0.05 |
|---|---|---|
| Step 2.2 | | |
| Chosen variant | v6 | $F^*_1$ =-0.27 $F^*_2$ =-0.11E-02 $F^*_3$ = 0.00 $F^*_4$ = 25.4 $F^*_5$ = 0.347 |
| Stage 3 | | |
| Step 3.1 | | |
| New ideal points | – | $F^{o1}_1$ = -0.27 $F^{o1}_2$ = -0.13E-02 $F^{o1}_3$ = 0.00 $F^{o1}_4$ = 18.5 $F^{o1}_5$ = 0.00<br>$F^{o2}_1$ = -0.33 $F^{o2}_2$ = -0.11E-02 $F^{o2}_3$ = 0.00 $F^{o2}_4$ = 18.5 $F^{o2}_5$ = 0.00<br>$F^{o3}_1$ = -0.33 $F^{o3}_2$ = -0.13E-02 $F^{o3}_3$ = 0.00 $F^{o3}_4$ = 18.5 $F^{o3}_5$ = 0.00<br>$F^{o4}_1$ = -0.33 $F^{o4}_2$ = -0.13E-02 $F^{o4}_3$ = 0.00 $F^{o4}_4$ = 25.4 $F^{o4}_5$ = 0.00<br>$F^{o5}_1$ = -0.33 $F^{o5}_2$ = -0.13E-02 $F^{o5}_3$ = 0.00 $F^{o5}_4$ = 18.5 $F^{o5}_5$ = 0.347 |
| Step 3.3 | | |
| New compromise variants | v10 | $F^{*1}_1$ =-0.25 $F^{*1}_2$ =-0.11E-02 $F^{*1}_3$ = 0.00 $F^{*1}_4$ = 23.9 $F^{*1}_5$ = 0.227 |
| | v8<br>v9 | $F^{*2}_1$ =-0.20 $F^{*2}_2$ =-0.10E-02 $F^{*2}_3$ = 0.00 $F^{*2}_4$ = 21.3 $F^{*2}_5$ = 0.057<br>$F^{*3}_1$ =-0.22 $F^{*3}_2$ =-0.10E-02 $F^{*3}_3$ = 0.00 $F^{*3}_4$ = 22.8 $F^{*3}_5$ = 0.114 |

## CONCLUSIONS

The five-criteria assessment of 18 variants obtained in the experiment showed that when adopting different values of the importance of individual criteria. some variants are never indicated by the multi-criteria decision support as compromise. The size of the subset of compromise variants was from 3 to 4. The variants that were repeatedly selected as compromise. with different values of the importance weights of individual criteria. were the variants for which the cavitation process time

**Table 8.** Compromise variants obtained during multi-criteria analysis

| Analysis number | Criteria weights | Variant |
|---|---|---|
| Analysis 1 | $\omega_1$=0.20; $\omega_2$=0.20; $\omega_3$=0.20; $\omega_4$=0.20; $\omega_5$=0.20 | v10 / v8, v15 |
| Analysis 2 | $\omega_1$=0.20; $\omega_2$=0.30; $\omega_3$=0.10; $\omega_4$=0.15; $\omega_5$=0.25 | v10 / v8, v15, v6 |
| Analysis 3 | $\omega_1$=0.10; $\omega_2$=0.10; $\omega_3$=0.10; $\omega_4$=0.10; $\omega_5$=0.60 | v10 / v8, v15 |
| Analysis 4 | $\omega_1$=0.10; $\omega_2$=0.50; $\omega_3$=0.05; $\omega_4$=0.10; $\omega_5$=0.25 | v11 / v6 / v10 |
| Analysis 5 | $\omega_1$=0.10; $\omega_2$=0.40; $\omega_3$=0.25; $\omega_4$=0.10; $\omega_5$=0.15 | v11 / v6 / v10 |
| Analysis 6 | $\omega_1$=0.20; $\omega_2$=0.50; $\omega_3$=0.05; $\omega_4$=0.20; $\omega_5$=0.05 | v6 / v10 / v8, v9 |

ranged from 5 to 20 minutes and the cavitation inlet pressure equalled 5 bar. The multi-criteria assessment showed that the best compromise variant that should be recommended for the coffee waste cavitation process is variant v10.

## Acknowledgements

The APC was funded by the Lublin University of Technology Scientific Fund FD-20/IT-3/001.

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