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Application of Coordinate Measuring Machines for Analysis of a Controlled Radius Based on Linear Regression

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ABSTRACT

The paper presents a new method of analysis of a controlled radius (CR). The presented method was verified based on simulations of coordinate measurements and during real measurements conducted on the selected coordinate measuring machine (CMM) – ACCURA II. In addition, the investigations were conducted by means of the Calypso metrology software and the Python programming environment. Computer simulations were performed for the selected dispersion of measurement points relating to nominal data of measured products. The actual coordinate measurements were conducted by using various measurement strategies. The created method takes into account the analysis of flattening of a considered curve, for which the measurement characteristic CR is determined. The analysis was conducted based on linear regression. The advantage of the presented method is the possibility of its implementation in software of a CMM and, consequently, the use of the above-mentioned method in industry. The new method detected flat fragments of measured objects regardless of the applied number of measurement points.

Keywords: controlled radius, coordinate measuring machine, coordinate metrology, metrology of geometrical quantities.

INTRODUCTION

Coordinate measurements are currently very popular both in industrial practice and research laboratories. Many different types of coordinate measuring systems can be distinguished. They are used to investigate products of various geometric shapes i.e., consisting of free-form surfaces and characterized by regular elements [1,2]. Coordinate measuring systems (CMSs) are used in various areas (e.g., in the aerospace industry). The examples of currently applied coordinate measuring systems are coordinate measuring machines (CMMs) [3,4] with touch and non-contact measuring probes, photogrammetry systems, computed tomographs [5], measuring arms [6] and CNC machine tools equipped with measuring probes [4]. The capabilities of the selected laser measuring system were verified by the authors of the work [7]. They compared the measurement results of the considered non-contact system to the results of measurements obtained by using a CMM. The use of a CMS based on a CNC machine tool is presented in [8]. However, CMMs are still characterized by better measurement accuracy [9] and versatility than the above-mentioned CNC machine tools. In addition, they are made in various measuring ranges enabling measurements of products of various sizes. The CMM user can perform coordinate measurements of various measurement characteristics in accordance with different ISO and ASME standards. The choice of a CMS for a given measurement task depends on the desired accuracy, expressed in the form of the measurement uncertainty, which should be matched to the tolerance of an investigated measurement characteristic.

Many factors influence the measurement uncertainty. They include errors of coordinate measuring systems, environmental conditions in a quality control department, the level of training of operators of measuring systems, a measurement strategy and product characteristics [10,11]. The measurement strategy includes, for example, the measurement points distribution on parts, measurement speed when using scanning measuring probes, filtration parameters of results of measurements and calculation algorithms that are the part of software of CMSs [10-12]. For example, the article [13] presents the impact of metrological software on the results of the probe radius compensation process, which consists in calculating corrected measurement points based on indicated points. The results of coordinate measurements obtained by using two software packages of leading metrological companies (Carl Zeiss and Mitutoyo) were compared.

In the case of some elements of a measurement strategy, manufacturers of CMSs offer the guidelines for their selection to reduce the uncertainty of measurements. The example of such a company is Carl Zeiss, which in the publication [14] presents exemplary values of measurement speed, distances between measurement points, types of associated elements and parameters of measurement data filtration that can be used during coordinate measurements and analysis of results of measurements of specific measurement characteristics. Unfortunately, in the case of the following measurement characteristic – the controlled radius, the above-mentioned publication does not provide any guidance on the assessment of this characteristic.

In most cases, the tolerances that have been defined in the ISO and ASME standards are also reflected in software of the mentioned advanced CMSs. The example of some popular metrological software is Calypso by the Carl Zeiss company. Unfortunately, not all measurement characteristics from the standards are included in this software. The controlled radius, defined by the symbol CR in technical documentation of a product, is the example of a characteristic not being a part of Calypso 2021.

The accuracy of the assessment of the controlled radius characteristic depends on the points distribution on a measured surface. This conclusion was drawn based on the preliminary research, which was the basis for the investigations, the results of which are presented in this article. So far, many articles have been published presenting various ways of positioning measurement points on measured objects, which mainly belong to two groups of methods (blind sampling and adaptive sampling methods) [15]. The measurement points distribution influences, among others, the time of coordinate measurements [16]. For example, Pagani and Scott [17] proposed the measurement point distribution strategy that includes the length of the curve under consideration and its complexity. The points positioning method was compared, among others, to the Latin Hypercube Sampling. The comparison of selected methods of measurement points distribution was also made in [18]. In turn, work [19] shows the method of locating points for coordinate measurements of blades. Moreover, Mian et al. [20] analysed the points positioning algorithms applicable to measurements of flatness deviations. In the case of the simulation and experimental tests, the results of which are presented in this paper, the uniform points distribution was used to assess the controlled radius CR.

Based on the review of the literature, a very high popularity of contact measurements carried out with the use of CMMs and the existence of many factors determining final results of coordinate measurements have been demonstrated. For example, measurement results depend on algorithms of metrological software. In addition, the lack of guidelines of the selected producer of CMMs in the field of coordinate measurements of a controlled radius was indicated. Therefore, based on the analysis of the literature regarding the coordinate measuring technique, it was decided to carry out research on coordinate measurements of a controlled radius.

This article presents the method of evaluating the controlled radius based on results of using a CMM. Coordinate measuring machines are successfully used to measure a radius. For example, Kawalec and Magdziak [21] and Kiran [22] presented research results based on which a radius of a hole can be calculated. The developed method can be used in commercial measurement software and thus in industrial practice. Measurements of the controlled radius conducted by a CMM can contribute to the increase in the automation of the analysis of the considered measurement characteristic. This, as a consequence, can improve the efficiency of measurements and the entire production process. At present, the controlled radius is assessed mainly manually in industry by using optical measuring systems. The disadvantage of measurements carried out in the manual mode is the dependence of results of measurements on the level of training of operators of measuring systems. The use of the developed method enables eliminating the negative operators' influence on results of measurements. The following sections of the paper concern the definition of the controlled radius CR, the proposed method algorithm of evaluating the considered measurement characteristic, the results of the numerical and experimental investigations and the conclusions drawn based on the research.

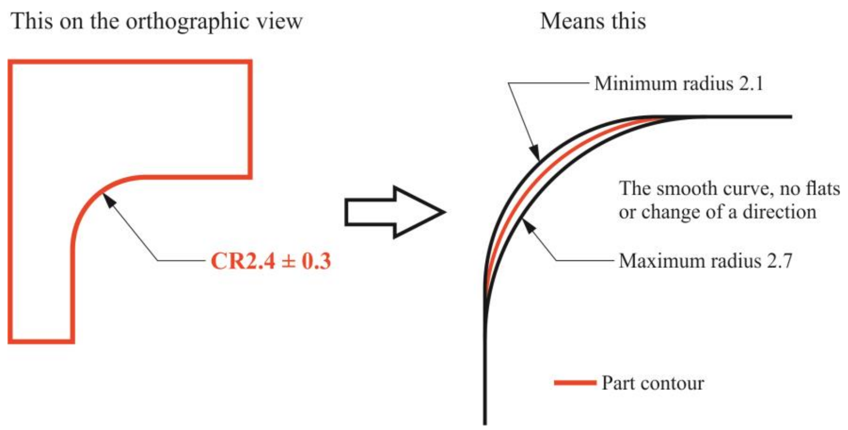
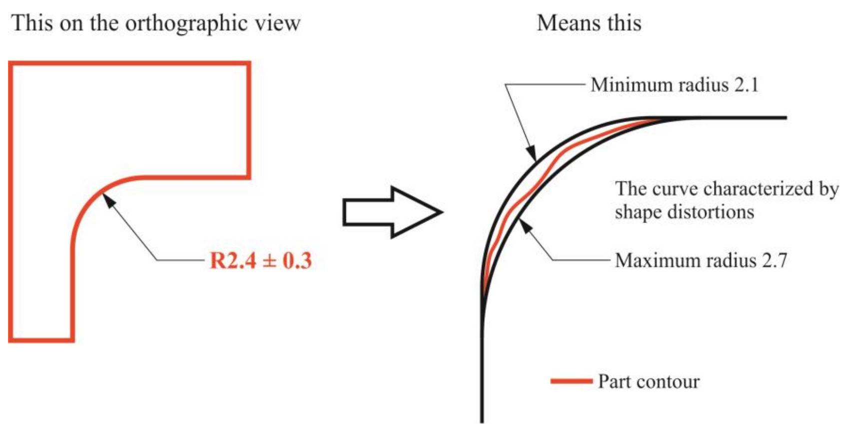
CONTROLLED RADIUS CR

The definition of the controlled radius is presented in the ASME Y14.5-2018 standard [23]. The value of the controlled radius in the technical documentation of an analysed product is preceded by the CR symbol. In the case of the controlled radius, for which the location of the centre of a circular arc is not defined by dimensions, the tolerance zone is formed by two arcs of circles that are tangent to the surfaces of a product adjacent to an evaluated circular arc. However, if the dimensions of the centre of the considered arc of a circle are specified, then the two arcs of circles forming a tolerance zone should be concentric.

As in the case of other geometrical tolerances, the profile representing a measured surface should be within a tolerance zone for a product in order to be considered to be made according to specification. In addition, a measured profile should be a smooth curve, without flattening and changing a direction of a curve. The radius determined on the basis of measurement points should be smaller than a maximum radius and larger than a minimum radius. The comparison of a standard and controlled radii, whose definitions are presented in the standard [23], is shown in Table 1.

In the case of a standard radius, a measured profile does not need to be a smooth curve. It may contain flat fragments and may be characterized by a change of a direction. Thus, the evaluation of a controlled radius is a much more difficult task than the assessment of the correctness of a standard

Table 1. Explanation of the controlled and standard radii based on the international standard [23]

Controlled radius

Standard radius


radius. The user of a measuring system, when measuring a controlled radius, must pay attention to the shape of a measured curve, which should not deviate from a smooth curve. Therefore, it is necessary to develop a method for assessing a controlled radius, which enables the detection of disturbances in the shape of a curve for which a controlled radius has been determined. Moreover, the need to conduct research regarding the considered measurement characteristic results from the fact that the standard [23] does not present any method of assessing a controlled radius that can be implemented in the coordinate metrology.

A NEW METHOD OF CONTROLLED RADIUS ASSESSMENT

The proposed method of assessing a controlled radius is divided into several stages. At the beginning, measurement points, exported from metrological software of a CMM and representing a measured profile of an object, are loaded to a program responsible for assessing the accuracy of the considered measurement characteristic – the controlled radius CR. When selecting the number of measurement points during coordinate measurements, the user can rely on the guidelines published in [14]. In the case of numerical and experimental research, the Python programming environment was used to develop a program responsible for the assessment of a controlled radius. The choice of Python resulted from the desire to use the new method in commercial metrology software.

In the next step, the filtration of measurement points located along the profile of a surface, for which the controlled radius measurement characteristic has been defined, is carried out. For this purpose, the low-pass filter and a specific value of the filter frequency should be used. The purpose of the filtration is to reduce the measurement noise, which may lead to the incorrect assessment of a controlled radius. Then, it is verified whether the measurement points representing an analysed controlled radius are within a tolerance zone formed by two arcs of circles. If this condition is met, the next stages of the controlled radius evaluation method are implemented.

Another stage of the developed method concerns the performance of linear regression for the assumed number of measurement points. The choice of the number of points is made by a CMM user. The result of the linear regression is the correlation coefficient R, which should be compared with the threshold value (R_{gr}) of the correlation coefficient. The definition of the correlation coefficient is presented in [24]. The correlation coefficient was assumed to vary from 0 to 1. As in the case of approximated points in the linear regression process, the choice of the R_{gr} parameter value depends on the person responsible for the analysis of coordinate measurement results. The method of selecting the value of the R_{gr} parameter is presented in the further part of this article. On the basis of this comparison, it is possible to verify the correctness of a considered profile and the measurement characteristics of a controlled radius, taking into account the presence of flattening of a profile. In the case of its existence, improper execution of a controlled radius should be stated. In addition, by comparing R to R_{gr}, it is possible to determine the fragments of a curve, which are characterized by the occurrence of profile flattening. The proposed method algorithm of analysis of a controlled radius is presented in Figure 1.

The program developed by using the Python programming environment, which enables the assessment of the accuracy of a controlled radius, includes all stages of the developed method. In the case of verification of the position of a measured profile within a tolerance zone, the matplotlib module of Python was applied. The developed shape of the tolerance zone for a controlled radius and the procedure for verifying the correctness of a measured surface are fully compliant with the requirements of the standard [23], which should also be the basis for the development of metrological software cooperating with coordinate measuring systems.

SIMULATION INVESTIGATIONS

The numerical research concerned the selected steps of the developed method, which is presented in the third section. The simulation studies were divided into several stages. In the first one, the method of selecting the limiting frequency value of the low-pass filter, by means of which measurement data representing a controlled radius are filtered, was presented. The third-order Butterworth filter was used for the calculations. The Python scipy module [25] was used to perform the filtration. The choice of the cut-off / critical frequency depends on the degree of randomness of measurement data. This process was carried out based on the filtration of measurement points, which were randomly generated in relation to the selected theoretical circle. The interval between the measurement and nominal data was ± 0.005 mm, which represents the measurement noise. The range of ± 0.005 mm is very large in the case of the noise during measurements conducted by touch probes. Therefore, such a value can only be used during simulation tests (verifying the possibility of using the created method in coordinate metrology and during real measurements), or in the case of using less accurate CMSs, for example based on non-contact probes. The measurement points were generated with the assumed range value by using the Calypso software operating in the simulation mode. The enlarged, randomly generated measurement points are shown in Figure 2. In turn, the simulated profile before filtering measurement data is presented in Figure 3.

Table 2 presents the profiles representing the simulated contours (for a measurement element of the circle type) after filtering, taking into account different values of the frequency of the low-pass filter. In addition, for comparison purposes, the figures in Table 2 also include the simulated profile, which is presented in Figure 3, before filtering the measurement data. The frequency of the low-pass filter was selected for further numerical research based on the analysis of the data presented in Table 2. Finally, the frequency of 5 Hz was used. The choice of frequency resulted from the comparison of the filtered profile with the error of the applied coordinate measuring machine. For the selected frequency, the form deviations after filtering correspond to the maximum permissible error of the CMM used during the experiments. In addition, the applicability of the filtration parameters proposed in [14] has been verified. However,

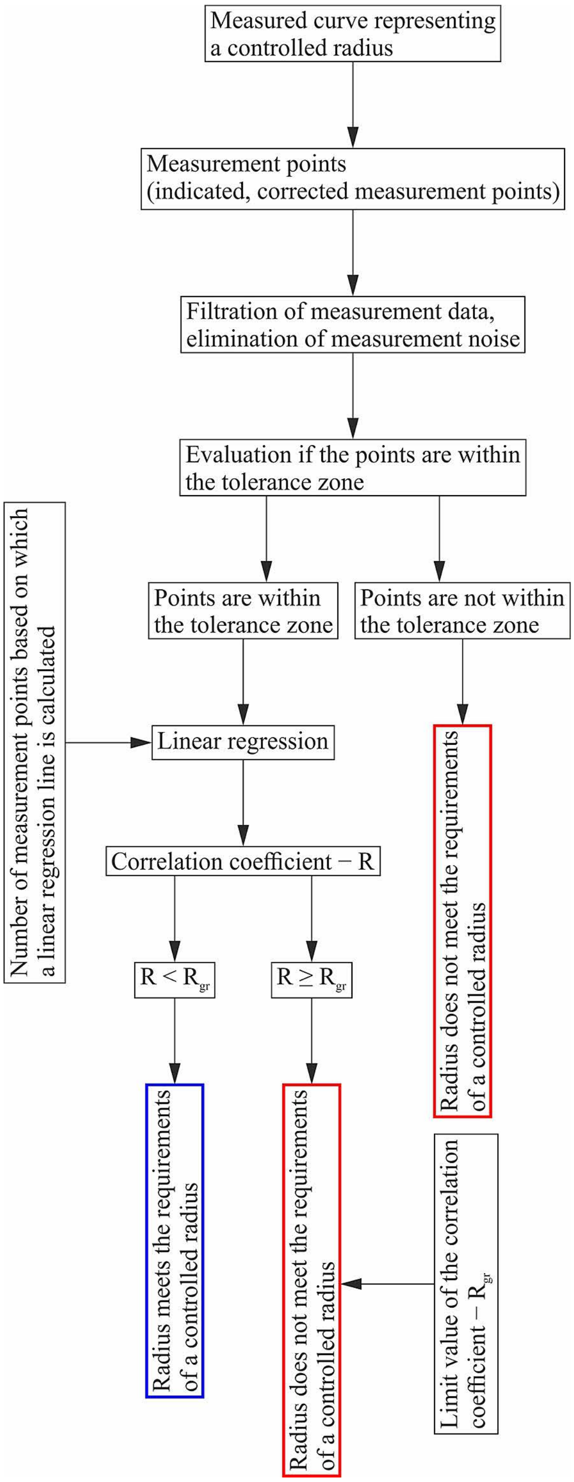


Fig. 1. The proposed method algorithm of estimating a controlled radius

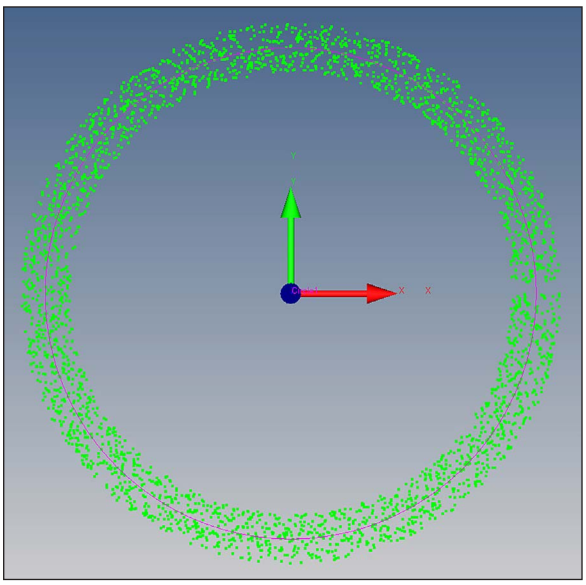


Fig. 2. The randomly generated measurement points for the circle measurement element

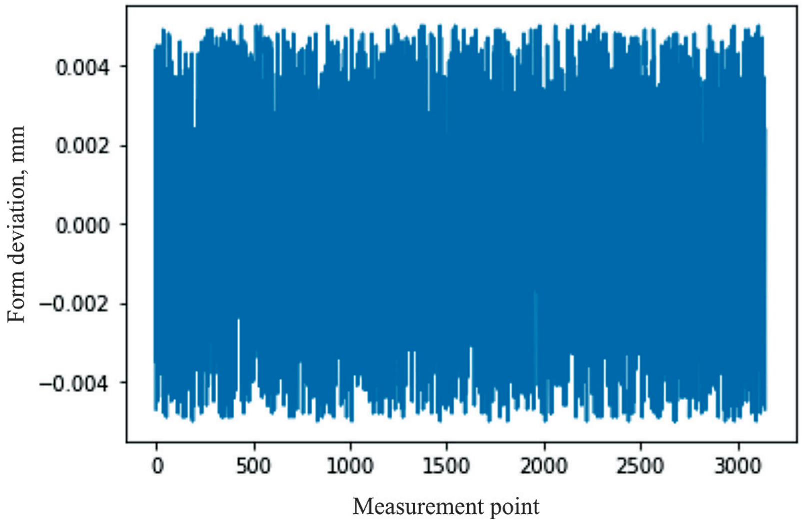


Fig. 3. The unfolded simulated profile before filtering the measurement data (for a measurement element of the circle type)

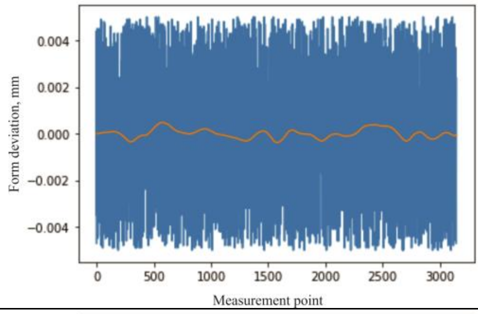
they did not lead to a satisfactory reduction of the measurement noise, which should be minimized for correct evaluation of a controlled radius.

The second stage of the simulations was carried out for two selected theoretical curves, which were imported to the Calypso software. Figures 4 and 5 show the considered curves, which represent the measured profiles of the object, for which the measurement characteristic of the controlled radius was defined. Both curves contain fragments of straight lines that should be detected as flattening by the developed method of assessing a controlled radius. The first curve contains two segments of straight lines, while the second curve consists of two arcs of circles and one line segment. In the case of both curves, the same dispersion of simulated measurement points, equal to ± 0.005 mm, in relation to the theoretical curves was used as in the first stage of the numerical research and for which the frequency of the low-pass filter was selected. The dispersion of points simulating the measurement data in relation to the theoretical curves was set in the Calypso software (Fig. 6). The dispersion of simulated measurement points was equal to ± 0.005 mm. Therefore, the Calypso software settings, presented in Figure 6, included the following values: from -0.005 to 0.005. In addition, the dispersion was activated.

In the next stage of the numerical research, the theoretical points (not including the measurement noise) and simulated measurement points were exported from Calypso to the calculation program responsible for the analysis of a controlled radius. Thus, the assessment of a controlled radius was carried out for both theoretical and simulated data. The number of measurement points was 50. The simulated data was filtered by using the low-pass filter with the selected frequency. The limit correlation coefficient R_{gr} of 0.99985 was adopted for the calculations. In the case of the numerical research, R_{gr} was selected arbitrarily based on the experience of the user of a CMS. The method of selecting the limiting correlation coefficient, regardless of the operator of a CMM, in the case of real coordinate measurements is presented in the next section of this article. This method was developed because the use of the incorrect value of the limit correlation coefficient R_{gr} may contribute to the erroneous evaluation of a controlled radius. The results of the calculations carried out for the first curve and the simulated points are shown in Figure 7. The calculations for two groups of data (theoretical and simulated) were aimed at checking whether the cut-off frequency of the applied low-pass filter was selected properly. The calculation results for the simulated data correspond to the results obtained on the basis of the theoretical points. This indicates the need to filter the measurement data.

It is possible to determine the fragments of the curve that are not arcs of circles and are characterized by the presence of flattening based on the analysis of the results of the calculations presented in Fig. 7. In the case of the first curve, flattening occurs around the points of the considered curve with numbers 11–21 and 33–43. At these points, the value of the parameter R is greater than the assumed limiting correlation coefficient R_{gr}. Thus, the controlled radius does not meet the requirements of the standard

Table 2. The simulated profiles after filtering by using different critical frequencies

<table>
<tr><th>The profile before and after filtration</th><th>The frequency of the low-pass filter, Hz</th></tr>
<tr><td>
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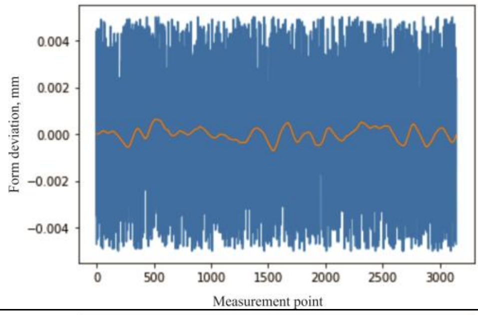
<tr><td>
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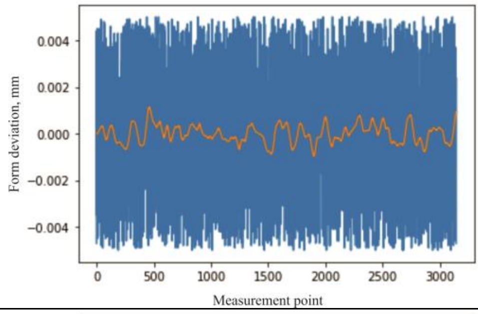
<tr><td>
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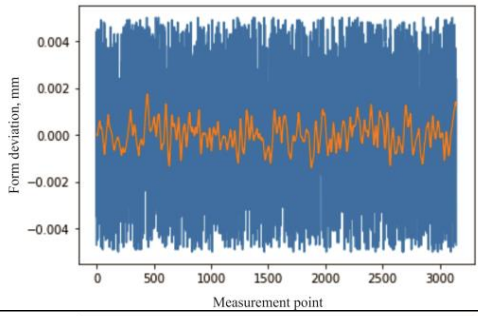
<tr><td>
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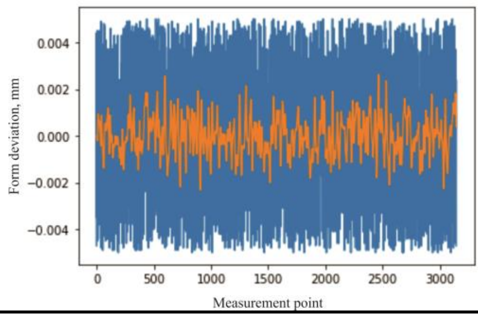
<tr><td>
</td><td>40</td></tr>
</table>

[23]. Similar calculations were carried out for the second considered curve. The same limiting correlation coefficient R_{gr} was used as for the first curve. In the case of the second curve, the results of the calculations carried out for the simulated data also correspond to the results obtained for the theoretical points. In the case of the considered curve, flattening occurs approximately at points 19–32 (Fig. 8).

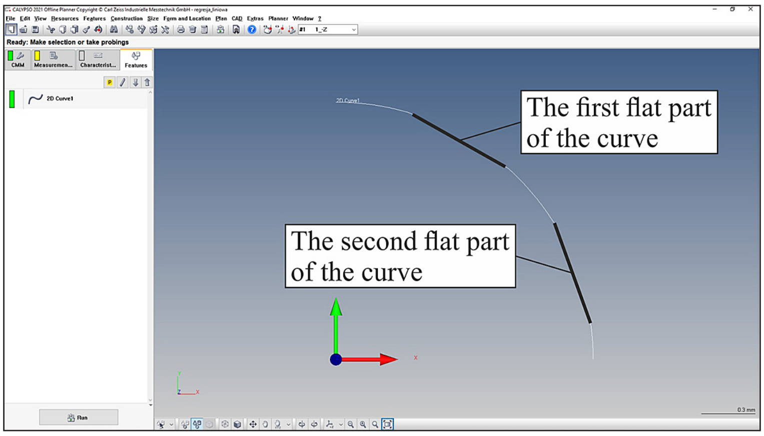


Fig. 4. The first theoretical curve to be analysed

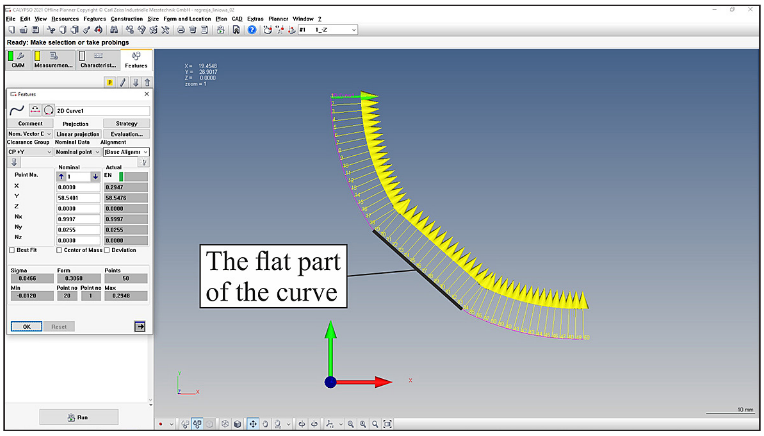


Fig. 5. The second theoretical curve to be analysed for which the controlled radius was defined

EXPERIMENTAL RESEARCH

To verify the results of the simulations, the experimental research was carried out. It was performed by using the ACCURA II CMM (Fig. 9), the VAST XXT probe (Fig. 10) and the Calypso metrological software. The applied CMM was calibrated. The machine is characterized by the following parameters:

- $E_{0,\ MPE} = 0.5 + L/152$, µm;
- $R_{0,\ MPL} = 0.5$ µm;
- $P_{Form.Sph.Scan:PP:Tact,MPE} = 2.3$ µm (for the VAST XT probe).

The investigations were conducted for three selected surfaces of the product obtained by using the rapid prototyping technology. The research was carried out, unlike the simulation studies, without filtering the measurement data. During the manufacturing process of the product, each of the measured surfaces was burdened with flattening, which should be detected by the developed method based on the results of the conducted experimental tests. Figure 11 shows the measured object with marked flat parts of its surface. Moreover, Figure 11 presents the fragment of the technical documentation of the product with the marked form tolerance – profile any line – equal to 0.3 mm. The measured value of the form deviation was equal to 0.27 mm and was smaller than the assumed tolerance. This means that the product is made in accordance with the specification, considering only the profile any line. Analogically to the numerical tests, the experimental investigations were performed for selected stages of the developed method of the controlled radius analysis. They are mainly related to the linear regression. The main purpose of the experimental research was not to determine the form deviations of the measured product. The experimental investigations were carried out to check whether the developed method correctly detects the places of the measured surfaces characterized by the presence of flat parts. During the experimental research, the limit correlation coefficient R_{gr} was adopted based

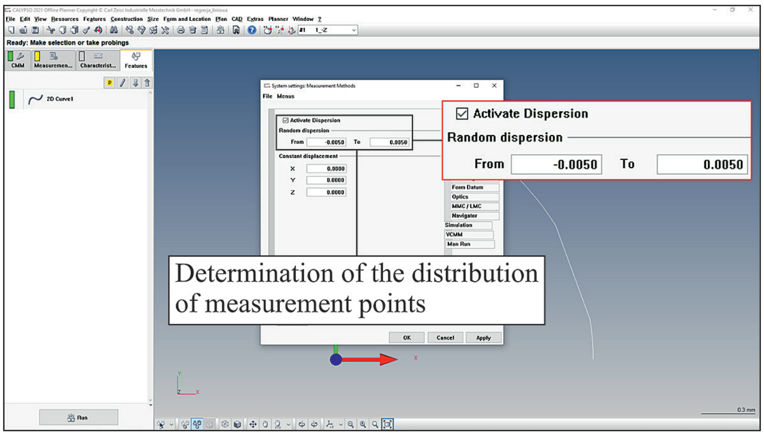


Fig. 6. Determining the measurement points distribution in Calypso

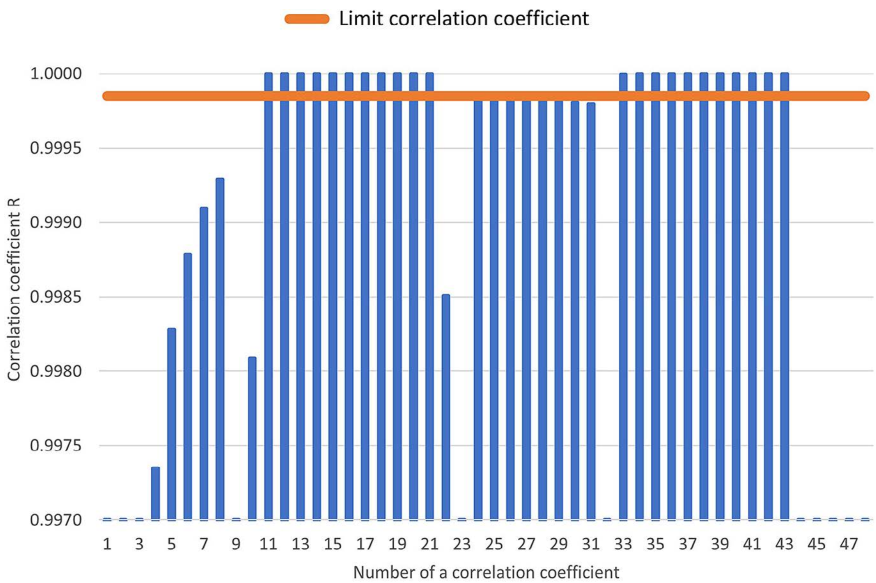


Fig. 7. The results of using the proposed method for the first profile

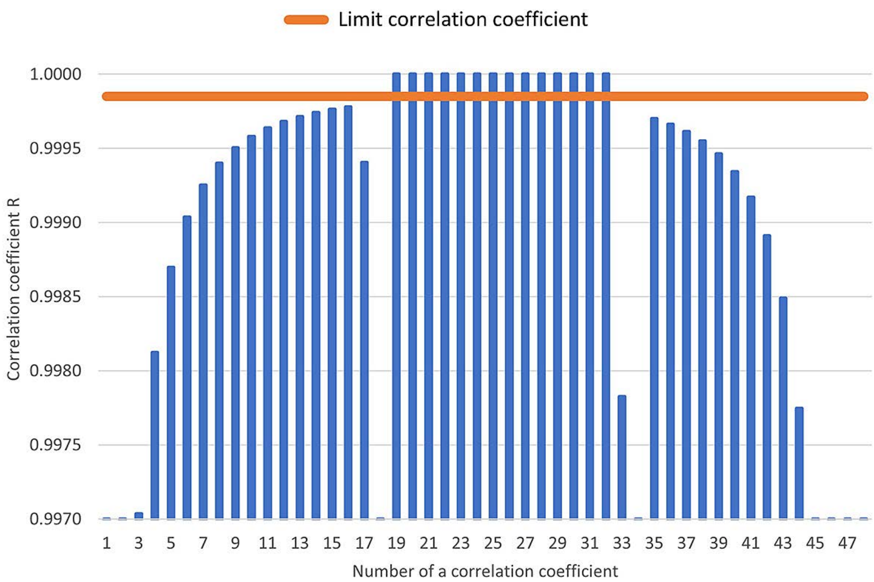


Fig. 8. The results of the application of the proposed method for the second profile

Fig. 9. The applied CMM – ACCURA II

Fig. 10. The probe applied for conducting experimental research

on the analysis of the nominal data of the measured object profiles. It was established that the limit correlation coefficient is the parameter calculated based on the nominal points. In order to detect the flattened fragments of the considered surface of the measured product, the correlation coefficients obtained by using the measurement points were compared with the limit correlation coefficients (calculated on the basis of nominal data, which do not take into account the flattening of the surfaces of the measured product). It was assumed that the flattening of the profile occurs in the place where the correlation coefficient calculated based on the measurement data most often exceeds the adopted limit correlation coefficient.

The linear regression was performed for successive groups of points consisting of ten nominal and measured points distributed along the considered product profiles. The choice of the number of the analysed points depends on the user of a CMS. This corresponds to the choice of the number of measurement points, which is also dependent on the decision of the user of a coordinate measuring system. The coordinate measurements for three surfaces of the product were carried out by using a different number of nominal and measurement points. For the first two curves, which are sections of the first two surfaces of the product, 100 points were used. On the other hand, for the third curve, the number of points was equal to 50. The applied numbers of measurement points

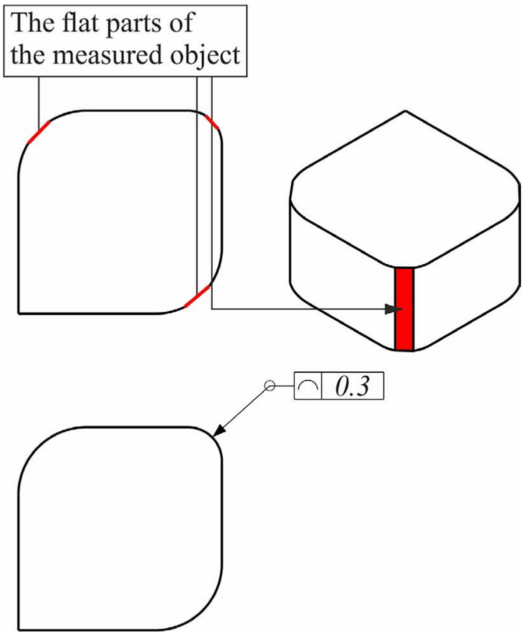


Fig. 11. The workpiece to be measured with the marked flatness of the considered surfaces

are in accordance with the recommendations of the manufacturer of the ACCURA II coordinate measuring machine published in [14]. The applied numbers of measurement points translate into a small distance between the measurement points of about 0.2 mm for the considered profiles. The use of the distance between points of 0.2 mm is also in line with the recommendations of Carl Zeiss [14]. The points were measured by the scanning probe, and they were uniformly distributed along the analysed surfaces.

Figure 12 shows the values of the correlation coefficients calculated by using the nominal points of the first curve. They were assumed to represent the limit correlation coefficients R_{gr} for the first curve. Figure 13 presents the values of the correlation coefficients, which were calculated based on the measurement points resulting from the actual coordinate measurements of the first profile of the investigated product. In the case of the first surface, the correlation coefficients with the following numbers 1–6, 12, 17–19, 35–55, 67–68, 75–79 and 84–87, calculated based on the measurement points are greater than the limiting correlation coefficients. It was assumed that the flattening of the profile occurs in its place represented by the coefficients 35-55. In this area of the measured surface, the number of the correlation coefficients calculated based on measurement points and greater than the limiting correlation coefficients R_{gr} is the largest. The selected coefficients approximately correspond to the following measurement points 35–64.

Similar experimental results were obtained in the case of two other considered profiles of the measured object. Figure 14 shows the values of the correlation coefficients calculated by using the second curve's nominal points, which represent the limit correlation coefficients R_{gr} of the second curve. In turn, Figure 15 presents the values of the correlation coefficients, which were obtained based on the measurement points being the result of the actual coordinate measurements of the second profile of the

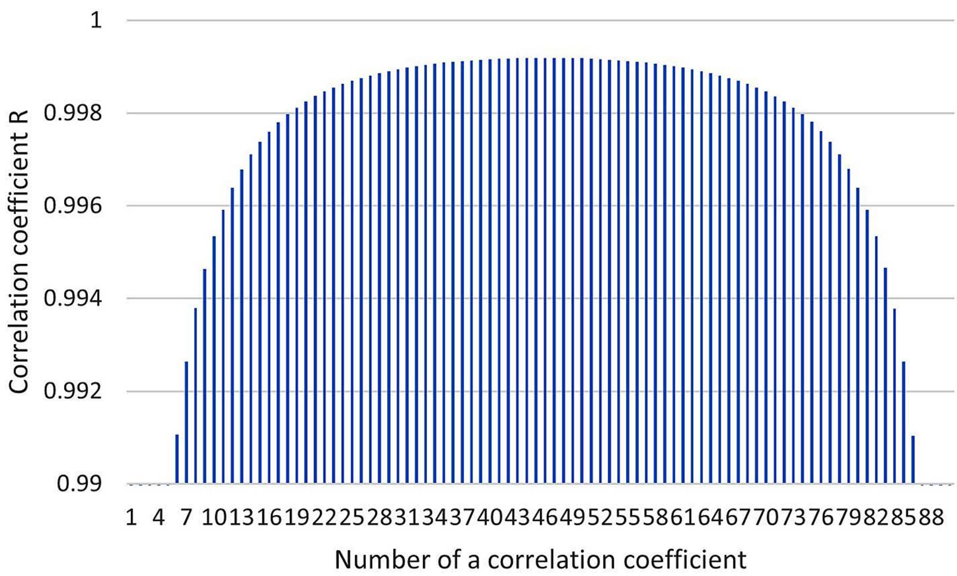


Fig. 12. The correlation coefficient values calculated by using the nominal points of the first curve

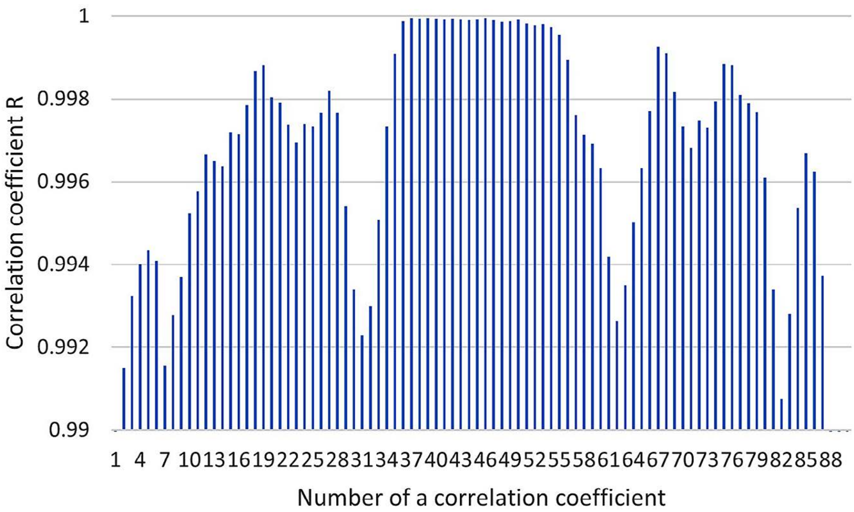


Fig. 13. The values of the correlation coefficients calculated on the basis of the measured points resulting from the actual coordinate measurements of the first profile

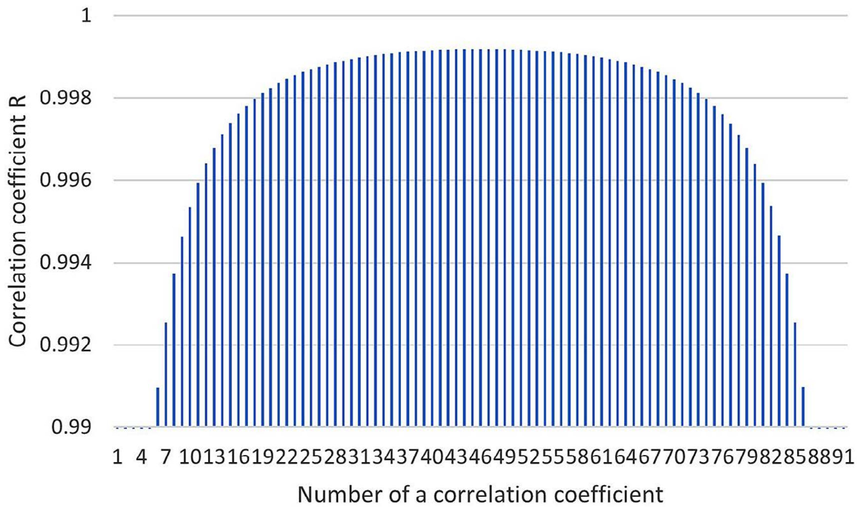


Fig. 14. The correlation coefficient values calculated by using the nominal points of the second curve

product. Based on the analysis of the nominal and measurement data, which was carried out analogically to the analysis of the first curve, it was concluded that the flattening of the second surface of the measured object occurs in the place of the investigated profile approximately represented by the points with the following numbers 42–71. In this part of the measured surface, the number of the points for which the correlation coefficients calculated based on the measurement data are greater than the limiting correlation coefficients R_{gr} is the largest. In the case of the third considered curve of the measured product, coordinate measurements were carried out for fewer measurement points than in the two previous curves (the number of points was equal to 50). Figure 16 shows the values of the correlation coefficients calculated by means of the nominal points of the third curve. In turn, Figure 17 presents the correlation coefficients, which were calculated based on the measurement points being the result of the real coordinate measurements of the third profile of the product. Based on the analysis of both graphs, it was found that the flattening of the third profile occurs in the place approximately represented by the following points 17–36. The summary of the results of the experimental investigations is presented in Table 3. The confirmation of the correctness of the calculations is Figure 18 presenting the third profile for which the flattening was correctly detected.

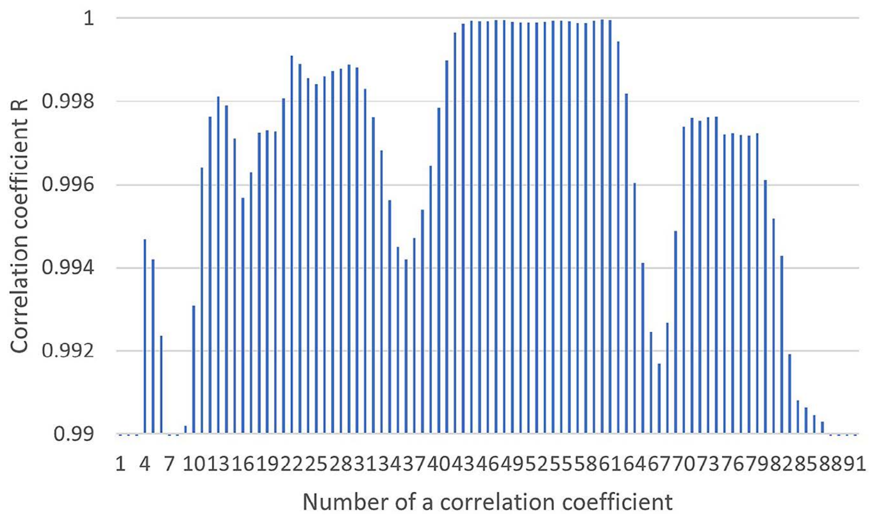


Fig. 15. The values of the correlation coefficients calculated on the basis of the measured points resulting from the actual coordinate measurements of the second profile

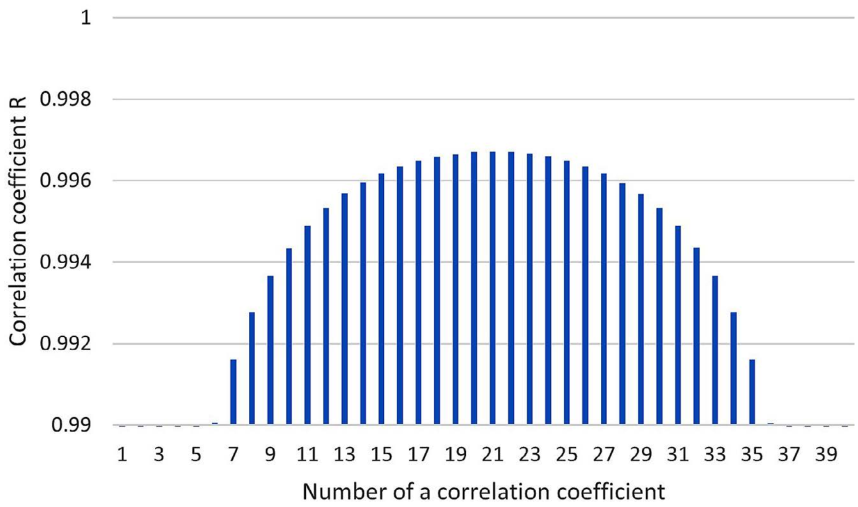


Fig. 16. The correlation coefficient values calculated by using the nominal points of the third curve

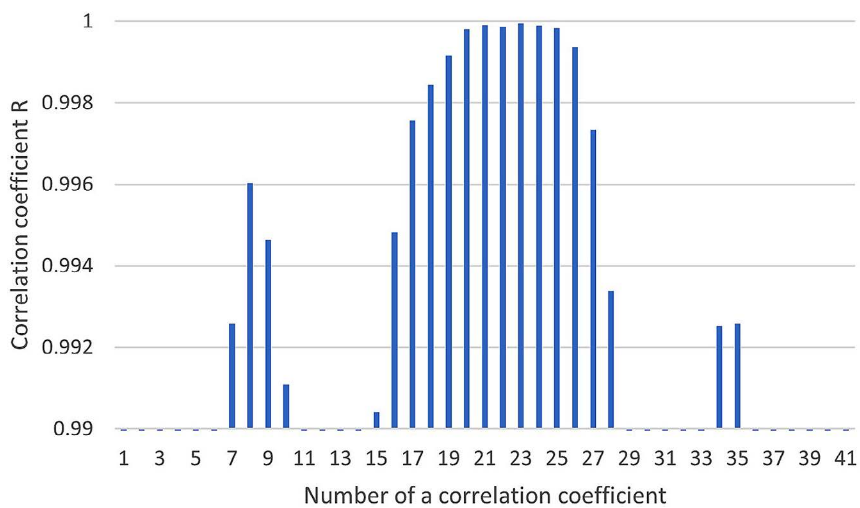


Fig. 17. The values of the correlation coefficients calculated based on the measured points resulting from the actual coordinate measurements of the third profile

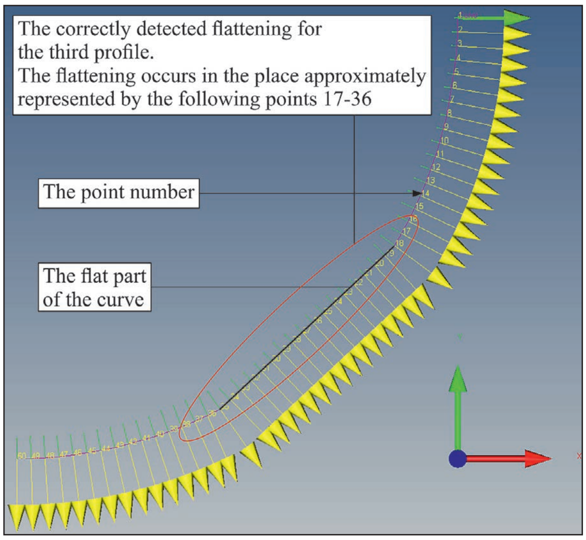


Fig. 18. The flattening of the third profile was correctly detected

Table 3. The results of the experimental investigations

Profile	The points that indicate flattening
1	35–64
2	42–71
3	17–36

In order to additionally check the proposed method of assessing a controlled radius, additional calculations for the first profile were performed. They consisted in calculating the correlation coefficients for other, different than ten, numbers of points. The additional calculations were made for eight and twelve points. In the case of eight points, the flattening of the first profile was detected approximately at the following measurement points 36–63. In turn, for twelve points, the flattening was found approximately for points 35–65. The results of the additional investigations correspond to the results of the tests carried out for ten points. Regardless of the number of the applied points, for which the correlation coefficients were calculated, the flattening of the first profile was detected in the same place of the product.

CONCLUSIONS

The proposed method of assessing a controlled radius enables quick, precise, and adapted to industrial conditions detection of flattening of an analysed surface profile, for which the measurement characteristic of a controlled radius is determined. The new method does not require much computing power. The calculation time is short and does not extend a measurement process. Another advantage of the developed method is the possibility of its implementation in commercial software of advanced CMSs. Thus, it is possible to use the new method of assessing a controlled radius in industry because production companies mainly use commercial solutions in the field of the coordinate measuring technique. The metrological software in which the created method can be used is Calypso. The implementation of the new method in the Calypso software is possible by using its Parameter-Coded Measurements (PCM) module.

The results of the simulation investigations were verified during experimental tests, the results of which confirmed the usefulness of the proposed method of analysis of a controlled radius in the coordinate metrology. The new method correctly detected the flat parts of the object measured during experimental investigations, regardless of the number of measurement points used, based on which the coordinate measurements were carried out. Further research should concern an additional analysis of surface profiles for which a controlled radius is determined, in terms of the occurrence of changes of directions of a curve, which, like flattening, disqualify the correctness of a controlled radius. In addition, further investigations may concern the linear regression carried out based on the numbers of measurement points other than those considered in the article. Moreover, further tests may also concern coordinate measurements of samples made by means of machining and the comparison of the developed method to the conventional method of measuring a controlled radius.

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