

## The Effect of Undercut Anchor Diameter on the Rock Failure Cone Area in Pullout Tests

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### ABSTRACT

The numerical analysis was conducted using the FEM ABAQUS software to establish the impact of various undercut anchor diameters on the rock breakout cone formation. The central focus of the investigations was on the rock breakout prism, which tends to be approximated to a cone or a quadrilateral pyramid, including its characteristic parameter, the angle of failure cone  $\alpha$ . Assuming that the embedment depth and the undercut anchor head angle were constant for the considered range of anchor head diameters, it remains unclear, however, precisely how the anchor head diameter affects the value of the failure cone angle, and thus the surface area of the full breakout prism. This conclusion stands in confirmation of our former considerations regarding the impact of the anchor head angle on the size of the breakout surface. Furthermore, it is supported by the results obtained from the mechanical model simulation of the anchor-rock system, where the anchor head angle and the effective embedment depth were determined as significant factors affecting the assumed rock breakout failure. The underlying aspect of the reported investigation was to evaluate the effectiveness of the non-conventional rock breakout technology performed with an undercut anchor, whose primary factors were both the pullout force and the assumed volume of the rock cone.

**Keywords:** rock mechanics, fracture mechanics, rock destruction, concrete breakout failure, numerical modelling of fracture

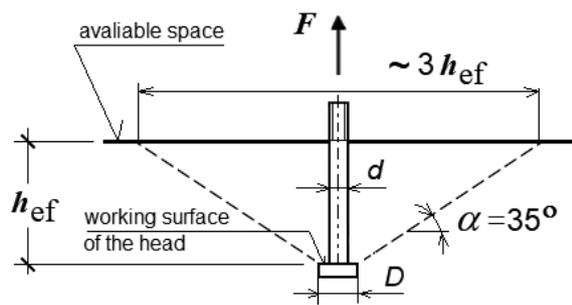
### INTRODUCTION

In engineering, anchors are primarily employed in fastening systems for fixing steel structures in concrete [1–4]. Their load-carrying capacity assessment typically involves an approximation model, assuming that the concrete breakout product formed during the anchor pullout is idealized as a prismatic cone. The breakout strength calculations are based on an idealized breakout prism [5], which is a convenient approximation. The prism angle is approximately  $35^\circ$  and its parameters are relative to the effective embedment depth of an anchor  $h_{ef}$  (Fig. 1).

Currently, however, according to updated standards, it is assumed that the failure surface is represented by a quadrilateral pyramid with its

apex situated in the anchor head, and the base on the concrete surface is equal to  $3h_{ef}$ .

By employing the computational prowess of numerical modeling methods [6–9] (e.g., Finite Element Method (FEM) [10–14] or Boundary Element Method (BEM) [15–17] or machine learning [18–20]) in conjunction with experimental research [21–24] we have developed a detailed understanding of the actual behavior of engineering structures and their optimization. What has emerged from the extensive laboratory research [25,26], theoretical analyzes [27–29] and FEM simulations [30–32] conducted to date on the subject is that the pullout anchor strength, also referred to as its load-carrying capacity, is affected by a number of other factors, such as mechanical parameters of concrete (e.g., [33–36]), effective



**Fig. 1.** Approximated concrete breakout cone for tension:  $h_{ef}$  – effective embedment depth,  $\alpha$  – breakout prism angle,  $d$  – anchor shaft diameter,  $D$  – anchor head diameter

embedment depth [37], the breakout anchor design [38,39], the anchor head geometry [40–44], or concrete reinforcement [45–50]. Numerous tests have been conducted in field and laboratory conditions that aimed to verify experimentally the results emerging from theoretical considerations [51,52]. These included analyzes devoted to the study of the group effect in anchorage systems on the value of the breakout force [53,54]. Concrete cracking mechanics has provided the basis for the development of mechanical models facilitating the interpretation of the anchorage effect on the material in which they are fastened [55,56]. Also, analytical models have significantly contributed to the understanding of the rock/concrete destruction mechanism induced by the pullout of mechanically [57,58] or chemically fastened anchors. These efforts have led to establishing new standards and recommendations for the estimation of the load capacity of anchors under specific application and load conditions [1].

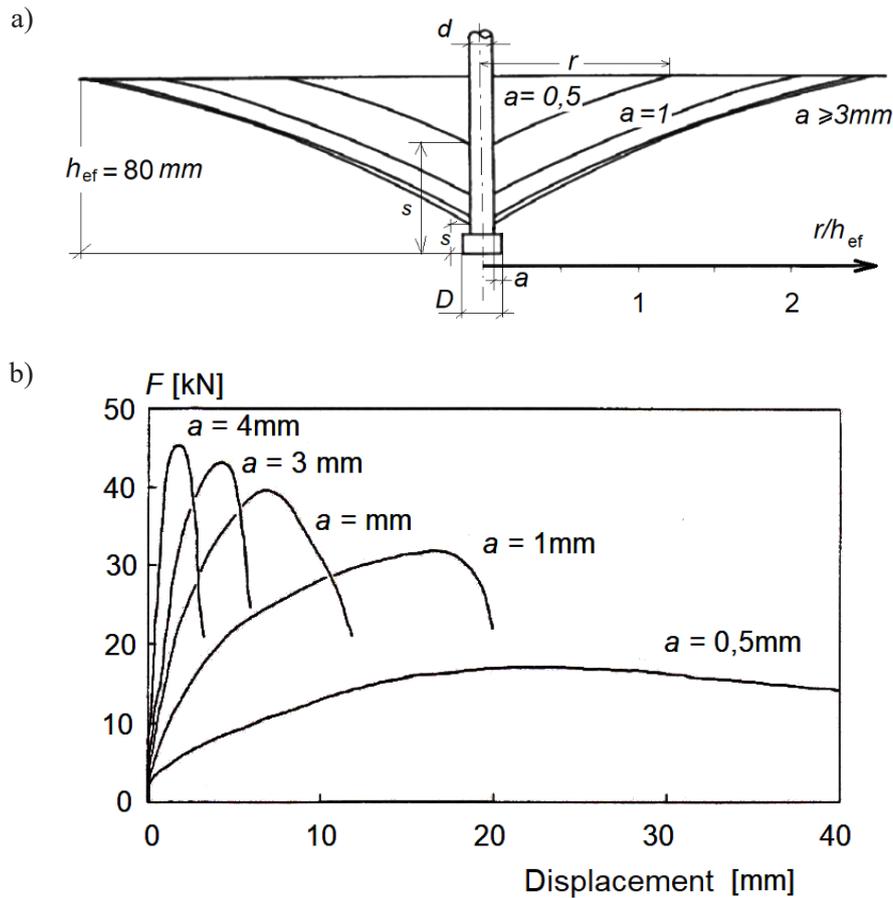
In his work, Furche [59] has taken on to describe the volume of concrete surrounding the anchor separating from the member under tensile loading as a function of pressure generated on the working/bearing area of the anchor head. In particular, if the surface is sufficiently large, the local crushing of concrete does not occur in this area. Conversely, at the initial stage, a smaller head size leads to a reduction in the effective embedment depth as a result of assumed concrete crushing under the head, only at a later stage to result in the prism detachment from the concrete member. The tests were carried out at a changing width of the working area of the anchor head  $a$  (Fig. 2a,  $a = (D-d)/2$ ) from 0.5 mm to 4 mm. Based on these data, for  $h_{ef} = 80$  mm, the failure cone angle can be estimated at  $26^\circ$ . The change in pullout force

as a function of its displacement is shown in Figure 2b [59].

A range of experimental and numerical studies describe the influence of the embedment depth or the anchor head size on anchor performance (e.g., [39,54]). In general terms, the mean cone angle to the concrete surface  $\alpha$  has been determined to range between  $25^\circ$  and  $30^\circ$  [43]. According to [28], the angle of the concrete breakout prism  $\alpha$  is in the range of  $25^\circ$  to  $40^\circ$  to the concrete surface. Although on average, it amounts to approximately  $35^\circ$ , it has been shown, by e.g., [28,56], to increase when the embedment depth is deeper. What is more, it exhibits a slightly rising tendency at higher anchor head angles [43–45]. Nonetheless, it has been numerically and experimentally proven [60] that the average concrete breakout prism angle decreases with the increase in the anchor head size, whereas the diameter of the cone base (on the concrete surface) increases with the size of the head.

Conclusions similar to Furche [59] have been reported in a different study [61], which provided the evidence that, at pullout, smaller-headed anchors are prone to higher displacements at maximum loading than their larger head size counterparts. This trend is even stronger in greater embedment depth scenarios. As the size of the anchor head increases, the pullout load is observed to rise significantly while the displacement values show exactly the opposite tendency. The comparative analysis of the post-failure anchor performance displayed that at pullout with large-head-size anchors concrete becomes very brittle – it succumbs to sudden failure without prior noticeable displacement of the anchor. In the case of smaller heads, the concrete exhibits plastic properties, thus undergoing extensive deformations at the concrete-anchor interface.

Elsewhere [38], it has been shown that the application of bigger diameter anchor heads (from 35 mm to 52 mm) is reflected in the decrease in the mean compressive stress under the anchor head, which in turn resulted in a minor displacement of the anchor head. Hence, the smaller the head diameter, the greater the displacement at anchor pullout. It was also found that the breakout of the base material is mainly triggered by the failure of the concrete in tension (circumferential cracking) and not in compression. The onset of circumferential cracking progression is observed at roughly 30% of the failure load and shows high stability until reaching the failure load. At the post-failure



**Fig. 2.** The effect of anchor head width on the failure cone formation – a) and the change in the pullout force as a function of its displacement – b), based on Furche [59],  $s$  – the head displacement in the hole at failure load,  $r$  – radius of the failure cone base on the free surface of the rock,  $a$  – the width of the bearing area of the anchor head

stage, cracking becomes unstable, progresses rapidly, and forms a final breakout cone. The failure load is mainly dependent on the concrete fracture energy  $G_F$ , which is approximately the function of the square root of  $G_F$ . On the other hand, it appears (36) that the peak crack length is shorter for smaller anchor heads and the prism is of a more acute angle than in bigger-size anchor heads. Stress gradients in the vicinity of the anchor head are considered to be the main factors responsible for these differences. In addition, notable differences are observed in the main fracture, whose propagation closely resembles mode I (large heads) rather than the mixed mode (small heads). Non-reinforced concrete failure cone angles were shown [62] to range between  $20^\circ$  and  $25^\circ$  for embedment depths of 40-80 mm.

It is important to note that there is a clear dependence of the failure cone angle on the effective embedment depth. The anchor load-carrying capacity estimation model based on the variable cone angle  $\alpha$  assumption [63] introduces the following equation defining the failure cone angle as a function of the embedment depth:

$$\alpha = 28 + 0,134h_{ef} \text{ for } h_{ef} \leq 127 \text{ mm}$$

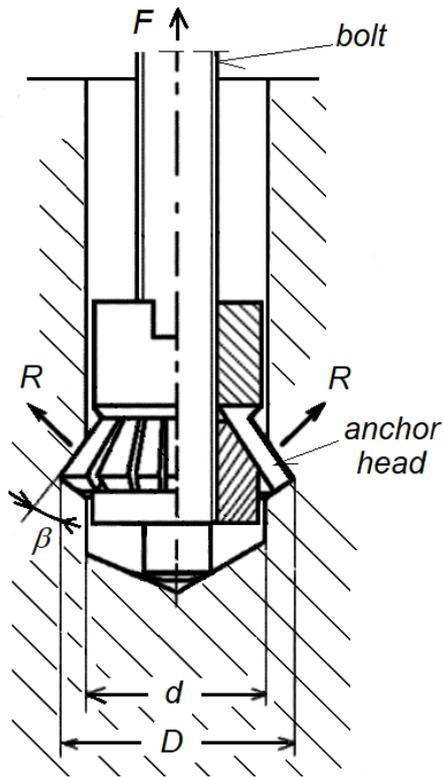
$$\alpha = 45^\circ \text{ for } h_{ef} > 127 \text{ mm}$$

Alternatively [56], the equation can take the following form:

$$\tan \alpha = 0.2h_{ef}^{0,25}$$

We propose the design of the undercut/breakout anchor for other-than-assembly applications, i.e., for the breakout of (solid) rock [64] where the anchor is initially fixed. In the tested breakout anchor, the bearing area of the undercut head is not perpendicular to the pullout load of the anchor, but is inclined at an angle  $\beta$  (see Figure 3).

Since the primary focus of the studies reported in the preceding paragraphs was on the effect of different diameters of the cylinder of headed studs (such as in [40] or Fig. 1), they do not fully explain the impact of the undercutting head diameter of the breakout anchor (Fig. 4) failure surface of the base material (including the failure cone angle  $\alpha$ ) where the anchorage is fixed. This factor may greatly alter the assumed breakout prism volume and, consequently, distort the



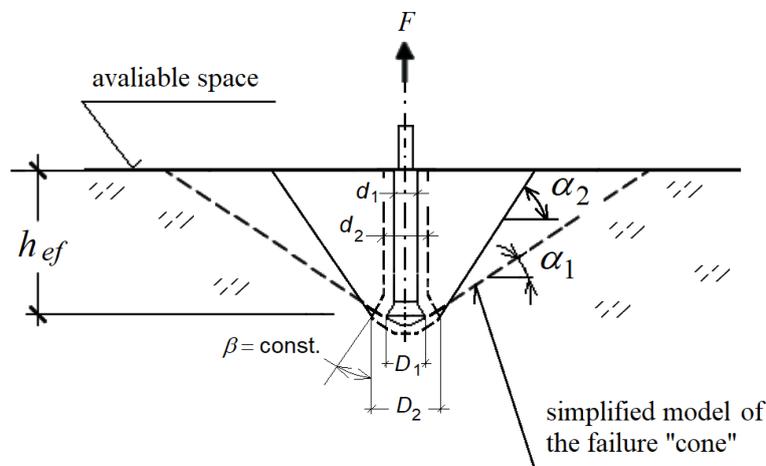
**Fig. 3.** The undercut/breakout anchor head:  $R$ , – anchor behavior in contact with the rock,  $\beta$  – angle of the cutting head,  $D$  – max. anchor diameter in the undercut,  $d$  – the diameter of the pre-drilled hole for the anchor,

calculations presenting the efficiency of the proposed method of detachment [37,64]. Hence, in order to address the knowledge gap, FEM numerical computations were involved. The results from these are presented in the following chapters of this study.

## MATERIALS AND METHODS

In an effort to explain how the application of undercut anchors of varying diameter affects the assumed surface of detachment (described by the failure cone angle  $\alpha$ ), it was initially established that only the following diameters would change:  $d$  (the diameter of the anchor hole) and  $D$  (the maximum undercut head diameter of the installed anchor), at a constant  $D/d$  ratio. It follows from the assumptions that the angle of the undercut head  $\beta$  is a constant value in all analyzed cases ( $\beta = \text{const.}$ ). The adopted dimensions  $d$  and  $D$  were characteristic for the considered undercut anchors (e.g., HDA-P Hilti anchor [4]) in the nominal range of dimensions M12, M16, and M20. Hence, the dimension  $d$  was equal to  $d = 22, 30, \text{ and } 37 \text{ mm}$  respectively. For the angle  $\beta = \text{const.} = 20^\circ$ , the dimension  $D$  is then  $D = 36.91, 44.91 \text{ and } 51.91$  respectively. Given the above assumptions, the mechanical model of the rock with the anchor hole adopted for simulations is shown in Figure 5. It is a flat axially-symmetric model, with the axis of symmetry along the anchor axis. The effective embedment depth was equal to  $h_{\text{ef}} = 50 \text{ mm}$ . The computations were performed using the FEM ABAQUS v.2022 software and its XFEM algorithm. Based on previous tests, the assumed surface of detachment on the free surface of the rock was approximately  $3h_{\text{ef}}$ . The dimensions of the base material model are the following:  $R = 500 \text{ mm}$  and  $H = 300 \text{ mm}$ .

The anchor head-rock contact area (the conical part of the undercut head (Fig. 6) was



**Fig. 4.** Assumed influence of the undercut head diameter on the rock breakout anchor performance:  $D_1, D_2$  – maximum head diameter,  $d_1, d_2$  – diameter of the anchor hole,  $\alpha_1, \alpha_2$  – assumed failure cone angle,  $\beta$  – undercut head angle.

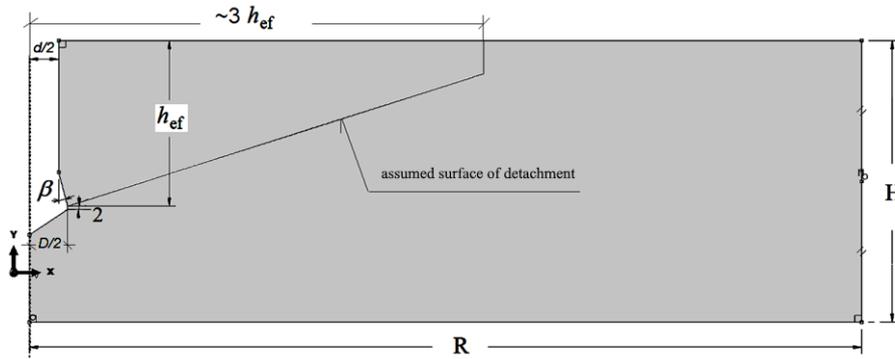


Fig. 5. Rock: a geometric model; H, R – model dimensions,  $h_{ef}$  – effective embedment depth,  $\beta$  – undercut head angle, d, D – considered characteristic head diameters

defined as the “Penalty contact,” derived from the ABAQUS library. The assumed coefficient of friction of steel against the rock (in the contact area) amounted to  $\mu = 0.2$ . The kinematic excitation of the anchor was applied along its Y axis. The anchor axis restraints, resulting from the model’s symmetric boundary conditions, were as follows: nodes on the right edge  $U1 = 0$  (deprived of all degrees of freedom in the OX axis), nodes in the model base  $U2 = 0$  (deprived of all degrees of freedom in the OY axis).

Material properties of the rock model: Young’s modulus – 14,275 MPa, Poisson’s number – 0.247, Failure mode – Max Principal Stress equal to  $f_t = 7.74$  MPa ( $f_t$  – tensile strength of the rock), Fracture Energy – 0.355. Stabilization factor 1E-06. Anchor model: Steel – material: Elastic, Isotropic, Elastic Modulus –  $E = 210,000$  MPa, Poisson’s Ratio –  $\nu = 0.3$ .

The finite element mesh of the model allowed for local densities. The global linear dimension of elements was 25 mm, in the contact area with the anchor – 2 mm, at the upper edges of the solid – the dimensions of the elements

vary from 3 to 10 mm, the line of the assumed fracture – 5 mm. The mesh was generated with the use of elements available in ABAQUS, such as Axisymmetric Stress, linear, Reduced integration, four-node mesh, and the element type CAX4R. The FE mesh for the rock base material itself is illustrated in Figure 7.

## RESULTS

Selected results from the numerical computations are shown in Figures 8÷9. Figure 8 demonstrates that the stress concentration  $\sigma_{max}$  is located in four areas of the model, i.e., the anchor head-rock contact area (compression), the area below the anchor in the model axis (tension), and the tensile stress in the area above the undercut and around the anchor hole can lead to propagation of radial cracks in the rock. What can be observed in the crack tip region is the typically strong concentration of tensile stresses that lead to fracture propagation at the crack tip.

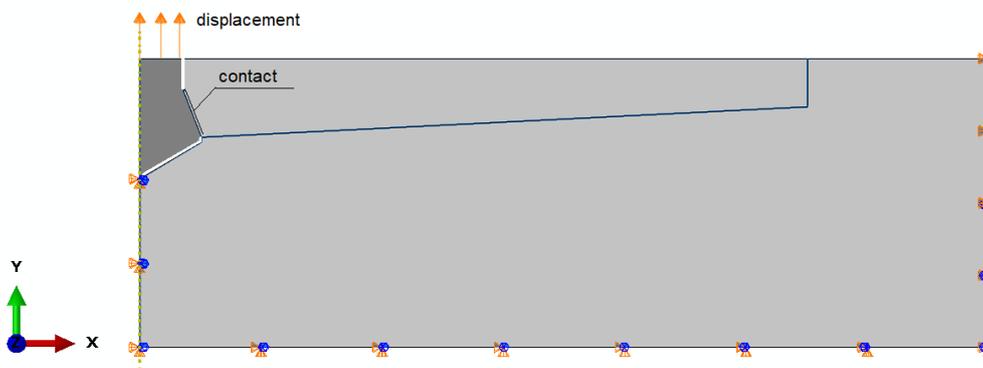
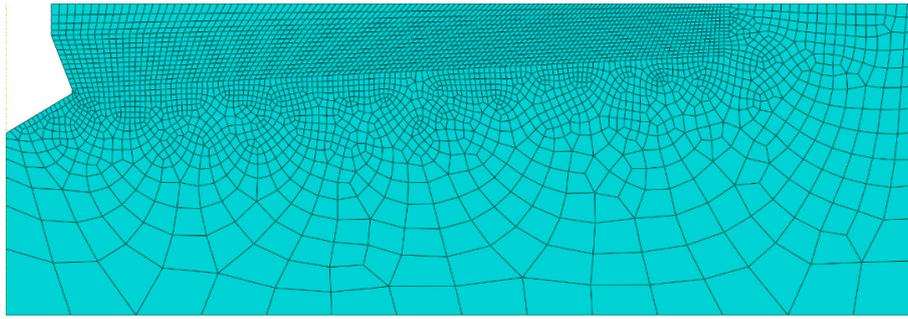
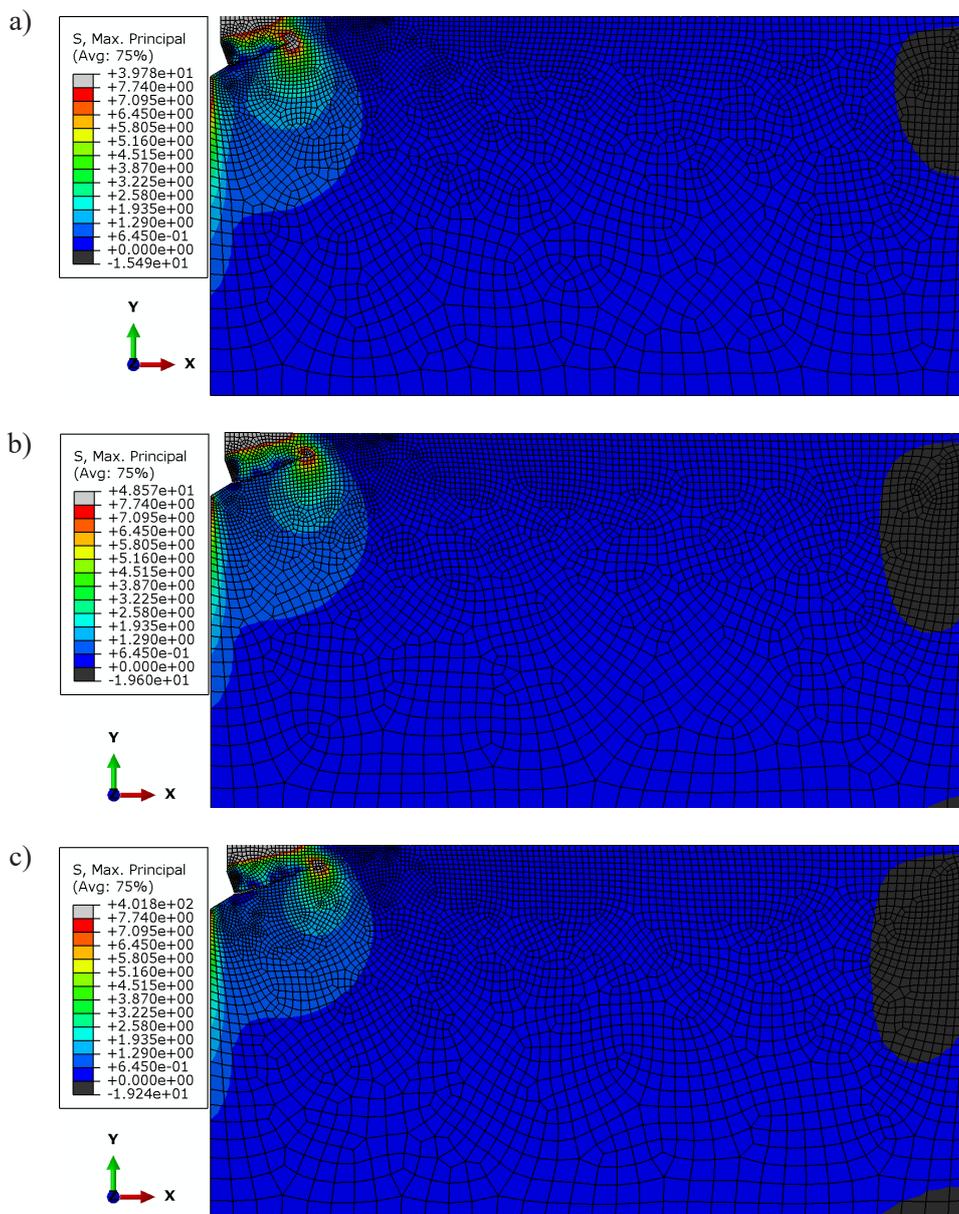


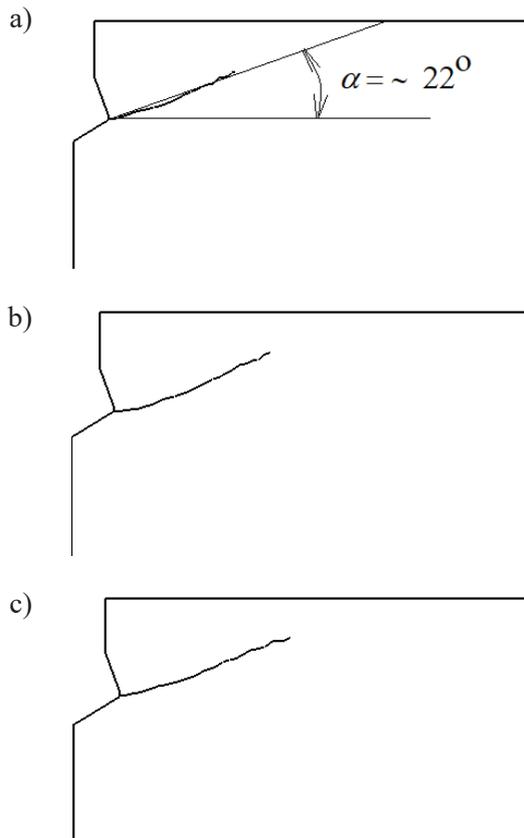
Fig. 6. Model boundary conditions, the location of the anchor head-rock contact area, and the kinematic excitation of the anchor head along the model’s OY axis



**Fig. 7.** The finite element mesh of the rock model with an undercut for the anchor head (the head model mesh turned off)



**Fig. 8.** Distribution of  $\sigma_{max}$  stress in the anchor head contact area. Anchor head diameter D equal to: a) 36.91, b) 44.91 c) 51.91mm (i.e., for M12, M16, and M20 anchors, respectively)

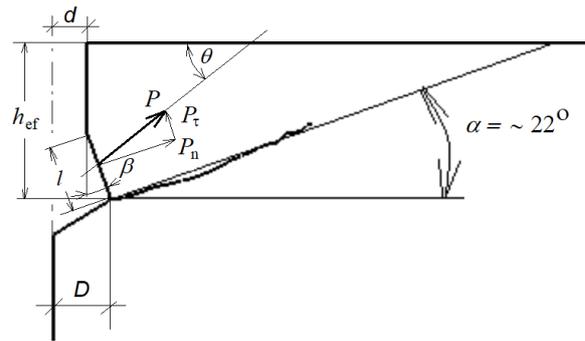


**Fig. 9.** The crack propagation trajectory under the pullout of an anchor head of a diameter  $D$ :  
 a) 36.91 mm, b) 44.91 mm, c) 51.91 mm  
 (i.e., for M12, M16, and M20 anchors respectively)

In the considered cases, the crack propagation angle  $\alpha$  at the initial phase amounted to approximately  $22^\circ$  relative to the free surface (or the plane perpendicular to the axis of the anchor), which is illustrated in Figure 9.

The failure mechanism that was recorded for the tested anchors in the numerical modeling tests above is confirmed by the one generated from the model of the anchor head impact on the rock base material presented below in Figure 10.

Due to the constants embedded in the model used in the simulation tests (i.e., the effective embedment depth  $h_{ef}$ , the undercut head angle  $\beta$ , the  $D/d$  ratio, the length of the undercut head cone element  $l$ ) and also given the fact that any alterations in the value of  $D$  and  $d$  were balanced by the adjustment of the anchor head size, the geometry of the undercut in the rock remained unchanged (and thus reflected the notch of constant geometric parameters). That is why, in the considered cases, neither the value of the angle  $\theta$  nor the anchor head bearing pressure on the rock  $P$  were affected. As a result, the crack propagated at an angle corresponding to the failure cone angle



**Fig. 10.** A simplified model of the anchor head impact:  $P$  – anchor head bearing pressure on the rock (including friction),  $P_n$ ,  $P_t$  – normal and tangential components to the rock undercut surface,  $l$  – contact zone length ( $l = \text{const.}$ ),  $\beta = \text{const.}$  – undercut head angle,  $\theta$  – load direction in relation to the free surface of the rock base material,  $h_{ef} = \text{const.}$  – effective embedment depth

$\alpha$ . The described breakout mechanism is similar to the mechanism accompanying the cutting of rocks with mining tools [65].

## CONCLUSIONS

This paper reports on the results from the numerical calculations that set out to determine the influence of different undercut anchor diameters on the failure surface of rock in pullout tests. The following constants were established: the undercut head angle  $\beta$ , the effective embedment depth  $h_{ef}$ , the cone element length, the rock friction coefficient  $\mu$ , and the hole  $d$ , and the head diameter  $D$  ratio. Only the particular values of parameters  $d$  and  $D$  were changed by simulating the operation of the undercut anchors (e.g., HDA-P Hilti) in the nominal dimensions M12, M16, and M20.

The investigations revealed that the change in the undercut anchor diameter (in practice, corresponding to applying a larger size anchor from a given range of anchors) does not produce a significant change regarding the failure cone angle  $\alpha$  at the initial stage of crack propagation. Therefore, the assumed volume of detached rock is not likely to vary between the considered anchor diameter variants.

Given the sheer number of factors that could potentially contribute to the final breakout effect (including the combination of mechanical parameters of the rock or the rock breakout technological process parameters). These findings must be then interpreted with caution because further analyzes and experimental verification tests are required.

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