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Impact of Using an Inerter on the Performance of Vehicle Active Suspension

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ABSTRACT

This paper investigates the effect of employing an inerter on the performance of active suspension systems. A quarter-car model with cubic-nonlinear spring is considered. The inerter is installed in parallel with the primary suspension spring and damper. First, feedback linearization is used to linearize the mathematical model. Then the linear quadratic regulator is adopted to control the suspension system. The proposed design is ride comfort-oriented and considers structural constraints. Numerical simulations are executed for passive systems with different values of inertance. Results show that employing an inerter to the passive suspension can improve the ride comfort performance by more than 32%. Employing an inerter to active suspension systems can also improve the ride comfort and reduce actuator force significantly. The actuator force can be reduced by 25%. However, the results also show that the uncaring selection of the inerter can dramatically degrade the performance of the suspension system.

Keywords: inerter, active suspension, nonlinear quarter-car model, ride comfort, feedback linearization.

INTRODUCTION

Vehicle suspension system is important to improve ride comfort, road holding, and vehicle handling and support the vehicle weight. Inerters are recently used in passive and semi-active suspension systems. The inerter was introduced on the basis of current−force analogy between mechanical and electrical networks [1]. The inerter was designed as a mechanical element with two terminals, in which the applied force at the two terminals is proportional to the relative acceleration between them, and the proportionality constant is called inertance [2]. Different types of inerters, such as ball−screw, rack-and-pinion, living-hinge, hydraulic, helical fluid, and electromagnetic, are available [3, 4]. Inerters have been used in passive vibration control for different mechanical and civil engineering systems [3]. The main application area of inerters is vehicle suspension systems. Extensive studies have investigated the effects of introducing inerters on the performance of passive and semi-active suspension systems [5]. For example, Smith and Wang [6] investigated different inerter-based configurations for passive suspension and found that introducing inerters can improve ride comfort, vehicle handling, and tire load indexes by more than 10%. Scheibe and Smith [7] analytically optimized the performances of ride comfort and tire grip. Hu et al. [8] investigated suspension deflection and other performance measures for passive suspension with different inerter-based configurations. They found that using inerters improved the mixed performance of ride comfort and tire grip and, simultaneously reduced the performance of suspension stroke. Shen et al. [9] studied the effect of spring, damper, and inerter on vibration transfer characteristics. The effects of nonlinearities of inerters on the performance of suspension systems were studied by Wang and Su [10], and Shen et al. [11, 12].

The semi-active inerter-based system was first introduced by Chen et al. [13, 14]. In semi-active inerter-based systems, the inertance can be adjusted by on-line control to achieve the desired performance. Chen et al. [13, 14] used force tracking approach in designing the semi-active suspension system. An active control law was designed, and then tracked on-line by varying the damping coefficient and the inertance. Li et al. [15] investigated the design of vehicle suspension based on a state feedback H_2 control method and using an adaptive inerter. Hu et al. [16] employed a skyhook inerter to improve the ride comfort by virtually increasing the sprung mass. They approximated the skyhook inerter-based suspension system by a semi-active inerter-based system. Soong et al. [17] investigated the design of semi-active suspension with on-off switchable inerter based on force cancelation strategy. Zhang et al. [18] realized a modified skyhook-inerter control by using a hydraulic system with an adjustable inertance between sprung and unsprung masses. They also considered the variation of the parasitic damping in the hydraulic system. Wang et al. [19] presented relative acceleration–relative velocity control method to switch the inertances of fluid inerter between maximum and minimum values based on the signs of relative acceleration and velocity. Li et al. [20] investigated the design of semi-active inerter and damper suspension system using combinations of different control methods. The inertance was controlled using skyhook control as in Hu et al. [16], and the damping coefficient was controlled using acceleration-driven and power-driven damping methods for on-off and continuous controls, respectively.

Instead of passive and semi-active suspensions, the conflicting purposes of suspension systems require using active suspension. Mechatronic or hydraulic-electric inerter was used as a control actuator which consists of a hydraulic inerter or a ball-screw inerter connected with a motor. The mechatronic inerter was proposed by Wang and Chan [21] to improve the performance of suspension systems. Shen et al. [22] designed a hydraulic-electric inerter and employed it in vehicle suspension systems. They described the suspension by a bicubic impedance function, which included mechanical and electrical parameters, and then optimized the parameters considering suspension performance. Liu et al. [23] designed a predictive controller to optimize the thrust force of a nonlinear hydraulic-electric inerter to improve ride comfort.

This article addresses the effects of applying an inerter on the performance of active suspension systems. Herein, the main objective of active control is to improve ride comfort by minimizing the sprung mass acceleration of a cubic nonlinear quarter-car model. Different control techniques have been utilized in the last two decades for the design of active suspension systems for nonlinear models without inerters. These techniques include the following: linear parameter varying control (LMI) [24-27], and gain scheduling control [28], backstepping control [29, 30], sliding mode control [31-33], energy sink control [34], feedback linearization (FBL) and PID control [35], FBL and H_∞ control [36], and FBL and linear quadratic regulator (LQR) [37]. Herein, the FBL combined with the LQR is adopted to design the active suspension system.

SUSPENSION MODEL

A nonlinear quarter-car model is adopted to study the effect of employing inerter on the performance of vehicle active suspension. The model has 2DOF (i.e., the bounces of the body and the wheel). The primary spring in the suspension system is assumed nonlinear. This nonlinearity cannot be ignored specially for off-road driving conditions. The equations of motion of the sprung mass (m_s) and unsprung mass (m_u) of the quarter-car model (Fig. 1) are respectively given by

$$(m_s + b)\ddot{x}_s - b\ddot{x}_u + c_s(\dot{x}_s - \dot{x}_u) + \\ + k_{s,L}(x_s - x_u) + k_{s,NL}(x_s - x_u)^3 = U \quad (1)$$

$$(m_u + b)\ddot{x}_u - b\ddot{x}_s - c_s(\dot{x}_s - \dot{x}_u) - \\ - k_{s,NL}(x_s - x_u)^3 + k_t x_u = k_t x_r - U \quad (2)$$

where: b represents the inertance; k_t is the tire stiffness; $k_{s,L}$, $k_{s,NL}$ are the primary spring linear and nonlinear stiffness, respectively; c_s is the damping coefficient; x_s, x_u are the displacements of the sprung and unsprung masses, respectively; x_r is the displacement of the road; and U is the actuator force (i.e., control law).

The main objective of the suspension system is to provide ride comfort while maintaining road holding. The ride comfort mainly depends on the sprung mass acceleration and can be quantified by $RMS(\ddot{x}_s)$, where $RMS(\ddot{x}_s)$ is the root-mean-square of the signal. The road holding (or tire grip) can be achieved by reducing the tire dynamic load (F_t) to become less than the static load, that is:

$$\frac{F_t}{(m_s+m_u)g} < 1 \quad (3)$$

where: $F_t = k_t(x_u - x_r)$.

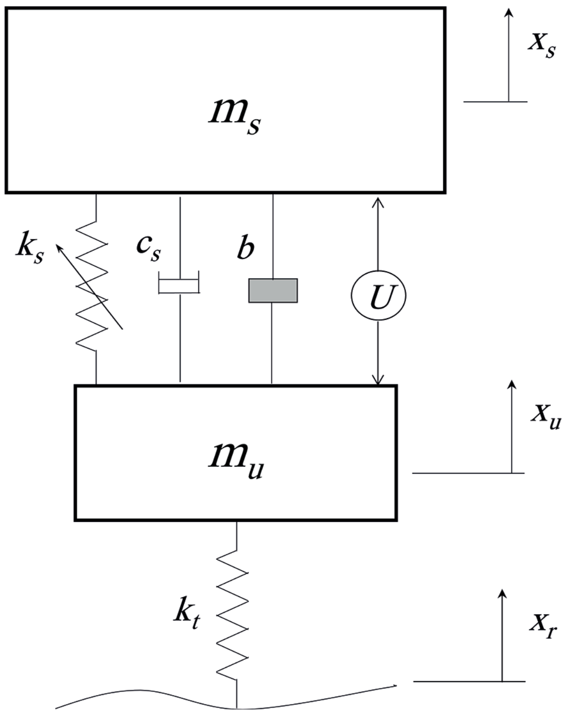

Fig. 1. Quarter-car model with an inerter

An important design requirement to prevent suspension system damage, is to constraint the suspension deflection, that is:

$$|x_s - x_u| \le \delta_{max} \tag{4}$$

where: δ_{max} is the maximum permissible suspension stroke. δ_{max} is assumed in this study to be 7.5 cm.

ACTIVE SUSPENSION DESIGN

The nonlinear quarter-car model in Eqs. (1) and (2) is a perfect choice for feedback linearization. The nonlinear terms can be cancelled immediately using FBL control law:

$$U = k_{s,NL}(x_s - x_u)^3 + u \tag{5}$$

The control law U includes two parts: the nonlinear control component $k_{s,NL}(x_s - x_u)^3$ and the linear control component u. Substituting the FBL control law (6) in Eqs. (1) and (2), the feedback linearized system gives the following:

$$(m_s + b)\ddot{x}_s - b\ddot{x}_u + c_s(\dot{x}_s - \dot{x}_u) + + k_{s,L}(x_s - x_u) = u \tag{6}$$

$$(m_u + b)\ddot{x}_u - b\ddot{x}_s - c_s(\dot{x}_s - \dot{x}_u) - - k_{s,L}(x_s - x_u) + k_t(x_u - x_r) = -u \tag{7}$$

Elimination by substitution of $\ddot{x}_s$ and $\ddot{x}_u$ in Eqs. (7) and (8), respectively, yields the following:

$$\left(m_s + b + \frac{m_s b}{m_u}\right)\ddot{x}_s + c_s(\dot{x}_s - \dot{x}_u) + + k_{s,L}(x_s - x_u) + \frac{k_t b}{m_u}(x_u - x_r) = u \tag{8}$$

$$\left(m_u + b + \frac{m_u b}{m_s}\right)\ddot{x}_u - c_s(\dot{x}_s - \dot{x}_u) - k_{s,L}(x_s - x_u) + \left(1 + \frac{b}{m_s}\right)k_t(x_u - x_r) = -u \tag{9}$$

The equivalent sprung mass ($m_{s,eq}$) and equivalent unsprung mass ($m_{u,eq}$), respectively, defined as follows:

$$m_{s,eq} = \left(m_s + b + \frac{m_s b}{m_u}\right) \tag{10}$$

$$m_{u,eq} = \left(m_u + b + \frac{m_u b}{m_s}\right) \tag{11}$$

Equations (9) and (10) can be represented in a state-space model as follows:

$$\dot{x} = Ax + Bu + Gw \tag{12}$$

$$y = Cx \tag{13}$$

where: the state vector is x=[(x_s $\dot{x}_s$ x_u $\dot{x}_u$)]^T, is the exogenous input (x_r), y is the measured output, and u is the controller effort. The state-space matrices are denoted as:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-k_{s,L}}{m_{s,eq}} & \frac{-c_s}{m_{s,eq}} & \frac{\left(k_{s,L} - \frac{k_t b}{m_u}\right)}{m_{s,eq}} & \frac{c_s}{m_{s,eq}} \\ 0 & 0 & 0 & 1 \\ \frac{k_{s,L}}{m_{u,eq}} & \frac{c_s}{m_{u,eq}} & \frac{-\left(k_{s,L} - \left(1 + \frac{b}{m_s}\right)k_t\right)}{m_{u,eq}} & \frac{-c_s}{m_{u,eq}} \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ \frac{1}{m_{s,eq}} \\ 0 \\ \frac{-1}{m_{u,eq}} \end{bmatrix}, G = \begin{bmatrix} 0 \\ \frac{\frac{k_t b}{m_u}}{m_{s,eq}} \\ 0 \\ \frac{\left(1 + \frac{b}{m_s}\right)k_t}{m_{u,eq}} \end{bmatrix}$$

and

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{14}$$

The LQR control design is a state-feedback control strategy that permits the development of a controller K while minimizing an objective function J. Shaqarin (2018) indicated that the objective function can be defined such that it incorporates sprung mass displacement, sprung mass acceleration, suspension stroke, and control effort; thus,

$$\int_0^\infty \left(x^T Q x + \ddot{x}_s{}^T Q_1 \ddot{x}_s + + (x_s - x_u)^T Q_2 (x_s - x_u) + u^T R u\right) dt \tag{15}$$

where: Q, Q_1 and Q_2 are positive semi-definite real Hermitian matrices, while R is a real positive definite Hermitian matrix.

The sprung mass acceleration and the suspension stroke are defined as follows,

$$\ddot{x}_s = \begin{bmatrix} \frac{-k_{s,L}}{m_{s,eq}} & \frac{-c_s}{m_{s,eq}} & \frac{\left(k_{s,L}-\frac{k_t b}{m_u}\right)}{m_{s,eq}} & \frac{c_s}{m_{s,eq}} \end{bmatrix} x + \\ + \frac{1}{m_{s,eq}} u + \frac{\frac{k_t b}{m_u}}{m_{s,eq}} w = vx + qu + zw \quad (16)$$

$$x_s - x_u = [1 \quad 0 \quad -1 \quad 0]x = fx \quad (17)$$

The optimal control effort minimizing the objective function (15) can be written as:

$$u = -Kx \quad (18)$$

where:

$$K = {R_0}^{-1}(N^T + B^T P) \quad (19)$$

The design of controller K requires solving the following algebraic Riccati equation to find the matrix P.

$$A^T P + PA + Q_0 - (N + PB)R^{-1}(N^T + B^T P) = 0 \quad (20)$$

where:

$$Q_0 = Q + v^T Q_1 v + f^T Q_2 f \quad (21)$$

$$R_0 = R + q^T Q_1 q \quad (22)$$

$$N = v^T Q_1 q \quad (23)$$

NUMERICAL SIMULATION

This section describes the performance of passive and active suspension systems incorporating inerters. Road bump is chosen as an input to the vehicle model with a height of 0.1 m. Through the simulation, the sprung mass, unsprung mass, and tire stiffness are fixed to m_s = 300 kg, m_u = 45 kg, and k_t = 150 kN/m, respectively. The ratio between the primary spring linear and nonlinear stiffness are fixed to 10 (i.e., $k_{s,NL}/k_{s,L}$ = 10). The simulation time for calculating the performance measures is 3 s.

Passive suspension

The effect of using inerter on the ride comfort performance of passive suspension systems is investigated. The variation of the root-mean-square of sprung mass acceleration, namely *RMS* ($\ddot{x}_s$), with inertance for different suspension stiffness k_s and damping coefficients c_s are respectively shown in Figures 2 and 3. In Figure 2, the damping coefficient is fixed to 2 kN·s/m with different linear stiffness (i.e. = 20, 30, 40, 50, and 60 kN/m). It is clear that adding an inerter to the suspension system with low stiffness has a negligible effect on enhancing the ride comfort (i.e., when the linear stiffness is 20 kN/m, the optimal inertance is 44 kg and the percentage reduction of the *RMS* ($\ddot{x}_s$) is 8%). For higher stiffness values, adding optimal inerters gives better reduction on the *RMS* ($\ddot{x}_s$). For example, the optimal inertance is 150 kg and the percentage reduction of the *RMS* ($\ddot{x}_s$) is 32.7% when the linear stiffness is 60 kN/m). However, increasing the inertance after the optimal values decreases the ride comfort significantly. For example, adding an inerter with $b = m_s$ to a low stiffness system (i.e. $k_{s,L}$ = 20 kN/m) increases the *RMS* ($\ddot{x}_s$) to 9.34 m/s^2 which is larger than that of the system without inerter by 275%. Furthermore, the values of optimal inertances increase with increasing the stiffness of the suspension system. Figure 3 shows the variation of the *RMS* ($\ddot{x}_s$) with inertance for a fixed linear stiffness (i.e. $k_{s,L}$ = 40 kN/m) and different damping coefficients (i.e. c_s = 1, 1.5, and 2 kN·s/m). It is clear that, changing the damping coefficient does not play a significant role in modifying the values of optimal inertances. Figure 4 shows a comparison between the sprung mass acceleration of passive suspension systems without inerter and with an optimal inerter (i.e. b_{opt} = 103 kg) for $k_{s,L}$ = 40 kN/m and c_s = 2 kN·s/m. Thus, the ride comfort performance is enhanced by using an optimal inerter in the passive suspension system.

Active suspension

Instead of passive suspension, the conflicting purposes of suspension system require using active suspension. This section discusses standard simulations of the nonlinear quarter-car response of the active suspension employing FBL with LQR controller. These simulations aim to assess the suggested approach performance against bump road profile with a 0.1 m peak height. The influence of inertance on the performance of the active suspension system ride comfort and control effort is explored. The inertance is varied from 0 to 300 kg, while the linear stiffness and damping coefficient are set to $k_{s,L}$ = 40 kN/m and c_s = 2 kN·s/m, respectively. The main objective of the proposed active suspension is to maximize the ride comfort while considering the design

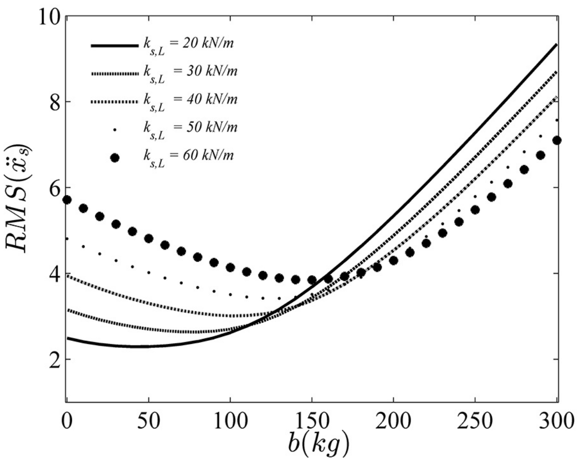

Fig. 2. Variation of the *RMS* ($\ddot{x}_s$) with inertance for different suspension stiffness (c_s = 2 kN·s/m)

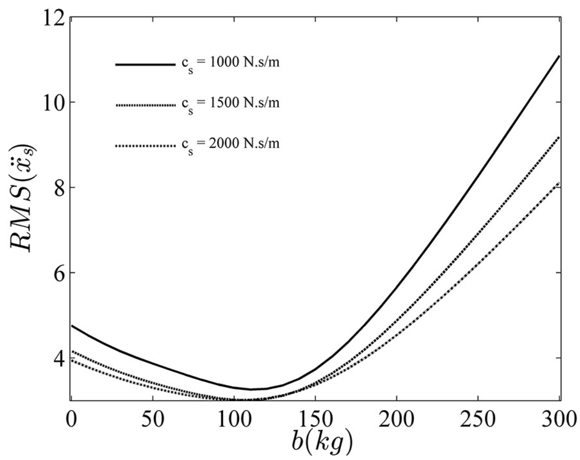

Fig. 3. Variation of the *RMS* ($\ddot{x}_s$) with inertance for different damping coefficients ($k_{s,L}$ = 40 kN/m)

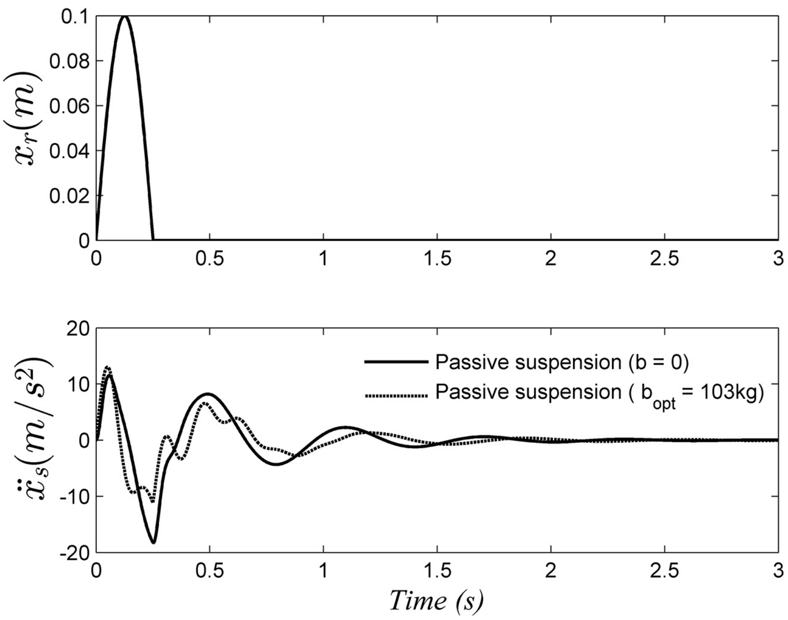

Fig. 4. Road bump and sprung mass acceleration $\ddot{x}_s$ for passive suspension system with optimal inerter and without inerter ($k_{s,L}$ = 40 kN/m and c_s = 2 kN·s/m)

constraints mentioned before. Thus, the LQR weights are carefully assumed and fixed for all the simulations as follows:

$$Q = \begin{bmatrix} 1e9 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Q_1 = 1e3, Q_2 = 1e7, and R = 0.5.

The variations of the *RMS* ($\ddot{x}_s$) and the *RMS* (u) with inertance are shown in Figures 5 and 6, respectively. The figures reveal that adding an inerter with kg reduces the *RMS* ($\ddot{x}_s$) and *RMS* (u) by 7% and 15.3%, respectively. Notably, at b = 110 kg, the *RMS* ($\ddot{x}_s$) is similar to that of the active suspension without inerter. The *RMS* (u) decreases by 25.2% using an inerter with b = 110 kg. Thus, the superiority of using the suitable inerter in reducing the actuator force is observed.

The simulation results for passive and active suspension systems with b = 60 kg and b = 110 kg are respectively shown in Figures 7 and 8. These figures compare between the active and passive suspensions of the nonlinear quarter-car model with and without inerter. The figures also show significant improvement in sprung mass acceleration and displacement, and the suspension stroke by using the proposed active suspension system compared with those of the passive suspension system. Table 1 summarizes the main results of the previous figures.

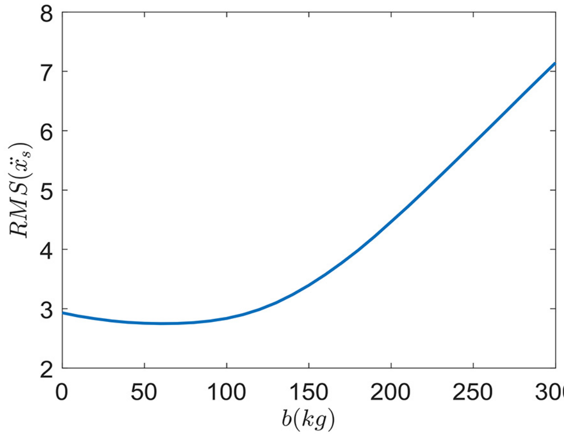

Fig. 5. Variation of the *RMS* ($\ddot{x}_s$) with inertance for the active suspension system ($k_{s,L}$ = 40 kN/m and c_s = 2 kN·s/m)

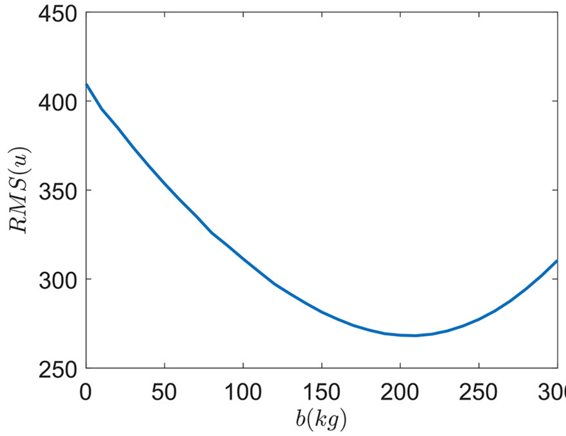

Fig. 6. Variation of the *RMS* (u) with inertance for the active suspension system ($k_{s,L}$ = 40 kN/m and c_s = 2 kN·s/m)

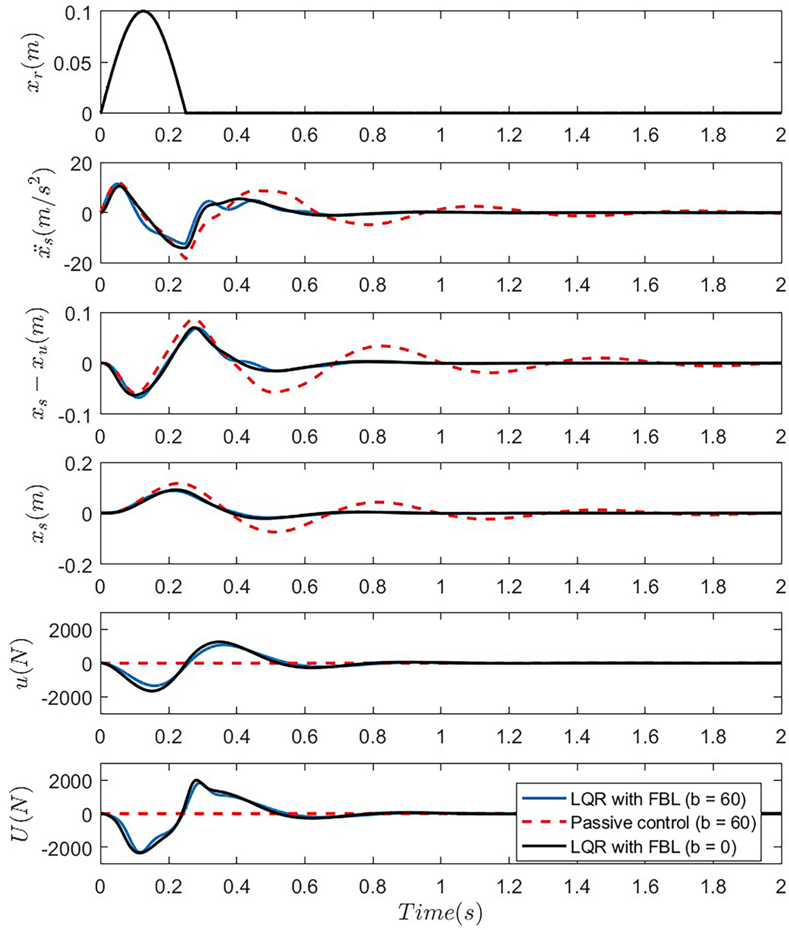

Fig. 7. Comparison between the passive and active suspension systems with bump input (b = 60 kg)

CONCLUSIONS

This paper investigated the effect of employing inerter on the performance of the active suspension system. A quarter-car model with cubic nonlinear primary spring was considered. The active suspension was built on the basis of the FBL combined with the LQR controller. The numerical simulations were carried out for bump profile input for passive and active suspension systems. The results show that employing inerters on the passive suspension system can improve

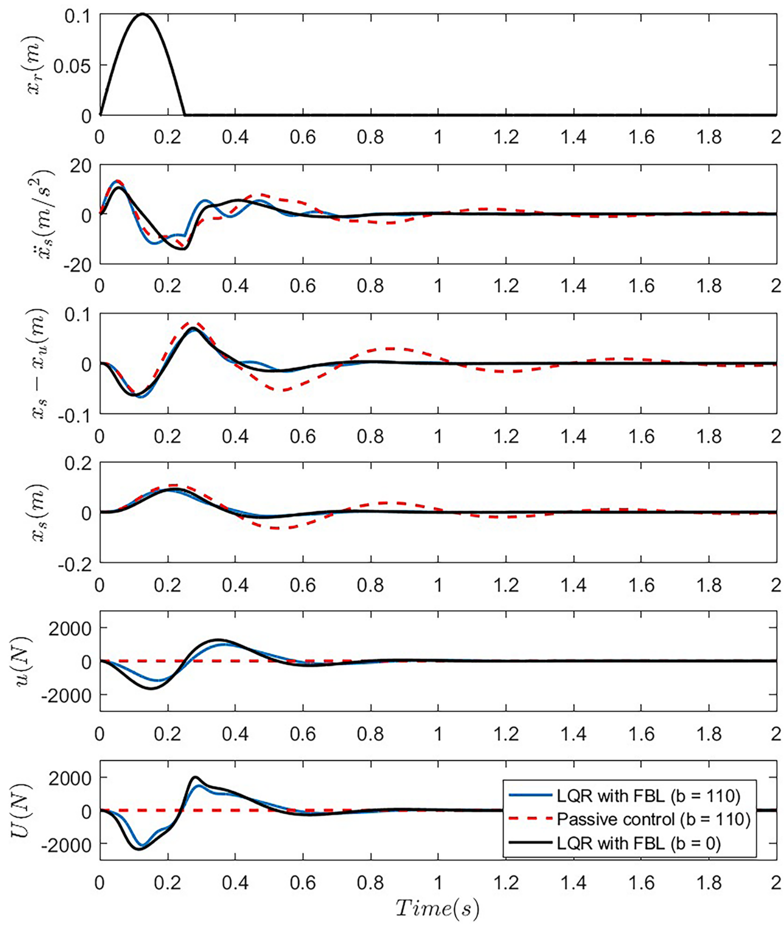

Fig. 8. Comparison between the passive and active suspension systems with bump input (*b* = 110 kg)

Table 1. The RMS of sprung mass acceleration and actuator force for passive and active suspension systems with different values of inertance

Suspension type	RMS ($\ddot{x}_s$)	RMS (u)
Passive (*b* = 0 kg)	3.94	–
Passive (*b* = 60 kg)	3.21	–
Passive (*b* = 110 kg)	3.02	–
Active (*b* = 0 kg)	2.93	408
Active (*b* = 60 kg)	2.75	345
Active (*b* = 110 kg)	2.93	305

the ride comfort significantly, particularly for systems with high-stiffness primary spring (i.e., the ride comfort may be improved by 32.7%). The inerters must be carefully selected to improve the ride comfort performance; otherwise, they may degrade the performance by increasing the sprung mass acceleration, particularly for high inertance values (i.e., the RMS of the sprung-mass acceleration may be dramatically increased by 275%). A similar trend can be observed in the active suspension system, and the suitable selection of the inerter can improve the ride comfort. Moreover, improvement was observed in reducing the actuator force (i.e., the RMS of the actuator force may be reduced by 25.2%) Therefore, employing an inerter to the active suspension system can be regarded as an energy saving technique. The numerical results proved the efficacy of the designed controller in improving the ride comfort of the nonlinear suspension system, with and without an inerter, while maintaining the validity of the structural and road holding limitations. Future extension of this work will study the impact of using an inerter on the active's suspension performance of vehicles with random road profile input.

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