

## Design Principles of Horizontal Drum Machines with Low Vibration

Ilona Drach<sup>1</sup>, Andrii Goroshko<sup>2\*</sup>, Renata Dwornicka<sup>3</sup>

<sup>1</sup> Software Engineering Department, Khmelnytskyi National University, 11 Institutaska st, 29016, Khmelnytskyi, Ukraine

<sup>2</sup> Physics and Electrical Engineering Department, Khmelnytskyi National University, 11 Institutaska st, 29016, Khmelnytskyi, Ukraine

<sup>3</sup> Department of Applied Computer Science, Faculty of Mechanical Engineering, Cracow University of Technology, 37 Jana Pawla II, 31-864 Krakow, Poland

\* Corresponding author's email: [iftomm@ukr.net](mailto:iftomm@ukr.net)

### ABSTRACT

Using the example of washing machines using the linear theory of vibrations, the dynamics of horizontal rotary drum machines is investigated and the basic requirements for their layout are formulated to reduce vibration activity. The mathematical equations of vibrations of the multiply connected system tub-drum on elastic suspensions are compiled for main types of washing machines and centrifuges with horizontal axis of rotation. The problem is solved in a linear setting based on the Lagrange equation of the second kind. The accuracy and adequacy of the mathematical model was tested directly on a full-scale object by measuring noise, vibrations, support forces and stress distribution in individual elements and units of the washing machine in the entire range of drum rotation frequencies. Investigations of the nature of system vibrations depending on changes in the position and attachment points of elastic and damping elements were carried out using simulation in the *Simulink* environment. As a result of the research, the basic requirements for the layout of horizontal rotary drum machines were experimentally confirmed. Experimental verification was carried out to confirm the results obtained. It has been experimentally proven that the improvement of the experimental setup to reduce its vibration activity increases the efficiency of using direct-acting liquid autobalancers.

**Keywords:** rotary drum machine, washing machine, imbalance, vibration activity, stiffness, self-balancing device

### INTRODUCTION

Rotary mechanisms are used in many areas of modern industry, including mechanical engineering and computer technology. Often these mechanisms must operate at high speeds. Therefore, strong vibrations caused by the displacement of the center of gravity of the rotor can become a serious problem and even lead to damage to the mechanism.

The problems of reducing vibration excited by rotating rotors and having the frequency of the first rotor harmonic (rotor vibration) are among the most important in the design, manufacture and operation of almost all types of modern rotary machines. Rotary vibration largely determines the reliability of such machines, which are

characterized by: high rotational speeds, relatively low structural rigidity, critical modes are often located within the operating ranges of angular velocities.

The washing machine is of particular interest as an object of study of dynamics and reduction of vibrations and noise, since due to the constant presence of randomly located and wandering imbalance of laundry in the drum and low requirements for the accuracy of its manufacture and assembly of parts and assemblies, so as not to increase the cost.

The solution of problems related to the problems of reducing rotor vibration is based on the linear theory of mechanical vibrations. The theory also indicates the main directions

of the vibration control: vibration isolation [1]; damping [2]; dynamic vibration damping [3]; improvement of methods for balancing rotors [4], including taking into account their flexibility [5].

Many reports investigate electromagnetic balancing devices in the form of electromagnetic bearings [6] and electromagnetic rings [7]. A synchronous radial force can be applied to the shaft to balance the imbalance [8]. Methods of optimal vibration control have been widely used [9]. Most of the publications are devoted to the study of the phenomenon of passive automatic balancing using ball, pendulum autobalancers [10] and due to the free movement of fluid. For example, in the robot [11], the phenomenon of self-balancing of the rotor is analyzed with the help of a fluid moving in hollow chambers partially filled with it, which function as passive regulators that do not need a power supply and control system. A mathematical model of the dynamic system of the washing machine was created, which was used to analyze the dynamics and optimize the parameters of the machine using the technology of rigid-flexible connection [12]. The report [13] investigates the rotor-dynamic effects for horizontal front-loading washing machines. Also noteworthy are the methods of damping washing machines using magneto and electro-rheological fluids [14].

Despite the fact that in recent years, researchers have made significant progress in creating active and passive methods and means of reducing the vibrations of washing machines, the level of vibrations can also be reduced due to the optimal arrangement of the components of the machine structure, which this work is aimed at studying.

### VIBRATION ISOLATION SYSTEM MATHEMATICAL MODEL

Consider the vibrations of an elastically suspended tub containing a cantilevered rotating unbalanced drum inside it. This scheme is inherent in machines with a horizontally located laundry tub, for example, LG, Electrolux, etc.

The assumptions in the course of schematization of the object of research are such that the real typical structure is replaced by a design scheme (dynamic model), in which an absolutely rigid body (tub of mass  $m_1$ ) is elastically connected to the body by an arbitrary number of

supports and is able to move in space, having 6 degrees of freedom (Fig. 1). In this body there is a cavity in which a rotor (drum) rotates with an angular velocity  $w$  with a mass of  $m_2$ , resting on absolutely rigid supports located in the same body.

For the generalized coordinates defining the position of this system in space, three Cartesian coordinates of the center of inertia of the tub of the machine are taken. Axes  $a_1, b_1, c_1$  are the main central axes. The angles  $\alpha, \beta, \gamma$  set the rotations of these coordinate axes relative to the fixed axes  $X_1, Y_1, Z_1$  associated with the body, or the axes  $X, Y, Z$  parallel to them, converging at the center of mass of the tub  $O$ . Center  $O$  generally does not lie on the axis of rotation of the drum. In such coordinates, the oscillations of the system can be represented as a superposition of six screw motions with fixed axes of the screws  $X_1, Y_1, Z_1$ . The tub-drum system generally performs six-link vibrations. To compose the differential equations of motion of the system, the Lagrange equation of the second kind is used.

Let  $m_1$  be the mass of tub;  $J_{a_1}^{(1)}, J_{b_1}^{(1)}, J_{c_1}^{(1)}$  – mass moments of tub inertia relative to the main central axes  $a_1, b_1, c_1$  respectively;  $m_2$  – drum mass;  $J_{a_2}^{(2)}, J_{b_2}^{(2)}, J_{c_2}^{(2)}$  – mass moments of drum inertia relative to the main central axes  $a_2, b_2, c_2$ .

The total kinetic energy of the system is equal to the sum:

$$T = T_1 + T_2, \tag{1}$$

where:  $T_1, T_2$  are the kinetic energies of the tub and drum, respectively.

Using Konig’s theorem, the kinetic energy of the tub is:

$$T_1 = \frac{1}{2} \cdot m_1 \cdot (\dot{x}_1^2 + \dot{y}_1^2 + \dot{z}_1^2) + \frac{1}{2} \cdot J_{a_1}^{(1)} \cdot \dot{\psi}_{a_1}^2 + \frac{1}{2} \cdot J_{b_1}^{(1)} \cdot \dot{\psi}_{b_1}^2 + \frac{1}{2} \cdot J_{c_1}^{(1)} \cdot \dot{\psi}_{c_1}^2, \tag{2}$$

where:  $\dot{\psi}_{a_1}^2, \dot{\psi}_{b_1}^2, \dot{\psi}_{c_1}^2$  are the projections of the angular velocity vector  $\vec{\psi} = \vec{\alpha} + \vec{\beta} + \vec{\gamma}$  on the axis  $a_1, b_1, c_1$ .

To obtain an expression for the kinetic energy of the drum, consider its motion in the general case (Fig. 2), when the center of mass of the drum, point  $S$ , does not coincide with the center of mass of the tub, point  $O$ . Let  $e$  be the eccentricity of the drum.

Consider additional coordinate systems. System  $X_2, Y_2, Z_2$  with the origin at point  $S$  and system  $X_3, Y_3, Z_3$  starting at point  $D$  move

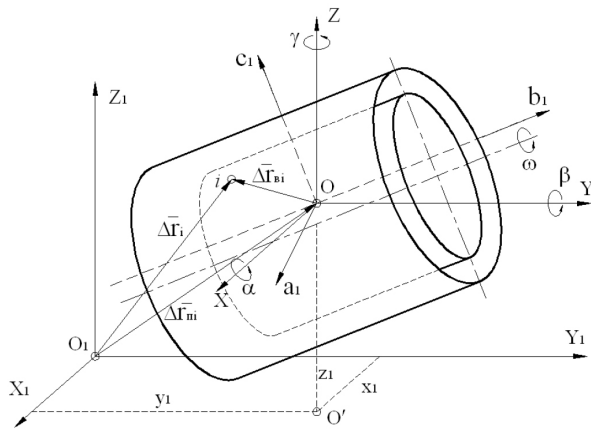


Fig. 1. Design diagram of the tub-drum system

translationally. System  $a_2, b_2, c_2$  is rigidly connected to the drum. Its axes are the main central axes of inertia of the drum.

The origin point  $D$  will be obtained as a result of the intersection of the axis of rotation of the drum with a plane drawn through the center of mass point  $S$  perpendicular to the axis of rotation of the drum. In the initial state and in the absence of eccentricity ( $e = 0$ ), point  $S$  coincides with point  $D$ . We assume that in the initial state, the  $Y, Y_1, Y_2, Y_3$  and  $b_1, b_2, b_3$  axes are parallel to the drum rotation axis. In the considered coordinate system, the movement of the drum in the general case can be represented as a complex one: translational movement with the center of mass at point  $S$  and rotation around this center of mass with an angular velocity  $\bar{\Omega} = \bar{\psi} + \bar{\omega}$ .

Using Konig's theorem, the kinetic energy of the drum is:

$$T_2 = \frac{1}{2} \cdot m_2 \cdot (V_{Sx_1}^2 + V_{Sy_1}^2 + V_{Sz_1}^2) + \frac{1}{2} \cdot J_{a_2}^{(2)} \cdot (\Omega_{a_2}^2 + \Omega_{c_2}^2) + \frac{1}{2} \cdot J_{b_2}^{(2)} \cdot \Omega_{b_2}^2. \quad (3)$$

The expression for the kinetic energy of the tub-drum system has the form (4), where:

$m = m_1 + m_2$  is the mass of tub-drum system;

$J_{a_1} = J_{a_1}^{(1)} + J_{a_2}^{(2)} + m_2 (y_D^2 + z_D^2)$  is the moment of inertia of the tube-drum system about the axis  $a_1$ ;

$$T = \frac{1}{2} m (\dot{x}_1^2 + \dot{y}_1^2 + \dot{z}_1^2) + \frac{1}{2} J_{a_1} \dot{\alpha}^2 + \frac{1}{2} J_{b_1} \dot{\beta}^2 + \frac{1}{2} J_{c_1} \dot{\gamma}^2 + m_2 \dot{x}_1 (\dot{\beta} z_D - \dot{\gamma} y_D + \omega e \cos \omega t) - m_2 \dot{y}_1 (\dot{\beta} z_D + \omega e \cos \omega t) + m_2 \dot{\beta} \omega e (z_D \cos \omega t + x_D \sin \omega t) + m_2 \omega^2 e^2 \cos \omega t + m_2 \dot{y}_1 (\dot{\gamma} x_D - \dot{\alpha} z_D) - m_2 \dot{\alpha} x_D z_D + m_2 \dot{z}_1 (\dot{\alpha} y_D - \dot{\beta} x_D - \omega e \sin \omega t) - m_2 \dot{\alpha} y_D (\dot{\beta} x_D + \omega e \sin \omega t) + \frac{1}{2} I_{b_2}^{(2)} \omega (\omega + 2(\dot{\beta} + \alpha \dot{\gamma} - \dot{\alpha} \gamma)), \quad (4)$$

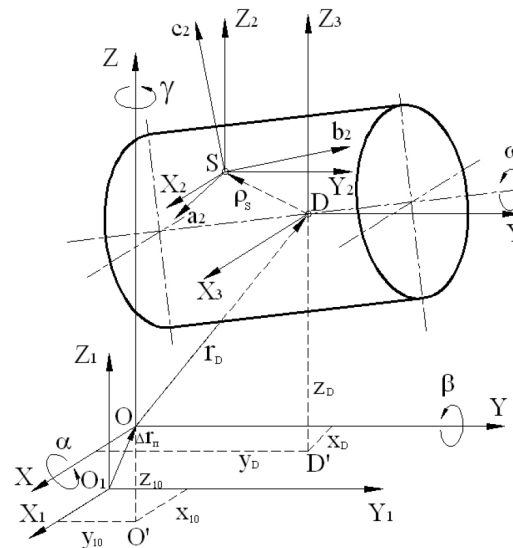


Fig. 2. Moving the drum in general view

$J_{b_1} = J_{b_1}^{(1)} + J_{b_2}^{(2)} + m_2 (z_D^2 + x_D^2)$  is the moment of inertia of the tube-drum system about the axis  $b_1$ ;

$J_{c_1} = J_{c_1}^{(1)} + J_{c_2}^{(2)} + m_2 (x_D^2 + y_D^2)$  is the moment of inertia of the tube-drum system about the axis  $c_1$ ;

Let us find expressions for the potential energy and dissipation energy in the dampers of the tub-drum system. The potential energy of the tub-drum system during vibrations is determined by the elastic deformations of the supports. Let the tub-drum system be connected to the washing machine frame using  $n$  elastic elements and  $m$  dampers. To simplify the dependencies, we assume that the main axes of stiffness and viscous friction constants of all elastic elements or dampers, respectively, are parallel to the main central axes of inertia of the tub-drum system. Then the projections of the stiffness vector of the  $i$ -th elastic element on to the coordinate axes  $X_1, Y_1, Z_1$ , which are their main stiffnesses are  $C_{x_i}, C_{y_i}, C_{z_i}$ . For each  $i$ -th damper, the projections of the vector of constant viscous friction are  $h_{x_i}, h_{y_i}$  and  $h_{z_i}$ , which are the principal constants of viscous friction. This assumption is practically consistent with the structural arrangement of the elastic elements of existing washing machines, and their other arrangement does not provide any advantages.

Then the potential energy of the tub-drum system will be equal to:

$$\dot{I} = \frac{1}{2} \sum_1^n (C_{x_i} \Delta r_{x_i}^2 + C_{y_i} \Delta r_{y_i}^2 + C_{z_i} \Delta r_{z_i}^2), \quad (5)$$

where:  $\Delta r_{x_i}, \Delta r_{y_i}, \Delta r_{z_i}$  are displacements along the  $X_1, Y_1, Z_1$  axes of the points of attachment to the moving system of elastic elements;  $n$  is the number of elastic elements of the tub-drum system.

The dissipation of energy in dampers due to viscous friction, depending on the speed of the points, is equal to:

$$D = \frac{1}{2} \sum_1^m (h_{x_i} \Delta \dot{r}_{x_i}^2 + h_{y_i} \Delta \dot{r}_{y_i}^2 + h_{z_i} \Delta \dot{r}_{z_i}^2), \quad (6)$$

where:  $\Delta \dot{r}_{x_i}, \Delta \dot{r}_{y_i}, \Delta \dot{r}_{z_i}$  are the speeds of points of connection of dampers to the tub-drum system along the axes  $X_1, Y_1, Z_1$ ;  $m$  is the number of dampers of the tub-drum system.

The differential equations of oscillations of the tub-drum system are based on the Lagrange equation of the second kind and take into account the energy dissipation during Rayleigh damping:

$$\frac{d}{dt} \left( \frac{\partial T}{\partial \dot{q}_j} \right) - \frac{\partial T}{\partial q_j} + \frac{\partial \Pi}{\partial q_j} + \frac{\partial D}{\partial \dot{q}_j} = 0, \quad (7)$$

where:  $j = 1, 2, \dots, 6$  is the number of generalized coordinates.

Performing mathematical operations for the Lagrange equation (7) in six generalized coordinates  $X_1, Y_1, Z_1, a, b, g$  and setting  $\omega = const$  and removing the indices at  $X_1, Y_1, Z_1$  we obtain a system of six differential equations in matrix form:

$$\mathbf{M}\ddot{\mathbf{q}} + (\mathbf{G} + \mathbf{D})\dot{\mathbf{q}} + \mathbf{A}\mathbf{q} = \mathbf{Q}, \quad (8)$$

where:  $\mathbf{M} = [P_{ij}]_1^6$  is the matrix of inertial coefficients;

$\mathbf{G} = [g_{ij}]_1^6$  is the matrix of gyroscopic coefficients;

$\mathbf{D} = [\alpha_{ij}]_1^6$  is the matrix of damping coefficients;

$\mathbf{A} = [\alpha_{ij}]_1^6$  is the matrix of stiffness coefficients;

$\mathbf{q} = [x, y, z, \alpha, \beta, \gamma]^T$  is the column vector of generalized coordinates;

$\mathbf{Q} = [Q_x, Q_y, Q_z, Q_\alpha, Q_\beta, Q_\gamma]^T$  is the column vector of generalized force factors.

## ANALYSIS OF A MATHEMATICAL MODEL

The system of equations (8) shows many connections between the motion along the selected coordinates. These connections are created by arbitrary placement of the points of attachment of elastic elements and dampers to the movable suspension part of the washing machine and the mismatch of the center of mass of the tub with the center of mass of the balanced drum. In the general case of movement, the tub-drum system performs six-link vibrations. Perturbation in one direction causes vibrations along and around all axes of any Cartesian coordinate system used to determine the position of the hanging tub of the washing machine.

It is known from the theory of vibrations that the higher the connectivity of vibrations, the wider, under the same other conditions, the spectrum of natural frequencies, that is, the greater the difference between the highest and lowest frequencies of free vibrations. When all six frequencies are linked, changing the value of one of them will change the values of all frequencies. This complicates the problem of vibration isolation and reducing the dynamic activity of the hanging tub, therefore, it is desirable to find out the practical possibilities of reducing the connectivity of vibrations that can be realized when designing drum-type machines.

Analyzing equations (8), it is possible to separate the motion directed along and around the  $X, Y, Z$  axes of the moving coordinate system, removing the connections between oscillations relative to the generalized coordinates  $q_i, i = \overline{1, 6}$  of the tub-drum system. Let the drum does not rotate and the matrix  $\mathbf{M}$  of inertial coefficients is diagonal, when the center of mass of the tub coincides with the center of mass of the balanced drum. Then the complete separation of free vibrations will take place under the condition when for all  $i=j$   $a_{ij} = 0$ , when the stiffness matrix  $\mathbf{A}$  and the damping matrix  $\mathbf{D}$  are diagonal. According to their structure, the elements  $a_{ij}$  of the matrix  $\mathbf{A}$  can be divided into four groups and named similarly to the components of the tensor of inertia.

The first group includes elements  $a_{ij}, i, j \leq 3, i=j$ . These are the total stiffnesses. Their value is significantly positive:

$$a_{11} = \sum_{i=1}^n c_{x_i}, \quad a_{22} = \sum_{i=1}^n c_{y_i}, \quad a_{33} = \sum_{i=1}^n c_{z_i},$$



where:  $c_{x_i}, c_{y_i}, c_{z_i}$ , are the the projection of the stiffness vector of the  $i$ -th elastic element on the coordinate axes  $X_1, Y_1, Z_1$ .

The second group includes elements  $a_{ij}, i, j > 3, i=j$ . These are the torsional stiffnesses of the hanging of the tub-drum system. In sign they are similar to the moments of inertia about the coordinate axes (always positive):

$$a_{44} = \sum_{i=1}^n c_{z_i} y_i^2 + \sum_{i=1}^n c_{y_i} z_i^2,$$

$$a_{55} = \sum_{i=1}^n c_{x_i} z_i^2 + \sum_{i=1}^n c_{z_i} x_i^2, \quad a_{66} = \sum_{i=1}^n c_{y_i} x_i^2 + \sum_{i=1}^n c_{x_i} y_i^2.$$

The third group includes elements  $a_{ij}, i \leq 3, j > 3, i=j$  and  $i > 3, j \leq 3, i=j$ . These are static moments of stiffness, relative to the coordinate planes of the system. In signs, they are similar to the static moments of masses relative to the coordinate planes and can be positive, negative and equal to zero:

$$a_{15} = a_{51} = \sum_{i=1}^n c_{x_i} z_i, \quad a_{16} = a_{61} = -\sum_{i=1}^n c_{x_i} y_i,$$

$$a_{24} = a_{42} = -\sum_{i=1}^n c_{y_i} z_i, \quad a_{26} = a_{62} = \sum_{i=1}^n c_{y_i} x_i,$$

$$a_{34} = a_{43} = \sum_{i=1}^n c_{z_i} y_i, \quad a_{35} = a_{53} = -\sum_{i=1}^n c_{z_i} x_i,$$

$$a_{12} = a_{21} = 0, \quad a_{13} = a_{31} = 0, \quad a_{14} = a_{41} = 0,$$

$$a_{23} = a_{32} = 0, \quad a_{25} = a_{52} = 0, \quad a_{36} = a_{63} = 0.$$

The fourth group includes elements  $i, j > 3, i=j$ . These are the centrifugal moments of stiffness relative to the pairs of coordinate planes. In sign they are similar to centrifugal moments of inertia and can be positive, negative and equal to zero:

$$a_{45} = a_{54} = -\sum_{i=1}^n c_{z_i} x_i y_i, \quad a_{46} = a_{64} = -\sum_{i=1}^n c_{y_i} x_i z_i,$$

$$a_{56} = a_{65} = -\sum_{i=1}^n c_{x_i} y_i z_i.$$

This analogy allows us to formulate simple rules under which the off-diagonal elements of the stiffness matrix: static and centrifugal moments of stiffness will be equal to zero. This is necessary to separate the vibrations of the system. In particular, the static and centrifugal moments of stiffness are equal to zero if the coordinate planes, relative to which they are determined, are the planes of symmetry of the tub-drum system suspension. The same conclusions are valid for the structure of the elements of the damping matrix **D**. However, in practice, due to the random nature

of the distribution of the laundry in the drum, it is not possible to achieve complete coincidence of the center of mass of the tub with the center of mass of the balanced drum. But to reduce the vibration activity of the washing machine, it is necessary that the center of mass of the tub lies on the axis of rotation of the drum and as close as possible to its center of mass with a uniform layout of the laundry.

The analysis of the differential equations of motion made it possible to formulate the requirements for the design of the washing machine. These requirements must be met during design. The center of mass of the tub must lie on the axis of rotation of the drum; the axis of rotation of the drum should be the main central axis of inertia of the tub; the center of mass of the tub must coincide with the center of mass of the drum; the center of stiffness of the system of elastic supports must coincide with the center of gravity of the tub, and the main axes of stiffness - with the main central axes of inertia of the tub. The principal axes of viscous friction constants must coincide with the principal central axes of inertia of the tub. The principal axes of viscous friction constants must coincide with the principal central axes of inertia of the tub.

The concept of the center and axes of stiffness of a system of elastic supports are based on the following. If we assume that the main elasticity axes of the dampers are parallel to the coordinate axes, then the displacement of the tub-drum system in the direction of one of the axes will cause the appearance of parallel reactions of the supports directed against the displacement. The system of these parallel forces has a resultant, the line of action of which is the axle of suspension stiffness. In general, the stiffness axes of the support system are three crossing straight lines. With a certain choice of stiffness and the location of the supports, it is possible to achieve that two or three axes intersect. The point of intersection of two axes of stiffness is called the center of stiffness. The point of intersection of the three axes is the main center of stiffness. In some cases, the stiffness axis can be positioned so that the centrifugal moments about the other two stiffness axes are equal to zero. These axes are mutually perpendicular and orthogonal to the considered one. Under this condition, the stiffness axis is called the main one.

The adequacy and accuracy of the developed mathematical model was tested using field

experiments. For this, a laboratory bench with a household washing machine was used. The stand is shown in Figure 3. The theoretically calculated natural frequencies of the oscillating system of the washing machine differed from the experimentally obtained ones by no more than 10%. The calculated amplitude of vertical vibrations of the hanging tub with an artificial imbalance of the drum  $D=1800 \text{ g}\cdot\text{mm}$  was 3,7 mm, and experimentally measured with the same unbalance was 4 mm.

**DYNAMIC SIMULATION AND ANALYSIS**

The formulated structural design requirements were verified by simulating the behavior of the hanging part of the washing machine using the interactive *Simulink* tool and the *Matlab* software package. The dynamic model of the machine is shown in Figure 4. The suspension part of the machine is secured with two springs and two dampers. The dynamic characteristics of the washing machine are presented in Table 1.

In the ideal case, the center of mass of a balanced drum coincides with the center of mass of the tub, and the axis of rotation of the drum passes through its center of mass, eccentricity  $e=0$ , the right side of equation (8)  $\mathbf{Q} = 0$ . Oscillations of the suspension part are absent at any values of the angular velocity of rotation  $\omega$ .



Fig. 3. General view of the experimental stand

**Table 1.** Dynamic characteristics of the washing machine

Tub weight $m_1$ , kg	48.2
Drum weight $m_2$ , kg	12.5
Stiffness of each of the two suspension springs $c$ , N/mm	6.0
Spring angle $\chi_1$	75°
Damping constant of each damper $h$ , N·s/m	175
Damper tilt angle $\chi_2$	70°
Eccentricity $e$ , mm	14.8
Drum length $L_d$ , mm	190

In practice, due to the random distribution of the loaded laundry in the drum, the eccentricity is not zero. The center of mass of the drum does not always lie on the axis of rotation. It was found that the greatest eccentricity in the drum is  $e = 0.08R_d$ , where  $R_d$  is the drum radius. For the investigated model of the washing machine, the value of the eccentricity is  $e = 14,8 \text{ mm}$ . The main dynamic characteristics of the washing machine are shown in Table 1.

The amplitude-frequency characteristic obtained for vertical vibrations of the suspension part, taking into account that the center of mass of the tub lies on the axis of rotation of the drum and the drum has only a static unbalance caused by the eccentricity  $e$  is shown in Figure 5. Generalized force factors are equal  $Q_y = Q_\alpha = Q_\beta = Q_\gamma = 0$ ,  $Q_x, Q_z \neq 0$ . Due to the force perturbation along the  $OX$  and  $OZ$  axes, there are transverse vibrations in the  $XOZ$  plane.

*Consideration of moment imbalance.* The condition  $x_D = z_D = 0$ , according to which the center of mass of the tub must lie on the axis of

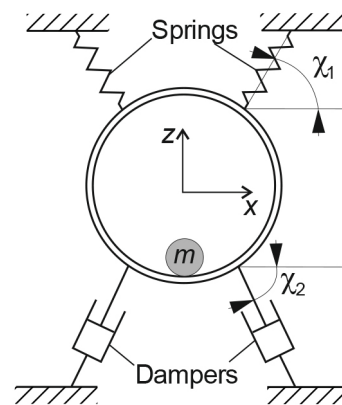


Fig. 4. Dynamical model of the washing machine

rotation of the drum, can be structurally and technologically fulfilled with sufficient accuracy. This does not change while the system is running. But due to the random arrangement of the laundry, the center of mass of the loaded drum, in addition to eccentricity, can have a longitudinal displacement  $y_D$  along the  $OY$  axis. The displacement causes the rotor to be moment unbalanced. The issue of calculating the value  $y_D$  is considered by the authors of [15]. It has been proven that the value of the longitudinal displacement increases with the increase in the load of the drum and the length of the drum. The maximum displacement is  $y_{Dmax} = 0,476L$ , where  $L_a$  is the length of the drum. For the model studied in this work, it was taken  $y_{Dmax} = 90$  mm.

The simulation was carried out at values of  $y_D = 0 \dots 90$  mm in the entire range of operating frequencies. The simulation showed little effect on the deflection of lateral vibrations in the stabilised zone. The relation of the amplitude of the vibration of the suspended part in the direction of the  $OY$  axis to the horizontal displacement in the range of operating frequencies is shown in Figure 6. The displacement of the center of mass of the loaded drum at operating frequencies in the supercritical region has almost no effect on the level of lateral vibrations, but significantly increases the longitudinal vibrations of the suspension part. The maximum value of longitudinal vibrations is observed at the critical frequency. This is explained by an increase in the moment of action of the centrifugal force, which leads to an increase in angular oscillations relative to the  $OX$  and  $OZ$  axes due to the action of generalized force factors along the coordinates  $\alpha$  and  $\gamma$ :  $Q_\alpha = m_2\omega^2ey_D \cos \omega t$  and  $Q_\gamma = -m_2\omega^2ey_D \sin \omega t$ . In this regards, The matrix

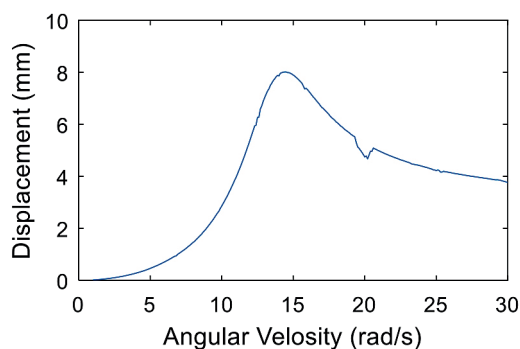


Fig. 5. Frequency response (lateral vibrations along the  $OZ$  axis)

of inertial coefficients  $\mathbf{M}$  is nondiagonal. An increase in angular vibrations causes an increase in the displacement of the suspension part due to the translational and rotational movement. These vibrations can negatively affect the performance of the bearing node and reduce its service life.

*Consideration of the influence of the support attachment coordinates on the displacement of the stiffness center.* Stiffness matrix  $\mathbf{A}$  is off-diagonal if static and centrifugal moments of stiffness are nonzero. Changing the coordinates  $x_i, z_i$   $i$ -th of the  $i$ -th spring does not affect the position of the resultant of the parallel reaction forces of the supports along the axes, and, consequently, the position of the suspension stiffness axis. Similar conclusions could be done regarding the damping matrix  $\mathbf{D}$ . However, the axle of the suspension stiffness does not coincide with the center of gravity of the tub if the main axes of stiffness do not coincide with the main central axes of symmetry of the tub. This is possible provided that the coordinates of the  $i$ -th spring is  $y_i \neq 0$ .

To study the effect of the mismatch between the center of stiffness of the elastic supports and the center of gravity of the tub, the model was simulated in the entire operating frequency range when the center of stiffness was changed along the  $OY$  axis. The results of the simulation of the tub-drum system at various values of deviation  $l_c$ , the position of the center of stiffness of the elastic supports of the suspension part from the center of gravity of the tub are shown in Figure 8 and 9. The dependence for the case  $y_D = 0$  when the center of mass of the loaded drum lies in the  $XOZ$  plane, where the center of gravity of the tub is located, is shown in Figure 7. The dependence for the case when the center of mass of the loaded drum is displaced along the  $OY$  axis by the

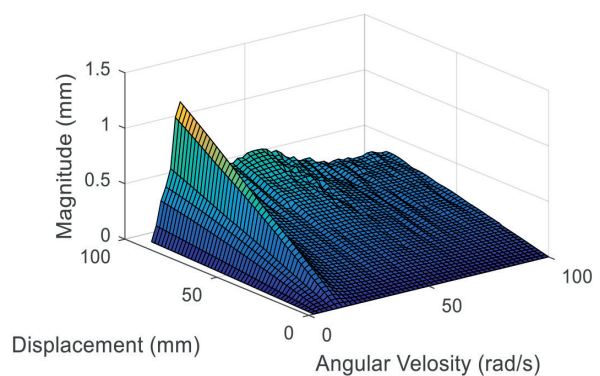
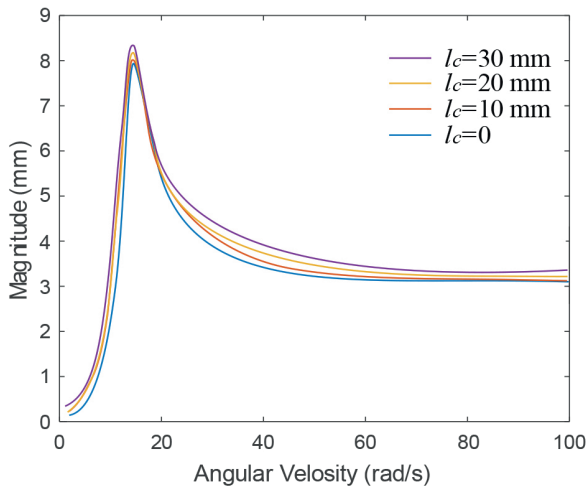
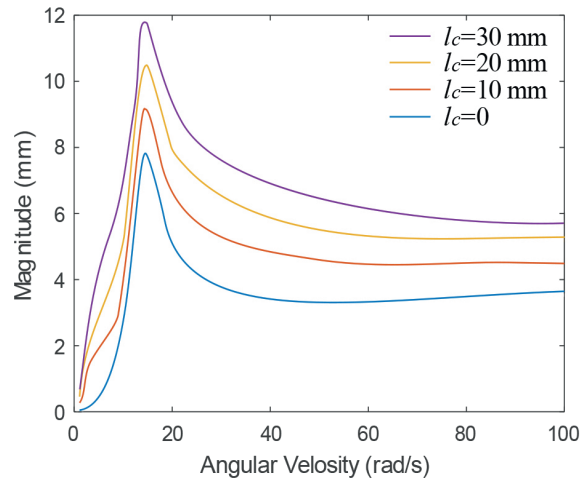


Fig. 6. Frequency response at variable displacement  $y_D$  (longitudinal vibration along the  $OY$  axis)



**Fig. 7.** Amplitude-frequency characteristics of the suspension part at  $y_D = 0$



**Fig. 8.** Amplitude-frequency characteristics of the suspension part  $y_D = 90$  mm

value  $y_{Dmax}$  is shown in Figure 8. The increase in the amplitude of vertical vibrations is caused by the appearance in the stiffness matrix  $\mathbf{A}$  torsional stiffnesses  $a_{44}$ ,  $a_{66}$  relative to the axes  $OX$  and  $OZ$ , static moments of stiffness  $a_{16} = a_{61}$ ,  $a_{34} = a_{43}$  relative to the  $XOZ$  and centrifugal moments of stiffness  $a_{45} = a_{54}$ ,  $a_{56} = a_{65}$  relative to pairs of coordinate planes.

Similar studies of the effect of the mismatch of the center of damping with the center of gravity of the tub and the main axes of constant viscous friction of dampers with the main central axes of inertia of the suspension part showed that this effect is insignificant.

*The influence of rotation of elastic and damping supports on the displacement of the position of the center of stiffness.* When the elastic (damper) support is rotated through an angle  $\varphi$  relative to the  $XOZ$  plane, the support system is non-equi-stiff, and the stiffness (damping) matrix is off-diagonal.

For the spring support of the tub with axial rigidity  $C$  shown in Figure 9, the following relation holds:

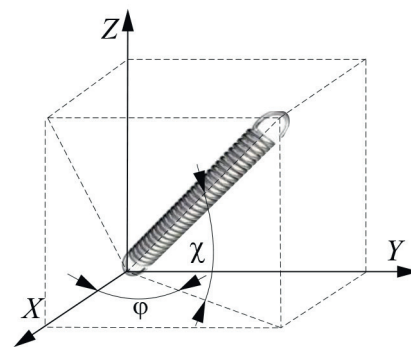
$C = \sqrt{C_x^2 + C_y^2 + C_z^2}$ . The stiffness of the spring along the coordinate axes is:

$$C_x = C \cos \chi \cos \varphi, \quad C_y = C \cos \chi \sin \varphi, \\ C_z = C \sin \chi \cos \varphi,$$

where:  $\chi$  is the angle of the spring,  $\varphi$  is the angle of rotation of the spring relative to the  $XOZ$  plane. For a spring with axial stiffness  $C$  and an angle of inclination  $\chi$  the projections  $C_x$  and  $C_z$  are maximum when the springs lie in the  $OXZ$  plane and  $\varphi = 0$ .

If the values of the angle  $\varphi$  are small and the longitudinal displacement of the center of gravity of the loaded drum is equal to zero ( $y_D = 0$ ), then when the drum rotates in the entire range of operating frequencies, the magnitude of the resulting vibrations changes insignificantly. The presence of horizontal displacement  $y_D \neq 0$  and the angle of rotation  $\varphi \neq 0$  of the spring leads to an increase in the amplitude of vibrations due to the additional oscillation connectivity. Analysis of the influence of the angle of rotation of the damper leads to similar results. The dependences of the amplitude of vertical vibrations on the drum rotation frequency at  $y_{Dmax}$  obtained for the angles of rotation of the springs and damper from 0 to 3° are shown in Figure 10.

The most sensitive to changes in the angles of rotation of the springs and dampers are lateral vibrations. The dependence of the amplitude and frequency of vibrations of the suspension part along the  $OY$  axis on the change in the angle of



**Fig. 9.** Spring support of the suspension part



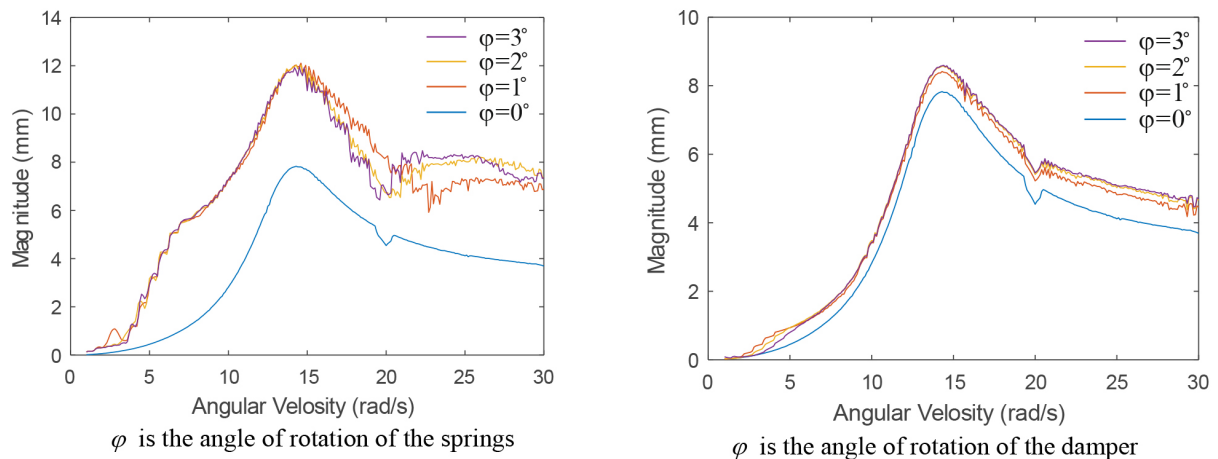


Fig. 10. The dependences of the amplitude of vertical vibrations for different angles of rotation

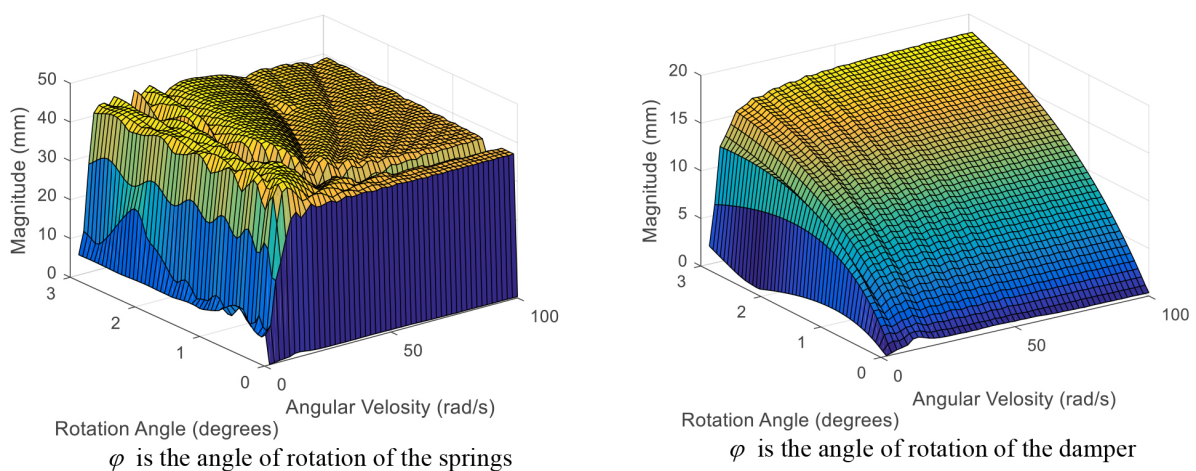


Fig. 11. Dependence of the amplitude of longitudinal vibrations on the frequency and angle of rotation

rotation of the spring  $\varphi$  and the damper when the center of gravity of the drum is displaced by  $y_{Dmax}$  along the axis of the drum are shown in Figure 11.

### EXPERIMENTAL VERIFICATION

The results of theoretical research can be applied to the problem of evaluating the effectiveness of a liquid self-balancing device (SBD) for a washer-extractor machine, depending on its dynamics.

Based on the preliminary results of the analysis of the dynamic model of the rotary machine, the ways of improving the design of the experimental plant are formulated and implemented. The experimental plant contains a rigid horizontal rotor mounted on elastic supports (Fig. 3). To reduce its vibroactivity, the center of mass of the drum by means of dynamic

balancing is almost aligned with the axis of rotation and the center of mass of the tub; elastic tub supports are selected so that their center of stiffness approximately coincides with the center of mass of the tub-drum system.

Experimental verification of the possibility of increasing the efficiency of self-balancing of a horizontal rotor with a liquid auto-balancer by improving the dynamics of a rotary machine was carried out using a device with two reservoirs. The experiments were carried out in four stages.

- 1) The plant with an initial design was used, the rotor was balanced without liquid in the chamber, by fixing corrective masses in the rotor.
- 2) A specific rotor unbalance was introduced, equal to  $D=1000 \text{ g}\cdot\text{cm}$ , by fixing an imbalance with a mass of  $m_D=50 \text{ g}$ . The rotor was set in rotation; the amplitude-frequency characteristic of the oscillations of the front edge of the tub of the plant was recorded.

- 3) One chamber of the reservoir was filled with a liquid with a volume of  $V = 25$  ml. The rotor was set in rotation; the amplitude-frequency characteristic of the oscillations of the front edge of the tub of the plant was recorded. The experiments were repeated using 50 ml, 100 ml and 150 ml working fluids.
- 4) The same studies were repeated on an improved installation, in each experiment the amplitude of oscillations of the front edge of the tub of the improved installation was measured.

The experiments were repeated using an SBD with two chambers filled with a working fluid. The research results are presented in the form of frequency response of oscillations of the front edge of the tub in Figure 12 and 13.

Figure 12 illustrates the following conclusions: the introduction of an imbalance using a simulation mass leads to the occurrence of oscillations of the front edge of the installation tub in the initial structure with an amplitude of 16.7 mm when the rotor rotates without liquid in the chamber with a resonant angular velocity. At the fourth stage, it was found that the installation of a simulated unbalance mass in the rotor leads to the appearance of oscillations of the front edge of the tub with an amplitude of 5.5 mm when the rotor rotates at a resonant angular velocity. A decrease in amplitudes was observed over the entire range of angular velocities by an average of 45–50%.

Considering that at the third stage, when installing imbalance, the optimal filling of the SBD chamber with working fluid ( $V = 50$  ml) leads to a decrease in the vibrations of the initial design rotor by almost 30% over the entire range

of the machine motion, the efficiency of automatic rotor balancing with liquid SBD is 1.65.

The auto-balancing efficiency value obtained using a liquid-type SBD for the advanced installation is 1.79. Thus it has been proved that the improvement of rotary machine dynamics significantly increases the efficiency of automatic balancing.

The experiments were repeated using filling of two chambers with liquid SBD. The results are shown in Figure 13. The efficiency value of the liquid multi-chamber SBD for the advanced machine is 2.10. This is almost the same value for the efficiency of the ABP, which was obtained for the machine of the initial design (2.09).

### CONCLUSIONS

The paper discusses a generalized dynamic model for a rotary machine with a horizontal axis of rotation. The proposed model has six degrees of freedom. Differential equations of free and forced oscillations of the tub-drum system were created. The equations take into account gyroscopic moments and damping oscillations.

It is shown that when considering the general case of oscillations of the tub-drum system, there is complete connectivity of oscillations along all generalized coordinates.

Based on the analysis of the connectivity of oscillations, basic requirements were formulated for improving the design of the experimental set-up to reduce its vibration activity: the center of mass of the tub must lie on the axis of rotation of the drum; the axis of rotation of the drum

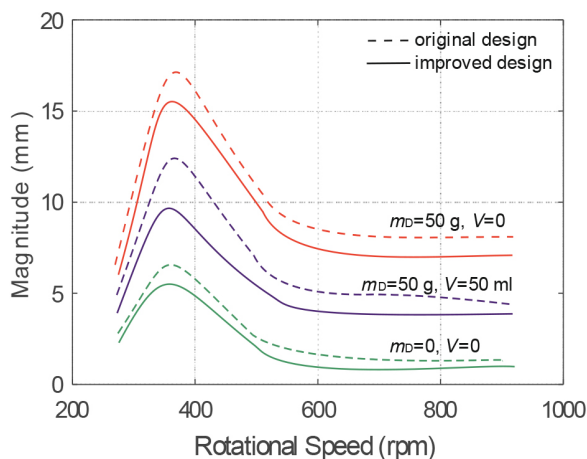


Fig. 12. Frequency response of the front edge of the tub when using the SBD with one filled chamber

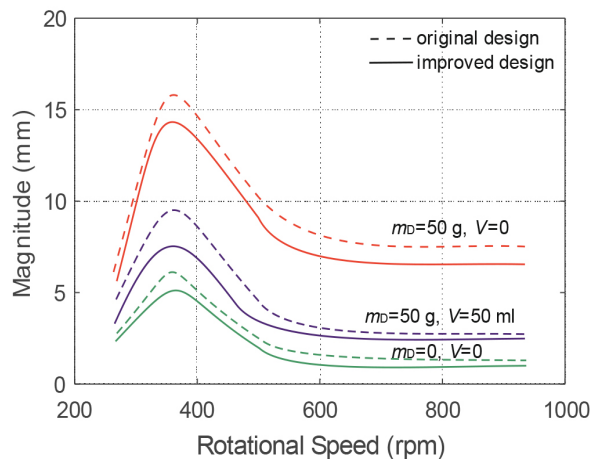


Fig. 13. Frequency response of the front edge of the tub when using the SBD with two filled chamber

should be the main central axis of inertia of the tub; the center of mass of the tub must coincide with the center of mass of the balanced drum with a uniformly distributed imitation imbalance; the center of stiffness of the system of elastic supports and viscous friction must coincide with the center of gravity of the tub, and the main axes of stiffness and viscous friction – with the main central axes of inertia of the tub. Compliance with these requirements made it possible to increase the efficiency of direct-acting liquid SBD to 30% over the entire range of speeds.

Thus, the effectiveness and prospects of using liquid SBD for reducing vibration of unbalanced horizontal rotors were proved. It is shown that liquid SBD effectively reduce machine vibrations not only after resonance, but also when passing through resonance.

### Acknowledgements

The research was funded by the Ministry of Education and Science of Ukraine (Project No. 0120U102067).

### REFERENCES

1. Ribeiro, E.A., Pereira, J.T., & Bavastri, C.A. (2015). Passive vibration control in rotor dynamics: optimization of composed support using viscoelastic materials. *Journal of Sound and Vibration*, 351, 43-56
2. Osinski, Z. (Ed.). (2018). *Damping of vibrations*. CRC Press.
3. Shen, Y., Chen, L., Yang, X., Shi, D., & Yang, J. (2016). Improved design of dynamic vibration absorber by using the inerter and its application in vehicle suspension. *Journal of Sound and Vibration*, 361, 148-158.
4. MacCamhaoil, M. (2016). Static and dynamic balancing of rigid rotors. *Bruel & Kjaer application notes*, BO, 0276-12.
5. Goroshko, A., Ostashevicius, V., & Royzman, V. (2016). Balancing of turbomachine rotors by increasing the eccentricity identification accuracy. *Mechanics*, 22(3), 206-211.
6. Wan, S.K., Li, X.H., Su, W.J. et al (2019). Active damping of milling chatter vibration via a novel spindle system with an integrated electromagnetic actuator. *Precision Engineering*, 57: 203-210.
7. Fan, H.W., Zhi, J.J., Shi, B.J. et al (2018). Adaptive rotor balancing algorithm and single-disk rotation test for electromagnetic balancer. *Journal of Xi'an Jiaotong University*, 52(8): 15-21, 29.
8. Pan, X., Lu, J., Huo, J. et al (2020). A review on self-recovery regulation (SR) technique for unbalance vibration of high-end equipment. *Chin. J. Mech. Eng.* 33, 89.
9. Peng, C., He, J.X., Zhu, M.T. et al (2019). Optimal synchronous vibration control for magnetically suspended centrifugal compressor. *Mechanical Systems and Signal Processing*, 132: 776-789.
10. Goncharov, V., Filimonikhin, G., Dumenko, K. et al. (2016). Studying the peculiarities of balancing of flexible double-support rotors by two passive automatic balancers placed near supports. *Eastern-European Journal of Enterprise Technologies*, 4(7): 4-9.
11. Royzman, V., Drach, I., Tkachuk, V., Pilkauskas, K., Čižauskas, G., & Šulgina, A. (2018). Operation of passive fluid self-balancing device at resonance transition regime. *Mechanics*, 24(6), 805-810.
12. Xiao, L., Zhang, S. (2017). Analysis and optimization of drum washing machine vibration isolation system based on rigid-flexible virtual prototype model. *Journal of Vibroengineering*, 19(3). 1653-1664.
13. Drüke, S., Bicker, R., Schuller, B., Henke, C., Trächtler, A. (2019) Rotordynamic instabilities in washing machines. In: Cavalca K., Weber H. (Eds). *Proceedings of the 10<sup>th</sup> International Conference on Rotor Dynamics, IFToMM 2018. Mechanisms and Machine Science*, vol. 61. Springer, Cham.
14. Ulasyar, A., Lazoglu, I. (2018) Design and analysis of a new magneto rheological damper for washing machine. *J Mech Sci Technol* 32, 1549–1561.
15. Fetisov, V.G., Alekhin, S.N., & Petrosov, S.P. (2012). Study of nonhomogeneous equations with variable coefficients describing washing machines vibration. *European Researcher*, (5-2), 609-612.