APPENDIX. FORMULATION OF THE PROBLEM

The study utilized classical laminate plate theory (CLPT) [30,31,33]. It was assumed that the plate materials were subject to Hooke's law.

According to CLPT's kinematic assumptions regarding the displacement of a thin laminate plate, the plate's arbitrary displacements can be written as:

$$\tilde{u}(x,y,z,t) = u(x,y,t) - zw_{,x}(x,y,t)$$
$$\tilde{v}(x,y,z,t) = v(x,y,t) - zw_{,y}(x,y,t)$$
$$\tilde{w}(x,y,z,t) \equiv w(x,y,t). \qquad (A.1)$$

where *u, v, w* – displacements of the plate middle surface in the *x, y* and *z* directions. A reduced expression $(\ldots)_{,x} = \partial(\ldots)/\partial x$, $(\ldots)_{,y} = \partial(\ldots)/\partial y$ was used.

For each of the plates, the following strain-displacement equations describing in-plane strains can be written

$$\widetilde{\varepsilon_x}(x,y,z,t) = \varepsilon_x + z\kappa_x \qquad \widetilde{\varepsilon_y}(x,y,z,t) = \varepsilon_y + z\kappa_y$$
$$2\widetilde{\varepsilon_{xy}}(x,y,z,t) = \widetilde{\gamma_{xy}}(x,y,z,t) = 2\varepsilon_{xy} + z\kappa_{xy} = \gamma_{xy} + z\kappa_{xy} \qquad (A.2)$$

where

$$\varepsilon_x = u_{,x} + 0.5w_{,x}^2 \qquad \varepsilon_y = v_{,y} + 0.5w_{,y}^2 \qquad 2\varepsilon_{xy} = \gamma_{xy} = u_{,y} + v_{,x} + w_{,x}w_{,y} \qquad (A.3)$$

and

$$\kappa_x = -w_{,xx} \qquad \kappa_y = -w_{,yy} \qquad \kappa_{xy} = -2w_{,xy} \qquad (A.4)$$

The in-plane strains $\varepsilon_x, \varepsilon_y, \gamma_{xy}$ vary linearly through the thickness *h*.

Constitutive equations describing the plate have the form [30,31,33,38]:

$$\begin{Bmatrix} \{N\} \\ \{M\} \end{Bmatrix} = \begin{bmatrix} \boldsymbol{A} & \boldsymbol{B} \\ \boldsymbol{B} & \boldsymbol{D} \end{bmatrix} \begin{Bmatrix} \{\varepsilon\} \\ \{\kappa\} \end{Bmatrix} \qquad (A.5)$$

where the stiffness matrix:

$$\begin{bmatrix} \boldsymbol{A} & \boldsymbol{B} \\ \boldsymbol{B} & \boldsymbol{D} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{21} & A_{22} & A_{26} & B_{21} & B_{22} & B_{26} \\ A_{61} & A_{62} & A_{66} & B_{61} & B_{62} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{21} & B_{22} & B_{26} & D_{21} & D_{22} & D_{26} \\ B_{61} & B_{62} & B_{66} & D_{61} & D_{62} & D_{66} \end{bmatrix} \qquad (A.6)$$

The submatrix **A** is known as extensional stiffness, the submatrix **D** is bending stiffness, and the submatrix **B** is coupling (or interaction) stiffness.

According to Hamilton's principle, out of all possible motion types a system can perform in a time interval (t_0, t_1), conservative systems perform a motion for which the variational operation Ψ reaches a stationary value, i.e.

$$\delta\Psi = \delta\int_{t_0}^{t_1} \Lambda dt = \delta\int_{t_0}^{t_1} (K - \Pi)dt = 0. \qquad (A.7)$$

The quantities in Eqs.(A.7) denote the following: Λ – Lagrange function, *K* – kinetic energy of the system, Π – total potential energy of the system. The total potential energy Π of the *i*-th thin rectangular composite plate can be expressed as:

$$\Pi = U - W = 0.5\int_\Omega (\sigma_x\varepsilon_x + \sigma_y\varepsilon_y + \tau_{xy}\gamma_{xy})\, d\Omega - \{\int_0^b [N_x^0(y)u + N_{xy}^0(y)\, v]dy \big|_{x=0}^{x=l} + \int_0^\ell [N_y^0(x)v + N_{xy}^0(x)u]dx \big|_{y=0}^{y=b}\} \qquad (A.8)$$

where *U* – internal energy of elastic deformation; W – work of external loads; N_x^0, N_y^0, N_{xy}^0 – pre-buckling external load in the plate middle surface, $\Omega = L\, b\, h = S\, h$ was employed in the above relation.

The kinetic energy of a thin inhomogeneous composite plate, considering Eqs.(A.1), is:

$$K = 0.5\int_\Omega \rho(\tilde{u}_{,tt}^2 + \tilde{v}_{,tt}^2 + \tilde{w}_{,tt}^2)d\Omega. \qquad (A.9)$$

The Lagrange function for the entire system is equal to the sum of functions Λ for all components. After grouping the terms containing the same variations and equating each group of terms to zero (due to the mutual independence of the variations), the following equations were obtained:

$\int_{t_0}^{t_1}\int_S \{[N_{x,x}+N_{xy,y}]+(-h\rho_0 u_{,tt}+h^2\rho_1 w_{,xtt})\}\delta u dSdt=\int_{t_0}^{t_1}\int_S X_1\delta u dSdt=0$
$\int_{t_0}^{t_1}\int_S \{[N_{y,y}+N_{xy,x}]+(-h\rho_0 v_{,tt}+h^2\rho_1 w_{,ytt})\}\delta v dSdt=\int_{t_0}^{t_1}\int_S X_2\delta v dSdt=0$ (A.10)
$\int_{t_0}^{t_1}\int_S \{[M_{x,xx}+M_{y,yy}+2M_{xy,xy}+(N_x w_{,x})_{,x}+(N_y w_{,y})_{,y}+(N_{xy}w_{,x})_{,y}+(N_{xy}w_{,y})_{,x}]+$
$[-h\rho_o w_{,tt}-h^2\rho_1(u_{,xtt}+v_{,ytt})+h^3\rho_2(w_{,xxtt}+w_{,yytt})]\}\delta w dSdt=\int_{t_0}^{t_1}\int_S X_3\delta w dSdt=0$
for *x=const*
$\int_{t_0}^{t_1}\int_0^b[N_x-N_x^0(y)]\delta u dydt|_{x=const}=0$
$\int_{t_0}^{t_1}\int_0^b[N_{xy}-N_{xy}^0(y)]\delta v dydt|_{x=const}=0$
$\int_{t_0}^{t_1}\int_0^b M_x\,\delta w_{,x} dydt|_{x=const}=0$ (A.11)
$\int_{t_0}^{t_1}\int_0^b[(M_{x,x}+2M_{xy,y}+N_x w_{,x}+N_{xy}w_{,y})+(h^2\rho_1 u_{,tt}-h^3\rho_2 w_{,xtt})]\delta w dydt|_{x=const}=0$
for *y=const*
$\int_{t_0}^{t_1}\int_0^L[N_y-N_y^0(x)]\delta v dxdt|_{y=const}=0$
$\int_{t_0}^{t_1}\int_0^L[N_{xy}-N_{xy}^0(x)]\delta u dxdt|_{y=const}=0$
$\int_{t_0}^{t_1}\int_0^L M_y\,\delta w_{,y} dxdt|_{y=const}=0$ (A.12)
$\int_{t_0}^{t_1}\int_0^L[(M_{y,y}+2M_{xy,x}+N_y w_{,y}+N_{xy}w_{,x})+(h^2\rho_1 v_{,tt}-h^3\rho_2 w_{,ytt})]\delta w dxdt|_{y=const}=0$
for the plate corner, i.e. for *x=const* and *y=const*
$\int_{t_0}^{t_1} 2M_{xy}\delta w dt|_{x=const}|_{y=const}=0$ (A.13)
If we impose a constraint that at t_0 and t_1 the displacement variations are equal to zero at all points of the structure, then the boundary conditions are automatically satisfied.
In Equations (A.10)-(A.13), the following denotations are used:
$\rho_0=\frac{1}{h}\int_{-h/2}^{h/2}\rho(z)\,\mathrm{dz}$ $\rho_1=\frac{1}{h^2}\int_{-h/2}^{h/2}\rho(z)\,\mathrm{z\,dz}$ $\rho_2=\frac{1}{h^3}\int_{-h/2}^{h/2}\rho(z)\,\mathrm{z^2\,dz}$ (A.14)
For a plate with symmetric transverse nonhomogeneity we have $\rho_1=0$, whereas for a symmetrically homogenous plate the following relationships are satisfied $\rho_0=\rho$, $\rho_1=0$ and $\rho_2=\rho/12$. The system of equations in Eqs.(A.10) is a system of variational equations of motion. In turn, Equations (A.11)-(A.13) describe the boundary conditions for *x=const*, *y=const* and for the plate corner (i.e. *x=const*, *y=const*), respectively.
Differential equations of motion for a multi-ply composite plate, resulting directly from Eqs.(A.10), have the form:
$N_{x,x}+N_{xy,y}+[-h\rho_0 u_{,tt}+h^2\rho_1 w_{,xtt}]=0$
$N_{xy,x}+N_{y,y}+[-h\rho_0 v_{,tt}+h^2\rho_1 w_{,ytt}]=0$
$M_{x,xx}+M_{y,yy}+2M_{xy,xy}+q+(N_x w_{,x})_{,x}+(N_y w_{,y})_{,y}+(N_{xy}w_{,x})_{,y}+(N_{xy}w_{,y})_{,x}+[-h\rho_o w_{,tt}-$
$h^2\rho_1(u_{,xtt}+v_{,ytt})+h^3\rho_2(w_{,xxtt}+w_{,yytt})]=0$ (A.15)
For a plate with symmetric transverse density, with rotational inertia omitted, the equations in Eqs. (A.15) become considerably simplified because $\rho_0=\rho$, $\rho_1=0$ and $\rho_2=0$. A further simplification of the equations in Eqs.(A.15) can be made in a case when the tangential inertial forces parallel to the plate's midplane are not taken into consideration, i.e., the terms $h\rho_0 u_{,tt}$ and $h\rho_0 v_{,tt}$ are omitted respectively in the first two equations in Eqs.(A.15). Such a simplification is possible for short plate structures.
In a natural frequency analysis, i.e. in a linear problem, Equations Eqs.(A.15), according to Eqs.(A.2)-(A.6), have the form:
$A_{11}\frac{\partial^2 u}{\partial x^2}+2A_{16}\frac{\partial^2 u}{\partial x\partial y}+A_{66}\frac{\partial^2 u}{\partial y^2}+A_{16}\frac{\partial^2 v}{\partial x^2}+(A_{12}+A_{66})\frac{\partial^2 v}{\partial x\partial y}+A_{26}\frac{\partial^2 v}{\partial y^2}-B_{11}\frac{\partial^3 w}{\partial x^3}-3B_{16}\frac{\partial^3 w}{\partial x^2\partial y}-$
$(B_{12}+2B_{66})\frac{\partial^3 w}{\partial x\partial y^2}-B_{26}\frac{\partial^3 w}{\partial y^3}+[-h\rho_0 u_{,tt}+h^2\rho_1 w_{,xtt}]=0$
$A_{16}\frac{\partial^2 u}{\partial x^2}+(A_{12}+A_{66})\frac{\partial^2 u}{\partial x\partial y}+A_{26}\frac{\partial^2 u}{\partial y^2}+A_{66}\frac{\partial^2 v}{\partial x^2}+2A_{26}\frac{\partial^2 v}{\partial x\partial y}+A_{22}\frac{\partial^2 v}{\partial y^2}-B_{16}\frac{\partial^3 w}{\partial x^3}-(B_{12}+$
$2B_{66})\frac{\partial^3 w}{\partial x^2\partial y}-3B_{26}\frac{\partial^3 w}{\partial x\partial y^2}-B_{22}\frac{\partial^3 w}{\partial y^3}+[-h\rho_0 v_{,tt}+h^2\rho_1 w_{,ytt}]=0$

$$D_{11}\frac{\partial^4 w}{\partial x^4}+4D_{16}\frac{\partial^4 w}{\partial x^3\partial y}+2(D_{12}+2D_{66})\frac{\partial^4 w}{\partial x^2\partial y^2}+4D_{26}\frac{\partial^4 w}{\partial x\partial y^3}+D_{22}\frac{\partial^4 w}{\partial y^4}-B_{11}\frac{\partial^3 u}{\partial x^3}-3B_{16}\frac{\partial^3 u}{\partial x^2\partial y}-(B_{12}+2B_{66})\frac{\partial^3 u}{\partial x\partial y^2}-B_{26}\frac{\partial^3 u}{\partial y^3}-B_{16}\frac{\partial^3 v}{\partial x^3}-(B_{12}+2B_{66})\frac{\partial^3 v}{\partial x^2\partial y}-3B_{26}\frac{\partial^3 v}{\partial x\partial y^2}-B_{22}\frac{\partial^3 v}{\partial y^3}-N_x\frac{\partial^2 w}{\partial x^2}-2N_{xy}\frac{\partial^2 w}{\partial x\partial y}-N_y\frac{\partial^2 w}{\partial x^2}+[-h\rho_o w_{,tt}-h^2\rho_1(u_{,xtt}+v_{,ytt})+h^3\rho_2(w_{,xxtt}+w_{,yytt})]=0 \tag{A.16}$$