Appendix A. Equations of flexible mechanicsm

*) Equations written for generalized coordinates φ_1:

$$\frac{\partial T}{\partial \varphi_1} = -\mu_1 l_1 \dot{\varphi}_1 \cos(\varphi_1 - \varphi_2)\sum_{k=1}^{N_2} H_k^{(1)} \dot{p}_k^{(1)} - \mu_1 l_1 \dot{\varphi}_1 \dot{\varphi}_2 \sin(\varphi_1 - \varphi_2)\left[\frac{l_2^2}{2} + \sum_{k=1}^{N_2} H_k^{(1)} p_k^{(1)}\right]$$
$$- \mu_1 l_1 \dot{\varphi}_1 \sin(\varphi_1 - \varphi_2)\sum_{i=1}^{N_1} C_i^{(1)} \dot{q}_i^{(1)} + \mu_1 l_1 \dot{\varphi}_1 \dot{\varphi}_2 \cos(\varphi_1 - \varphi_2)\sum_{i=1}^{N_1} C_i^{(1)} q_i^{(1)}$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\varphi}_1}\right) = \left(I_{O_1} + \mu_1 l_1^2 l_2\right)\ddot{\varphi}_1 + \frac{\mu_1 l_1 l_2^2}{2}\ddot{\varphi}_2 \cos(\varphi_1 - \varphi_2) + \mu_1 l_1 \ddot{\varphi}_2 \cos(\varphi_1 - \varphi_2)\sum_{k=1}^{N_2} H_k^{(1)} p_k^{(1)}$$
$$+\mu_1 l_1 \ddot{\varphi}_2 \sin(\varphi_1 - \varphi_2)\sum_{i=1}^{N_1} C_i^{(1)} q_i^{(1)} - \mu_1 l_1 \sin(\varphi_1 - \varphi_2)\sum_{k=1}^{N_2} H_k^{(1)} \ddot{p}_k^{(1)} + \mu_1 l_1 \cos(\varphi_1 - \varphi_2)\sum_{i=1}^{N_1} C_i^{(1)} \ddot{q}_i^{(1)}$$
$$-\frac{\mu_1 l_1 l_2^2}{2}\dot{\varphi}_2(\dot{\varphi}_1 - \dot{\varphi}_2)\sin(\varphi_1 - \varphi_2) - \mu_1 l_1(\dot{\varphi}_1 - \dot{\varphi}_2)\cos(\varphi_1 - \varphi_2)\sum_{k=1}^{N_2} H_k^{(1)} \dot{p}_k^{(1)} + \mu_1 l_1 \dot{\varphi}_2 \cos(\varphi_1 - \varphi_2)\sum_{k=1}^{N_2} H_k^{(1)} \dot{p}_k^{(1)} \quad \frac{\partial \Pi}{\partial \varphi_1} = 0\,;$$
$$-\mu_1 l_1 \dot{\varphi}_2(\dot{\varphi}_1 - \dot{\varphi}_2)\sin(\varphi_1 - \varphi_2)\sum_{k=1}^{N_2} H_k^{(1)} p_k^{(1)} - \mu_1 l_1(\dot{\varphi}_1 - \dot{\varphi}_2)\sin(\varphi_1 - \varphi_2)\sum_{i=1}^{N_1} C_i^{(1)} \dot{q}_i^{(1)}$$
$$+\mu_1 l_1 \dot{\varphi}_2(\dot{\varphi}_1 - \dot{\varphi}_2)\cos(\varphi_1 - \varphi_2)\sum_{i=1}^{N_1} C_i^{(1)} q_i^{(1)} + \mu_1 l_1 \dot{\varphi}_2 \sin(\varphi_1 - \varphi_2)\sum_{i=1}^{N_1} C_i^{(1)} \dot{q}_i^{(1)}$$

$$\lambda_1 \frac{\partial f_1}{\partial \varphi_1} + \lambda_2 \frac{\partial f_2}{\partial \varphi_1} + \lambda_3 \frac{\partial f_3}{\partial \varphi_1} + \lambda_4 \frac{\partial f_4}{\partial \varphi_1} = -l_1 \lambda_1 \sin\varphi_1 + l_1 \lambda_2 \cos\varphi_1\ ;$$

$$Q_{\varphi_1} = \tau - \tau_{f_1}$$

Substitute the above derivatives into equation (2), simplify and we get the equation :

$$\begin{aligned}
&\left(I_{O_1} + \mu_1 l_1^2 l_2\right)\ddot{\varphi}_1 + \frac{\mu_1 l_1 l_2^2}{2}\ddot{\varphi}_2 \cos(\varphi_1 - \varphi_2) + \mu_1 l_1 \ddot{\varphi}_2 \cos(\varphi_1 - \varphi_2)\sum_{k=1}^{N_2} H_k^{(1)} p_k^{(1)} \\
&+\mu_1 l_1 \ddot{\varphi}_2 \sin(\varphi_1 - \varphi_2)\sum_{i=1}^{N_1} C_i^{(1)} q_i^{(1)} - \mu_1 l_1 \sin(\varphi_1 - \varphi_2)\sum_{k=1}^{N_2} H_k^{(1)} \ddot{p}_k^{(1)} + \mu_1 l_1 \cos(\varphi_1 - \varphi_2)\sum_{i=1}^{N_1} C_i^{(1)} \ddot{q}_i^{(1)} \\
&+\frac{\mu_1 l_1 l_2^2}{2}\dot{\varphi}_2^2 \sin(\varphi_1 - \varphi_2) + 2\mu_1 l_1 \dot{\varphi}_2 \cos(\varphi_1 - \varphi_2)\sum_{k=1}^{N_2} H_k^{(1)} \dot{p}_k^{(1)} + \mu_1 l_1 \dot{\varphi}_2^2 \sin(\varphi_1 - \varphi_2)\sum_{k=1}^{N_2} H_k^{(1)} p_k^{(1)} \\
&+2\mu_1 l_1 \dot{\varphi}_2 \sin(\varphi_1 - \varphi_2)\sum_{i=1}^{N_1} C_i^{(1)} \dot{q}_i^{(1)} - \mu_1 l_1 \dot{\varphi}_2^2 \cos(\varphi_1 - \varphi_2)\sum_{i=1}^{N_1} C_i^{(1)} q_i^{(1)} = l_1 \sin\varphi_1 \lambda_1 - l_1 \cos\varphi_1 \lambda_2 + \tau
\end{aligned} \tag{1}$$

*) Equation written for generalized coordinates φ_2, we have :

$$\frac{\partial T}{\partial \varphi_2} = \mu_1 l_1 \dot{\varphi}_1 \cos(\varphi_1 - \varphi_2)\sum_{k=1}^{N_2} H_k^{(1)} \dot{p}_k^{(1)} + \mu_1 l_1 \dot{\varphi}_1 \dot{\varphi}_2 \sin(\varphi_1 - \varphi_2)\left[\frac{l_2^2}{2} + \sum_{k=1}^{N_2} H_k^{(1)} p_k^{(1)}\right]$$
$$+ \mu_1 l_1 \dot{\varphi}_1 \sin(\varphi_1 - \varphi_2)\sum_{i=1}^{N_1} C_i^{(1)} \dot{q}_i^{(1)} - \mu_1 l_1 \dot{\varphi}_1 \dot{\varphi}_2 \cos(\varphi_1 - \varphi_2)\sum_{i=1}^{N_1} C_i^{(1)} q_i^{(1)}$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial\dot{\varphi}_2}\right)=\frac{\mu_1 l_1 l_2^2}{2}\ddot{\varphi}_1\cos(\varphi_1-\varphi_2)+\mu_1 l_1\ddot{\varphi}_1\cos(\varphi_1-\varphi_2)\sum_{k=1}^{N_2}H_k^{(1)}p_k^{(1)}+\mu_1 l_1\ddot{\varphi}_1\sin(\varphi_1-\varphi_2)\sum_{i=1}^{N_1}C_i^{(1)}q_i^{(1)}$$
$$+\frac{\mu_1 l_2^3}{3}\ddot{\varphi}_2+\mu_1\ddot{\varphi}_2\sum_{i=1}^{N_1}\sum_{j=1}^{N_1}m_{ij}^{(1)}q_i^{(1)}q_j^{(1)}+\mu_1\sum_{i=1}^{N_1}D_i^{(1)}\ddot{q}_i^{(1)}+\mu_1\sum_{i=1}^{N_1}\sum_{k=1}^{N_2}n_{ik}^{(1)}\,p_k^{(1)}\ddot{q}_i^{(1)}$$
$$+2\mu_1\dot{\varphi}_2\sum_{i=1}^{N_1}\sum_{j=1}^{N_1}m_{ij}^{(1)}\dot{q}_i^{(1)}q_j^{(1)}+\mu_1\ddot{\varphi}_2\left(2\sum_{k=1}^{N_2}F_k^{(1)}p_k^{(1)}+\sum_{k=1}^{N_2}\sum_{l=1}^{N_2}b_{kl}^{(1)}p_k^{(1)}p_l^{(1)}\right)$$
$$+2\mu_1\dot{\varphi}_2\left(\sum_{k=1}^{N_2}F_k^{(1)}\dot{p}_k^{(1)}+\sum_{k=1}^{N_2}\sum_{l=1}^{N_2}b_{kl}^{(1)}\dot{p}_k^{(1)}p_l^{(1)}\right)-\frac{\mu_1 l_1 l_2^2}{2}\dot{\varphi}_1(\dot{\varphi}_1-\dot{\varphi}_2)\sin(\varphi_1-\varphi_2)+\mu_1 l_1\dot{\varphi}_1\cos(\varphi_1-\varphi_2)\sum_{k=1}^{N_2}H_k^{(1)}\dot{p}_k^{(1)}$$
$$-\mu_1 l_1\dot{\varphi}_1(\dot{\varphi}_1-\dot{\varphi}_2)\sin(\varphi_1-\varphi_2)\sum_{k=1}^{N_2}H_k^{(1)}p_k^{(1)}+\mu_1 l_1\dot{\varphi}_1(\dot{\varphi}_1-\dot{\varphi}_2)\cos(\varphi_1-\varphi_2)\sum_{i=1}^{N_1}C_i^{(1)}q_i^{(1)}$$
$$+\mu_1 l_1\dot{\varphi}_1\sin(\varphi_1-\varphi_2)\sum_{i=1}^{N_1}C_i^{(1)}\dot{q}_i^{(1)}-\mu_1\sum_{i=1}^{N_1}\sum_{k=1}^{N_2}n_{ik}^{(1)}\dot{q}_i^{(1)}\dot{p}_k^{(1)}-\mu_1\sum_{i=1}^{N_1}\sum_{k=1}^{N_2}n_{ik}^{(1)}q_i^{(1)}\ddot{p}_k^{(1)}+\mu_1\sum_{i=1}^{N_1}\sum_{k=1}^{N_2}n_{ik}^{(1)}\dot{p}_k^{(1)}\dot{q}_i^{(1)}$$

$$\frac{\partial T}{\partial\dot{\varphi}_2}=\frac{\mu_1 l_2^3}{3}\dot{\varphi}_2+\mu_1\dot{\varphi}_2\sum_{i=1}^{N_1}\sum_{j=1}^{N_1}m_{ij}^{(1)}q_i^{(1)}q_j^{(1)}+\mu_1\left(2\sum_{k=1}^{N_2}F_k^{(1)}p_k^{(1)}+\sum_{k=1}^{N_2}\sum_{l=1}^{N_2}b_{kl}^{(1)}p_k^{(1)}p_l^{(1)}\right)\dot{\varphi}_2+\frac{\mu_1 l_1 l_2^2}{2}\dot{\varphi}_1\cos(\varphi_1-\varphi_2)$$
$$+\mu_1 l_1\dot{\varphi}_1\cos(\varphi_1-\varphi_2)\sum_{k=1}^{N_2}H_k^{(1)}p_k^{(1)}+\mu_1 l_1\dot{\varphi}_1\sin(\varphi_1-\varphi_2)\sum_{i=1}^{N_1}C_i^{(1)}q_i^{(1)}-\mu_1\sum_{i=1}^{N_1}\sum_{k=1}^{N_2}n_{ik}^{(1)}q_i^{(1)}\dot{p}_k^{(1)}\qquad\frac{\partial\Pi}{\partial\varphi_2}=0;$$
$$+\mu_1\sum_{i=1}^{N_1}D_i^{(1)}\dot{q}_i^{(1)}+\mu_1\sum_{i=1}^{N_1}\sum_{k=1}^{N_2}n_{ik}^{(1)}\,p_k^{(1)}\dot{q}_i^{(1)}$$
$$\lambda_1\frac{\partial f_1}{\partial\varphi_2}+\lambda_2\frac{\partial f_2}{\partial\varphi_2}+\lambda_3\frac{\partial f_3}{\partial\varphi_2}+\lambda_4\frac{\partial f_4}{\partial\varphi_2}=-(l_2+u_{1B})\sin\varphi_2\lambda_1+(l_2+u_{1B})\cos\varphi_2\lambda_2$$

From (12) and (15) we have:

$$u_{1B}=u_1(l_2,t)=\sum_{k=1}^{N_2}p_k^{(1)}\sin\left(\frac{2k-1}{2}\pi\right)\qquad(2)$$

$$Q_{\varphi_2}=-\tau_{f_2}$$

Substitute the derivatives into (2), simplify to get the equation :

$$\frac{\mu_1 l_1 l_2^2}{2}\ddot{\varphi}_1\cos(\varphi_1-\varphi_2)+\mu_1 l_1\ddot{\varphi}_1\cos(\varphi_1-\varphi_2)\sum_{k=1}^{N_2}H_k^{(1)}p_k^{(1)}+\mu_1 l_1\ddot{\varphi}_1\sin(\varphi_1-\varphi_2)\sum_{i=1}^{N_1}C_i^{(1)}q_i^{(1)}\varphi_3=\varphi_3^*+\beta$$
$$+\frac{\mu_1 l_2^3}{3}\ddot{\varphi}_2+\mu_1\ddot{\varphi}_2\sum_{i=1}^{N_1}\sum_{j=1}^{N_1}m_{ij}^{(1)}q_i^{(1)}q_j^{(1)}+\mu_1\ddot{\varphi}_2\left(2\sum_{k=1}^{N_2}F_k^{(1)}p_k^{(1)}+\sum_{k=1}^{N_2}\sum_{l=1}^{N_2}b_{kl}^{(1)}p_k^{(1)}p_l^{(1)}\right)+\mu_1\sum_{i=1}^{N_1}D_i^{(1)}\ddot{q}_i^{(1)}$$
$$+\mu_1\sum_{i=1}^{N_1}\sum_{k=1}^{N_2}n_{ik}^{(1)}\,p_k^{(1)}\ddot{q}_i^{(1)}-\mu_1\sum_{i=1}^{N_1}\sum_{k=1}^{N_2}n_{ik}^{(1)}q_i^{(1)}\ddot{p}_k^{(1)}+2\mu_1\dot{\varphi}_2\sum_{i=1}^{N_1}\sum_{j=1}^{N_1}m_{ij}^{(1)}\dot{q}_i^{(1)}q_j^{(1)}\qquad(3)$$
$$+2\mu_1\dot{\varphi}_2\left(\sum_{k=1}^{N_2}F_k^{(1)}\dot{p}_k^{(1)}+\sum_{k=1}^{N_2}\sum_{l=1}^{N_2}b_{kl}^{(1)}\dot{p}_k^{(1)}p_l^{(1)}\right)-\frac{\mu_1 l_1 l_2^2}{2}\dot{\varphi}_1^2\sin(\varphi_1-\varphi_2)-\mu_1 l_1\dot{\varphi}_1^2\sin(\varphi_1-\varphi_2)\sum_{k=1}^{N_2}H_k^{(1)}p_k^{(1)}$$
$$+\mu_1 l_1\dot{\varphi}_1^2\cos(\varphi_1-\varphi_2)\sum_{i=1}^{N_1}C_i^{(1)}q_i^{(1)}=(l_2+u_{1B})\sin\varphi_2\lambda_1-(l_2+u_{1B})\cos\varphi_2\lambda_2$$

*) Equation written for generalized coordinates φ_3^*:

with attention , infer $\dot{\varphi}_3=\dot{\varphi}_3^*,\ddot{\varphi}_3=\ddot{\varphi}_3^*$, instead of writing to φ_3 I write for φ_3^*:

$$\frac{\partial T}{\partial\varphi_3^*}=-\mu_2 l_3^*\dot{\varphi}_3^*\cos(\varphi_3^*-\varphi_4)\sum_{k=1}^{N_4}H_k^{(2)}\dot{p}_k^{(2)}-\mu_2 l_3^*\dot{\varphi}_3^*\dot{\varphi}_4\sin(\varphi_3^*-\varphi_4)\left[\frac{l_4^2}{2}+\sum_{k=1}^{N_4}H_k^{(2)}p_k^{(2)}\right]$$
$$-\mu_2 l_3^*\dot{\varphi}_3^*\sin(\varphi_3^*-\varphi_4)\sum_{i=1}^{N_3}C_i^{(2)}\dot{q}_i^{(2)}+\mu_2 l_3^*\dot{\varphi}_3^*\dot{\varphi}_4\cos(\varphi_3^*-\varphi_4)\sum_{i=1}^{N_3}C_i^{(2)}q_i^{(2)}$$

$$\frac{\partial T}{\partial\dot{\varphi}_3^*}=I_{O_2}\dot{\varphi}_3^*+\mu_2 l_3^{*2}l_4\dot{\varphi}_3^*-\mu_2 l_3^*\sin(\varphi_3^*-\varphi_4)\sum_{k=1}^{N_4}H_k^{(2)}\dot{p}_k^{(2)}+\mu_2 l_3^*\dot{\varphi}_4\cos(\varphi_3^*-\varphi_4)\left(\frac{l_4^2}{2}+\sum_{k=1}^{N_4}H_k^{(2)}p_k^{(2)}\right)$$
$$+\mu_2 l_3^*\cos(\varphi_3^*-\varphi_4)\sum_{i=1}^{N_3}C_i^{(2)}\dot{q}_i^{(2)}+\mu_2 l_3^*\dot{\varphi}_4\sin(\varphi_3^*-\varphi_4)\sum_{i=1}^{N_3}C_i^{(2)}q_i^{(2)}$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\varphi}_3^*}\right)=\left(I_{O_2}+\mu_2 l_3^{*2} l_4\right)\ddot{\varphi}_3^*-\mu_2 l_3^*\left(\dot{\varphi}_3^*-\dot{\varphi}_4\right)\cos\left(\varphi_3^*-\varphi_4\right)\sum_{k=1}^{N_4}H_k^{(2)}\dot{p}_k^{(2)}-\mu_2 l_3^*\sin\left(\varphi_3^*-\varphi_4\right)\sum_{k=1}^{N_4}H_k^{(2)}\ddot{p}_k^{(2)}$$
$$+\mu_2 l_3^*\ddot{\varphi}_4\cos\left(\varphi_3^*-\varphi_4\right)\left(\frac{l_4^2}{2}+\sum_{k=1}^{N_4}H_k^{(2)}p_k^{(2)}\right)-\mu_2 l_3^*\dot{\varphi}_4\left(\dot{\varphi}_3^*-\dot{\varphi}_4\right)\sin\left(\varphi_3^*-\varphi_4\right)\left[\frac{l_4^2}{2}+\sum_{k=1}^{N_4}H_k^{(2)}p_k^{(2)}\right]$$
$$+\mu_2 l_3^*\dot{\varphi}_4\cos\left(\varphi_3^*-\varphi_4\right)\sum_{k=1}^{N_4}H_k^{(2)}\dot{p}_k^{(2)}+\mu_2 l_3^*\cos\left(\varphi_3^*-\varphi_4\right)\sum_{i=1}^{N_3}C_i^{(2)}\ddot{q}_i^{(2)}-\mu_2 l_3^*\left(\dot{\varphi}_3^*-\dot{\varphi}_4\right)\sin\left(\varphi_3^*-\varphi_4\right)\sum_{i=1}^{N_3}C_i^{(2)}\dot{q}_i^{(2)}$$
$$+\mu_2 l_3^*\ddot{\varphi}_4\sin\left(\varphi_3^*-\varphi_4\right)\sum_{i=1}^{N_3}C_i^{(2)}q_i^{(2)}+\mu_2 l_3^*\dot{\varphi}_4\left(\dot{\varphi}_3^*-\dot{\varphi}_4\right)\cos\left(\varphi_3^*-\varphi_4\right)\sum_{i=1}^{N_3}C_i^{(2)}q_i^{(2)}+\mu_2 l_3^*\dot{\varphi}_4\sin\left(\varphi_3^*-\varphi_4\right)\sum_{i=1}^{N_3}C_i^{(2)}\dot{q}_i^{(2)}$$

$$\frac{\partial \Pi}{\partial \varphi_3^*}=0\,;\; Q_{\varphi_3^*}=-\tau_{f_3}$$

$$\lambda_1\frac{\partial f_1}{\partial \varphi_3^*}+\lambda_2\frac{\partial f_2}{\partial \varphi_3^*}+\lambda_3\frac{\partial f_3}{\partial \varphi_3^*}+\lambda_4\frac{\partial f_4}{\partial \varphi_3^*}=l_3\lambda_1\sin\left(\varphi_3^*+\beta\right)-l_3\lambda_2\cos\left(\varphi_3^*+\beta\right)-l_3^*\lambda_3\sin\varphi_3^*+l_3^*\lambda_4\cos\varphi_3^*$$

Substitute into (2) we get the third equation:

$$\left(I_{O_2}+\mu_2 l_3^{*2} l_4\right)\ddot{\varphi}_3^*+\mu_2 l_3^*\ddot{\varphi}_4\cos\left(\varphi_3^*-\varphi_4\right)\left(\frac{l_4^2}{2}+\sum_{k=1}^{N_4}H_k^{(2)}p_k^{(2)}\right)+\mu_2 l_3^*\ddot{\varphi}_4\sin\left(\varphi_3^*-\varphi_4\right)\sum_{i=1}^{N_3}C_i^{(2)}q_i^{(2)}$$
$$-\mu_2 l_3^*\sin\left(\varphi_3^*-\varphi_4\right)\sum_{k=1}^{N_4}H_k^{(2)}\ddot{p}_k^{(2)}+\mu_2 l_3^*\cos\left(\varphi_3^*-\varphi_4\right)\sum_{i=1}^{N_3}C_i^{(2)}\ddot{q}_i^{(2)}+2\mu_2 l_3^*\dot{\varphi}_4\cos\left(\varphi_3^*-\varphi_4\right)\sum_{k=1}^{N_4}H_k^{(2)}\dot{p}_k^{(2)}$$
$$+\mu_2 l_3^*\dot{\varphi}_4^2\sin\left(\varphi_3^*-\varphi_4\right)\left[\frac{l_4^2}{2}+\sum_{k=1}^{N_4}H_k^{(2)}p_k^{(2)}\right]+2\mu_2 l_3^*\dot{\varphi}_4\sin\left(\varphi_3^*-\varphi_4\right)\sum_{i=1}^{N_3}C_i^{(2)}\dot{q}_i^{(2)} \quad (4)$$
$$-\mu_2 l_3^*\dot{\varphi}_4^2\cos\left(\varphi_3^*-\varphi_4\right)\sum_{i=1}^{N_3}C_i^{(2)}q_i^{(2)}=-l_3\lambda_1\sin\left(\varphi_3^*+\beta\right)+l_3\lambda_2\cos\left(\varphi_3^*+\beta\right)+l_3^*\lambda_3\sin\varphi_3^*-l_3^*\lambda_4\cos\varphi_3^*-\tau_{f_3}$$

*) Equations written for generalized coordinates φ_4 :

$$\frac{\partial T}{\partial \dot{\varphi}_4}=\mu_2\dot{\varphi}_4\sum_{i=1}^{N_3}\sum_{j=1}^{N_3}m_{ij}^{(2)}q_i^{(2)}q_j^{(2)}+\mu_2\dot{\varphi}_4\left[\frac{l_4^3}{3}+2\sum_{k=1}^{N_4}F_k^{(2)}p_k^{(2)}+\sum_{k=1}^{N_4}\sum_{l=1}^{N_4}b_{kl}^{(2)}p_k^{(2)}p_l^{(2)}\right]$$
$$+\mu_2 l_3^*\dot{\varphi}_3^*\cos\left(\varphi_3^*-\varphi_4\right)\left[\frac{l_4^2}{2}+\sum_{k=1}^{N_4}H_k^{(2)}p_k^{(2)}\right]+\mu_2 l_3^*\dot{\varphi}_3^*\sin\left(\varphi_3^*-\varphi_4\right)\sum_{i=1}^{N_3}C_i^{(2)}q_i^{(2)}$$
$$-\mu_2\sum_{i=1}^{N_3}\sum_{k=1}^{N_4}n_{ik}^{(2)}q_i^{(2)}\dot{p}_k^{(2)}+\mu_2\left[\sum_{i=1}^{N_3}D_i^{(2)}\dot{q}_i^{(2)}+\sum_{i=1}^{N_3}\sum_{k=1}^{N_4}n_{ik}^{(2)}p_k^{(2)}\dot{q}_i^{(2)}\right]$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\varphi}_4}\right)=\mu_2\ddot{\varphi}_4\sum_{i=1}^{N_3}\sum_{j=1}^{N_3}m_{ij}^{(2)}q_i^{(2)}q_j^{(2)}+2\mu_2\dot{\varphi}_4\sum_{i=1}^{N_3}\sum_{j=1}^{N_3}m_{ij}^{(2)}\dot{q}_i^{(2)}q_j^{(2)}+\mu_2\ddot{\varphi}_4\left[\frac{l_4^3}{3}+2\sum_{k=1}^{N_4}F_k^{(2)}p_k^{(2)}+\sum_{k=1}^{N_4}\sum_{l=1}^{N_4}b_{kl}^{(2)}p_k^{(2)}p_l^{(2)}\right]$$
$$+2\mu_2\dot{\varphi}_4\left[\sum_{k=1}^{N_4}F_k^{(2)}\dot{p}_k^{(2)}+\sum_{k=1}^{N_4}\sum_{l=1}^{N_4}b_{kl}^{(2)}\dot{p}_k^{(2)}p_l^{(2)}\right]+\mu_2 l_3^*\ddot{\varphi}_3^*\cos\left(\varphi_3^*-\varphi_4\right)\left[\frac{l_4^2}{2}+\sum_{k=1}^{N_4}H_k^{(2)}p_k^{(2)}\right]$$
$$-\mu_2 l_3^*\dot{\varphi}_3^*\left(\dot{\varphi}_3^*-\dot{\varphi}_4\right)\sin\left(\varphi_3^*-\varphi_4\right)\left[\frac{l_4^2}{2}+\sum_{k=1}^{N_4}H_k^{(2)}p_k^{(2)}\right]+\mu_2 l_3^*\dot{\varphi}_3^*\cos\left(\varphi_3^*-\varphi_4\right)\sum_{k=1}^{N_4}H_k^{(2)}\dot{p}_k^{(2)}$$
$$+\mu_2 l_3^*\ddot{\varphi}_3^*\sin\left(\varphi_3^*-\varphi_4\right)\sum_{i=1}^{N_3}C_i^{(2)}q_i^{(2)}+\mu_2 l_3^*\dot{\varphi}_3^*\left(\dot{\varphi}_3^*-\dot{\varphi}_4\right)\cos\left(\varphi_3^*-\varphi_4\right)\sum_{i=1}^{N_3}C_i^{(2)}q_i^{(2)}+\mu_2 l_3^*\dot{\varphi}_3^*\sin\left(\varphi_3^*-\varphi_4\right)\sum_{i=1}^{N_3}C_i^{(2)}\dot{q}_i^{(2)}$$
$$-\mu_2\sum_{i=1}^{N_3}\sum_{k=1}^{N_4}n_{ik}^{(2)}q_i^{(2)}\ddot{p}_k^{(2)}-\mu_2\sum_{i=1}^{N_3}\sum_{k=1}^{N_4}n_{ik}^{(2)}\dot{q}_i^{(2)}\dot{p}_k^{(2)}+\mu_2\left[\sum_{i=1}^{N_3}D_i^{(2)}\ddot{q}_i^{(2)}+\sum_{i=1}^{N_3}\sum_{k=1}^{N_4}n_{ik}^{(2)}p_k^{(2)}\ddot{q}_i^{(2)}\right]+\mu_2\sum_{i=1}^{N_3}\sum_{k=1}^{N_4}n_{ik}^{(2)}\dot{p}_k^{(2)}\dot{q}_i^{(2)}$$

$$\frac{\partial T}{\partial \varphi_4}=\mu_2 l_3^*\dot{\varphi}_3^*\cos\left(\varphi_3^*-\varphi_4\right)\sum_{k=1}^{N_4}H_k^{(2)}\dot{p}_k^{(2)}+\mu_2 l_3^*\dot{\varphi}_3^*\dot{\varphi}_4\sin\left(\varphi_3^*-\varphi_4\right)\left[\frac{l_4^2}{2}+\sum_{k=1}^{N_4}H_k^{(2)}p_k^{(2)}\right]$$
$$+\mu_2 l_3^*\dot{\varphi}_3^*\sin\left(\varphi_3^*-\varphi_4\right)\sum_{i=1}^{N_3}C_i^{(2)}\dot{q}_i^{(2)}-\mu_2 l_3^*\dot{\varphi}_3^*\dot{\varphi}_4\cos\left(\varphi_3^*-\varphi_4\right)\sum_{i=1}^{N_3}C_i^{(2)}q_i^{(2)}$$

$$\frac{\partial \Pi}{\partial \varphi_4}=0\,;\;\lambda_1\frac{\partial f_1}{\partial \varphi_4}+\lambda_2\frac{\partial f_2}{\partial \varphi_4}+\lambda_3\frac{\partial f_3}{\partial \varphi_4}+\lambda_4\frac{\partial f_4}{\partial \varphi_4}=-\left(l_4+u_{2D}\right)\lambda_3\sin\varphi_4+\left(l_4+u_{2D}\right)\lambda_4\cos\varphi_4$$

where : $u_{2D}=u_2(l_4,t)=\sum_{k=1}^{N_4}p_k^{(2)}\sin\left(\frac{2k-1}{2}\pi\right)$ (5)

$$Q_{\varphi_4}=0$$

Substitute the derivatives into equation (2), simplify and we get the equation:

$$\mu_2 l_3^* \ddot{\varphi}_3^* \cos\left(\varphi_3^* - \varphi_4\right)\left(\frac{l_4^2}{2} + \sum_{k=1}^{N_4} H_k^{(2)} p_k^{(2)}\right) + \mu_2 l_3^* \ddot{\varphi}_3^* \sin\left(\varphi_3^* - \varphi_4\right)\sum_{i=1}^{N_3} C_i^{(2)} q_i^{(2)} + \mu_2 \ddot{\varphi}_4 \sum_{i=1}^{N_3}\sum_{j=1}^{N_3} m_{ij}^{(2)} q_i^{(2)} q_j^{(2)}$$
$$+\mu_2 \ddot{\varphi}_4 \left(\frac{l_4^3}{3} + 2\sum_{k=1}^{N_4} F_k^{(2)} p_k^{(2)} + \sum_{k=1}^{N_4}\sum_{l=1}^{N_4} b_{kl}^{(2)} p_k^{(2)} p_l^{(2)}\right) - \mu_2 \sum_{i=1}^{N_3}\sum_{k=1}^{N_4} n_{ik}^{(2)} q_i^{(2)} \ddot{p}_k^{(2)} + \mu_2\left(\sum_{i=1}^{N_3} D_i^{(2)} \ddot{q}_i^{(2)} + \sum_{i=1}^{N_3}\sum_{k=1}^{N_4} n_{ik}^{(2)} p_k^{(2)} \ddot{q}_i^{(2)}\right) \quad (6)$$
$$+2\mu_2 \dot{\varphi}_4 \sum_{i=1}^{N_3}\sum_{j=1}^{N_3} m_{ij}^{(2)} \dot{q}_i^{(2)} q_j^{(2)} + 2\mu_2 \dot{\varphi}_4 \left(\sum_{k=1}^{N_4} F_k^{(2)} \dot{p}_k^{(2)} + \sum_{k=1}^{N_4}\sum_{l=1}^{N_4} b_{kl}^{(2)} \dot{p}_k^{(2)} p_l^{(2)}\right) - \mu_2 l_3^* \dot{\varphi}_3^{*2} \sin\left(\varphi_3^* - \varphi_4\right)\left[\frac{l_4^2}{2} + \sum_{k=1}^{N_4} H_k^{(2)} p_k^{(2)}\right]$$
$$+\mu_2 l_3^* \dot{\varphi}_3^{*2} \cos\left(\varphi_3^* - \varphi_4\right)\sum_{i=1}^{N_3} C_i^{(2)} q_i^{(2)} = \left(l_4 + u_{2D}\right)\lambda_3 \sin\varphi_4 - \left(l_4 + u_{2D}\right)\lambda_4 \cos\varphi_4$$

*) Equations written for generalized coordinates φ_5:

$$\frac{\partial T}{\partial \dot{\varphi}_5} = I_{O_3}\dot{\varphi}_5 \Rightarrow \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\varphi}_5}\right) = I_{O_3}\ddot{\varphi}_5 ;\ \frac{\partial T}{\partial \varphi_5} = 0 ;\ \frac{\partial \Pi}{\partial \varphi_5} = 0$$

$$\lambda_1 \frac{\partial f_1}{\partial \varphi_5} + \lambda_2 \frac{\partial f_2}{\partial \varphi_5} + \lambda_3 \frac{\partial f_3}{\partial \varphi_5} + \lambda_4 \frac{\partial f_4}{\partial \varphi_5} = \lambda_1 l_5 \sin\varphi_5 - \lambda_2 l_5 \cos\varphi_5 ;\ Q_{\varphi_5} = -\tau_{f_5}$$

Substitute into (2), we get the 5th equation:

$$I_{O_3}\ddot{\varphi}_5 + l_5 \sin\left(\varphi_5\right)\lambda_1 - l_5\cos\left(\varphi_5\right)\lambda_2 = -\tau_{f_5} \quad (7)$$

*) Equations written for generalized coordinates $q_i^{(1)}$ (i = 1, 2, ... N_1):

$$\frac{\partial T}{\partial \dot{q}_i^{(1)}} = \mu_1 \sum_{j=1}^{N_1} m_{ij}^{(1)} \dot{q}_j^{(1)} + \mu_1 l_1 \dot{\varphi}_1 \cos\left(\varphi_1 - \varphi_2\right) C_i^{(1)} + \mu_1 \dot{\varphi}_2 \left(D_i^{(1)} + \sum_{k=1}^{N_2} n_{ik}^{(1)} p_k^{(1)}\right)$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i^{(1)}}\right) = \mu_1 \sum_{j=1}^{N_1} m_{ij}^{(1)} \ddot{q}_j^{(1)} + \mu_1 l_1 \ddot{\varphi}_1 \cos\left(\varphi_1 - \varphi_2\right) C_i^{(1)} - \mu_1 l_1 \dot{\varphi}_1 \left(\dot{\varphi}_1 - \dot{\varphi}_2\right)\sin\left(\varphi_1 - \varphi_2\right) C_i^{(1)}$$
$$+ \mu_1 \ddot{\varphi}_2 \left(D_i^{(1)} + \sum_{k=1}^{N_2} n_{ik}^{(1)} p_k^{(1)}\right) + \mu_1 \dot{\varphi}_2 \sum_{k=1}^{N_2} n_{ik}^{(1)} \dot{p}_k^{(1)}$$

$$\frac{\partial T}{\partial q_i^{(1)}} = \mu_1 \dot{\varphi}_2^2 \sum_{j=1}^{N_1} m_{ij}^{(1)} q_j^{(1)} + \mu_1 l_1 \dot{\varphi}_1 \dot{\varphi}_2 \sin\left(\varphi_1 - \varphi_2\right) C_i^{(1)} - \mu_1 \dot{\varphi}_2 \sum_{k=1}^{N_2} n_{ik}^{(1)} \dot{p}_k^{(1)}$$

$$\frac{\partial \Pi}{\partial q_i^{(1)}} = E_1 I_1 \sum_{j=1}^{N} k_{ij}^{(1)} q_j^{(1)} ;\ \lambda_1 \frac{\partial f_1}{\partial q_i^{(1)}} + \lambda_2 \frac{\partial f_2}{\partial q_i^{(1)}} + \lambda_3 \frac{\partial f_3}{\partial q_i^{(1)}} + \lambda_4 \frac{\partial f_4}{\partial q_i^{(1)}} = 0 ;\ Q_{q_i^{(1)}} = 0$$

Substitute into equation (2), simplify and we get equation:

$$\mu_1 l_1 \ddot{\varphi}_1 \cos\left(\varphi_1 - \varphi_2\right) C_i^{(1)} + \mu_1 \ddot{\varphi}_2 \left(D_i^{(1)} + \sum_{k=1}^{N_2} n_{ik}^{(1)} p_k^{(1)}\right) + \mu_1 \sum_{j=1}^{N_1} m_{ij}^{(1)} \ddot{q}_j^{(1)}$$
$$-\mu_1 \dot{\varphi}_2^2 \sum_{j=1}^{N_1} m_{ij}^{(1)} q_j^{(1)} - \mu_1 l_1 \dot{\varphi}_1^2 \sin\left(\varphi_1 - \varphi_2\right) C_i^{(1)} + 2\mu_1 \dot{\varphi}_2 \sum_{k=1}^{N_2} n_{ik}^{(1)} \dot{p}_k^{(1)} + E_1 I_1 \sum_{j=1}^{N} k_{ij}^{(1)} q_j^{(1)} = 0 \quad (8)$$

*) Equations written for generalized coordinates $p_k^{(1)}$ (i = 1, 2, ... N_2):

$$\frac{\partial T}{\partial \dot{p}_k^{(1)}} = \mu_1 \sum_{l=1}^{N_2} b_{kl}^{(1)} \dot{p}_l^{(1)} - \mu_1 l_1 \dot{\varphi}_1 \sin\left(\varphi_1 - \varphi_2\right) H_k^{(1)} - \mu_1 \dot{\varphi}_2 \sum_{k=1}^{N_2} n_{ik}^{(1)} q_i^{(1)}$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{p}_k^{(1)}}\right) = \mu_1 \sum_{l=1}^{N_2} b_{kl}^{(1)} \ddot{p}_l^{(1)} - \mu_1 l_1 \ddot{\varphi}_1 \sin\left(\varphi_1 - \varphi_2\right) H_k^{(1)} - \mu_1 l_1 \dot{\varphi}_1 \left(\dot{\varphi}_1 - \dot{\varphi}_2\right)\cos\left(\varphi_1 - \varphi_2\right) H_k^{(1)}$$
$$- \mu_1 \ddot{\varphi}_2 \sum_{k=1}^{N_2} n_{ik}^{(1)} q_i^{(1)} - \mu_1 \dot{\varphi}_2 \sum_{k=1}^{N_2} n_{ik}^{(1)} \dot{q}_i^{(1)}$$

$$\frac{\partial T}{\partial p_k^{(1)}} = \mu_1 \left[F_k^{(1)} + \sum_{l=1}^{N_2} b_{kl}^{(1)} p_l^{(1)}\right]\dot{\varphi}_2^2 + \mu_1 l_1 \dot{\varphi}_1 \dot{\varphi}_2 \cos\left(\varphi_1 - \varphi_2\right) H_k^{(1)} + \mu_1 \dot{\varphi}_2 \sum_{i=1}^{N_1} n_{ik}^{(1)} \dot{q}_i^{(1)}$$

$$\frac{\partial \Pi}{\partial p_k^{(1)}} = E_1 A_1 \sum_{l=1}^{N_2} g_{kl}^{(1)} p_l^{(1)}$$

$$\lambda_1 \frac{\partial f_1}{\partial p_k^{(1)}} + \lambda_2 \frac{\partial f_2}{\partial p_k^{(1)}} + \lambda_3 \frac{\partial f_3}{\partial p_k^{(1)}} + \lambda_4 \frac{\partial f_4}{\partial p_k^{(1)}} = \lambda_1 \frac{\partial u_{1B}}{\partial p_k^{(1)}} \cos\varphi_2 + \lambda_2 \frac{\partial u_{1B}}{\partial p_k^{(1)}} \sin\varphi_2$$
$$= \lambda_1 \sin\left(\frac{2k-1}{2}\pi\right)\cos\varphi_2 + \lambda_2 \sin\left(\frac{2k-1}{2}\pi\right)\sin\varphi_2 = \left(\lambda_1\cos\varphi_2 + \lambda_2\sin\varphi_2\right)\alpha_k$$

where : $\alpha_k = \sin\left(\frac{2k-1}{2}\pi\right) = \begin{cases} 1 & khi\ \ k = 2j+1,\ j = 1,2,... \\ -1 & khi\ \ k = 2j,\ j = 1,2,... \end{cases}$

$$Q_{p_k^{(1)}} = 0$$

Substituting into equation (2), we get the equation:

$$\begin{aligned} &-\mu_1 l_1 \ddot{\varphi}_1 \sin(\varphi_1 - \varphi_2) H_k^{(1)} - \mu_1 \ddot{\varphi}_2 \sum_{k=1}^{N_2} n_{ik}^{(1)} q_i^{(1)} + \mu_1 \sum_{l=1}^{N_2} b_{kl}^{(1)} \ddot{p}_l^{(1)} - \mu_1 l_1 \dot{\varphi}_1^2 \cos(\varphi_1 - \varphi_2) H_k^{(1)} \\ &-2\mu_1 \dot{\varphi}_2 \sum_{k=1}^{N_2} n_{ik}^{(1)} \dot{q}_i^{(1)} - \mu_1 \left(F_k^{(1)} + \sum_{l=1}^{N_2} b_{kl}^{(1)} p_l^{(1)} \right) \dot{\varphi}_2^2 + E_1 A_1 \sum_{l=1}^{N_2} g_{kl}^{(1)} p_l^{(1)} + \left(\lambda_1 \cos\varphi_2 + \lambda_2 \sin\varphi_2\right)\alpha_k = 0 \end{aligned} \tag{9}$$

*) Equations written for generalized coordinates $q_i^{(2)}$ (i = 1, 2, ... N_3):

$$\frac{\partial T}{\partial \dot{q}_i^{(2)}} = \mu_2 \sum_{j=1}^{N_3} m_{ij}^{(2)} \dot{q}_j^{(2)} + \mu_2 l_3^* \dot{\varphi}_3^* \cos\left(\varphi_3^* - \varphi_4\right) C_i^{(2)} + \mu_2 \dot{\varphi}_4 \left(D_i^{(2)} + \sum_{k=1}^{N_4} n_{ik}^{(2)} p_k^{(2)} \right)$$

$$\begin{aligned} \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i^{(2)}}\right) &= \mu_2 \sum_{j=1}^{N_3} m_{ij}^{(2)} \ddot{q}_j^{(2)} + \mu_2 l_3^* \ddot{\varphi}_3^* \cos\left(\varphi_3^* - \varphi_4\right) C_i^{(2)} - \mu_2 l_3^* \dot{\varphi}_3^* \left(\dot{\varphi}_3^* - \dot{\varphi}_4\right) \sin\left(\varphi_3^* - \varphi_4\right) C_i^{(1)} \\ &+ \mu_2 \ddot{\varphi}_4 \left(D_i^{(2)} + \sum_{k=1}^{N_4} n_{ik}^{(2)} p_k^{(2)} \right) + \mu_2 \dot{\varphi}_4 \sum_{k=1}^{N_4} n_{ik}^{(2)} \dot{p}_k^{(2)} \end{aligned}$$

$$\frac{\partial T}{\partial q_i^{(1)}} = \mu_2 \dot{\varphi}_4^2 \sum_{j=1}^{N_3} m_{ij}^{(2)} q_j^{(2)} + \mu_2 l_3^* \dot{\varphi}_3^* \dot{\varphi}_4 \sin\left(\varphi_3^* - \varphi_4\right) C_i^{(2)} - \mu_2 \dot{\varphi}_4 \sum_{k=1}^{N_4} n_{ik}^{(2)} \dot{p}_k^{(2)}$$

$$\frac{\partial \Pi}{\partial q_i^{(2)}} = E_2 I_2 \sum_{j=1}^{N_3} k_{ij}^{(2)} q_j^{(2)}\ ;\ \ \lambda_1 \frac{\partial f_1}{\partial q_i^{(2)}} + \lambda_2 \frac{\partial f_2}{\partial q_i^{(2)}} + \lambda_3 \frac{\partial f_3}{\partial q_i^{(2)}} + \lambda_4 \frac{\partial f_4}{\partial q_i^{(2)}} = 0$$

$$Q_{q_i^{(2)}} = 0$$

Substituting equation (2), we get the equation:

$$\begin{aligned} &\mu_2 l_3^* \ddot{\varphi}_3^* \cos\left(\varphi_3^* - \varphi_4\right) C_i^{(2)} + \mu_2 \ddot{\varphi}_4 \left(D_i^{(2)} + \sum_{k=1}^{N_4} n_{ik}^{(2)} p_k^{(2)} \right) + \mu_2 \sum_{j=1}^{N_3} m_{ij}^{(2)} \ddot{q}_j^{(2)} \\ &-\mu_2 l_3^* \dot{\varphi}_3^{*2} \sin\left(\varphi_3^* - \varphi_4\right) C_i^{(2)} + 2\mu_2 \dot{\varphi}_4 \sum_{k=1}^{N_4} n_{ik}^{(2)} \dot{p}_k^{(2)} - \mu_2 \dot{\varphi}_4^2 \sum_{j=1}^{N_3} m_{ij}^{(2)} q_j^{(2)} + E_2 I_2 \sum_{j=1}^{N_3} k_{ij}^{(2)} q_j^{(2)} = 0 \end{aligned} \tag{10}$$

*) Equations written for generalized coordinates $p_k^{(2)}$ (i = 1,2,...N_4):

$$\frac{\partial T}{\partial \dot{p}_k^{(2)}} = \mu_2 \sum_{l=1}^{N_4} b_{kl}^{(2)} \dot{p}_l^{(2)} - \mu_2 l_3^* \dot{\varphi}_3^* \sin\left(\varphi_3^* - \varphi_4\right) H_k^{(2)} - \mu_2 \dot{\varphi}_4 \sum_{k=1}^{N_4} n_{ik}^{(2)} q_i^{(2)}$$

$$\begin{aligned} \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{p}_k^{(2)}}\right) &= \mu_2 \sum_{l=1}^{N_4} b_{kl}^{(2)} \ddot{p}_l^{(2)} - \mu_2 l_3^* \ddot{\varphi}_3^* \sin\left(\varphi_3^* - \varphi_4\right) H_k^{(2)} - \mu_2 l_3^* \dot{\varphi}_3^* \left(\dot{\varphi}_3^* - \dot{\varphi}_4\right) \cos\left(\varphi_3^* - \varphi_4\right) H_k^{(2)} \\ &- \mu_2 \ddot{\varphi}_4 \sum_{k=1}^{N_4} n_{ik}^{(2)} q_i^{(2)} - \mu_2 \dot{\varphi}_4 \sum_{k=1}^{N_4} n_{ik}^{(2)} \dot{q}_i^{(2)} \end{aligned}$$

$$\frac{\partial T}{\partial p_k^{(2)}} = \mu_2 \left[F_k^{(2)} + \sum_{l=1}^{N_4} b_{kl}^{(2)} p_l^{(2)} \right] \dot{\varphi}_4^2 + \mu_2 l_3^* \dot{\varphi}_3^* \dot{\varphi}_4 \cos\left(\varphi_3^* - \varphi_4\right) H_k^{(2)} + \mu_2 \dot{\varphi}_4 \sum_{i=1}^{N_1} n_{ik}^{(2)} \dot{q}_i^{(2)}$$

$$\frac{\partial \Pi}{\partial p_k^{(2)}} = E_1 A_1 \sum_{l=1}^{N_4} g_{kl}^{(2)} p_l^{(2)}$$

$$\lambda_1 \frac{\partial f_1}{\partial p_k^{(2)}} + \lambda_2 \frac{\partial f_2}{\partial p_k^{(2)}} + \lambda_3 \frac{\partial f_3}{\partial p_k^{(2)}} + \lambda_4 \frac{\partial f_4}{\partial p_k^{(2)}} = \lambda_1 \frac{\partial u_{2D}}{\partial p_k^{(2)}} \cos\varphi_4 + \lambda_2 \frac{\partial u_{2D}}{\partial p_k^{(2)}} \sin\varphi_4$$
$$= \lambda_1 \sin\left(\frac{2k-1}{2}\pi\right)\cos\varphi_4 + \lambda_2 \sin\left(\frac{2k-1}{2}\pi\right)\sin\varphi_4 = \left(\lambda_1\cos\varphi_4 + \lambda_2\sin\varphi_4\right)\alpha_k$$

where : $\alpha_k = \sin\left(\frac{2k-1}{2}\pi\right) = \begin{cases} 1 & khi\ \ k = 2j+1,\ j = 1,2,... \\ -1 & khi\ \ k = 2j,\ j = 1,2,... \end{cases}$

$$Q_{p_k^{(2)}} = 0$$

Substituting equation (2), we get the equation :

$$\begin{aligned} &-\mu_2 l_3^* \ddot{\varphi}_3^* \sin\left(\varphi_3^* - \varphi_4\right) H_k^{(2)} - \mu_2 \ddot{\varphi}_4 \sum_{k=1}^{N_4} n_{ik}^{(2)} q_i^{(2)} + \mu_2 \sum_{l=1}^{N_4} b_{kl}^{(2)} \ddot{p}_l^{(2)} - \mu_2 l_3^* \dot{\varphi}_3^{*2} \cos\left(\varphi_3^* - \varphi_4\right) H_k^{(2)} \\ &-2\mu_2 \dot{\varphi}_4 \sum_{k=1}^{N_4} n_{ik}^{(2)} \dot{q}_i^{(2)} - \mu_2 \dot{\varphi}_4^2 \left(F_k^{(2)} + \sum_{l=1}^{N_4} b_{kl}^{(2)} p_l^{(2)} \right) + E_1 A_1 \sum_{l=1}^{N_4} g_{kl}^{(2)} p_l^{(2)} = -\left(\lambda_1 \cos\varphi_4 + \lambda_2 \sin\varphi_4\right)\alpha_k \end{aligned} \qquad (11)$$

Appendix B. Equations for solid structures

From the above equations, removing the deformation components, we obtain the equations written for the solid structure:

$$\left(I_{O_1}+\mu_1 l_1^2 l_2\right)\ddot{\varphi}_1+\frac{\mu_1 l_1 l_2^2}{2}\ddot{\varphi}_2\cos\left(\varphi_1-\varphi_2\right)+\frac{\mu_1 l_1 l_2^2}{2}\dot{\varphi}_2^2\sin\left(\varphi_1-\varphi_2\right) = l_1\sin\varphi_1\lambda_1-l_1\cos\varphi_1\lambda_2+\tau-\tau_{f_1} \tag{12}$$

$$\frac{\mu_1 l_1 l_2^2}{2}\ddot{\varphi}_1\cos\left(\varphi_1-\varphi_2\right)+\frac{\mu_1 l_2^3}{3}\ddot{\varphi}_2-\frac{\mu_1 l_1 l_2^2}{2}\dot{\varphi}_1^2\sin\left(\varphi_1-\varphi_2\right)=l_2\sin\varphi_2.\lambda_1-l_2\cos\varphi_2.\lambda_2 \tag{13}$$

$$\left(I_{O_2}+\mu_2 l_3^{*2} l_4\right)\ddot{\varphi}_3+\frac{\mu_2 l_3^* l_4^2}{2}\ddot{\varphi}_4\cos\left(\varphi_3^*-\varphi_4\right)+\frac{\mu_2 l_3' l_4^2}{2}\dot{\varphi}_4^2\sin\left(\varphi_3^*-\varphi_4\right) = -l_3\lambda_1\sin\left(\varphi_3^*+\beta\right)+l_3\lambda_2\cos\left(\varphi_3^*+\beta\right)+l_3^*\lambda_3\sin\varphi_3^*-l_3^*\lambda_4\cos\varphi_3^*-\tau_{f_3} \tag{14}$$

$$\frac{\mu_2 l_3^* l_4^2}{2}\ddot{\varphi}_3\cos\left(\varphi_3^*-\varphi_4\right)+\frac{\mu_2 l_4^3}{3}\ddot{\varphi}_4-\frac{\mu_2 l_3^* l_4^2}{2}\dot{\varphi}_3^2\sin\left(\varphi_3^*-\varphi_4\right) = l_4\lambda_3\sin\varphi_4-l_4\lambda_4\cos\varphi_4 \tag{15}$$

where: $\varphi_3=\varphi_3^*+\beta$

$$I_{O_3}\ddot{\varphi}_5+l_5\sin\left(\varphi_5\right)\lambda_1-l_5\cos\left(\varphi_5\right)\lambda_2=-\tau_{f_5} \tag{16}$$

Link equations :

$$f_1 = l_1\cos\varphi_1 + l_2\cos\varphi_2 - l_3\cos\varphi_3 - l_0\cos\theta_1 = 0 \tag{17}$$

$$f_2 = l_1\sin\varphi_1 + l_2\sin\varphi_2 - l_3\sin\varphi_3 - l_0\sin\theta_1 = 0 \tag{18}$$

$$f_3 = l_3^*\cos(\varphi_3 - \beta) + l_4\cos\varphi_4 - l_5\cos\varphi_5 - l_0^*\cos\theta_2 = 0 \tag{19}$$

$$f_4 = l_3^*\sin(\varphi_3 - \beta) + l_4\sin\varphi_4 - l_5\sin\varphi_5 - l_0^*\sin\theta_2 = 0 \tag{20}$$

Appendix C. Calculate the integrals

+) $C_i^{(1)} = \int_0^{l_2} X_i^{(1)} dx_1 = \int_0^{l_2} \sin\left(\frac{i\pi x}{l_2}\right) dx_1$, $C_i^{(2)} = \int_0^{l_4} X_i^{(2)} dx_2 = \int_0^{l_4} \sin\left(\frac{i\pi x_2}{l_4}\right) dx_2$:

$$C_1^{(1)} = \frac{2l_2}{\pi};\ C_2^{(1)} = 0;\ C_3^{(1)} = \frac{2l_2}{3\pi};\ C_1^{(2)} = \frac{2l_4}{\pi};\ C_2^{(2)} = 0;\ C_3^{(2)} = \frac{2l_4}{3\pi} \quad (21)$$

+) $D_i^{(1)} = \int_0^{l_2} x_1 X_i^{(1)} dx_1 = \int_0^{l_2} x_1 \sin\left(\frac{i\pi x_1}{l_2}\right) dx_1$, $D_i^{(2)} = \int_0^{l_4} x_2 X_i^{(2)} dx_1 = \int_0^{l_4} x_2 \sin\left(\frac{i\pi x_2}{l_4}\right) dx_2$:

$$D_1^{(1)} = \frac{l_2^2}{\pi}; D_2^{(1)} = -\frac{l_2^2}{2\pi}; D_3^{(1)} = \frac{l_2^2}{3\pi}; D_1^{(2)} = \frac{l_4^2}{\pi}; D_2^{(2)} = -\frac{l_4^2}{2\pi}; D_3^{(2)} = \frac{l_4^2}{3\pi} \quad (22)$$

+) $m_{ij}^{(1)} = \int_0^{l_2} X_i^{(1)} X_j^{(1)} dx_1 = \int_0^{l_2} \sin\left(\frac{i\pi x_1}{l_2}\right)\sin\left(\frac{j\pi x_1}{l_2}\right) dx_1$, $m_{ij}^{(2)} = \int_0^{l_4} X_i^{(2)} X_j^{(2)} dx_2 = \int_0^{l_4} \sin\left(\frac{i\pi x_2}{l_4}\right)\sin\left(\frac{j\pi x_2}{l_4}\right) dx_2$:

$$m_{11}^{(1)} = m_{22}^{(1)} = m_{33}^{(1)} = \frac{l_2}{2};\ m_{12}^{(1)} = m_{21}^{(1)} = m_{23}^{(1)} = m_{32}^{(1)} = m_{13}^{(1)} = m_{31}^{(1)} = 0 \quad (23)$$

$$m_{11}^{(2)} = m_{22}^{(2)} = m_{33}^{(2)} = \frac{l_4}{2};\ m_{12}^{(2)} = m_{21}^{(2)} = m_{23}^{(2)} = m_{32}^{(2)} = m_{13}^{(2)} = m_{31}^{(2)} = 0 \quad (24)$$

+) $k_{ij}^{(1)} = \int_0^{l_2} X_i'' X_j'' dx_1 = \int_0^{l_2} \left(\frac{i\pi}{l_2}\right)^2 \left(\frac{j\pi}{l_2}\right)^2 \sin\left(\frac{i\pi x_1}{l_2}\right)\sin\left(\frac{j\pi x_1}{l_2}\right) dx_1$, $k_{ij}^{(2)} = \int_0^{l_4} X_i'' X_j'' dx_2 = \int_0^{l_4} \left(\frac{i\pi}{l_4}\right)^2 \left(\frac{j\pi}{l_4}\right)^2 \sin\left(\frac{i\pi x_2}{l_4}\right)\sin\left(\frac{j\pi x_2}{l_4}\right) dx_2$:

$$k_{11}^{(1)} = \frac{\pi^4}{2l_2^3}; k_{22}^{(1)} = \frac{8\pi^4}{l_2^3};\ k_{33}^{(1)} = \frac{81\pi^4}{2l_2^3};\ k_{12}^{(1)} = k_{21}^{(1)} = k_{23}^{(1)} = k_{32}^{(1)} = k_{13}^{(1)} = k_{31}^{(1)} = 0 \quad (25)$$

$$k_{11}^{(2)} = \frac{\pi^4}{2l_4^3}; k_{22}^{(2)} = \frac{8\pi^4}{l_4^3};\ k_{33}^{(2)} = \frac{81\pi^4}{2l_4^3};\ k_{12}^{(2)} = k_{21}^{(2)} = k_{23}^{(2)} = k_{32}^{(2)} = k_{13}^{(2)} = k_{31}^{(2)} = 0 \quad (26)$$

+) $n_{ik}^{(1)} = \int_0^{l_2} X_i Y_k dx_1 = \int_0^{l_2} \sin\left(\frac{i\pi x_1}{l_2}\right)\sin\left(\frac{2k-1}{2}\times\frac{\pi x_1}{l_2}\right) dx_1$, $n_{ik}^{(2)} = \int_0^{l_4} X_i Y_k dx_1 = \int_0^{l_4} \sin\left(\frac{i\pi x_2}{l_4}\right)\sin\left(\frac{2k-1}{2}\times\frac{\pi x_2}{l_4}\right) dx_2$:

$$n_{11}^{(1)} = \frac{4l_2}{3\pi}; n_{12}^{(1)} = \frac{4l_2}{5\pi}; n_{13}^{(1)} = -\frac{4l_2}{21\pi}; n_{21}^{(1)} = -\frac{8l_2}{15\pi}; n_{22}^{(1)} = \frac{8l_2}{7\pi}; n_{23}^{(1)} = \frac{8l_2}{9\pi}; n_{31}^{(1)} = \frac{12l_2}{35\pi}$$
$$n_{32}^{(1)} = -\frac{4l_2}{9\pi}; n_{33}^{(1)} = \frac{12l_2}{11\pi}; n_{11}^{(2)} = \frac{4l_4}{3\pi}; n_{12}^{(2)} = \frac{4l_4}{5\pi}; n_{13}^{(2)} = -\frac{4l_4}{21\pi}; n_{21}^{(2)} = -\frac{8l_4}{15\pi} \quad (27)$$

$$n_{22}^{(2)} = \frac{8l_4}{7\pi}; n_{23}^{(2)} = \frac{8l_4}{9\pi}; n_{31}^{(2)} = \frac{12l_4}{35\pi}; n_{32}^{(2)} = -\frac{4l_4}{9\pi}; n_{33}^{(2)} = \frac{12l_4}{11\pi}$$

+) $H_k^{(1)} = \int_0^{l_2} Y_k dx_1 = \int_0^{l_2} \sin\left(\frac{2k-1}{2}\times\frac{\pi x_1}{l_2}\right) dx_1$, $H_k^{(2)} = \int_0^{l_4} Y_k dx_2 = \int_0^{l_4} \sin\left(\frac{2k-1}{2}\times\frac{\pi x_2}{l_4}\right) dx_2$:

$$H_1^{(1)} = \frac{2l_2}{\pi}; H_2^{(1)} = \frac{2l_2}{3\pi}; H_3^{(1)} = \frac{2l_2}{5\pi};\ H_1^{(2)} = \frac{2l_4}{\pi}; H_2^{(2)} = \frac{2l_4}{3\pi}; H_3^{(2)} = \frac{2l_4}{5\pi} \quad (28)$$

+) $F_k^{(1)} = \int_0^{l_2} x_1 Y_k dx_1 = \int_0^{l_2} x_1 \sin\left(\frac{2k-1}{2}\frac{\pi x_1}{l_2}\right) dx_1$, $F_k^{(2)} = \int_0^{l_4} x_2 Y_k dx_2 = \int_0^{l_4} x_2 \sin\left(\frac{2k-1}{2}\frac{\pi x_2}{l_4}\right) dx_2$:

$$F_1^{(1)} = \frac{4l_2^2}{\pi^2};\ F_2^{(1)} = -\frac{4l_2^2}{9\pi^2};\ F_3^{(1)} = \frac{4l_2^2}{25\pi^2}; F_1^{(2)} = \frac{4l_4^2}{\pi^2};\ F_2^{(2)} = -\frac{4l_4^2}{9\pi^2};\ F_3^{(2)} = \frac{4l_4^2}{25\pi^2} \quad (29)$$

+) $b_{kl}^{(1)} = \int_0^{l_2} Y_k^{(1)} Y_l^{(1)} dx_1 = \int_0^{l_2} \sin\left(\frac{2k-1}{2}\times\frac{\pi x_1}{l_2}\right)\sin\left(\frac{2l-1}{2}\times\frac{\pi x_1}{l_2}\right) dx_1$,

$$b_{11}^{(1)} = b_{22}^{(1)} = b_{33}^{(1)} = \frac{l_2}{2};\ b_{12}^{(1)} = b_{21}^{(1)} = b_{23}^{(1)} = b_{32}^{(1)} = b_{13}^{(1)} = b_{31}^{(1)} = 0 \quad (30)$$

+) $b_{kl}^{(2)}=\int_0^{l_4} Y_k^{(2)}Y_l^{(2)}dx_1=\int_0^{l_4}\sin\left(\frac{2k-1}{2}\times\frac{\pi x_2}{l_4}\right)\sin\left(\frac{2l-1}{2}\times\frac{\pi x_2}{l_4}\right)dx_2$:

$$b_{11}^{(2)}=b_{22}^{(2)}=b_{33}^{(2)}=\frac{l_4}{2};\ b_{12}^{(2)}=b_{21}^{(2)}=b_{23}^{(2)}=b_{32}^{(2)}=b_{13}^{(2)}=b_{31}^{(2)}=0 \quad (31)$$

+) $g_{kl}^{(1)}=\int_0^{l_2} Y_k'Y_l'dx_1=\int_0^{l_2}\left(\frac{(2k-1)\pi}{2l_2}\right)\left(\frac{(2l-1)\pi}{2l_2}\right)\cos\left(\frac{2k-1}{2}\times\frac{\pi x_1}{l_2}\right)\cos\left(\frac{2l-1}{2}\times\frac{\pi x_1}{l_2}\right)dx_1$,

$$g_{kl}^{(2)}=\int_0^{l_4} Y_k'Y_l'dx_2=\int_0^{l_4}\left(\frac{(2k-1)\pi}{2l_4}\right)\left(\frac{(2l-1)\pi}{2l_4}\right)\cos\left(\frac{2k-1}{2}\times\frac{\pi x_2}{l_4}\right)\cos\left(\frac{2l-1}{2}\times\frac{\pi x_2}{l_4}\right)dx_2 : \quad (32)$$

$$g_{11}^{(1)}=\frac{\pi^2}{8l_2};\ g_{22}^{(1)}=\frac{9\pi^2}{8l_2};\ g_{33}^{(1)}=\frac{25\pi^2}{8l_2};\ g_{12}^{(1)}=g_{21}^{(1)}=g_{23}^{(1)}=g_{32}^{(1)}=g_{13}^{(1)}=g_{31}^{(1)}=0 \quad (33)$$

$$g_{11}^{(2)}=\frac{\pi^2}{8l_4};\ g_{22}^{(2)}=\frac{9\pi^2}{8l_4};\ g_{33}^{(2)}=\frac{25\pi^2}{8l_4};\ g_{12}^{(2)}=g_{21}^{(2)}=g_{23}^{(2)}=g_{32}^{(2)}=g_{13}^{(2)}=g_{31}^{(2)}=0 \quad (34)$$