INTRODUCTION

In helical gears, the width of gear wheels ranges from 0.2 to 1.4 of the pitch diameter of a rack and it depends on the position of the gear wheels relative to supports and the hardness of the gear teeth. An increase in the width of the gears leads to an increase in the length of the contact line and the total tooth contact ratio, which results in a higher load-carrying capacity and durability of the gear. The optimal condition width in involute helical gears is indicated ensuring constant length of the line of contact between the meshing gears. As a result, it was possible to determine variations in the parameters for the optimized gear describing the meshing gears at different values of the profile correction coefficients.

FORMULATION OF THE PROBLEM AND ITS SOLUTION

The total tooth contact ratio of a helical gear is $\varepsilon_\gamma = \varepsilon_\alpha + \varepsilon_\beta$, where:

\[ \varepsilon_\alpha = \frac{e_1 + e_2}{2\pi b_1}, \quad \varepsilon_\beta = \frac{b_w \sin \beta}{\pi m}, \quad (1) \]

Where $e_\alpha$ is the end-face tooth contact ratio; $e_\beta$ is the overlap ratio; $e_1 = \sqrt{r_1^2 - r_{bl}^2 - r_{w1} \sin \alpha_w}$ is the length of tooth contact at the end of engagement; $e_2 = \sqrt{r_{20}^2 - r_{b2}^2 - r_{w2} \sin \alpha_w}$ is the length of tooth contact at the start of engagement; $r_{w1}$, $r_{w2}$ are the radii of the pitch circles of the pinion and gear, respectively; $r_{bl} = r_{a1} - r$, $r_{20} = r_{a2} - r$; $r = 0.2$ m is the rounding radius of the top land of a gear tooth; $r_{b1} = r \cos \alpha_i$, is the radius of the base circle in the pinion; $r_{b2} = r_2 \cos \alpha_i$, is the radius of the base circle in the gear; $r_1 = m_1 z_1 / 2 \cos \beta$ is the radius of the pitch circle in the pinion;
The angle $\Delta \varphi_{1E}$ describing the end of engagement is

$$\Delta \varphi_{1E} = \varphi_{10} + \varphi_{1E},$$

where

$$\varphi_{1E} = \tan \alpha_E - \tan \alpha_w, \quad \alpha_E = \arccos\left(\frac{r_0}{r}\right)$$

In the case of triple-double-triple engagement

$$\tan \alpha_{F_z} = \frac{r_{w1} \sin \alpha_w - (p_b - e_z) + 0.5 p_b (e_{\beta} - 1)}{r_1 \cos \alpha},$$

$$\tan \alpha_{F_1} = \frac{r_{w1} \sin \alpha_w - (p_b - e_z) - 0.5 p_b (e_{\beta} - 1)}{r_1 \cos \alpha},$$

$\varphi = \pi m \cos \alpha_w / \cos \beta$ is the tooth pitch;

$\alpha = 20^\circ$ is the pressure angle.

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$\alpha = 20^\circ$ is the pressure angle.
\[ \rho_{ri, j} = r_{bi} \tan \alpha_{ri, j}, \]
\[ \rho_{r2, j} = r_{w2} \sqrt{\left(\frac{r_{j}}{r_{w2}}\right)^2 - \cos^2 \alpha_{w}}, \]
\[ \alpha_{ri, j} = \arctan \left( \tan \alpha_{ri, 0} + j \Delta \varphi \right), \]
\[ \tan \alpha_{ri, 0} = \left( 1 + u \right) \tan \alpha_{w} - \frac{u}{\cos \alpha_{w}} \sqrt{r_{o2}/r_{w2}} \sqrt{\cos^2 \alpha_{w} - \cos^2 \alpha_{w}}, \]
\[ r_{2, j} = \sqrt{\left( \alpha_{w}^2 + r_{1, j}^2 - 2 \alpha_{w} r_{1, j} \cos\left( \alpha_{w} - \alpha_{ri, 0} \right) \right)^2 - \cos^2 \alpha_{w}}, \]
\[ r_{1, j} = r_{w1} \cos \alpha_{w} / \cos \alpha_{ri, 0}, \]
\[ a_{w} = \left( z_1 + z_2 \right) m / 2 \cos \beta. \]

\( \Delta \varphi \) is the angle of revolution of the pinion teeth at the start of engagement (p. 0) at p. 1, and so on; \( u \) is the gear ratio; \( a_{ri} \) is the distance between the axes; \( \alpha_{ri, 0} \) is the angle describing the position of the first point of engagement of the pinion teeth on the line of contact.

A simplified way to calculate the durability \( t_* \) of the gear when the teeth reach the maximum allowable wear \( h_{t*} \), taking account of the initial contact pressures \( p_{j, max} \), is to use the formula:
\[ t_* = h_{t*} / \bar{h}_{ji}, \quad k = 1; 2, \quad (8) \]
where \( \bar{h}_{ji} = 60 \eta_i h_{ji} \) is the linear wear of teeth at selected \( j \)-th points of their profiles during one hour of operation of the gear; \( k = 1 \) – pinion, \( k = 2 \) – gear; \( n_j = n_i / u \) is the number of revolutions of the gear; \( h_{ji}^* \) is the linear wear of the teeth at \( j \)-th point of their profile during single engagement; the minimum durability \( t_{min} \) of the gear will be observed at the point where the profile reaches the highest wear.

According to [16, 18]:
\[ h_{ji}^* = v_{j1}^* \left( t_{p, max} \right)^{m_k} / C_i \left( 0.35 R_m \right)^{m_k}, \quad (9) \]
where \( v_1 = v \) is the sliding velocity at \( j \)-th points of the tooth profiles; \( t_{1}^* = 2 \eta_i v_1 \) is the time of meshing during the displacement of \( j \)-th point of tooth contact along their profile per the width of tooth contact area; \( v_1 = \omega_j r_j \sin \alpha_i \) is the velocity of shift of the contact point along the tooth profile; \( \omega_i \) is the angular velocity of the pinion; \( \varphi^* \) is the sliding friction factor; \( R_m \) is the immediate tensile strength of the material; \( C_i \) and \( m_i \) are the factors of frictional wear resistance of gear materials at limit friction determined in compliance with the methodology presented in [16] based on the results of experimental tribological tests; \( 2b_j = 3.044 \sqrt{N \rho_j / E} \) is the width of tooth contact area.

The sliding velocity is determined in the following way:
\[ v_j = \omega_j r_{j1} \left( tg \alpha_{ri, j} - tg \alpha_{ri, 2, j} \right), \quad (10) \]
where \( \alpha_{ri, 2, j} = \arccos \left[ \left( r_{2, j} / r_{w2} \right) \cos \alpha \right] \).

Due to the wear of the gear teeth, the curvature radii \( \rho_{1, j} \), \( \rho_{2, j} \) of their profiles increase, which results in a change of the initial contact pressures \( p_{j, max} \) to the pressures \( p_{j, max} \), while the width of the contact area \( 2b_j \) of the teeth changes to \( 2b_{j, j} \). Accordingly, based on the modified Hertz formulas
\[ p_{j, max} = 0.418 \sqrt{N E / \rho_{j, j}}, \]
\[ 2b_{j, j} = 3.044 \sqrt{N E / \rho_{j, j}}, \quad (11) \]
where \( \rho_{j, j} = \rho_{1, j} + \rho_{2, j} \) is the change in reduced radius of tooth profile curvature due to tooth wear.

Changes in the radii of tooth profile curvature can be measured after every revolution of the gear. This, however, leads to the extending of computational time. To avoid this, we applied a authors block method ([19] – Fig. 1), which consists in measuring changes in the process parameters \( (h_{ji}, h_{ji}^*, \alpha_{ri, 0}, p_{ji, max}, p_{ji, 2, 1}, b_{ji, 1}, t_{ji}^*) \) after a certain number of gear revolutions (engagement block \( B \)). Accordingly, a change in the radii of the curvature \( \rho_{ji, j} \) is determined in the following way [16]:
\[ \rho_{ji, j} = \rho_{ji} + E_1 \sum_{h_{ji}} D_{ji, h} K_{ji, h}^{-1}, \quad k = 1; 2, \quad (12) \]
where \( D_{ji, h} = K_{ji, h}^{2} \); the size of block can be proportionate to the number of revolutions of the pinion – \( B = 1 \) revolution (accurate solution), \( B = n_1 \) (revolutions per hour), \( B = n_1 \) (revolutions per 10, 20, ... hours); \( E_1 = 3(h_{ji} + b_{ji}) \).

Changes in the profile curvature of the teeth due to their wear during every block of their engagement is determined in the following way:
\[ K_{ji} = 8 \sum_{h_{ji}} h_{ji}^* / l_{ji}^2. \quad (13) \]
The length of the chord replacing the involute between the points \(j - 1, j + 1\) is calculated in the following way:

\[
I_{kj} = 2\rho_{kj} \sin \varepsilon_{kj} = \text{const.}
\]

where:

\[
S_{kj} = \frac{mz}{4} \left( \frac{1}{\cos^2 \alpha_{dkj}} - \frac{1}{\cos^2 \alpha_{dk,j+1}} \right) \cos \alpha.
\]

\[
\alpha_{j,j+1} = \arctan \left( \tan \alpha_{j0} + (j+1) \Delta \phi \right),
\]

\[
\alpha_{2j} = \arccos \left[ \frac{r_{r2}/r_{rj}}{\cos \alpha_u} \right],
\]

\[
\alpha_{1,j+1} = \arccos \left[ \frac{r_{r1}/r_{rj}}{\cos \alpha_u} \right].
\]

The linear wear \(h'_{kj}\) of the teeth at every \(j\)-th point of their profile is calculated in this case after every block in the time \(t'_{jk}\) of their engagement. Accordingly,

\[
h'_{kj} = \frac{V_{rjk}}{C_4} \left( \frac{h_{j\phi}}{0.35R_m} \right)^{m_x},
\]

where \(t'_{jk} = 2b_{jk}/v_c\).

The total wear \(h_{1,jn}\) and \(h_{2,jn}\) of the gear teeth at the \(j\)-points \(j\) of their profiles for a selected number of pinion rotations \(n_{1j}\) or gear rotations \(n_{2j}\) is determined by the following formulas:

\[
h_{1,jn} = \sum_{i=1}^{n_{1j}} h'_{ij}, \quad h_{2,jn} = \sum_{i=1}^{n_{2j}} h'_{ij}.
\]

where: \(n_{2j} = n_{1j}/u\); \(h'_{ij} = \sum_{j=1}^{j\phi} h_{ij}\).

Given the change in the type of engagement and in the initial maximum contact pressures \(p_{j\phi}\) due to tooth wear, the gear durability \(t_{B_{\text{min}}}\) for the revolutions \(n_{1s}\) or \(n_{2s}\) of the gear wheels is calculated:

\[
t_{B_{\text{min}}} = n_{1s}/60n_1 = n_{2s}/60n_2.
\]

**NUMERICAL SOLUTION**

The input data included: \(z_1 = 20; z_2 = 80; m = 3\) mm; \(u = 4; n_1 = 700\) rpm; \(P = 5\) kW; \(f = 0.05\); \(\beta = 10^\circ\); \(K_g = 1.6\). The following materials were used: the pinion was made of 38HMJA steel after nitriding with 58 HRC; \(R_m = 1040\) MPa, \(C_1 = 3.5\cdot10^6\); \(m = 2\); the gear was made of 40H steel after bulk heat treatment with 53 HRC, \(R_m = 981\) MPa, \(C_2 = 0.17\cdot10^6\); \(m_2 = 2.5\); \(E = 2.1\cdot10^6\) MPa.

Lubrication involved the use of an oil described by the kinematic viscosity \(v_{C50} = 15\) cSt; \(h_{w} = 0.5\) mm; \(\Delta \phi = 4^\circ\). The profile correction coefficients were: \(x_1 = -x_2 = 0; 0.2; 0.4; 0.6; 0.8; a_w = 152.314\) mm, \(\alpha_j = 20.28^\circ\). Calculations were done using a block calculation method from the quantity of the interaction block \(B = 2100000\).

The results of the numerical solution are given in the figures below. Figure 1a shows the maximum initial contact pressures \(p_{j\phi}\) occurring during triple-double-triple tooth engagement when the wheel width is \(h_w = 54.275\) mm, while Figure 1b illustrates the variations in their \(p_{j\phi}\) caused by the tooth wear \(h_{w}\) = 0.5 mm.

The contact pressures \(p_{j\phi}\) are the highest at the start of double tooth engagement with the exception of the case when \(x_1 = -x_2 = 0\). During the gear operation, the highest tooth wear can be observed at the start of triple engagement, which leads to a decrease in the pressures \(p_{j\phi}\) even by two times \((x_1 = -x_2 = 0)\). With increasing the profile correction coefficients, the difference between \(p_{j\phi}\) and \(p_{j\phi}\) decreases or is maintained to the minimum \((x_1 = -x_2 = 0.8)\).

Figures 2 and 3 illustrate the linear wear of the gear \(h_{2j}\) and the pinion \(h_{ij}\).

The gear teeth are the first to reach the maximum allowable wear at different characteristic points of tooth contact depending on the coefficient of profile correction at the start of triple engagement or at the end of the double engagement. Similar observations with respect to the points marking maximum wear can be made about the pinion teeth. Figure 4 illustrates the relationship between minimum gear durability and profile correction.

The maximum durability is exhibited by the gear with profile correction when \(x_1 = -x_2 = 0.2\) – its durability is higher by 1.55 times than that of the gear without profile correction.

To determine the effect of gear wheel width on the type of engagement and the above contact and tribological parameters, two types of helical gear were tested: one described by the gear wheel width \(h_w = 30\) mm and double-single-double engagement, and the other described by \(h_w = 54.275\) mm and triple-double-triple engagement. Accordingly, Figure 5 shows the results of the maximum contact pressure \(p_{j\phi}\) for the two tested width of gear wheels.

As a result, increasing the gear width by 1.81 times leads to a nearly proportionate de-
Fig. 1. Variations in the initial contact pressures $p_{j_{\text{max}}}$ during the meshing of teeth due to wear

Fig. 2. Linear wear of the gear teeth along their profile
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crease (by 1.92 ÷ 1.8 times) in the initial contact pressures at the start of the engagement zone and by 1.95 times at the place where the type of engagement changes. The minimum durability of the gear is significantly more affected by the width of the gear wheels and the type of engagement, which is illustrated in Figure 6. In the case of the gear without profile correction, the minimum gear durability increases by over 10 times, and when the optimal values of the correction coefficients are applied \( x_i = -x_2 = 0.2 \) – by 9.6 times.

CONCLUSIONS

1. We determined the optimal gear width in helical cylindrical gears which ensures that the length of the contact line is maintained constant.

2. Using a new method for the determination of wear and durability of helical gears, depending on the profile correction and the type of engagement, a numerical solution block method was proposed to the problem of determining the maximum contact pressures, linear wear of the teeth and durability of the gear.
Fig. 5. Maximum contact pressures (a) and their variations (b) due to tooth wear: $b_W = 30$ mm (top), $b_W = 54.275$ mm (down)

Fig. 6. Profile correction versus minimum gear durability when $b_W = 54.275$ mm (top), $b_W = 30$ mm (down)
3. The study was performed on a helical gear with optimized gear width ensuring a constant meshing force and on a helical gear with decreased wheel width.

4. It has been found that the increase in the gear width in the range between 30 and 54.275 mm results in an almost proportionate decrease in the maximum contact pressures.

5. The increase in the gear width by 1.81 times leads to a significant increase in the minimum gear durability – by $10.24 \div 8$ times, depending on the applied coefficients of profile correction.

REFERENCES