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Computer Simulation of the Impact of Optimization of Width in the Helical Cylindrical Gear on Bearing and Durability Part 1. Height Correction of the Gear Profile

Myron Czerniec[1*]

[1] Institute of Information Technology Systems, Lublin University of Technology, ul. Nadbystrzycka 36, 20-618 Lublin, Poland

* Corresponding author's e-mail: m.czerniec@pollub.pl

ABSTRACT

Based on the elaborated calculation method he authors method for determining the wear and durability of gears was employed to measure the maximum contact pressures, linear wear of teeth and durability of the gear with height correction of the profile. The optimal condition width in involute helical gears is indicated ensuring constant length of the line of contact between the meshing gears. As a result, it was possible to determine variations in the parameters for the optimized gear describing the meshing gears at different values of the profile correction coefficients.

Keywords: block calculation method, helical cylindrical gear, height correction of engagement, optimal gear width, maximum contact pressures, tooth wear, gear durability

INTRODUCTION

In helical gears, the width of gear wheels ranges from 0.2 to 1.4 of the pitch diameter of a rack and it depends on the position of the gear wheels relative to supports and the hardness of the gear teeth. An increase in the width of the gears leads to an increase in the length of the contact line and the total tooth contact ratio, which results in a higher load-carrying capacity and durability of the gear. The overlap ratio ε_β depends on the width of the gear wheel and tooth inclination angle. To ensure constant load on the teeth during gear operation, in practice the constant load on the teeth during gear operation, in practice the width of the gear wheel must be selected such that the minimum length of the contact line can be maintained constant. The literature [1–15] of the subject offers hardly any studies on this problem, particularly when it comes to gears with profile correction.

FORMULATION OF THE PROBLEM AND ITS SOLUTION

The total tooth contact ratio of a helical gear is $\varepsilon_\gamma = \varepsilon_\alpha + \varepsilon_\beta$, where:

$$\varepsilon_\alpha = \frac{e_1 + e_2}{2\pi r_{b1}},\ \varepsilon_\beta = \frac{b_W \sin\beta}{\pi m}, \tag{1}$$

Where ε_α is the end-face tooth contact ratio; ε_β is the overlap ratio; $e_1 = \sqrt{r_{1s}^2 - r_{b1}^2} - r_{w1}\sin\alpha_w$ is the length of tooth contact at the end of engagement; $e_2 = \sqrt{r_{20}^2 - r_{b2}^2} - r_{w2}\sin\alpha_w$ is the length of tooth contact at the start of engagement; r_{w1}, r_{w2} are the radii of the pitch circles of the pinion and gear, respectively; $r_{1s} = r_{a1} - r$, $r_{20} = r_{a2} - r$; $r = 0.2$ m is the rounding radius of the top land of a gear tooth; $r_{b1} = r_1 \cos\alpha_t$ is the radius of the base circle in the pinion; $r_{b2} = r_2 \cos\alpha_t$ is the radius of the base circle in the gear; $r_1 = mz_1 / 2\cos\beta$ is the radius of the pitch circle in the pinion;

$r_2 = mz_2 / 2\cos\beta$ is the radius of the pitch circle in the gear; $r_{a1} = r_1 + m$ is the addendum radius of the pinion; $r_{a2} = r_2 + m$ is the addendum radius of the gear; β is the inclination angle of the teeth; b_W is the width of the pinion; m is the engagement modulus; z_1, z_2 are the numbers of gear teeth; α_t is the end-face pressure angle; α_w is the pressure angle of the corrected profile.

Hence, the length of the contact line during teeth engagement will be constant if ε_β = const (1 or 2). Accordingly, using the above formula of ε_β, the width of gear teeth is calculated as:

$$b_W = \frac{\pi m}{\sin\beta} \text{ when } \varepsilon_\beta = 1. \quad (2)$$

The minimum length of contact between a pair of teeth is calculated using the formula [18]:

$$l_{\min} = \frac{b_W \varepsilon_\alpha}{\cos\beta_b}\left[1 - \frac{n_\alpha n_\beta}{\varepsilon_\alpha \varepsilon_\beta}\right] \text{ when } n_\alpha + n_\beta \le 1, \quad (3)$$

where n_α, n_β are the fractional parts of the coefficients $\varepsilon_\alpha, \varepsilon_\beta$.

What follows is the analysis of two helical cylindrical gears with profile correction described by two different widths of gear wheels: $l_{\min}$ = const when $\varepsilon_\beta = 1$ and b_W = 54.275 mm (according to (2)); $l_{\min} \neq$ const when $\varepsilon_\beta < 1$ and b_W = 30 mm. The gear with b_W = 54.275 mm is described by triple-double-triple engagement while that with b_W = 30 mm by double-single-double engagement.

The angles of transition from double engagement ($\Delta\varphi_{1F_2}$) to single and then, again, to double engagement ($\Delta\varphi_{1F_1}$) are calculated in the following way:

$$\Delta\varphi_{1F_2} = \varphi_{10} - \varphi_{1F_2},\ \Delta\varphi_{1F_1} = \varphi_{10} + \varphi_{1F_1}, \quad (4)$$

where:

$$\varphi_{1F_2} = \tan\alpha_{F_2} - \tan\alpha_w,\ \varphi_{1F_1} = \tan\alpha_{F_1} - \tan\alpha_w,$$

$$\varphi_{10} = \tan\alpha_{t10} - \tan\alpha_w;$$

$$\tan\alpha_{F_2} = \frac{r_{w1}\sin\alpha_w - (p_b - e_1) + 0.5 n_\beta p_b}{r_1 \cos\alpha},$$

$$\tan\alpha_{F_1} = \frac{r_{w1}\sin\alpha_w - (p_b - e_2) - 0.5 n_\beta p_b}{r_1 \cos\alpha},$$

$p_b = \pi m \cos\alpha_w / \cos\beta$ is the tooth pitch;

$\alpha = 20^\circ$ is the pressure angle.

The angle $\Delta\varphi_{1E}$ describing the end of engagement is

$$\Delta\varphi_{1E} = \varphi_{10} + \varphi_{1E},$$

where

$$\varphi_{1E} = \tan\alpha_E - \tan\alpha_w,\ \alpha_E = \arccos(r_{b1} / r_{1s})$$

In the case of triple-double-triple engagement

$$\tan\alpha_{F_2} = \frac{r_{w1}\sin\alpha_w - (p_b - e_1) + 0.5 p_b(\varepsilon_\beta - 1)}{r_1 \cos\alpha},$$

$$\tan\alpha_{F_1} = \frac{r_{w1}\sin\alpha_w - (p_b - e_2) - 0.5 p_b(\varepsilon_\beta - 1)}{r_1 \cos\alpha}.$$

After the height correction of engagement, the addendum radii:

$$r_{a1} = r_1 + (1 + x_1)m,\quad r_{a2} = r_2 + (1 + x_2)m \quad (5)$$

where $x_1 = -x_2$ are the coefficients of displacement (correction) of the profile.

Other parameters of the gear are the same as those in the gear without profile correction.

The changes in the initial maximum contact pressures $p_{f\max}$ during one cycle of tooth engagement are determined using the method for determining the wear and durability of toothed gears [16, 17], considering the profile correction and the type of engagement [18]. This is done using the Hertz formula:

$$p_{j\max} = 0.418\sqrt{N'E / \rho_j}, \quad (6)$$

where $N' = N / l_{\min} w$, $N = T_{nom} K_g / r_{w1} \cos\alpha_w$ is the force acting in tooth engagement; $T_{nom} = 9550 P / n_1$ is the rated torque on the drive shaft; P is the power on the drive shaft; n_1 is the number of revolutions of the drive shaft; K_g is the dynamic coefficient; E is Young's modulus of steel teeth; $l_{\min}$ is the minimum length of a line of contact; j = 0, 1, 2, …, s are the points of contact on the tooth profile; w is the number of engagement pairs which transmit power simultaneously; ρ_j is the reduced radius of curvature of the tooth profile in normal section; ρ_{1j}, ρ_{2j} are the radii of curvature of the side profiles of the teeth of the pinion and gear, respectively

$$\rho_j = \frac{\rho_{1j}\rho_{2j}}{\rho_{1j} + \rho_{2j}}\quad \rho_{1j} = \frac{\rho_{t1j}}{\cos\beta_b}\quad \rho_{2j} = \frac{\rho_{t2j}}{\cos\beta_b}, \quad (7)$$

where

$$\beta_b = arc\left(\tan\beta\cos\alpha_t\right),\quad \alpha_t = \arctan\left(\frac{\tan\alpha}{\cos\beta}\right)$$

$$\rho_{t1j} = r_{b1} \tan \alpha_{t1j} ,$$

$$\rho_{t2j} = r_{w2} \sqrt{\left(r_{2j} / r_{w2}\right)^2 - \cos^2 \alpha_w} ,$$

$$\alpha_{t1j} = \arctan\left(\tan \alpha_{t10} + j\Delta\varphi\right),$$

$$\tan \alpha_{t10} = \left(1+u\right)\tan \alpha_w - \frac{u}{\cos \alpha_w}\sqrt{\left(r_{20} / r_{w2}\right)^2 - \cos^2 \alpha_w}$$

$$r_{2j} = \sqrt{a_w^2 + r_{1j}^2 - 2a_w r_{1j} \cos\left(\alpha_w - \alpha_{t1j}\right)} ,$$

$$r_{1j} = r_{w1} \cos \alpha_w / \cos \alpha_{t1j} ,$$

$$a_W = \left(z_1 + z_2\right) m / 2\cos\beta ,$$

$\Delta\varphi$ is the angle of revolution of the pinion teeth at the start of engagement (p. 0) at p.1, and so on; u is the gear ratio; a_w is the distance between the axes; α_{t10} is the angle describing the position of the first point of engagement of the pinion teeth on the line of contact.

A simplified way to calculate the durability t_* of the gear when the teeth reach the maximum allowable wear h_{k*}, taking account of the initial contact pressures $p_{j\max}$, is to use the formula:

$$t_* = h_{k*} / \overline{h}_{kj} ,\ k = 1;\ 2, \tag{8}$$

where $\overline{h}_{kj} = 60 n_k h'_{kj}$ is the linear wear of teeth at selected j-th points of their profiles during one hour of operation of the gear; $k = 1$ – pinion, $k = 2$ – gear; $n_2 = n_1 / u$ is the number of revolutions of the gear; h'_{kj} is the linear wear of the teeth at j-th point of their profile during single engagement; the minimum durability $t_{\min}$ of the gear will be observed at the point where the profile reaches the highest wear.

According to [16, 18]:

$$h'_{kj} = \frac{v_j t'_j \left(f p_{j\max}\right)^{m_k}}{C_k \left(0.35 R_m\right)^{m_k}} , \tag{9}$$

where $v_j = v$ is the sliding velocity at j-th points of the tooth profiles; $t'_j = 2_{bj}/v_0$ is the time of meshing during the displacement of j-th point of tooth contact along their profile per the width of tooth contact area; $v_0 = \omega_1 r_1 \sin \alpha_t$ the velocity of shift of the contact point along the tooth profile; ω_1 is the angular velocity of the pinion; f is the sliding friction factor; R_m is the immediate tensile strength of the material; C_k, m_k are the factors of frictional wear resistance of gear materials at limit friction determined in compliance with the methodology presented in [16] based on the results of experimental tribological tests; $2b_j = 3.044\sqrt{N'\rho_j / E}$ is the width of tooth contact area.

The sliding velocity is determined in the following way:

$$v_j = \omega_1 r_{b1}\left(tg\alpha_{t1j} - tg\alpha_{t2j}\right) \tag{10}$$

where $\alpha_{t2j} = \arccos\left[\left(r_2 / r_{2j}\right)\cos\alpha\right]$.

Due to the wear of the gear teeth, the curvature radii ρ_{1jh}, ρ_{2jh} of their profiles increase, which results in a change of the initial contact pressures $p_{j\max}$ to the pressures $p_{jh\,max}$, while the width of the contact area $2b_j$ of the teeth changes to $2b_{jh}$. Accordingly, based on the modified Hertz formulas

$$p_{jh\max} = 0.418\sqrt{N'E / \rho_{jh}} ,$$
$$2b_{jh} = 3.044\sqrt{N'\rho_{jh} / E} , \tag{11}$$

where $\rho_{jh} = \dfrac{\rho_{1jh}\rho_{2jh}}{\rho_{1jh} + \rho_{2jh}}$

is the change in reduced radius of tooth profile curvature due to tooth wear.

Changes in the radii of tooth profile curvature can be measured after every revolution of the gear. This, however, leads to the extending of computational time. To avoid this, we applied a autors block method ([19] – Fig. 1), which consists in measuring changes in the process parameters (h_{1j}, h_{2j}, ρ_{1jh}, ρ_{2jh}, ρ_{jh}, $p_{jh\max}$, $2b_{jh}$, t'_{jh}) after a certain number of gear revolutions (engagement block B). Accordingly, a change in the radii of the curvature ρ_{kjh} is determined in the following way [16]:

$$\rho_{kjh} = \rho_{kj} + E_k \sum_{B_1}^{B_{\max}} D_{kjB} K_{kjB}^{-1} ,\ \mathrm{k} = 1;\ 2, \tag{12}$$

where $D_{kjB} = K_{kjB}^2$; the size of block can be proportionate to the number of revolutions of the pinion – B = 1 revolution (accurate solution), $B = n_1$ (revolutions per hour), $B = n_1$ (revolutions per hour), $B = n_1$ (revolutions per 10, 20, ... hours); $E_1 = 3(h_{1*} + h_{2*})$

Changes in the profile curvature of the teeth due to their wear during every block of their engagement is determined in the following way:

$$K_{kjB} = 8\sum^{B} h'_{kjn} / l_{kj}^2 . \tag{13}$$

The length of the chord replacing the involute between the points $j – 1$, $j + 1$ is calculated in the following way:

$$l_{kj} = 2\rho_{kjh} \sin \varepsilon_{kjh} = \text{const}, \qquad (14)$$

where:

$$S_{kj} = \left| \frac{mz_k}{4} \left(\frac{1}{\cos^2 \alpha_{tkj}} - \frac{1}{\cos^2 \alpha_{tk,j+1}} \right) \cos\alpha \right|,$$

$$\alpha_{t1,j+1} = \arctan\left(\tan\alpha_{t10} + (j+1)\Delta\varphi\right),$$

$$\alpha_{t2j} = \arccos\left[\left(r_{w2}/r_{2j}\right)\cos\alpha_w\right],$$

$$\alpha_{t2,j+1} = \arccos\left[\left(r_{w2}/r_{2,j+1}\right)\cos\alpha_w\right].$$

The linear wear h'_{kjn} of the teeth at every j-th point of their profile is calculated in this case after every block in the time t'_{jh} of their engagement. Accordingly,

$$h'_{kjn} = \frac{v_j t'_{jh} \left(f p_{jh\max} \right)^{m_k}}{C_k \left(0.35 R_m \right)^{m_k}}, \qquad (15)$$

where $t'_{jh} = 2b_{jh}/v_0$.

The total wear h_{1jn} and h_{2jn} of the gear teeth at the j-points j of their profiles for a selected number of pinion rotations n_{1s} or gear rotations n_{2s} is determined by the following formulas:

$$h_{1jn} = \sum_{1}^{n_{1s}} h_{1jB}, \; h_{2jn} = \sum_{1}^{n_{2s}} h_{2jB}, \qquad (16)$$

where: $n_{2s} = n_{1s}/u$; $h_{kjB} = \sum h'_{kj}$.

Given the change in the type of engagement and in the initial maximum contact pressures $p_{j\max}$ due to tooth wear, the gear durability $t_{B\min}$ for the revolutions n_{1s} or n_{2s} of the gear wheels is calculated:

$$t_{B\min} = n_{1s}/60n_1 = n_{2s}/60n_2. \qquad (17)$$

NUMERICAL SOLUTION

The input data included: z_1 = 20; z_2 = 80; m = 3 mm; u = 4; n_1 =700 rpm; P = 5 kW; f = 0.05; β = 10°; K_g = 1.6. The following materials were used: the pinion was made of 38HMJA steel after nitriding with 58 HRC; R_m = 1040 MPa, C_1 = 3.5·10^6, m_1 = 2; the gear was made of 40H steel after bulk heat treatment with 53 HRC, R_m = 981 MPa, C_2 = 0.17·10^6, m_2 = 2.5; E =2.1·10^5 MPa. Lubrication involved the use of an oil described by the kinematic viscosity $v_{+50^o} \approx 15$ cSt; h_{k*} = 0.5 mm; $\Delta\varphi$ = 4°. The profile correction coefficients were: $x_1 = -x_2$ = 0; 0.2; 0.4; 0.6; 0.8; a_w = 152.314 mm, α_t = 20.283°. Calculations were done using a block calculation method from the quantity of the interaction block B = 2100000.

The results of the numerical solution are given in the figures below. Figure 1a shows the maximum initial contact pressures $p_{j\max}$ occurring during triple-double-triple tooth engagement when the wheel width is b_W = 54.275 mm, while Figure 1b illustrates the variations in their $p_{jh\,max}$ caused by the tooth wear h_{2*} = 0.5 mm.

The contact pressures $p_{j\max}$ are the highest at the start of double tooth engagement with the exception of the case when $x_1 = -x_2$ = 0. During the gear operation, the highest tooth wear can be observed at the start of triple engagement, which leads to a decrease in the pressures $p_{j\max}$ even by two times ($x_1 = -x_2$ = 0). With increasing the profile correction coefficients, the difference between $p_{j\max}$ and $p_{jh\,max}$ decreases or is maintained to the minimum ($x_1 = -x_2$ = 0.8).

Figures 2 and 3 illustrate the linear wear of the gear h_{2j} and the pinion h_{1j}.

The gear teeth are the first to reach the maximum allowable wear at different characteristic points of tooth contact depending on the coefficient of profile correction at the start of triple engagement or at the end of the double engagement. Similar observations with respect to the points marking maximum wear can be made about the pinion teeth. Figure 4 illustrates the relationship between minimum gear durability and profile correction.

The maximum durability is exhibited by the gear with profile correction when $x_1 = -x_2$ = 0.2 – its durability is higher by 1.55 times than that of the gear without profile correction.

To determine the effect of gear wheel width on the type of engagement and the above contact and tribological parameters, two types of helical gear were tested: one described by the gear wheel width b_W = 30 mm and double-single-double engagement, and the other described by b_W = 54.275 mm and triple-double-triple engagement. Accordingly, Figure 5 shows the results of the maximum contact pressure $p_{j\max}$ for the two tested width of gear wheels.

As a result, increasing the gear width by 1.81 times leads to a nearly proportionate de-

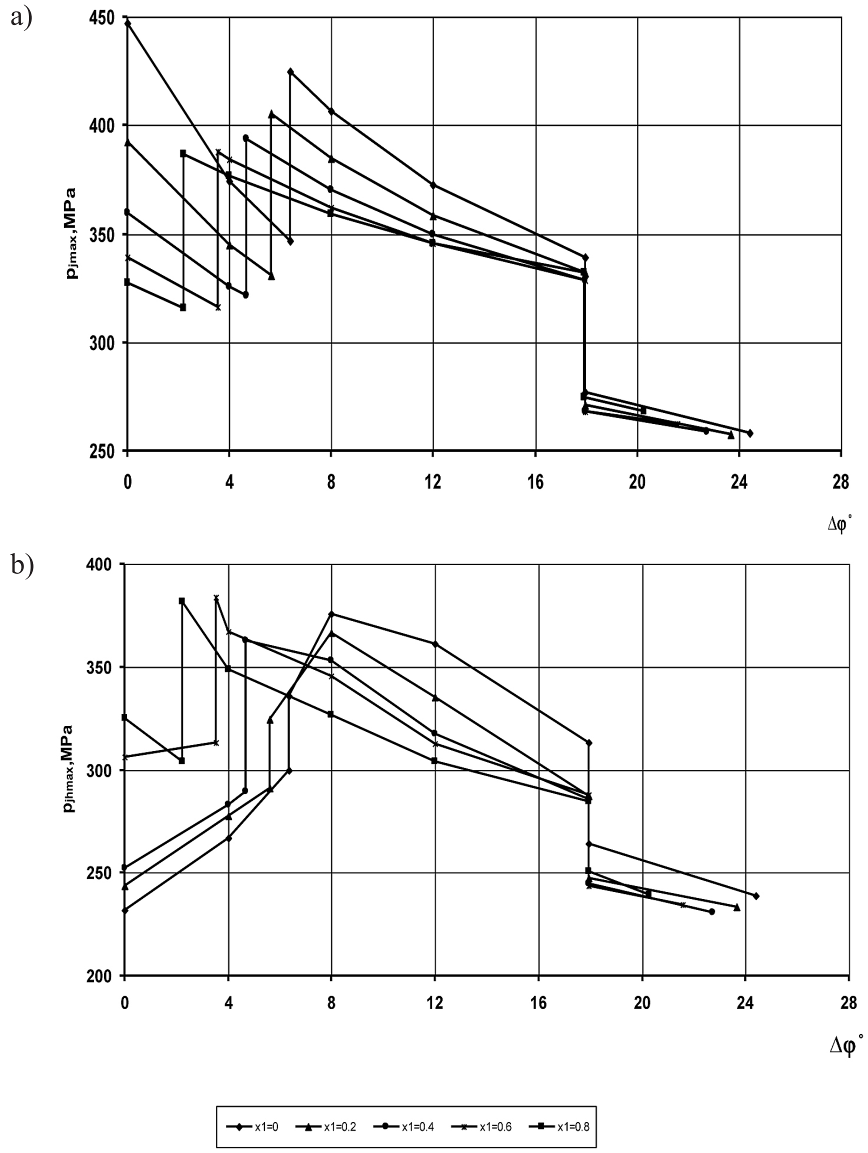

Fig. 1. Variations in the initial contact pressures $p_{j\max}$ during the meshing of teeth due to wear

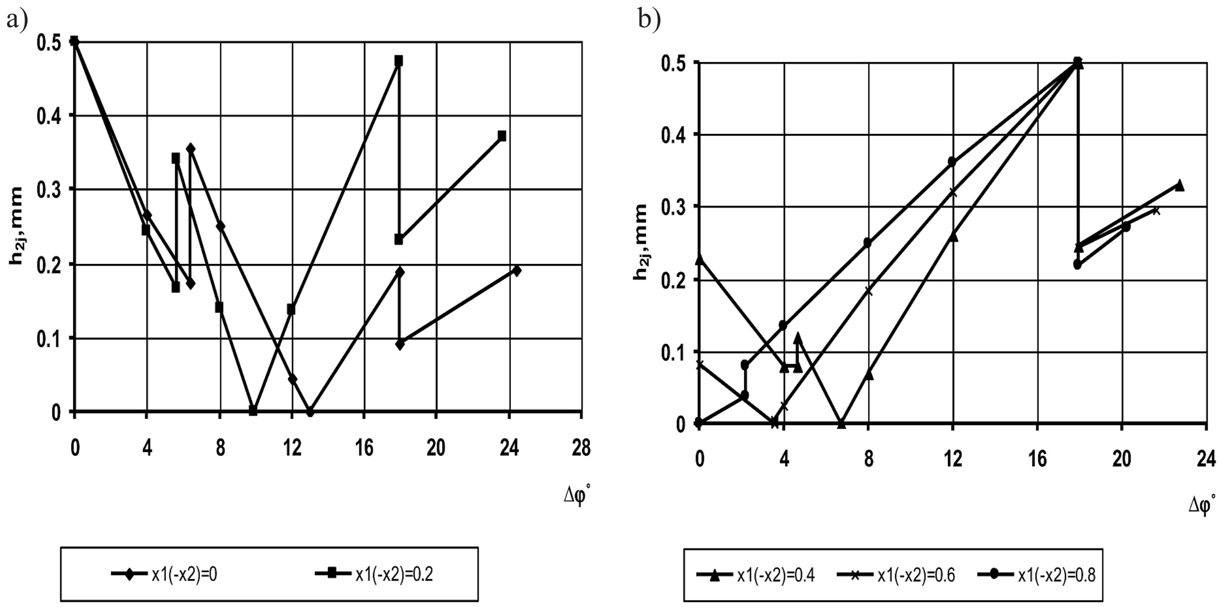

Fig. 2. Linear wear of the gear teeth along their profile

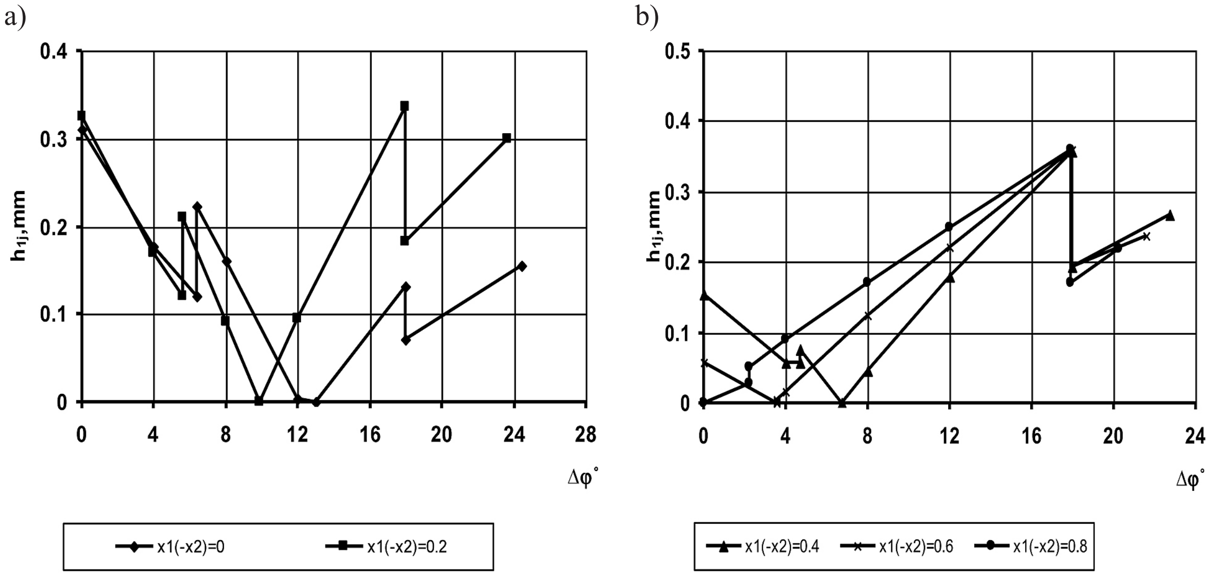

Fig. 3. Linear wear of the pinion teeth along their profile

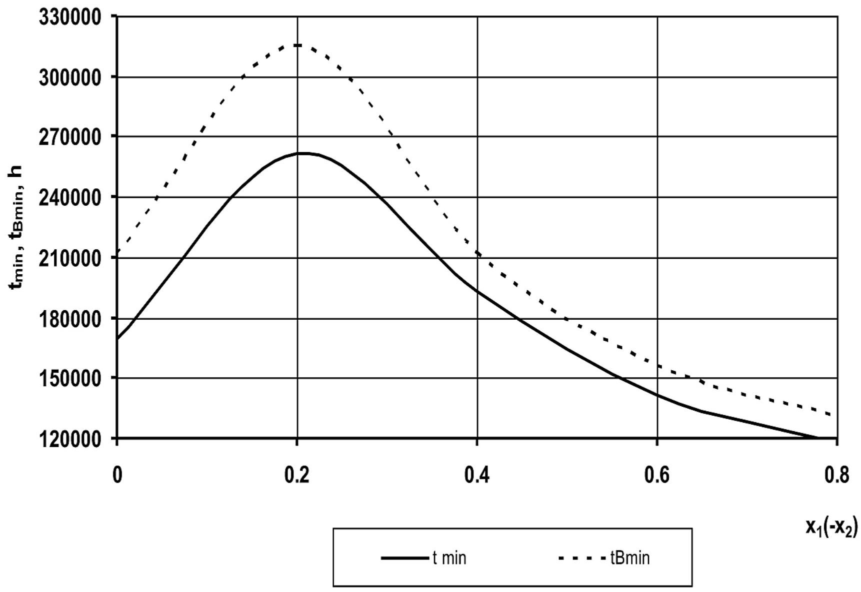

Fig. 4. Minimum durability of the gear with profile correction: solid line – $t_{\min}$ when $p_{j\max}$ = const, broken line – $t_{B\min}$ when $p_{jh\max}$ = **var**

crease (by 1.92 ÷ 1.8 times) in the initial contact pressures at the start of the engagement zone and by 1.95 times at the place where the type of engagement changes. The minimum durability of the gear is significantly more affected by the width of the gear wheels and the type of engagement, which is illustrated in Figure 6. In the case of the gear without profile correction, the minimum gear durability increases by over 10 times, and when the optimal values of the correction coefficients are applied $x_1 = -x_2$ = 0.2 – by 9.6 times.

CONCLUSIONS

1. We determined the optimal gear width in helical cylindrical gears which ensures that the length of the contact line is maintained constant.
2. Using a new method for the determination of wear and durability of helical gears, depending on the profile correction and the type of engagement, a numerical solution block method was proposed to the problem of determining the maximum contact pressures, linear wear of the teeth and durability of the gear.

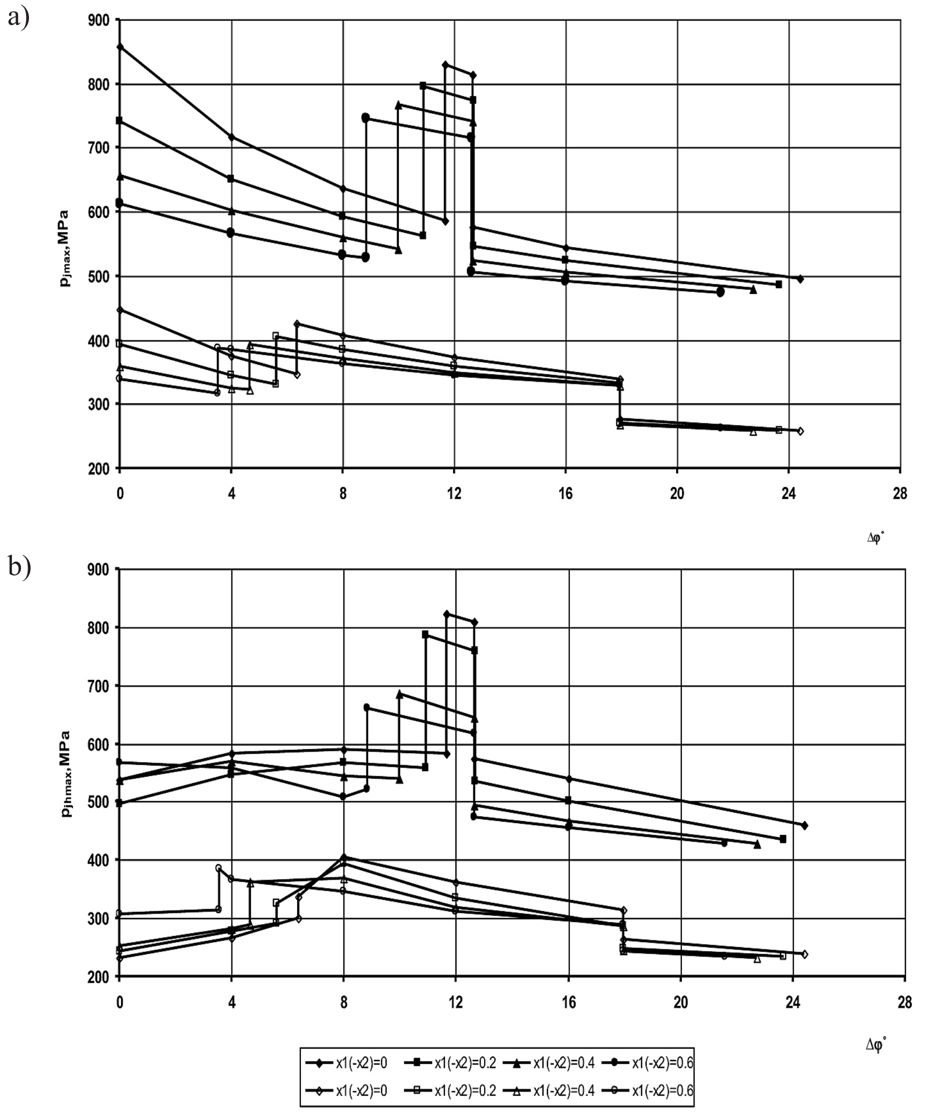

Fig. 5. Maximum contact pressures (a) and their variations (b) due to tooth wear: b_W = 30 mm (top), b_W = 54.275 mm (down)

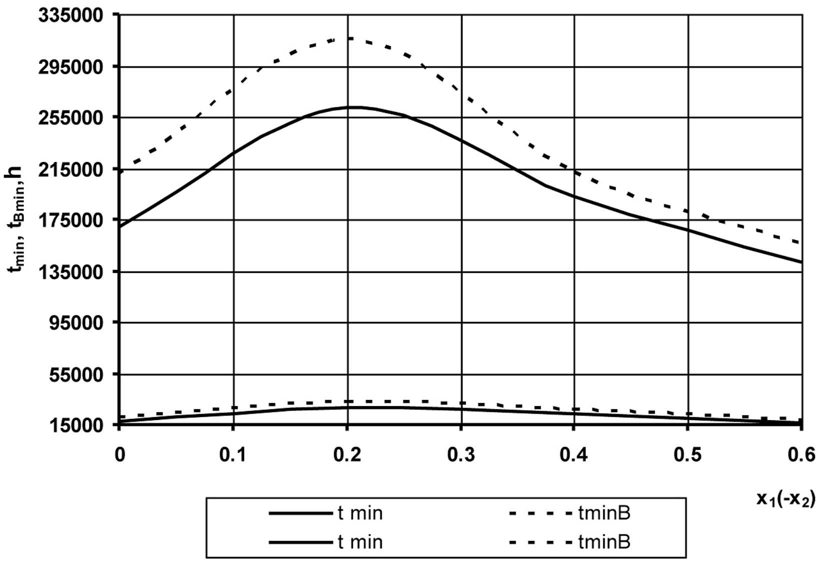

Fig. 6. Profile correction versus minimum gear durability when b_W = 54.275 mm (top), b_W = 30 mm (down)

3. The study was performed on a helical gear with optimized gear width ensuring a constant meshing force and on a helical gear with decreased wheel width.
4. It has been found that the increase in the gear width in the range between 30 and 54.275 mm results in an almost proportionate decrease in the maximum contact pressures.
5. The increase in the gear width by 1.81 times leads to a significant increase in the minimum gear durability – by 10.24÷8 times, depending on the applied coefficients of profile correction.

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