

ELIMINATION OF THERMAL DRIFT IN MEASURING THE POSITIONING ACCURACY OF A THREE AXIS MILLING MACHINE

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ABSTRACT

The aim of this paper is to draw attention to more reliable verification method of positioning accuracy. The improvement lies in mathematical elimination of thermal impact during the measurement process. This thermal impact, always present during the motion of movable parts, is of a special systemic character. It increases the indefiniteness of measurement of the classic measurement method. This measurement uncertainty can be reduced by implementing the procedure introduced in this paper. The reduction can be achieved by separating the temperature impact from other sources of inaccuracy. Such separation is a very new solution. The methodology relies on mathematical processing and does not depend on the manner of accuracy measurement. The evaluation method also yields diagnostic information on the machine's condition. The experiments were performed with the use of laser interferometer.

Keywords: accuracy, laser interferometer, positioning, machine tool.

INTRODUCTION

The overall positioning accuracy of the moving carriage of a machine tool is determined by many factors, of which proportional impact on the final accuracy can be represented differently. These factors can be detected from the machine structure, thermal fields of the measured area and positioning of interferometer's components. Typical sources of positioning inaccuracy are kinematic errors, thermal-mechanical errors, loads, dynamic powers, motions control and control software [1]. The accuracy may also be defined on the basis of technological conditions and the shape of the machined material [2]. The issue of dynamic Tool-center Point (TCP) adjustment is further dealt with in [3].

The positioning accuracy is significantly influenced by temperature. Thermal impact during the machining can be simulated by a mathematical model. Simulation may determine places where TCP displacement can be minimized by means of structural changes [4].

Time duration of the measurement significantly influences relative accuracy of measured positions. Reduction of measurement time is not important only economically. Shorter measurement time decreases thermal deformations and indirectly increases measurement accuracy. Reduction of measurement time is nowadays one of the main trends the research teams are concerned with. Measuring time can be saved by the method of dynamic calibration of production machines. More details are provided by Castro and Burdekin [5]. Articulated Arm Coordinate Measuring Machine (AACMM) system uses increase in number of measured positions over shorter time [6]. Positioning accuracy is also influenced by the direction of impacting forces. This is a rule applicable to machinery in general. The topic is discussed in [7, 8, 9, 10, 11]. Experiment planning can also significantly contribute to saving time in obtaining suitable results [12]. The principle of indirect measurement is based on measurements taken by a laser tracker [13, 14].

New way of increasing the precision and speed of measurement is deployment of a stereo camera system in position measurement system for levitated motion stages. In this case, kinematic errors are done away with and so are the structural errors due to thermal deformation. We speak about direct measurement between two fixed points in space [15].

Overall kinematic errors of multi-axial machining tools are better detectable with a 3D probe-ball attached to the spindle. This also constitutes a direct measurement of the tool's precision [16, 17].

If five-axis machine tools are dealt with, calculation of geometric errors heavily depends on the calculation formula. The new method of calculation yields better results in offsetting geometric errors. The method is based on gradual offset of errors of the rotating axis, with subsequent offset of errors of the linear axis. [18].

Various research cases confirm that spatial errors result from complex influence of various factors and it is more suitable to measure them directly rather than calculate them from partial error sources. Evaluation of geometric errors present in multi-axial tools is researched in [19, 20, 21, 22, 23, 24]. Nevertheless, measuring only a single axis offers significant information on a tool's condition, as well as on the accuracy of its positioning. Again, we are dealing here with a system changing in time, but it offers a better option of separating individual error types than the spatial model. Measurement uncertainty calculations are elaborated on in [25]. Volumetric calibration of large tools is conveniently done by deploying a laser tracker. Analysis of measurement uncertainties has significantly increased calibration precision. The methodology makes use of the Monte Carlo evaluation. The analysis was based on evaluating sources of uncertainty, especially the thermal drift and recurrence [26]. The greatest progress is achieved by a measurement that adjusts the tool's movement simultaneously with using the data from laser tracker in real time [27].

Temperature impact analysis is important not only during the machining but also during the measurement process of positioning accuracy. Inaccurate error map determination is a primary source of the overall work inaccuracy of highly accurate production machines. This paper points out that wrong accuracy assessments may occur even while observing the measurement rules.

Laser interferometry is one the most accurate

distance measurement methods. Ordinary working procedure of positioning measurement is performed in accordance with the ISO 230-2 [28] standard. Our experimental positioning accuracy measurements were not focused on standardized procedures in accordance with the ISO standard. We used measuring procedures beyond the mentioned standard.

MATERIAL AND METHODS

Positioning accuracy of the machine aims to eliminate systemic errors during the positioning. Current measurement results are a part of the machines' acceptance report, in which the producer presents the relevant machine accuracy parameters. Certain conditions for tests concerning the environment, the tested machine and heating must be met to duly determine the positioning accuracy parameters.

Brief description of standard procedure

Standard procedure defines the change of position between the part carrying the tool and the part carrying the workpiece. General formula for target position is as follows:

$$P_i = (i-1)p + r \quad (1)$$

where i is the number of the current target position, p is an interval based on a uniform spacing of target points over the measurement travel, r is random number in \pm amplitude of possible periodic error.

At least 5 positions must be chosen for tests in linear axis up to 2000 mm and at least 5 repetitive starts of the testing cycles. Assessment of the results is given by the calculation of deviation boundaries (2) and (3). The symbol \uparrow signifies a parameter derived from a measurement made after an approach in the positive direction, and \downarrow one in the negative direction.

$$\bar{x}_i \uparrow + 2s_i \text{ and } \bar{x}_i \uparrow - 2s_i \quad (2)$$

$$\bar{x}_i \downarrow + 2s_i \text{ and } \bar{x}_i \downarrow - 2s_i \quad (3)$$

where \bar{x}_i [μm] is an average of unidirectional positional deviation, s_i [μm] is estimation of standard positioning deviation.

The next test log shall include the highest value of environmental temperature gradient in degrees per hour over 12 hours before measurement and during the measurement, in addition to

the following items: the feed rate between target positions, the position of the measurement travel, the dwell time at each target position and the location of first and last target positions.

The most important output parameters are reversal value of the axis B_i [μm], bidirectional repeatability of the R_i [μm] positioning, the range of the mean bidirectional positional deviation M [mm], bidirectional systemic positional deviation E [mm] and bidirectional accuracy of positioning A [mm].

New procedure

Great number of repetitive measurement cycles are performed contrary to the standard procedure. This reveals all sources of inaccuracy. The main sources are progressive thermal deformations which change the result of repetitive measurements. Thermal change shows a special systemic character of an error. The systematic error is not constant, but changes by a certain value with every cycle. This change is detectable from the sequence of multiple measurements.

Thermal trend can be eliminated from other inaccuracy components by application of the suggested mathematical procedure.

The main principle lies in the fact that the least thermal change occurs between two consecutive position measurements. In that case, the systemic error caused by the change of temperature is minimal. Most clearly demonstrated shall be the random error component. For this reason, the average mutual deviation \bar{x}_i is determined from two neighbouring positions x_i and x_{i+1} in formula (4):

$$\bar{x}_i = \frac{\sum_{j=1}^n (x_{i,j} - x_{i+1,j})}{n} \tag{4}$$

Calculation of sample standard deviation between two adjacent positions s_i (6) is based on the standard deviation formula (5).

$$\bar{s} = \sqrt{\frac{\sum_{i=1}^n (\bar{x} - x_i)^2}{n(n-1)}} \tag{5}$$

$$s_i = \sqrt{\frac{\sum_{j=1}^n (\bar{x}_i - (x_{i,j} - x_{i+1,j}))^2}{n(n-1)}} \tag{6}$$

where n is the number of repetitive measurements, j is the serial number of a repetitive mea-

surement and i is the serial number of a position. The first reference position does not count. For example, we have 15 positions for calculating $i = 2, 3, \dots, 16$ out of 16 measured positions.

In formula (4), the mean is calculated from derivative sequence of random values. In general, we work with the sequence $\{a_n\}_{n=1}^\infty = \{a_1, a_2, a_3, \dots\}$, from which we get the sequence $\{b_n\}_{n=1}^\infty = \{b_1, b_2, b_3, \dots\}$. If there is one random distribution, the derivate sequence is obtained by subtracting the subsequent values from the previous ones (7).

$$b_n = a_n - a_{n+1} \tag{7}$$

If the value occurrence probability is normal, the sequences a_n and a_{n+1} are independent of each other. These sequences have the same dispersion (8).

$$s_a^2 = s_{a+1}^2 = s^2 \tag{8}$$

Dispersion D_b of the sequence b_n is a sum of dispersions of independent random sequences a_n and a_{n+1} [29]:

$$D_b = D_a + D_{a+1} = 2s^2 = s_b^2 \tag{9}$$

then:

$$\frac{s_b}{s_a} = \sqrt{2} = 1.414 \tag{10}$$

The result has been verified by the Monte Carlo method on 1000 random samples generated by the random numbers generator. A standard deviation ratio from the Monte Carlo experiment is 1.4076. Such match is a good one. Confidence interval of derived sequence b_n is 1.4 times greater (Fig. 1).

The Monte Carlo method is applied to evaluate measurement uncertainty in spatial applications [30]. Simulation methods are discussed in [31]. In this case, it is used to evaluate uncertainty

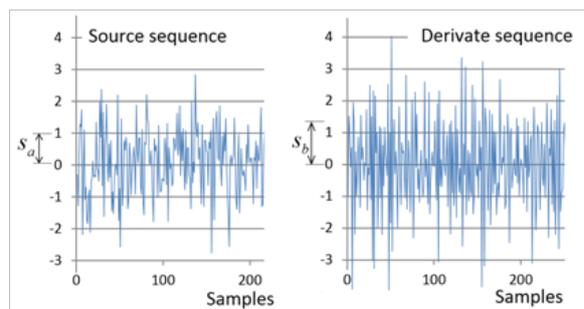


Fig. 1. Normal distribution of source and derivate sequence

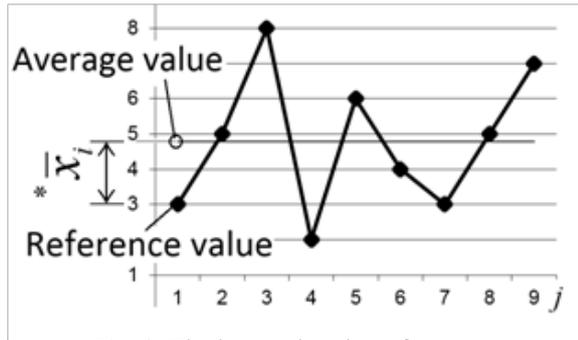


Fig. 2. The best estimation of average mutual deviation

in two subsequent measurements under the assumption that the errors are distributed normally. Therefore, it is the evaluation of the statistical, not the measuring method.

Deriving a sequence from two different sequences is an analogous procedure. For calculation of the average mutual and the sample standard deviation, the (4) and (6) formulas are applied respectively. In general, two random value distributions need not have equally large standard deviations. In distribution with greater standard deviation, the ratio of standard deviation in derived sequence and standard deviation in original sequence shall continue to be less than 1.4. This fact can also be reliably verified by the Monte Carlo method.

The best estimate of average deviation is achieved by adding the final value of the mutual average deviation (4) to the values measured in the first cycle (for $j = 1$). The explanation of this phenomenon is illustrated in Figure 2.

This is the most important step in the calculation. Common method compares deviations between the same points after running all other

measured points. Significant thermal changes occur within the scope of one cycle. Each measured point is thus significantly influenced by the systemic component of thermal deformation. This component cannot be mathematically separated from the random component. Thermal error thus counts in determining positioning accuracy and can show high values compared to other components of resulting positioning error.

To establish normal distribution parameters, at least 30 measurements of a single position should be performed. Applying the formulas no. (4), (5) and (6) respectively on a smaller number of measurements is incorrect. In case of a lesser number of measurements, the correction of measurement uncertainty by Student distribution must be applied. In this case, the confidence interval increases. Standard ISO 230-2 does not account for this fact at all.

The following graphic model of position accuracy detection is presented to emphasize the novelty of the method (Fig. 3).

EXPERIMENT SEQUENCE

Analysis of particular impacts on positioning accuracy was performed on the school's milling machine in two perpendicular plane axes X and Y. A Renishaw laser interferometer XL 80 was used for measuring the accuracy.

The aim was to find out if it is possible to go as far as to retroactively analyse the next additional impacts of positioning accuracy on the basis of increased repetitive measurement. Measurements of thermal fields in the upper part of the table were performed to determine the im-

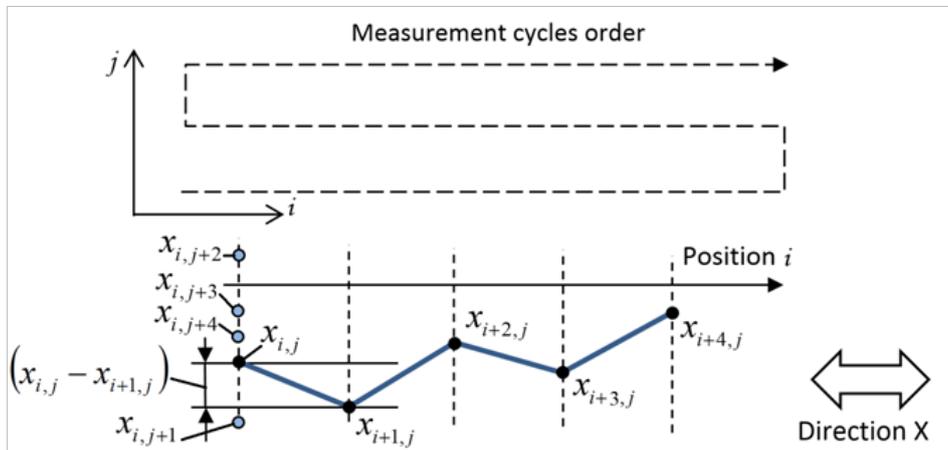


Fig. 3. Positioning model

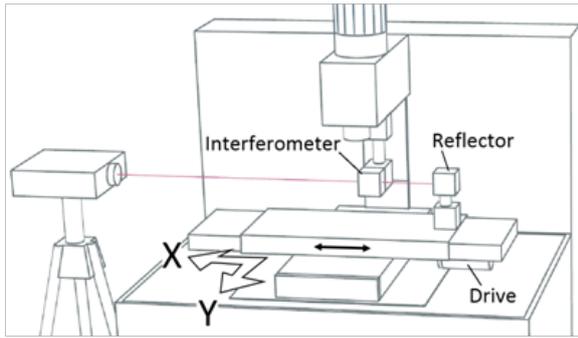


Fig. 4. First measurement arrangement (axis X, 100 cycles)

pacts, with the number of measurement repetitions in one position between 50 and 100. Conditions were changing for particular measurements, which made it possible to eliminate or calculate specific sources of inaccuracy.

ORDER OF MEASUREMENTS

The number of repetitive cycles of the first measurement was 100, which corresponds to approximately seven hours of measuring. The average ambient temperature was 24°C, the speed of the feed to the position was 300 mm/min, dwell time in the target positions was 2 seconds. The range of measured axes was determined by the maximum motion range. I.e. 150 mm in direction of axis Y and 300 mm in direction of axis X. 16 positons were measured on each axis (Fig. 4) and (Fig. 5).

Displayed results of positioning measurement over the large number of cycles show typical thermal feeds that bring systematic error into the overall measurement. Figure 6 shows clearly visible trend of measured positions feed in space.

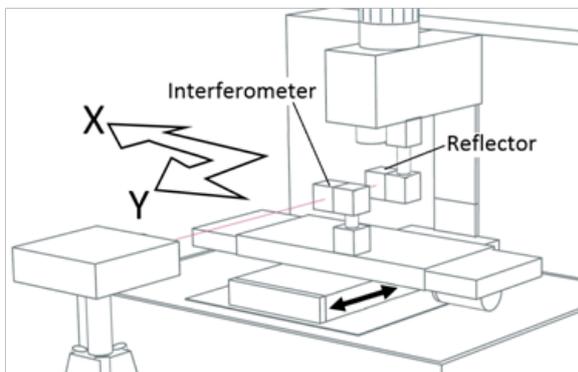


Fig. 5. Second measurement arrangement (axis Y, 50 cycles)

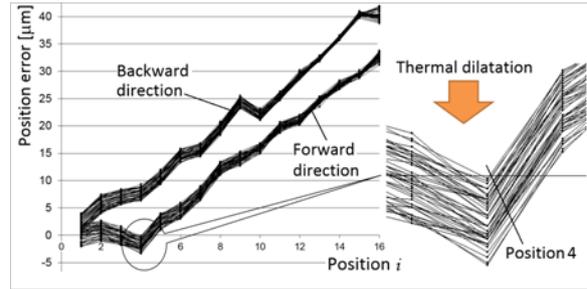


Fig. 6. First measurement deviation chart after 100 cycles

It is necessary to say that thermal dilata-tions in particular positions of a moving part do not have unified direction in either space or time. A measured positon can sometimes move forward in space and after some time it changes its direction the other way (Fig. 7). These unpredictable changes are caused by the machine structure and by thermal sources in the machine. Thermal sources have various courses of temperature balancing. It is a considerably complicated situation. Total temperature balancing does not occur even in thermally compensated systems. Process of changes lasts for many hours even under steady external conditions. Classic measuring method cannot eliminate these changes, so they are added to measurement errors. This alone does not pose a substantial practical problem. The problem is that a measurement result depends on the measurement time and the number of measurement repetitions. Range of errors will be small in the small number of measurements. In such case, the measurement is insufficient for determining random components. Only systemic components and thermal dilatation are demonstrated.

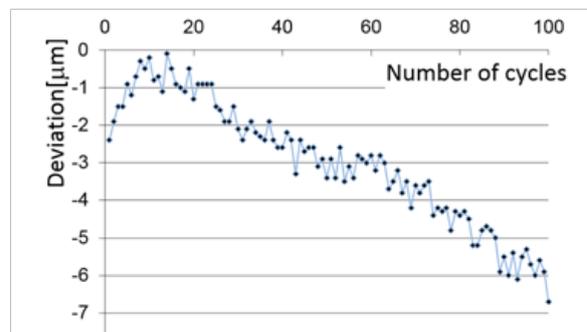


Fig. 7. Example of a change in the measured error in a single positon during 100 cycles (forward direction, positon 4, axis X)

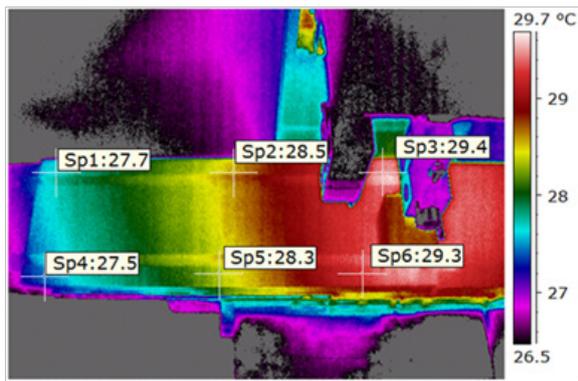


Fig. 8. Thermal fields of the table after 100 measurement cycles in the X axis direction (plan view)

The new method of measurement and evaluation proposed here reduces the influence of temperature and better determines the random components. It is a more time-consuming but nevertheless a more reliable method.

Not even the thermal fields of a machine show the real motion of the positioning point in space. This also depends on the machine's structure. The thermal fields of the milling machine table were observed throughout all measurements. Their changes influenced the change in thermal dilatations. However, this is not a precondition. Thermal dilatations occurring in the remotest parts of the machine can nonetheless influence positioning accuracy.

For example, differences in the thermal fields of the table are clearly visible in direction of the X axis after 100 cycles (Fig. 8). Changes in the thermal fields of the table were not demonstrated in measurements in the Y axis direction. Despite the fact, thermal dilatations did occur (Fig. 9). The thermal source of the engine was placed further from the table in this case.

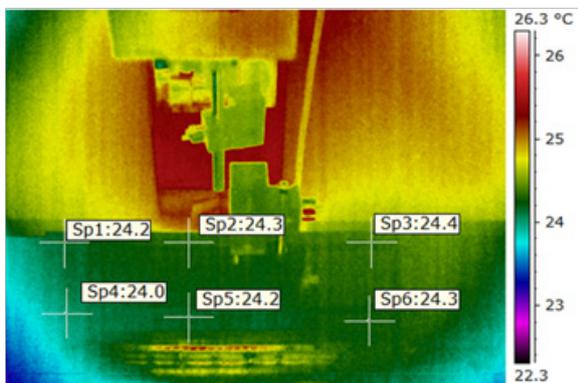


Fig. 9. Thermal fields of the table after 50 measurement cycles in the Y axis direction (plan view)

A measurement was performed with the use of a FLIR SC-660 thermal imaging camera (FLIR, Sweden). Special colour was used to obtain a homogeneous emissivity of the table surface, which was measured at 0.97. Referential thermal differentiation is 0.05°C.

THERMAL TREND ELIMINATION

Thermal trend determination draws on the idea that changes in deviation between two adjacent positions show systemic error, least influenced by the change in temperature. This is due to the fact that the distance between adjacent positions is the shortest one and the time between two measurements is also the minimum time. Group of pair deviations between adjacent positions during repetitive cycles can determine the relative systemic error between positions least influenced by the thermal trend. Estimation of the common uncertainty is determined in a similar way; it is, too, least influenced by the thermal trend.

Deviation chart can be created by choosing a reference value for the first position, corresponding with some measurement time. Position of this point is given by the current temperature changes occurring just over the given time. It is suitable to choose some value after at least two hours of measurements, when it is possible to predict less intense changes in thermal dilatations of components.

Points assumed in other positions, obtained by the statistical analysis, will be used to design reference values of the first cycle (Fig. 12).

Figure 10 shows sample standard deviation in particular positions, obtained by application of the formula (6) in the forward direction. We can see greater error in position 3.

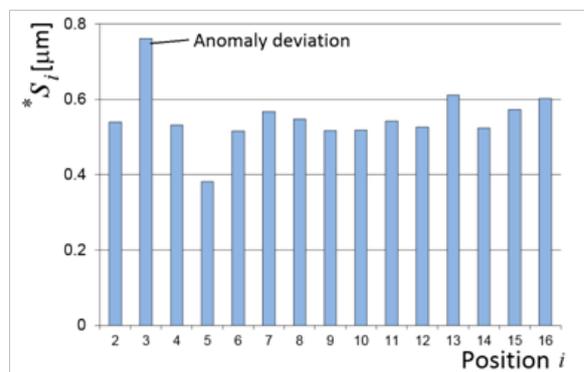


Fig. 10. Sample standard deviations between two adjacent positions

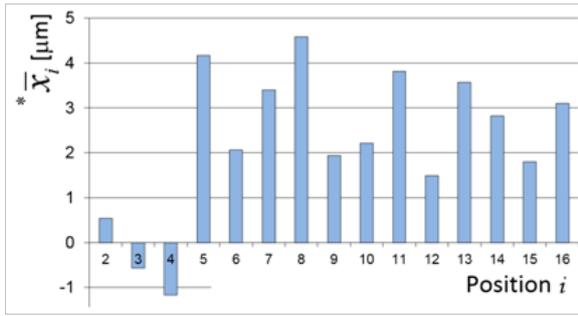


Fig. 11. Average mutual deviation between two adjacent positions

This anomaly is caused by the local temperature impact or by some structural error. Thus the thermal trend elimination method can be used for troubleshooting purposes. Designed deviation chart will correspond with specific thermal conditions at given time only in case an anomaly occurs at some position. This anomaly does not show when classic measurement procedure is followed.

Change in the average mutual deviation is represented in Figure 11. These values shall be used to correct the deviation chart (calculated deviation) (Fig. 12).

PRINCIPLE OF DRAWING THE DEVIATION CHART

The obtained deviation chart is the result of progressive addition of relative deviations to the reference value (Fig. 12). Deviation borders can be also roughly estimated according to formula (6). They are shown in dashed lines $\pm 2s$. Since the deviations are relative, certain influencing by the adjacent values is present. However, this estimation is still better than ignoring the thermal trend.

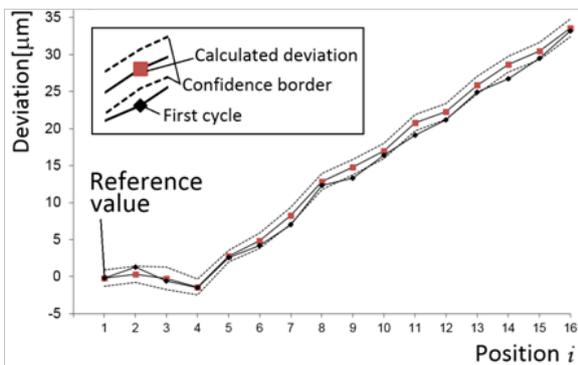


Fig. 12. Deviation chart of average values in relation to reference position 1

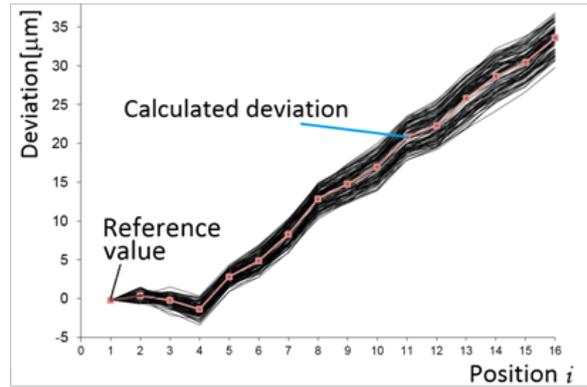


Fig. 13. Deviation chart in relation to reference position 1 after 100 cycles

Therefore, values of positions 1 and 2 respectively are the same. For next positions, results based on the formula (6) will be used.

The value of a standard deviation calculated by standard procedure according to formula (5), for example, in position 4 in the forward movement equals $s=1.64$ mm. This value is 3.09 times greater than the value calculated by the new method. This is a marked difference in spite of the fact that the novel method applies a confidence interval expanded 1.4 times.

Figure 13 illustrates expansion of value distribution over seven hours of measure taking when thermal drift is not eliminated. All cycles are shifted to the common reference point in the first position.

CONCLUSION

The current trend aims to reduce measuring time. This fact, of course, improves measurement results. On one hand, more measurements can be performed. On the other hand, the overall time becomes shorter, so that there is no significant demonstration of changes in temperature. The time constraint is entirely due to positioning reasons. Higher positioning speed translates into greater dynamic load. That worsens positioning accuracy. Therefore, it is desirable to increase the number of measurements and thus achieve greater accuracy of position determination through application of the new method.

This method of measurement is especially suitable for high precision machines, where systemic errors are almost on par with random errors. Less accurate machines with greater systemic errors can continue to be measured in a standard way.

New methodology of measurement and evaluation of positioning accuracy can quite effectively reveal the errors of machine structure and installation. Sudden large deviations in values that shall eliminate thermal impacts point to a hidden machine failure. This type of errors cannot be revealed in course of commonly applied measurement procedure.

Time-dependent sources of random positioning errors, such as vibrations, were below one tenth of the micrometer.

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REFERENCES

1. Chen, X. B., Geddam, A., & Yuan, Z. J. Accuracy improvement of three-axis CNC machining centers by quasi-static error compensation. *Journal of Manufacturing Systems*, 16(5), 1997, 323-336.
2. Král, J. jr. Král, J. Verification of a three axis milling machine accuracy in the process of complex shaped part production. In: *Applied Mechanics and Materials* (Vol. 474, 2014, 261-266). Trans Tech Publications.
3. Lin, M. T., & Wu, S. K. Modeling and improvement of dynamic contour errors for five-axis machine tools under synchronous measuring paths. *International Journal of Machine Tools and Manufacture*, 72, 2013, 58-72.
4. Mayr, J., Ess, M., Weikert, S., & Wegener, K. Comparing different cooling concepts for ball screw systems. In: *Proceedings ASPE Annual Meeting*. 2010, 978-981.
5. Castro, H. F. F., & Burdekin, M. Dynamic calibration of the positioning accuracy of machine tools and coordinate measuring machines using a laser interferometer. *International Journal of Machine Tools and Manufacture*, 43(9), 2003, 947-954.
6. Acero, R., et al. Verification of an articulated arm coordinate measuring machine using a laser tracker as reference equipment and an indexed metrology platform, *Measurement*, 69, 2015, 52-63.
7. Debski, H., Teter, A., Kubiak, T., & Samborski, S. Local buckling, post-buckling and collapse of thin-walled channel section composite columns subjected to quasi-static compression. *Composite Structures*, 136, 2016, 593-601.
8. Jachowicz T.: Construction of clamping units of injection molding machines. *Polimery*, 50 (2), 2005, 110-117.
9. Molnár, V., Fedorko, G., Stehlíková, B., Tomášková, M., & Hulínová, Z. Analysis of asymmetrical effect of tension forces in conveyor belt on the idler roll contact forces in the idler housing. *Measurement*, 52, 2014, 22-32.
10. Molnár, V., Fedorko, G., Stehlíková, B., Kudelás, L., & Husáková, N. Statistical approach for evaluation of pipe conveyor's belt contact forces on guide idlers. *Measurement*, 46(9), 2013, 3127-3135.
11. Molnár, V., Fedorko, G., Stehlíková, B., Michalik, P., & Weiszer, M. A regression model for prediction of pipe conveyor belt contact forces on idler rolls. *Measurement*, 46(10), 2013, 3910-3917.
12. Parkinson, S., & Longstaff, A. P. Multi-objective optimisation of machine tool error mapping using automated planning. *Expert Systems with Applications*, 42(6), 2015, 3005-3015.
13. Aguado, Sergio, et al. Improving a real milling machine accuracy through an indirect measurement of its geometric errors. *Journal of Manufacturing Systems*, 40, 2016, 26-36.
14. Schwenke, H., Franke, M., Hannaford, J., & Kunzmann, H. Error mapping of CMMs and machine tools by a single tracking interferometer. *CIRP Annals-Manufacturing Technology*, 54(1), 2005, 475-478.
15. Lu, X., & Rao, N. Six-axis position measurement system for levitated motion stages. *CIRP Annals-Manufacturing Technology*, 62(1), 2013, 507-510.
16. Lei, W. T., & Hsu, Y. Y. Accuracy test of five-axis CNC machine tool with 3D probe-ball. Part I: design and modeling. *International Journal of Machine Tools and Manufacture*, 42(10), 2002, 1153-1162.
17. Lei, W. T., & Hsu, Y. Y. Accuracy test of five-axis CNC machine tool with 3D probe-ball. Part II: errors estimation. *International Journal of Machine Tools and Manufacture*, 42(10), 2002, 1163-1170.
18. Hsu, Y. Y., & Wang, S. S. . A new compensation method for geometry errors of five-axis machine tools. *International Journal of Machine Tools and Manufacture*, 47(2), 2007, 352-360.
19. Fu, G., Fu, J., Xu, Y., Chen, Z., & Lai, J. Accuracy enhancement of five-axis machine tool based on differential motion matrix: geometric error modeling, identification and compensation. *International Journal of Machine Tools and Manufacture*, 89, 2015, 170-181.

20. He, Z., Fu, J., Zhang, L., & Yao, X. A new error measurement method to identify all six error parameters of a rotational axis of a machine tool. *International Journal of Machine Tools and Manufacture*, 88, 2015, 1-8.
21. Chen, D., Dong, L., Bian, Y., & Fan, J. Prediction and identification of rotary axes error of non-orthogonal five-axis machine tool. *International Journal of Machine Tools and Manufacture*, 94, 2015, 74-87.
22. Chen, J. X., Lin, S. W., & He, B. W. Geometric error measurement and identification for rotary table of multi-axis machine tool using double ballbar. *International Journal of Machine Tools and Manufacture*, 77, 2014, 47-55.
23. Ding, S., Huang, X., Yu, C., & Liu, X. Identification of different geometric error models and definitions for the rotary axis of five-axis machine tools. *International Journal of Machine Tools and Manufacture*, 100, 2016, 1-6.
24. Lee, J. H., Liu, Y., & Yang, S. H. Accuracy improvement of miniaturized machine tool: geometric error modeling and compensation. *International Journal of Machine Tools and Manufacture*, 46(12), 2006, 1508-1516.
25. Palencar, R., Duris, S., & Ranostaj, J. Conclusions and some comments on the calculation of uncertainty when constructing a temperature scale. *Measurement Techniques*, 54(8), 2011, 910-920.
26. Liu, Y., Gao, D., & Lu, Y. Volumetric calibration in multi-space in large-volume machine based on measurement uncertainty analysis. *The International Journal of Advanced Manufacturing Technology*, 76(9-12), 2015, 1493-1503.
27. Wang, Z., & Maropoulos, P. G. Real-time laser tracker compensation of a 3-axis positioning system – dynamic accuracy characterization. *The International Journal of Advanced Manufacturing Technology*, 84(5-8), 2016, 1413-1420.
28. ISO 230-2 Machine tools - Test code for machine tools - Part 2: Determination of accuracy and repeatability of positioning of numerically controlled axes.
29. Kay, S. M. *Fundamentals of statistical signal processing*. Prentice Hall PTR, 1993.
30. Andolfatto, L., Mayer, J. R. R., & Lavernhe, S. Adaptive Monte Carlo applied to uncertainty estimation in five axis machine tool link errors identification with thermal disturbance. *International Journal of Machine Tools and Manufacture*, 51(7), 2011, 618-627.
31. Honus, S., & Juchelková, D. Comparison of Numeric Methods that Simulate Energy Transfer by Radiation. In: *Applied Mechanics and Materials* (Vol. 260, 2013, pp. 605-610). Trans Tech Publications.