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Original Article

DETECTION OF CHANGES OF THE SYSTEM TECHNICAL STATE USING STOCHASTIC SUBSPACE OBSERVATION METHOD

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ABSTRACT

System diagnostics based on vibroacoustics signals, carried out by means of stochastic subspace methods was undertaken in the hereby paper. Subspace methods are the ones based on numerical linear algebra tools. The considered solutions belong to diagnostic methods according to data, leading to the generation of residuals allowing failure recognition of elements and assemblies in machines and devices. The algorithm of diagnostics according to the subspace observation method was applied – in the paper – for the estimation of the valve system of the spark ignition engine.

Keyworda: system diagnostic, subspace method, vibroacoustic.

INTRODUCTION

The methods of identification of dynamic models by means of subspace, known from the control theory [1–4]: N4SID (Numerical Algorithms for Subspace State Space System IDentification), MOESP (Multivariable Output-Error State Space), CVA (Canonical Variate Analysis) or the method of principal components analysis (PCA) are also used for diagnostics of technical objects.

The idea of identification by subspace methods is applied in structural health monitoring (SHM) [5]. The SHM range, aimed at improving a broadly considered safety and at decreasing maintenance costs, encompasses low-frequency methods originated from vibroacoustic diagnostics of machines, as well as high-frequency methods (the so-called non-destructive inspections).

The condition monitoring is used in the automotive and aircraft industry, power industry (mainly rotating machines) and in civil engineering [6–10]. The result of the vibroacoustic analysis is often a modal model in a form of a natural frequencies set, damping ratios and mode shapes used in various detection procedures of structural defects based on these parameters.

Algorithms of vibroacoustic diagnostics of mechanical objects by subspace methods lead to the determination of the signature (pattern) of the system in a serviceable state [11–14]. This approach requires neither knowledge of modal parameters nor finite elements method (FEM) in relation to the identification of the system state.

Both direct measuring data and covariances of these data or impulse characteristics of investigated systems can be used in constructing subspace matrices. As the result of the decomposition of the subspace observation matrix the vectors of residuals – assessed as ‘null’ in a nominal (serviceable) state – are obtained.

BASIC PROBLEMS OF STOCHASTIC FILTRATION OF EMPIRICAL DATA

The assessment method of vectors of residuals, by means of the theory of local approach to the statistic identification of changes in dynamic systems, is used in the hereby paper [15–17].

The vector of random variables with the observation space Y= $\{Y_1,\ldots,Y_N\}$, $Y_k \in R^m$, encompassing sampling periods k = 1,…,N in a form of measurement data collection (time window) and parameters space Θ generating residual $R_N(\theta \in \Theta)$ – is considered.

The subset of the parameters space in a nominal state is marked as Θ_0, while the subset of the parameters space in a damage state as Θ_1. In addition: $\Theta_0 \cap \Theta_1 = 0$ and $\Theta_0 \cup \Theta_1 = \Theta$.

It is assumed that the distance between the investigated models or experiments, represented by the error vector:

$$\delta\theta = \frac{const}{\sqrt{N}} \tag{1}$$

is sufficiently small when the observation data record *N* is large. The local approach to the identification of the nominal model means that the deviation of the error vector – determined by equation (1) – is of the order of $\frac{1}{\sqrt{N}}$.

The statistics of diagnostics parameters constructed on the basis of observation of the system in a serviceable state constitutes, in turn, grounds for formulating the zero hypothesis, which is tested against the alternative hypothesis, corresponding to small deviations of parameter values.

These assumptions allow to transfer the problem of detection of changes in the system into the plane of investigating the expected value of the residual in a form of random vector of the normal Gaussian distribution in a serviceable state as well as during further monitoring.

Applying generalized likelihood ratio (GLR) the rejection criterion of the verified hypothesis, in a form of the chi-square test with the assumed critical level for the given confidence interval, is obtained.

CONSTRUCTING THE SIGNATURE OF THE VIBRATING STRUCTURES

The model of the linear mechanical system – given by equations [18] – is considered.

$$M\ddot{q}(t) + D\dot{q}(t) + Kq(t) = f(t) \tag{2}$$

$$y(t) = Lq(t) \tag{3}$$

where: *M, D, K, L* are matrices of mass, damping, stiffness and monitored states filter – respectively, *q(t)* is the displacement vector, *f(t)* is the external forces vector (modelled in a general case as the non-stationary Gaussian white noise) and *y(t)* is the vector of the monitored states.

Modes of the system μ (representing natural frequencies and damping ratios corresponding to them) are the solution of the equation:

$$\det(M\mu^2 + D\mu + K) = 0 \tag{4}$$

while mode shapes:

$$\psi_\mu = L\varphi_\mu \tag{5}$$

satisfy the formula:

$$(M\mu^2 + D\mu + K)\varphi_\mu = 0 \tag{6}$$

Assuming forcing in a white noise form and discretising the above model with a sampling time T, the system description is obtained by means of differential equations of state and observations:

$$x_k = Ax_{k-1} + w_{k-1} \tag{7a}$$

$$y_k = Cx_k + v_k \tag{7b}$$

where:

$$x_k = \begin{bmatrix} q(kT) \\ \dot{q}(kT) \end{bmatrix}, \; A = e^{\mathcal{A}T}, \quad \mathcal{A} = \begin{pmatrix} 0 & I \\ -M^{-1}K & -M^{-1}D \end{pmatrix}, \; C = [L \quad 0], \tag{8}$$

$x_k \in R^n$ – state vector, $y_k \in R^m$ – response (observation) vector in the k-ts sampling moment, $A \in R^{nxn}$, $C \in R^{mxn}$, w_k and v_k represent white Gaussian noises of mean values being zero and covariance matrices Q_k and R_k – respectively.

Discrete eigenvalues and eigenvectors $(\lambda, \varphi_\lambda)$ of matrix *A* complying with the equations:

$$\det(\lambda I - A) = 0, \; (\lambda I - A)\varphi_\lambda = 0 \tag{9}$$

represent the modal description for a continuous time, as follows:

$$e^{T\mu =}\lambda \, , \psi_\mu = C\varphi_\lambda \tag{10}$$

Equations (9) and (10) indicate that the matrix pair (*A, C*), and thus also the observability matrix of the system described by equations (7), allows to calculate modal parameters of the object.

The problem of detection of defects in a mechanical system on the bases of vibration monitoring is thus equivalent to the problem of analysing changes in the structure of its linear model of state variables with an unstable process noise.

Defining various ways of signature determination and generation the investigated system residual will be the main subject of further considerations.

When the modal model is available, the signature of the nominal system $S^T(\theta_0)$ corresponding to the vector of parameters $\theta_0 \overset{\text{def}}{=} \binom{\Lambda}{vec\,\Phi} \in R^{n(m+1)}$ ($\Lambda \in R^n$ is the vector of eigenvalues λ of matrix *A*, $\Phi \in R^{mxn}$ is the matrix, whose columns are vectors $C\varphi_\lambda$ and φ_λ is the matrix eigenvector corresponding to eigenvalue λ) is constructed as the left null space of the observability matrix $\mathcal{O}_{p+1}(\theta)$ [19]:

$$\mathcal{O}_{p+1}(\theta) = \begin{pmatrix} \Phi \\ \Phi\Delta \\ \vdots \\ \Phi\Delta^p \end{pmatrix} \in R^{m(p+1)xn}, \quad (11)$$

where $\Delta = diag(\Lambda)$.

In case of diagnostics of systems for which the modal model is not available, the observability matrix can be determined by means of singular value decomposition (SVD) of the Hankel matrix $\mathcal{H}_{p+1,q}$ of covariances of experimental data obtained for the serviceable system.

In order to do this, the series of covariances of the output signals vector Y_k: $R_i \overset{\text{def}}{=} \boldsymbol{E}(Y_k Y_{k-i}^T)$ will be considered and the Hankel matrix of data covariances, for the selected constants p and q, will be defined as follows:

$$\mathcal{H}_{p+1,q} = Hank(R_i) \overset{\text{def}}{=} \begin{pmatrix} R_1 & R_2 & \cdots & R_q \\ R_2 & R_3 & \cdots & R_{q+1} \\ \vdots & \vdots & & \vdots \\ R_{p+1} & R_{p+2} & \cdots & R_{p+q} \end{pmatrix} \quad (12)$$

The empirical Hankel matrix $\hat{R}_i$ is determined on the basis of the covariance estimator in a form:

$$\hat{R}_i \overset{\text{def}}{=} \frac{1}{N-i} \sum_{k=i+1}^{N} y_k y_{k-i}^T \quad (13)$$

Then the SVD of the Hankel matrix of data covariance is considered:

$$\mathcal{H}_{p+1,q} = U\Sigma V^T \approx (U_1 \quad U_2) \begin{pmatrix} \Sigma_1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} V_1^T \\ V_2^T \end{pmatrix} = U_1 \Sigma_1 V_1^T \quad (14)$$

where $U \in R^{(p+1)x(p+1)}$ and $V \in R^{qxq}$ are orthogonal matrices, and $\Sigma_1 = diag[\sigma_i]$ is squared diagonal matrix of n – singular values σ_i of the Hankel matrix $\mathcal{H}_{p+1,q}$.

It is known, from the theory of realisation by subspace methods, that the observability matrix and the Hankel matrix are linked by the equation:

$$\mathcal{H}_{p+1,q} = \mathcal{O}_{p+1}\mathcal{Z}, \quad (15)$$

where $\min(pm, qm) \geq n$ and $\mathcal{Z}$ is the matrix of a form resulting from the assumed identification algorithm. In addition the observability matrix of the system is determined by:

$$\mathcal{O}_{p+1} = U_1 \Sigma_1^{\frac{1}{2}} \quad (16)$$

Matrices $\mathcal{H}_{p+1,q}$ and $\mathcal{O}_{p+1}$ have the same left null space. The left null space S^T determined for the observability matrix $\mathcal{O}_{p+1}$ satisfies asymptotically – under serviceable conditions – the equation:

$$S^T(\theta_0)\mathcal{H}_{p+1,q} = 0 \quad (17)$$

for the empirical Hankel matrix $\mathcal{H}_{p+1,q}$.

DIAGNOSTICS PROCEDURE

The described procedure can be presented in a form of the algorithm of the dynamic system diagnostics, based on the observed vibrations output signals:

- Step 1. To calculate of the averaged Hankel matrix of data covariance, on the basis of J data sets recorded under various working and environmental conditions of the serviceable system:

$$\hat{\mathcal{H}}_{p+1,q}^{0} \overset{\text{def}}{=} \frac{1}{J} \sum_{j=1}^{J} \hat{\mathcal{H}}_{p+1,q}^{0,j} \quad (18)$$

- Step 2. To perform SVD of the matrix $\hat{\mathcal{H}}_{p+1,q}^{0}$ according to equation (14).
- Step 3. To determine the observability matrix according to equation (16).
- Step 4. To determine the left null space S_0^T of the observability matrix $\mathcal{O}_{p+1}$ of the data obtained in the serviceable state:

$$S_0 = null(\mathcal{O}_{p+1}^T) \quad (19)$$

- Step 5. To determine the residual vector in a form:

$$R_N \overset{\text{def}}{=} \sqrt{N} vec[S^T(\theta_0)\hat{\mathcal{H}}_{p+1,q}] \quad (20)$$

for the empirical matrix $\hat{\mathcal{H}}_{p+1,q}$ obtained for the current experimental data

- Step 6. To determine the estimator $\hat{\Sigma}$ of the covariance matrix of the residual vector R_N, after dividing data into K segments of the same dimension l acc. to:

$$\hat{\Sigma} = \frac{1}{Kl} \sum_{k=1}^{K} R_N R_N{}^T \quad (21)$$

- Step 7. To calculate the index of defects detection χ^2 in a form of the chi-square test:

$$\chi^2 = R_N{}^T \hat{\Sigma}^{-1} R_N \quad (22)$$

The algorithm is illustrated in Figure 1 and verified on the example of data sets obtained in measurements of the acceleration of the IC engine head vibrations, at cylinder 1 in a horizontal direction.

The analysis was performed on the basis of the data originated from road tests of 4-cylinder IC engine of a cubic capacity 1.4 l and a mileage of 400 000 km described in details in [20, 21].

Data collections formed from the recorded and properly transformed vibration responses

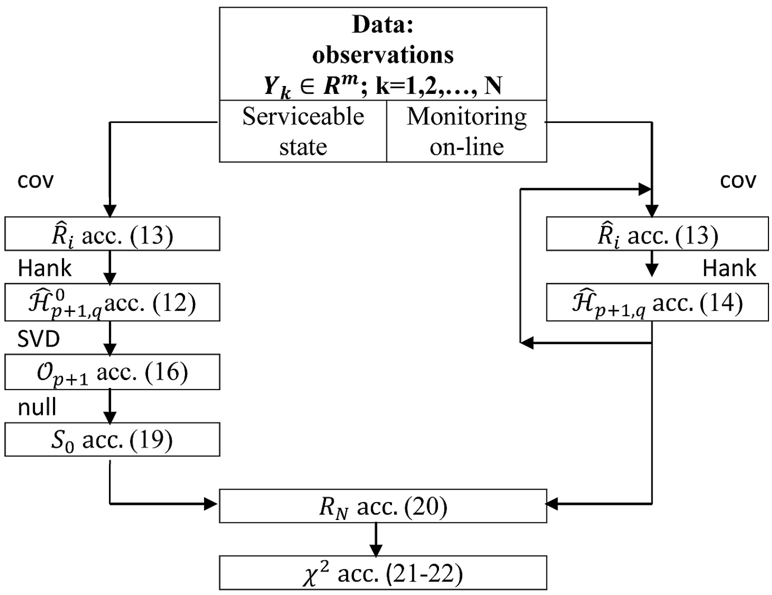

Fig. 1. Algorithm of the index χ^2 determination for the monitored object

for closing the exhaust valve were utilised. Each observation contained 26 signal samples, recorded at a constant rotational speed of 3000 rpm, at states:

- Non-defective valve, a proper valve clearance (0.25 mm),
- Defective valve type I (valve head cut 3 mm), a proper valve clearance.

Values of detection indices in a form of chi-square test of the discussed defects for the successive observation collections of non-defective and defective systems are presented in Figure 2.

CONCLUSIONS

The presented example of a non-parametric diagnostics method of the IC engine valve system, on the basis of data subspace, belongs to algorithms of filtration realised according to statistic properties. Original results of the acceleration vibrations signal, obtained in several road tests for various maintenance states of the valve system, proved the algorithm efficiency.

The analysis of results indicates the possibility of applying this method for fault detection and isolation. However, the method does not allow for defect identification, i.e. for the determination of its size and discriminating the reason of such a state.

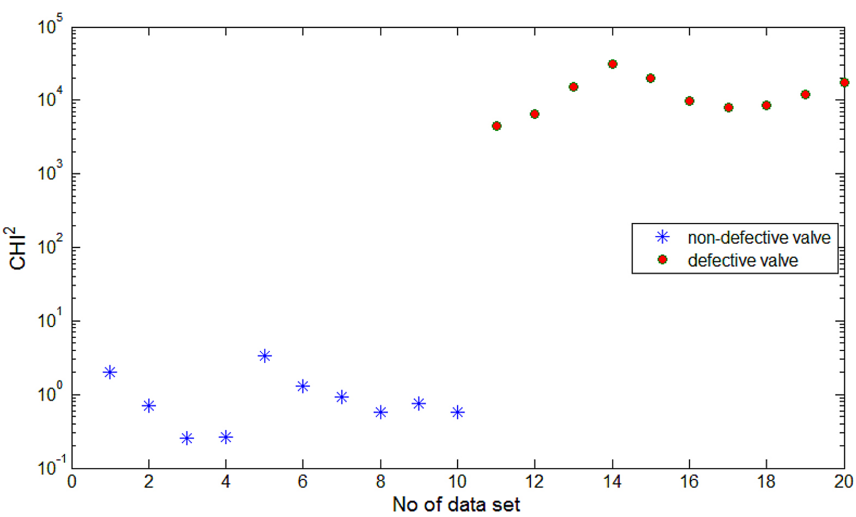

Fig. 2. Chi-square statistic values of the diagnosed valve system as a function of successive observation collections

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