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The Study of Geometry of the Selected Transition Curves in the Design of Circular Roads

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ABSTRACT

Transition curves connecting straight sections with circular curves and vice versa are present on all types of roads. They are designed to smooth the transition from rectilinear to circular motion. In numerous instances, they are hardly noticeable by road use. Although adequately selected, they increase the safety and comfort of driving. The research aimed to investigate the effectiveness of three most popular curves, in the design of road routes: clothoid, a parabola of the 3rd-degree and Bernoulli lemniscate. Starting from the permissible values of the increment of centripetal acceleration on the transition curves, the minimum lengths of the transition curves for the assumed speeds of the wheeled vehicle were determined. Then, based on the formulas of individual curves, their lengths were calculated, depending on the design speed and the radius of the circular arc. As a result of the research, the differences between the individual curves were obtained. Finally, it was found that due to the unconstrained movement of the vehicle on the road, all curves could be used as transition curves. By slightly adjusting their lengths, we can achieve the appropriate value of the increment of centripetal acceleration. The legitimacy of using different curves as transition curves lies primarily in their different geometric appearance. The choice of the curve to be used is based primarily on the terrain conditions and the type of the planned section of the route (serpentines, turnouts, etc.).

Keywords: transition curve, clothoid, 3rd-degree parabola, Bernoulli lemniscate, incremental centripetal acceleration

INTRODUCTION

Public roads are places where the transition curves are applied [1, 2]. The requirements to be met by the transition curves in Poland can be found in the "Regulation of the Minister of Transport and Maritime Economy of March 2, 1999, on technical conditions to be met by appropriate public roads and their location" [3] and in the "Regulation of the Minister of Infrastructure of January 16, 2002, on technical and construction regulations for toll motorways" [4].

According to the aforementioned legal acts, transition curves should be used when connecting two road sections with different, constant curvature values. It is allowed to replace the transition curve with the transition straight line only in justified cases on L and D class roads and Z class streets [3].

The transition curve has a variable radius, from $r = \infty$ ($k = 0$) at contact with the straight line to the value of R equal to the size of the arc radius [5, 6]. Thanks to this, the continuity of the route is maintained [7]. A vehicle moving along a curve is influenced by centrifugal force, the value of which increases with increasing curvature. In the transition curve, there is a combination of translational and rotary motion [8, 9]. There have been many studies verifying the suitability of the curves for the routing of circular roads [10–13], in which emphasis was placed on the dynamics of the vehicle movement along this curve and on their compliance with the boundary conditions [14]. The track-vehicle system is often considered crucial for road safety [15].

This study focuses on the appropriate selection of the transition curve length.. According to the literature the three transition curves are most frequently used in the design of circular roads: clothoid [16, 17], the 3rd-degree parabola [18] and Bernoulli lemniscate [18]. The above curves are also the most frequently used in designing and building roads in Poland [19, 20]. The study aimed to investigate whether the selected transition curves will not exceeding the allowable increase in centripetal acceleration and the resulting geometric condition of reaching at least the minimum length.

Research methodology

The primary condition for the selection of the transition curve is the maximum increase in centripetal acceleration acting on the vehicle moving along this curve, which depends on the speed on a given road section. The turning angle along the transition curve should be in the range of 3°–30°. It does not apply when the angle of the route change is less than 9° and the serpentine (Table 1) [3, 4].

The roadway should have a transverse slope enabling the free flow of water, and its value, except for a few exceptions, depends on the surface type and amounts to 2.0%, 3.0% or 4.0% [3]. The transverse slope of the motorway roadway should not be less than 2% [4]. There are three cases when the transition curve may not be used [3]:

- the radius of the arc in the plane is higher than 2,000 m on the road outside the built-up area at a design speed of 33.33 m/s and 27.78 m/s;
- the radius of the arc in the plane is higher than 1,000 m at a design speed of 22.22 m/s and less;
- the road on the building site has a transverse slope on the curve in the plane, as on a straight section.

On the basis of this regulation, 37 cases were selected for which the lengths of the transition curves were tested:

- for design radii of circular curves with values: 300, 500, 700 and 900 metres, transition curves were calculated for seven design vehicle speeds (13.89, 16.67, 19.44, 22.22, 25.00, 27.78 and 33.33 [m/s]);
- for design circular curve radii with values: 1100, 1300 and 1500 metres, transition curves were calculated for three design vehicle speeds (25.00, 27.78 and 33.33 [m/s]).

Based on the said regulation [3], using the formula [21]:

$$a = \sqrt{\frac{v^3}{\psi_d}}\ [m] \tag{1}$$

where: a – transition curve parameter [m];
v – design speed of the vehicle [m/s];
Ψd – acceleration [m/s³].

Seven minimum values for the parameter a_{min} were calculated (Table 2), applicable to G class roads [3].

Using the formula [14]:

$$L_{min} = \frac{v^3}{R\psi}\ [m] \tag{2}$$

where: L_{min} – minimum length of the transition curve [m];
v – design speed of the vehicle [m/s];
R – radius of a circular arc [m];
Ψ – acceleration.

The minimum transition curve lengths L_{min} have been calculated depending on the adopted length of the radii of the circular arc R.

RESULTS

First, graphs showed the transition curve's length variability depending on the circular arc's assumed radius.

From Figures 1 and 2 we can see that the minimum length of the transition curve always increases with the increase in the design speed. We observe an inverse relationship between a circular arc's speed and projected radius. The larger the circular arc radius, we can use a shorter transition curve for a given vehicle speed. Although the

Table 1. Allowable increments of centripetal acceleration Ψd on transition curves [3, 4]

Design speed v [m/s]	33.33–27.78	22.22	19.44	16.67	13.89	11.11
Centripetal acceleration gain ψ_d [m/s²]	0.3	0.5	0.6	0.7	0.8	0.9

Table 2. Values of parameter a_{min} depending on speed

Speed v [m/s]	Acceleration ψ_d [m/s²]	Transition curve parameter a_{min} [m]
13.89	0.8	57.870
16.67	0.7	81.325
19.44	0.6	110.692
22.22	0.5	148.148
25.00	0.4	197.642
27.78	0.3	267.292
33.33	0.3	351.364

analysed speeds are designed at equal intervals of 2.78 m/s between 13.89 m/s and 27.78 m/s, the differences between the minimum lengths of the transition curves are not equal. The most considerable differences are observed at the smallest circular arc at a constant vehicle speed. Within one designed route, with increasing speed, the differences between the minimum lengths of the transition curves are more significant (they change logarithmically). The larger the circular arc radius towards which the transition curve tends, the more flattened the diagram in this relationship. As expected, the differences in the curve lengths for the successive design speeds are the smallest for the lowest speed.

The clothoid

Based on the formula [22]:

$$L = \frac{a^2}{R}[m] \tag{3}$$

where: L – length of clothoid [m];
a – transition curve parameter [m];
R – radius of a circular arc [m].

The parameter values a_{min} presented in Table 3, the lengths of the clothoid were calculated depending on the speed and the adopted radius of the circular arc. We can see that the minimum length of the transition curve obtained for the clothoid coincide. It results from the fact that the

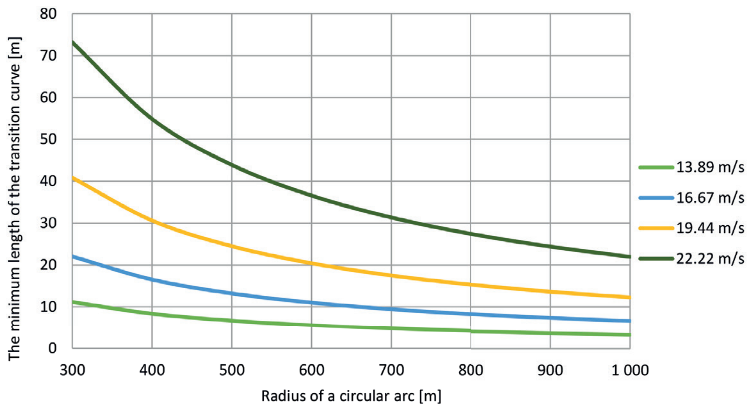

Figure 1. The minimum length of the transition curve depending on the radius of the circular arc for four design speeds: 13.89, 16.67, 19.44 and 22.22 m/s.

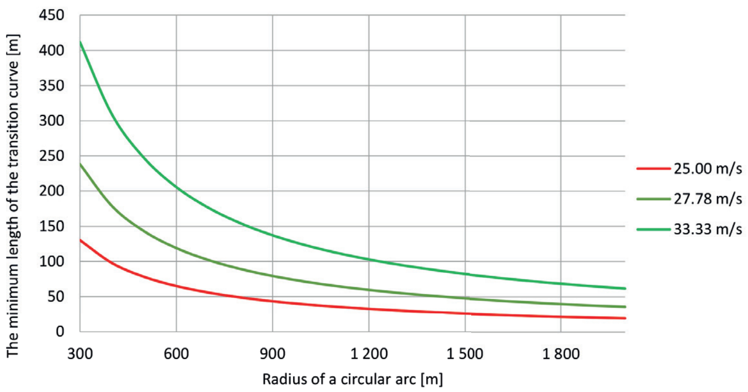

Figure 2. The minimum length of the transition curve depending on the radius of the circular arc for three design speeds: 25.00, 27.78 and 33.33 m/s.

calculated parameter a_{min} is the same as the proportionality coefficient of the clothoid [22].

$$\frac{l}{k} = a^2 = const \tag{4}$$

where: l – length of transition curve [m];
k – curvature of the transition curve $[\frac{1}{m}]$.

Substituting for k with the following relationship defining the curvature:

$$k = \frac{1}{r} \tag{5}$$

where: k – curvature of the transition curve $[\frac{1}{m}]$;
r – radius of curvature [m].

we get:

$$lr = a^2 \tag{6}$$

where: l – length of transition curve [m];
r – radius of curvature [m];
a – transition curve parameter [m]

and thus:

$$\sqrt{lr} = a \tag{7}$$

Therefore, the clothoid was not analysed further.

Cubic parabola

Starting from the equations for curvature [23] and curve length [24] written in the explicit form:

$$k = \frac{|f''(x)|}{[1 + f'(x)^2]^{\frac{3}{2}}} \tag{8}$$

$$L = \int_a^b \sqrt{1 + \left(f'(x)\right)^2} dx \tag{9}$$

where: k – curvature of the transition curve $[\frac{1}{m}]$;
x – value of the OX axis in the Cartesian coordinate system [m];
L – length of the transition curve [m] ;
a – initial value of the length of the transition curve [m];
b – final value of the length of the transition curve [m].

Substituting the formula for a cubic parabola [18]:

$$y = \frac{x^3}{6a^2} \tag{10}$$

where: y – value of the OY axis in the Cartesian coordinate system [m];
x – value of the OX axis in the Cartesian coordinate system [m];
a – transition curve parameter [m]

and using the fundamental relationship for curvature [22]:

$$k = \frac{1}{r} \tag{11}$$

where: k – curvature of the transition curve $[\frac{1}{m}]$;
r – radius of curvature [m].

Table 3. Clothoid lengths

Radius of the circular R [m]		300	500	700	900	1100	1300	1500
Speed v [m/s]	Transition curve parameter a_{min} [m]	Clothoid lengths L [m]						
13.89	57.870	11.163	6.698	4.784	3.721			
16.67	81.325	22.046	13.228	9.448	7.349			
19.44	110.692	40.843	24.506	17.504	13.614			
22.22	148.148	73.160	43.896	31.354	24.387			
25.00	197.642	130.208	78.125	55.804	43.403	35.511	30.048	26.042
27.78	267.292	238.150	142.890	102.064	79.383	64.950	54.958	47.630
33.33	351.364	411.523	246.914	176.367	137.174	112.233	94.967	82.305

the radius and curve length formulas for the discussed transition curve were derived:

$$R = \frac{a^2(1 + \frac{x^4}{4a^4})^{3/2}}{x} \tag{12}$$

$$L = \int_0^x \sqrt{1 + \frac{x^4}{4a^4}}\,dx \tag{13}$$

where: *R* – radius of a circular arc [m].
a – transition curve parameter [m]
x – value of the OX axis in the Cartesian coordinate system [m];
L – length of the transition curve [m].

Based on the above formulas, using the bisection and the trapezoidal method, the lengths of a cubic parabola were calculated depending on the speed and the adopted radius of the circular arc (Table 4).

During the calculations, it turned out that for two cases: R = 300 m and v = 27.78 m/s and for R = 300 m and v = 33.33 m/s, the function:

$$f(x) = \frac{a^2(1 + \frac{x^4}{4a^4})^{3/2}}{x} - R \tag{14}$$

where: *a* – transition curve parameter [m]
x – value of the OX axis in the Cartesian coordinate system [m]
R – radius of a circular arc [m],

is positive in the range $x \in \langle 0, R \rangle$. Therefore, it was necessary to find the minimum radius of the circular arc so that it would be possible to use a cubic parabola as the transition curve, with the assumed values of the minimum curve length from the equation a – being 267.292 and 351.364, respectively.

After using the bisection method again, the desired values were read out graphically and were determined with an accuracy of 1 m. The results are presented in Table 5.

After modifying the length of circular arcs, a cubic parabola always meets the essential condition when designing a road route $L_{calc} \geq L_{min}$. It should also be noted that the lengths obtained for the cubic parabola are usually very close to the minimum sizes. From the road designer's perspective, that proportionally increases the radius of the circular curve in relation to the designed vehicle speed, we are most interested in the values located on the main diagonal of Table 6. The most considerable differences are marked in blue.

When calculating the length of the parabola, it was first necessary to obtain the abscissa of x. Compared to L, it turned out that the most significant difference obtained for v = 33.33 m/s and R = 300 m does not exceed 5.7 m (Table 7). It shows the correctness of the simplification. For road construction, the formulas for a cubic parabola, where the equality L = x is assumed.

Bernoulli lemniscate

For this analysis, the Bernoulli lemniscate equation has been transformed into a polar form [18]:

$$\rho^2 = 3a^2\sin(2\varphi) \qquad 0 \leq \varphi \leq \frac{\pi}{2} \quad \text{and} \quad \pi \leq \varphi \leq \frac{3\pi}{2} \tag{15}$$

where: *ρ* – the leading radius of a lemniscate point [rad];

Table 4. Cubic parabola lengths

Radius of the circular R [m]		300	500	700	900	1100	1300	1500
Speed v [m/s]	Transition curve parameter a_{min} [m]	Cubic parabola lengths L [m]						
13.89	57.870	11.169	6.698	4.784	3.721			
16.67	81.325	22.094	13.231	9.449	7.349			
19.44	110.692	41.155	24.529	17.508	13.615			
22.22	148.148	75.088	44.033	31.379	24.394			
25.00	197.642	145.361	78.919	55.947	43.443	35.526	30.055	26.045
27.78	267.292	no data	148.295	102.962	79.633	65.041	54.997	47.649
33.33	351.364	no data	296.526	181.368	138.498	112.709	95.171	82.404

Table 5. Corrected radii of a circular arc

Speed v [m/s]	Transition curve parameter a_{min} [m]	Radius of the circular R [m]	$L_{parabola}$ [m]	L_{min} [m]	$L_{parabola} – L_{min}$ [m]	$L_{parabola} – x$ [m]
27.78	267.292	372	250.071	192.056	58.014	4.290
33.33	351.364	489	328.780	252.468	76.312	5.643

φ – the angle that the leading radius ρ of a lemniscate point makes with the axis Ox (point amplitude) [rad];
a – transition curve parameter.

By using dependence [14]:

$$R = \frac{a^2}{\rho} \tag{16}$$

where: *R* – radius of a circular arc [m];
a – transition curve parameter [m]
ρ – the leading radius of a lemniscate point [rad].

The formula for the amplitude of the point *φ* was derived:

$$\varphi = \frac{\arcsin(\frac{a^2}{3R^2})}{2} \tag{17}$$

where: *φ* – the angle that the leading radius ρ of a lemniscate point makes with the axis Ox (point amplitude) [rad];
a – transition curve parameter [m].
R – radius of a circular arc [m].

Based on the obtained angle values and the formula [16]:

$$L = a\sqrt{3} \cdot \int_0^{\varphi} \frac{d\varphi}{\sqrt{\sin(2\varphi)}} \tag{18}$$

where: *L* – length of the transition curve [m];
a – transition curve parameter [m];
φ – the angle that the leading radius ρ of a lemniscate point makes with the axis Ox (point amplitude) [rad];

Using the trapezoidal method, the length of the lemniscate was calculated depending on the speed and the adopted radius of the circular arc Table 8).

A lemniscate, unlike a cubic parabola, does not satisfy the condition at all: $L_{calc} \geq L_{min}$. However, due to the differences $L_{calc} – L_{min}$ (presented in Table 9) in the vast majority of cases, do not exceed 1 meter (this value is exceeded in three cases: v = 27.78 m/s and R = 300 m, v = 33.33 m/s and R = 300 m and v = 33.33 m/s and R = 500 m) consider that it can be used as a transition curve due to the accuracy of construction works and the specificity of the movement of a wheeled vehicle on the road (unconstrained trajectory). As in the case of the cubic parabola, the most considerable difference was obtained for v = 33.33 m/s, R = 300 m.

Satisfying the condition $L_{calc} \geq L_{min}$ is motivated by the requirement that the permissible value of centripetal acceleration should not be exceeded along the entire length of the transition curve [3, 4]. Therefore, the influence of the transition

Table 6. Differences between the minimum length and the calculated length of a cubic parabola

Radius of the circular R [m]		300	500	700	900	1100	1300	1500
Speed v [m/s]	Transition curve parameter a_{min} [m]	Differences between the minimum length and the calculated length of a cubic parabola $\Delta L = L_{parabola} – L_{min}$ [m]						
13.89	57.870	0.006	0.000	0.000	0.000			
16.67	81.325	0.048	0.004	0.001	0.000			
19.44	110.692	0.312	0.024	0.004	0.001			
22.22	148.148	1.929	0.137	0.025	0.007			
25.00	197.642	15.152	0.794	0.143	0.041	0.015	0.006	0.003
27.78	267.292		5.405	0.898	0.250	0.091	0.039	0.019
33.33	351.364		49.613	5.001	1.324	0.475	0.204	0.100

curve length described by the Bernoulli lemniscate equation was checked. The values presented in Table 10 were calculated based on the formula:

$$\psi = \frac{v^3}{RL} \tag{19}$$

where: Ψ – acceleration;
v – design speed of the vehicle [m/s];
R – radius of a circular arc [m];
L – length of the transition curve [m]

From the Table 11 we can see that the difference $\Psi_{lemniscate} - \Psi_d$ in most cases, does not exceed 0.001 m/s². For v = 13.89 m/s it is 0.002 m/s², the same as for the design speed of 60.80 and 25.00 m/s with a circular arc radius of 300 m. The most significant differences are noticeable where the calculated length of the transition curve differed the most from the minimum value: for v = 27.78 m/s and v = 33.33 m/s for the smallest analysed radius of the circular curve, respectively 0.004 m/s² and 0.009 m/s². Table 12 showing the values from the main diagonals for the 3rd-degree parabola and the lemniscate.

Table 7. Differences between the calculated length of the cubic parabola and the abscissa

Radius of the circular R [m]		300	500	700	900	1100	1300	1500
Speed v [m/s]	Transition curve parameter a_{min} [m]	Differences between the calculated length of the cubic parabola and the abscissa $\Delta = L_{parabola} - x$ [m]						
13.89	57.870	0.000	0.000	0.000	0.000			
16.67	81.325	0.003	0.000	0.000	0.000			
19.44	110.692	0.020	0.001	0.000	0.000			
22.22	148.148	0.123	0.009	0.002	0.000			
25.00	197.642	1.017	0.050	0.009	0.003	0.001	0.000	0.000
27.78	267.292		0.346	0.056	0.016	0.006	0.002	0.001
33.33	351.364		3.487	0.318	0.083	0.030	0.013	0.006

Table 8. Lemniscate lengths

Radius of the circular R [m]		300	500	700	900	1100	1300	1500
Speed v [m/s]	Transition curve parameter a_{min} [m]	Lemniscate lengths L [m]						
13.89	57.870	11.139	6.684	4.774	3.713			
16.67	81.325	21.997	13.199	9.428	7.333			
19.44	110.692	40.744	24.453	17.467	13.585			
22.22	148.148	72.933	43.797	31.287	24.335			
25.00	197.642	129.529	77.928	55.680	43.309	35.436	29.984	25.986
27.78	267.292	235.232	142.395	101.812	79.205	64.809	54.840	47.528
33.33	351.364	398.934	245.403	175.809	136.831	111.977	94.758	82.126

Table 9. Differences between the minimum length of the transition curve and the calculated length of the lemniscate

Radius of the circular R [m]		300	500	700	900	1100	1300	1500
Speed v [m/s]	Transition curve parameter a_{min} [m]	Differences between the minimum length of the transition curve and the calculated length of the lemniscate $\Delta L = L_{lemniscate} - L_{min}$ [m]						
13.89	57.870	-0.024	-0.014	-0.010	-0.008			
16.67	81.325	-0.049	-0.028	-0.020	-0.016			
19.44	110.692	-0.099	-0.053	-0.037	-0.029			
22.22	148.148	-0.227	-0.098	-0.067	-0.052			
25.00	197.642	-0.679	-0.197	-0.124	-0.093	-0.076	-0.064	-0.055
27.78	267.292	-2.917	-0.495	-0.252	-0.178	-0.141	-0.118	-0.101
33.33	351.364	-12.588	-1.510	-0.558	-0.343	-0.257	-0.209	-0.178

Table 10. The values of the increment of centripetal acceleration for the Bernoulli lemniscate

Radius of the circular R [m]		300	500	700	900	1100	1300	1500
Speed v [m/s]	Transition curve parameter a_{min} [m]	The values of the increment of centripetal acceleration for the Bernoulli lemniscate $\psi_{lemniscate}$ [m/s^2]						
13.89	57.870	0.802	0.802	0.802	0.802			
16.67	81.325	0.702	0.701	0.701	0.701			
19.44	110.692	0.601	0.601	0.601	0.601			
22.22	148.148	0.502	0.501	0.501	0.501			
25.00	197.642	0.402	0.401	0.401	0.401	0.401	0.401	0.401
27.78	267.292	0.304	0.301	0.301	0.301	0.301	0.301	0.301
33.33	351.364	0.309	0.302	0.301	0.301	0.301	0.301	0.301

Table 11. Differences between the value of the increment of centripetal acceleration calculated for the Bernoulli lemniscate and the limit value

Radius of the circular R [m]		300	500	700	900	1100	1300	1500
Speed v [m/s]	Transition curve parameter a_{min} [m]	Differences between the value of the increment of centripetal acceleration calculated for the Bernoulli lemniscate and the limit value $\psi_{lemniscate} - \psi_d$ [m/s^2]						
13.89	57.870	0.002	0.002	0.002	0.002			
16.67	81.325	0.002	0.001	0.001	0.001			
19.44	110.692	0.001	0.001	0.001	0.001			
22.22	148.148	0.002	0.001	0.001	0.001			
25.00	197.642	0.002	0.001	0.001	0.001	0.001	0.001	0.001
27.78	267.292	0.004	0.001	0.001	0.001	0.001	0.001	0.001
33.33	351.364	0.009	0.002	0.001	0.001	0.001	0.001	0.001

It is easy to notice from Table 12 that with the optimal adaptation of the vehicle speed to the radius of the circular arc, the length of the transition curve, depending on the equation chosen for it (clothoid, 3rd-degree parabola, Bernoulli lemniscate), will vary by up to ±20 cm, compared to the minimum value. As shown on the example of a lemniscate, the mentioned difference does not significantly affect the driving comfort, the more so as in the case of roads, we deal with a free path.

CONCLUSIONS

The primary condition that transition curves in road engineering should meet is: $\Psi_{calc} \leq \Psi_d$, from which the condition results: $L_{calc} \geq L_{min}$. The calculations presented in this paper show that both the clothoid and the 3rd-degree parabola meet these conditions. If we compare the minimum lengths of the spiral curves to those calculated for the 3rd order parabola, the calculated length of the parabola is always higher, and almost 80% of the

Table 12. Values obtained on the main diagonal

Speed v [m/s]	Minimum curve length L_{min} [m]	Length of cubic parabola curve $L_{parabola}$ [m]	$\Delta L_{parabola}$ [m]	Length of Bernoulli Lemniscate curve $L_{lemniscate}$ [m]	$\Delta L_{lemniscate}$ [m]	Radius of the circular R [m]
13.89	11.163	11.169	0.006	11.139	-0.024	300
16.67	13.228	13.231	0.004	13.199	-0.028	500
19.44	17.504	17.508	0.004	17.467	-0.037	700
22.22	24.387	24.394	0.007	24.335	-0.052	900
25.00	35.511	35.525	0.014	35.436	-0.076	1100
27.78	54.958	54.995	0.037	54.840	-0.118	1300
33.33	82.305	82.398	0.093	82.126	-0.178	1500

differences fall within the range < 0,000; 1,000 >. The lengths obtained for the lemniscate do not meet the condition in question at all. Received differences $L_{calc} - L_{min}$ are between -0.008 m for v = 13.89 m/s and R = 900 m, and -12.588 m for v = 33.33 m/s and R = 300 m. This difference exceeds 1 m for only three cases, which, due to the accuracy of construction works and the specificity of the movement of a wheeled vehicle on the road (unconstrained trajectory), does not disqualify a lemniscate as a transition curve.

Looking at the obtained results of calculations, it can be noticed that when designing roads, it is crucial to properly adjust the vehicle's speed to the circular curve's radius. With the increase in the design speed, the circular arc radius towards which the transition curve tends should be increased.

Based on results, it should be stated that the choice of the curves in question as the transition curve is of no great importance. However, due to their different geometrical course, they are applicable in various terrain conditions. For example, lemniscate is very commonly used in serpentine and road junctions. Clothoid, on the other hand, is popular due to the ease of calculations.

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