

Dynamics of Railway Vehicles Movement on Transition Curves

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ABSTRACT

Transition curves are an integral part of a route. They connect a straight section with a circular curve or two circular curves with different radii. Due to the forced path, the size and stiffness of the rail vehicle, their correct design is particularly important when constructing rail roads. Basing on the limits of lateral acceleration increment, non-equilibrium centripetal acceleration and wheel lift velocity, set in accordance with the model of motion of a material point moving along a trajectory determined by the track axis, the initial parameters of the considered transition curves were selected: radius of the curve, design speed of the train, length of the transition curve and track cant. The scope of research covered basic dynamic parameters: curvature and its increment, speed and acceleration of wheel lift, track twist and lateral acceleration increment. As a result of conducted analysis it can be stated that polynomial curves deserve special attention in the design of railway routes. By appropriate modification of their equation, it is possible to change easily positions of extremes of particular dynamic parameters and thus to adjust their values to terrain conditions, direction of movement and type of rail vehicle.

Keywords: railway, transition curve, dynamics of vehicle movement.

INTRODUCTION

The transition curve, i.e. the section of the route connecting a straight section with a circular curve or two circular curves with different radii [1] is a very important element of any road, including a railway road, although it is almost invisible to traffic participants.

The question of transition curves is being addressed by many industries. These range from designers [2], architects [3], construction workers [4] and geodesists [5] to physicists [6] and mathematicians [7]. Of which the former are primarily concerned with the shape of curves, their proper construction and location in the field [8]. The others focus on the movement of vehicles along curves [9] and the forces acting on them [10]. Although many publications address the issue of transition curves in railway roads, there are few studies that combine both approaches (geometry and movement). The dynamics of railway vehicles movement in the context of

the shape of transition curves is described by Krzysztof Zboński in his publications [11, 12] and by Andrzej Grzyb and Roman Bogacz [9].

The basic condition to be fulfilled in the design of a railway route is its continuity. Therefore, in order to be able to use a given curve as a transition curve, it must meet the appropriate boundary conditions [10]. Starting from the geometrical feature of the curve, i.e. its curvature, it is possible to examine the forces acting on a vehicle moving along a curvilinear route [3]. One of them is the centrifugal force. In order to counteract it, track cant is used. In addition to the speed of the vehicle, the direction of motion and three basic kinematical parameters must be taken into account: unbalanced acceleration, acceleration build-up, wheel velocity climbing up the cant ramp [13]. Ride quality is a very important factor to be taken into account in the design of railway routes. It is influenced, apart from the mentioned parameters, by the vibrations occurring during the movement of a railway vehicle on a curvilinear track [14].

While designing railway roads in Poland, strictly defined conditions must be observed, which are included in the Regulation of the Minister of Transport and Maritime Economy of 10 September 1998 on technical conditions to be met by railway structures and their location [15]. Due to Poland’s accession to the European Union in 2004, Polish law had to be adapted to EU regulations. In 2009, technical standards entitled: “Detailed technical conditions for modernisation or construction of railway lines for speeds $V_{max} \leq 200$ km/h (for conventional rolling stock)/250 km/h (for tilting rolling stock)” were developed [16]. Additionally, in 2014 the Regulation of the Minister of Infrastructure and Development of 5 June 2014 amending the Regulation on technical conditions to be met by railway structures and their location was issued [17]. The limit values specified in the regulations are not uniform across the European Union [13].

Polish regulations to date do not take into account high-speed rail, where the maximum speed of a vehicle is 350 km/h. The technical standards for this type of infrastructure can be found in Commission Regulation (EU) No 1299/2014 of 18 November 2014 on the technical specifications for interoperability relating to the ‘infrastructure’ subsystem of the rail system in the European Union [1]. Therefore, when conducting research on the possibility of constructing this type of railway line in Poland, it is necessary to base it on the regulations of other European countries, for instance Spain, which has an extensive HSR network [18].

The objective of the research was to compare the distribution of values of dynamic parameters on the length of transition curves defined by equations of various curves, the parameters of which were selected to comply with the law in force in Poland [17, 15]. The considerations were carried out in accordance with the model of motion of a material point moving along a trajectory determined by the track axis [19].

RESEARCH METHODOLOGY

According to the Regulation of the Minister of Transport and Maritime Economy of 10 September 1998 [15], the transition curve should be defined by the third-degree parabola with the equation:

$$y = \frac{x^3}{6 \cdot R \cdot L} \tag{1}$$

where: R - radius of circular curve [m]; L - length of the transition curve with straight cant ramp [m]; x - distance from the beginning of the transition curve [m].

There is a range of curves that can also be used as transition curves in railway road design. For the purpose of this study, the third-degree parabola was compared with seven other curves: the fourth-degree parabola, the sine curve, the Bloss curve, the Auberlen curve [20] and three polynomial curves [21, 22] with a parameter C equal to 0.4, 0.5 and 0.6, respectively (Table 1).

The fixed values for the parameters of the transition curves (Table 2) were established under Polish law in accordance with the following assumptions:

- the track parameters shall be adapted to a magistral railway line with passenger transport;
- terrain conditions shall be convenient and the line is still to be built;
- the rolling stock shall have a non-tilting body [13];
- all transition curves shall connect a straight section with a circular curve of radius R ;
- the length of the transition curve shall be equal to the length of the straight cant ramp.

Table 1. Summary of the formulae of the analysed curves as a function of curvature [20]

Curve type	Formula
The third-degree parabola	$k(x) = \frac{x}{RL}$
The fourth-degree parabola	$k(x) = \begin{cases} \frac{2}{RL^2}x^2 & dla x \in (0, \frac{L}{2}) \\ \frac{1}{R} - \frac{2}{RL^2}(L-x)^2 & dla x \in (\frac{L}{2}, L) \end{cases}$
The sine curve	$k(x) = \frac{1}{R} \left(\frac{x}{L} - \frac{1}{2\pi} \sin\left(\frac{2\pi}{L}x\right) \right)$
The Bloss curve	$k(x) = \frac{1}{R} \left(\frac{3x^2}{L^2} - \frac{2x^3}{L^3} \right)$
The Auberlen curve	$k(l) = \frac{1}{2R} \left[1 - \cos\left(\pi \frac{x}{L}\right) \right]$
Polynomial curves	– for $C = 0.4$ $k(x) \cong \frac{1}{R} \left(\frac{4}{L^3}x^3 - \frac{3}{L^4}x^4 \right)$ – for $C = 0.5$ $k(x) \cong \frac{1}{R} \left(\frac{3}{L^2}x^2 - \frac{2}{L^3}x^3 \right)$ – for $C = 0.6$ $k(x) \cong \frac{1}{R} \left(\frac{6}{L^2}x^2 - \frac{8}{L^3}x^3 + \frac{3}{L^4}x^4 \right)$

Table 2. Constant values for the parameters of the transition curves

Parameter	Denotation	Value	Unit
Radius of circular curve	R	4000	m
Designed speed of trains	V	160	km/h
Length of transition curve	L	126	m
Track cant	h	79	mm

The following dynamic parameters were analysed:

- curvature and its increment;
- wheel lift velocity and acceleration;
- track twist;
- lateral acceleration increment.

The values of the mentioned parameters were calculated depending on the ordinate x . Due to the very small route return angles on the transition curves, it was assumed that the length of the transition curve is the same as the x variable of the Cartesian coordinate system $x \cong L$. This issue was discussed in detail by Zboiński [12].

RESULTS

By analysing the changes in curvature of the curves with their length, shown in the form of a graph in Figure 1, the transition curves in question can be divided into three groups:

1. The third-degree parabola is the only one of the curves discussed whose curvature diagram is linear.
2. Another group of curves are the Auberlen curve, the Bloss curve and the polynomial curve with parameter $C = 0.5$, of which the curvature diagrams of the last two coincide, and the fourth-degree parabola and the sine curve, for which the diagrams are similar to the previous ones but show smoother changes in curvature at the start and end points.
3. The last group consists of two polynomial curves with parameters $C = 0.4$ and $C = 0.6$. It is easy to notice that the graphs of their curvature are their reflections with the point of symmetry in the middle of the length of the transition curve. The curvature graph of the polynomial transition curve with parameter $C = 0.4$ is slightly smoother from the side of the starting point, and the curvature graph of the polynomial transition curve with parameter $C = 0.6$ from the side of the end point.

Additionally, it needs to be noted that the curvature of all the transition curves in question at the end point reaches the value of the curvature of the circular curve i.e. $250 \cdot 10^{-6} \frac{1}{m}$ (the continuity condition of the route [1] is met). Except for the two polynomial curves with parameter C equal to 0.6 and 0.4, the other curves in the middle of the

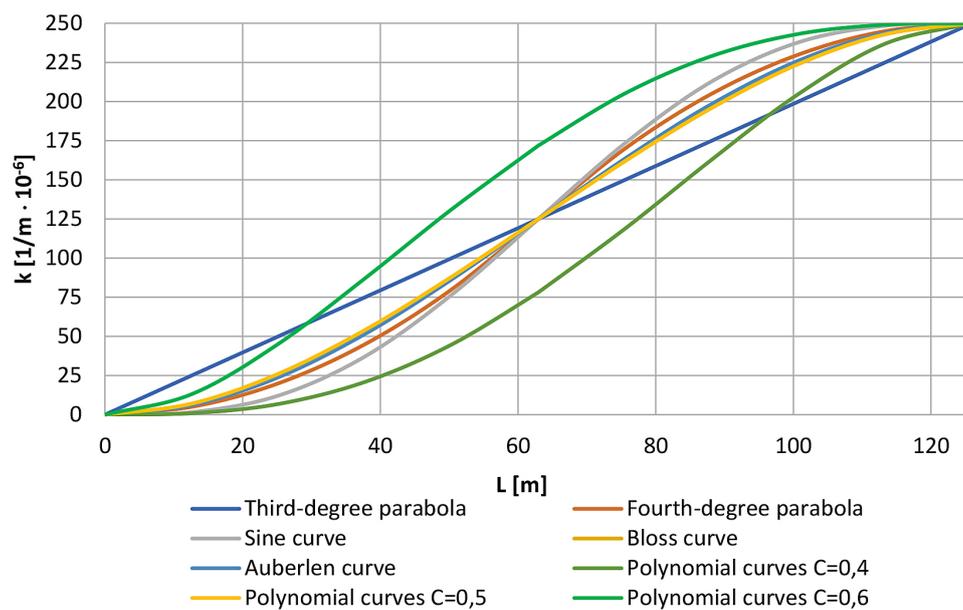


Fig. 1. Curvature $k \left[\frac{1}{m} \cdot 10^{-6} \right]$

transition curve length ($\frac{L}{2} \approx 62.963 \text{ m}$) obtain the same curvature value equal to $125 \cdot 10^{-6} \frac{1}{m}$.

By calculating:

$$\frac{dk(x)}{dx} \tag{2}$$

the increment of curvature on the transition curve was obtained [20].

By analysing the results of the curvature increment calculation, shown in Figure 2, we can observe that, apart from the third-degree parabola (whose curvature increment has a constant value, equal to $1.985 \cdot 10^{-6} \frac{1}{m^2}$) and the two polynomial curves with parameter $C = 0.4$ and $C = 0.6$, the maximum curvature increment occurs at the midpoint of the transition curve. Furthermore, it should also be noted that:

- the graphs for the Bloss curve and the polynomial curve with parameter $C = 0.5$ overlap. Their graphs and the graph of the Auberlen curve are similar and have a smooth course;
- the maximum value for sine curve and the fourth-degree parabola is the same and amounts to $3.591 \cdot 10^{-6} \frac{1}{m^2}$. It is also the highest value of curvature increment from all examined curves;

- the curvature diagram of the fourth-degree parabola for $l \in \langle 0; \frac{L}{2} \rangle$ increments linearly and for $l \in \langle \frac{L}{2}; L \rangle$ decreases linearly;
- graphs of polynomial curves with parameter $C = 0.4$ and $C = 0.6$ are their mirror images with the axis of symmetry determined for the middle value of the transition curve length. The graphs shown for them (Figure 2) intersect at a point with a curvature increment value of $2.978 \cdot 10^{-6} \frac{1}{m^2}$;
- the maximum of the polynomial curve with parameter $C = 0.4$ is shifted towards the starting point and the maximum of the polynomial curve with parameter $C = 0.6$ towards the end point.

Further dynamic parameters investigated are the velocity f_1 and acceleration f_2 of wheel lift [20].

$$f_1 = v h_0 R \frac{dk(l)}{dl} \tag{3}$$

$$f_2 = v^2 h_0 R \frac{d^2k(l)}{dl^2} \tag{4}$$

When analysing the graphs of the wheel lift velocity (Figure 3), it can be seen that their course is the same as for the curvature increment. As it was already mentioned, the highest wheel lift velocity occurs in the places where the curvature

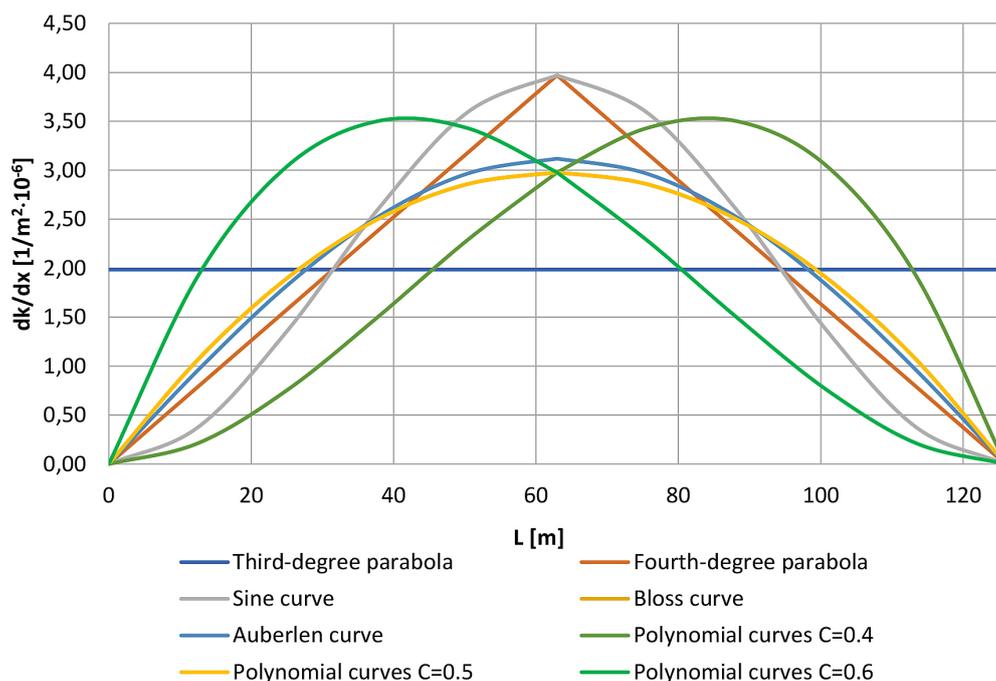


Fig. 2. Increment in curvature $\frac{dk}{dx} [\frac{1}{m^2} \cdot 10^{-6}]$

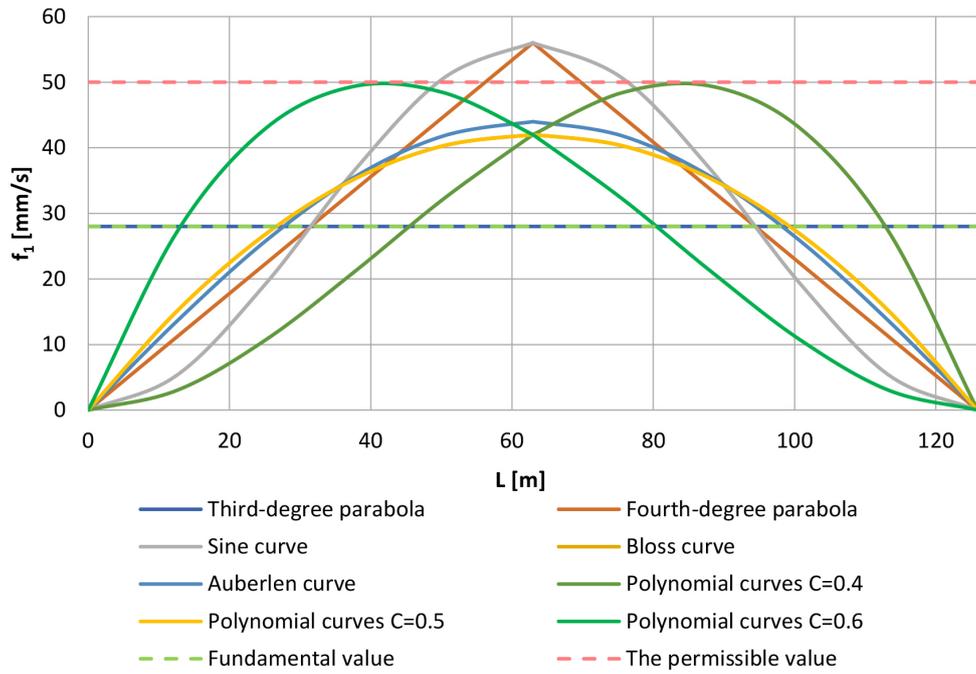


Fig. 3. Wheel lift velocity $f_1 \left[\frac{\text{mm}}{\text{s}} \right]$

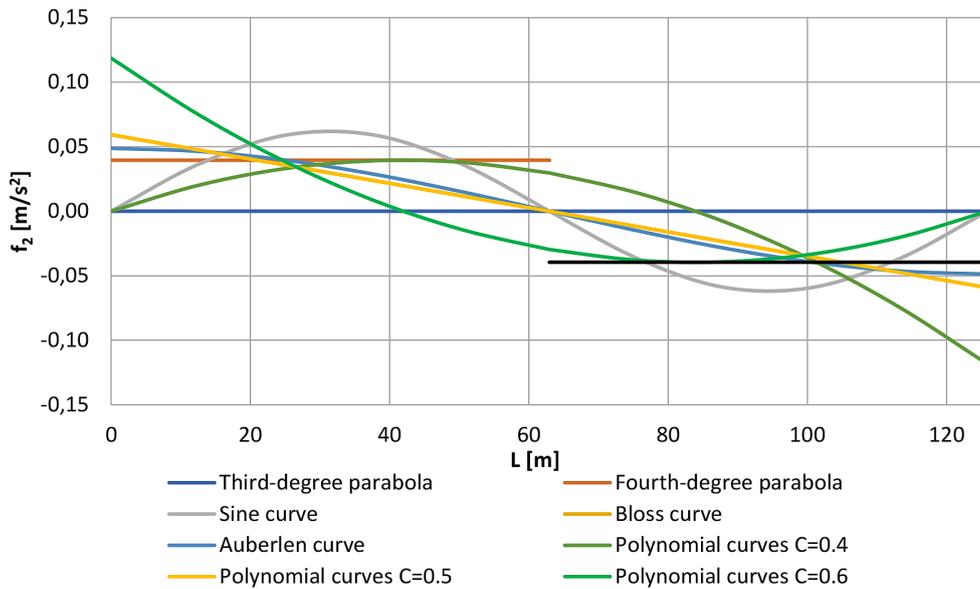


Fig. 4. Wheel lift acceleration $f_2 \left[\frac{\text{m}}{\text{s}^2} \right]$

increment reaches its maximum. Therefore, the course of the obtained graphs needs no additional comment. It should be noted, however, that there are two significant values of the wheel lift velocity adopted in the Polish law. One of them is the fundamental value, which is 28 mm/s [15]. This value is not exceeded by only one transition curve: the third-degree parabola. Another is the permissible value. Due to the rigidity of the rolling stock, the wheel lift velocity must not be

greater than the permissible 50 mm/s [15], which guarantees that the vehicle is stable and its wheels do not detach from the rails. This condition is fulfilled for the transition curves in question except for the fourth-degree parabola and the sine curve.

The wheel lift acceleration graphs (Figure 4) are very different:

- for the third-degree parabola, the wheel lift acceleration has a constant value equal to zero;

- for the fourth-degree parabola, the sine curve, the Bloss curve, the Auberlen curve and the polynomial curve with parameter $C = 0.5$, the wheel lift acceleration values are positive in the interval $\langle 0; \frac{L}{2} \rangle$, and for $L \in (\frac{L}{2}; L \rangle$ they are negative. Whereby, for the fourth-degree parabola, the wheel lift acceleration has constant values in both cases, equal in absolute value to $0.140 \frac{m}{s^2}$. In the other cases they are decreasing curves;
- for the polynomial curves with $C = 0.4$ and $C = 0.6$, the sign change points are shifted respectively to the right and to the left by $1/6$ of the length of the transition curve.

For the curves in question, apart from the third-degree and fourth-degree parabolas, the obtained graphs of the wheel lift acceleration are convex in the positive part and concave in the negative part.

The track twist i_w , which is the algebraic difference between two cant values measured at a specific distance from each other, is another dynamic parameter affecting the ride quality. It is a feature of the cant ramp [20].

$$i_w = \frac{f_1}{v} \tag{5}$$

The course of the twist diagrams (Figure 5) depending on the length of the transition curve is the same as for the diagrams of the wheel lift

velocity and curvature increment. The third-degree parabola has a constant value of the twist along its entire length equal to 0.630 ‰ . As before, two polynomial transition curves with the maximum and minimum value of the parameter C reach the extremum of the dynamic parameter in $1/3$ and $2/3$ of the length of the transition curve, respectively. The graphs of the Bloss curve and the polynomial curve with parameter $C = 0.5$ coincide and are very close to the graph of the Auberlen curve. The fourth-degree parabola and the sine curve in the middle of the transition curve reach the highest value equal to 1.260 ‰ .

The last dynamic parameter analysed is lateral acceleration increment ψ . As in the case of wheel lift velocity, the obtained results were confronted with the permissible value defined by Polish law [20].

$$\psi = \frac{v^2}{h_0 R} f_1 \tag{6}$$

Analysing the results obtained for the dynamic parameter (Fig. 6), which is the lateral acceleration increment, we notice that half of the curves under discussion do not meet the most important condition $\psi_{calculated} \leq \psi_{permissible}$ (the permissible value is $0.30 \frac{m}{s^3}$) [15]. The lowest value of the parameter in question, equal to $0.174 \frac{m}{s^3}$ has the third-degree parabola, which is the same along the entire length of the curve. The permissible value is

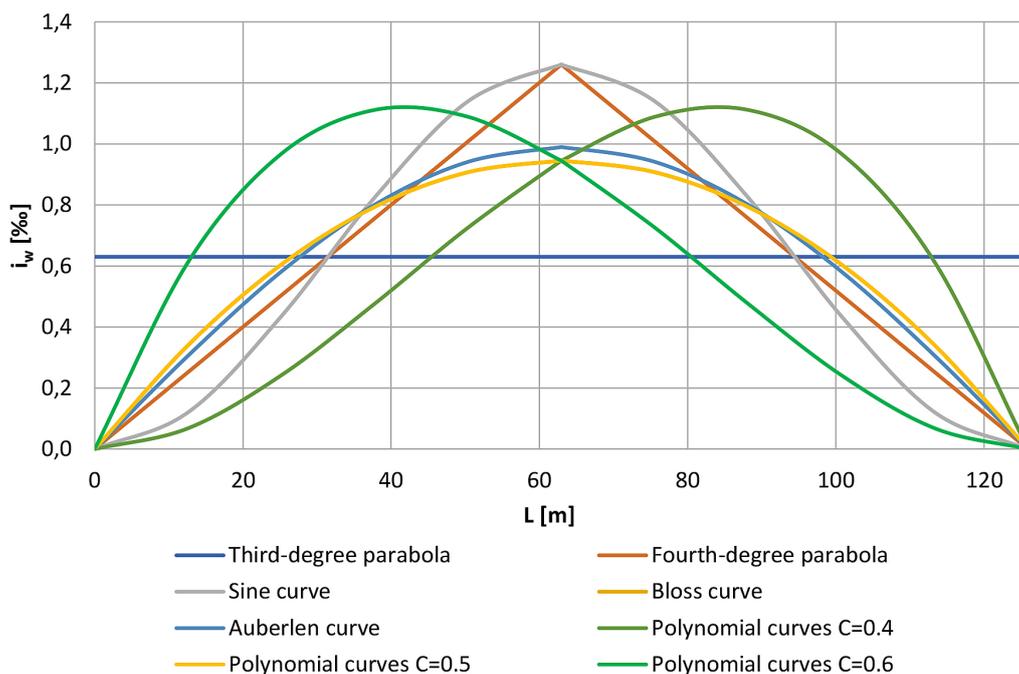


Fig. 5. Track twist i_w [‰]

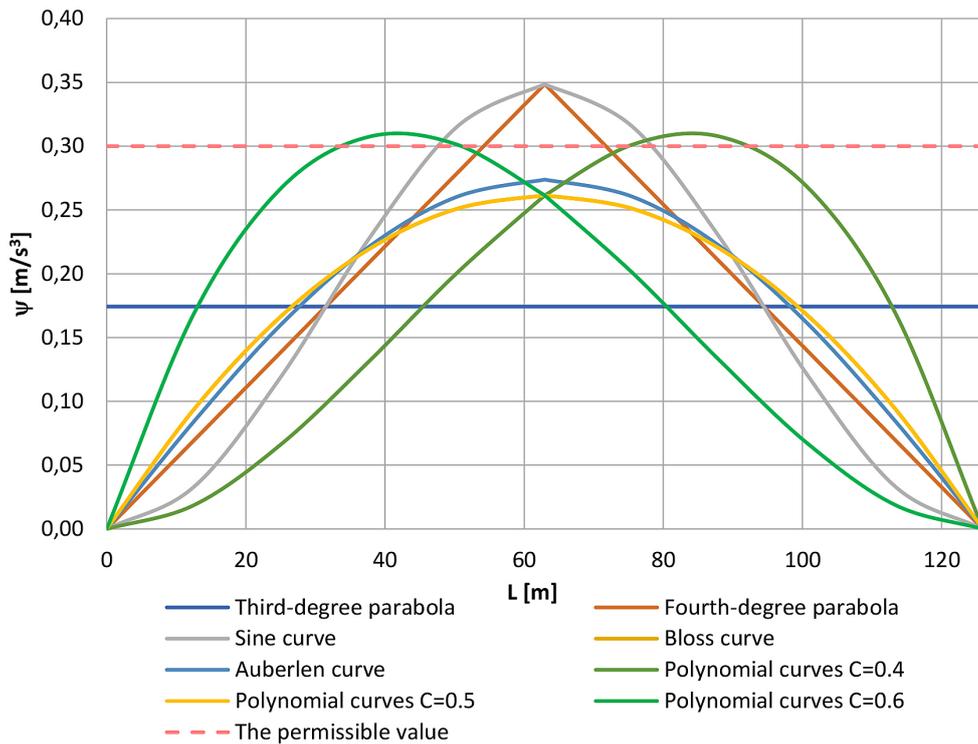


Fig. 6. Lateral acceleration increment $\psi \left[\frac{m}{s^3}\right]$

not exceeded also for the Bloss curve, the Auberlen curve and the polynomial curve with the parameter C equal to 0.5. The other two polynomial curves slightly exceed the permissible value. The maximum value of the twist for the fourth-degree parabola and the sine curve is $0.349 \frac{m}{s^3}$, which is $0.049 \frac{m}{s^3}$ higher than the permissible value.

CONCLUSIONS

The first issue analysed was the change in curvature with the length of the transition curve. Only for one curve, the third-degree parabola, this relation is linear. The values and graphs of the remaining dynamic parameters discussed i.e. curvature increment, wheel lift velocity and acceleration, track twist and lateral acceleration increment are constant values for this curve.

The curvature diagrams of the other curves analysed are, like the third-degree parabola, increasing. However, the increments observed in the initial and final parts of these transition curves are milder. The smallest increment of the curvature at the transition from a straight section of the road to the transition curve occurs for the polynomial curve with the parameter $C = 0.4$. At the transition from the transition curve to the

circular curve, the smallest increment of the curvature occurs for the polynomial curve with the parameter $C = 0.6$.

The obtained diagrams of the following dynamic parameters: curvature increment $\frac{dk}{dx}$, wheel lift velocity f_1 , track twist i_w and lateral acceleration increment ψ , for the curves under consideration, have the same monotonicity. The graphs of these dynamic parameters for the Bloss curve and the polynomial curve with parameter $C = 0.5$ coincide in each case. Together with the fourth-degree parabola, the sine curve and the Auberlen curve, they reach their maximum at the midpoint of the transition curve and are symmetric to the straight line perpendicular to the OL axis passing through this point. The fourth-degree parabola and the sine curve always reach, for a given dynamic parameter, equal maximum values, greater than the other transition curves. It should be noted that the graphs representing the mentioned parameters in the case of the fourth-degree parabola are always for $l \in \langle 0; \frac{L}{2} \rangle$ linearly increasing and for $l \in \langle \frac{L}{2}; L \rangle$ symmetrically, linearly decreasing. The visible breaking point of the graphs indicates a rapid change of dynamic parameters of railway vehicle movement, which is unfavourable from the point of view of ride quality. Not counting the third-degree

parabola, the smallest values of dynamic parameters are obtained by the Bloss curve and polynomial curve with parameter $C = 0.5$. The graphs obtained for the two polynomial curves with extreme values of the parameter C are their mirror images in relation to the straight line perpendicular to the OL axis passing through the centre of the transition curve. The dynamic parameters of the polynomial curve with parameter $C = 0.4$ have the maximum value at the point $l = \frac{2}{3}L$, and for $C = 0.6$ at the point $l = \frac{1}{3}L$.

The graph of the wheel lift acceleration f_2 has quite a different course. It is linear not only for the third-degree parabola, but also for the fourth-degree parabola, with the difference that for the latter, in the middle point of the curve, its sign changes (the absolute value remains the same). At the same point the sign of other curves also changes, except for polynomial curves with $C = 0.4$ and $C = 0.6$, for which this point is shifted to the right and left respectively.

On the basis of the obtained results, the development of research on various types of transition curves, in particular polynomial curves, can be confirmed. They allow to freely shape the distribution of values of dynamic parameters along the entire length of the transition curve, which makes it possible to adapt its course to terrain conditions, direction of movement and type of rail vehicle.

Acknowledgements

Funded by the “Excellent Science” Program of The Ministry of Science and Higher Education, as a part of the contract no. dnk/Sp/465641/2020, “The Role of the Agricultural Engineering and Environmental Engineering in the Sustainable Agriculture Development”.

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