Advances in Science and Technology Research Journal

Advances in Science and Technology Research Journal 2021, 15(2), 176–183
https://doi.org/10.12913/22998624/136178
ISSN 2299-8624, License CC-BY 4.0

Received: 2021.02.18
Accepted: 2021.04.06
Published: 2021.04.30

Study of Contact Pressures in Total Hip Replacement

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ABSTRACT

Total hip arthroplasty is a complex procedure. The achievements of implantology enabled the development of a faithful representation of hip joint physiology as well as the production of materials that can successfully replace damaged natural tissues. The challenge is to correctly select the geometry of the endoprosthesis adequate to the load of the joint. Materials used for endoprosthesis are a metal head and a polymer cup (e.g. PE-UHMW). The main interactions in the endoprosthesis are friction and contact pressure, which must not exceed their limit. Exceeding them causes the destruction of the biomechanical system - plastic deformation of the polymer that is too large and the formation of unacceptable radial clearances. The paper presents the author's empirical method of determining the contact pressures in the tribological pair of the acetabulum - the head of the hip joint endoprosthesis. Based on the obtained research results, it was shown that the developed method gives correct solutions to the contact problem and gives reliable results. The assumption for the work was to prove that empirical methods give correct solutions to contact problems on a par with simulation methods such as FEM. The aim of the work was to demonstrate the correctness of the author's empirical method for determining the maximum contact pressures. Based on the author's developed calculation method of hip joint endoprosthesis contact parameters, the impact on maximum contact pressure and the angle of contact of the joint load was estimated depending on the diameter of the endoprosthesis and radial clearance. The correctness of changing the values of maximum contact pressure from the mentioned parameters was determined. Correspondingly: an increase in joint load causes a linear increase in the maximum contact pressure; increasing the diameter of the endoprosthesis head - their non-linear decrease, and increasing radial clearance - their increase.

Keywords: hip endoprosthesis, calculation method, contact mechanics, contact pressure, diameter of the endoprosthesis head, radial clearance

INTRODUCTION

Hip endoprosthesis are considered the greatest stride forward in orthopedic surgery in the last 100 years. Every year carried over 1 000 000 operations, of which only 300 000 in the United States. Hip arthroplasty is an operation involving the replacement of a diseased hip joint with an artificial one. During the operation, the damaged femoral head and the inside of the acetabulum are removed and replaced with artificial elements. Each endoprosthesis consists of an acetabulum, acetabular insert, stem and metal head. The artificial cup is mounted in place of the natural acetabulum. Most often, it is made of titanium, and inside it, there is an insert made of polyethylene or ceramics. The endoprosthesis mandrel is fixed in the femur. It is also made of titanium and a metal or ceramic head is placed on it. In this way, the metal head and acetabular insert form a new joint, capable of making movements.

A very important issue when designing, implanting and using endoprosthesis is the assessment of contact pressure depending on the load, diameter and the size of the gap (radial clearance) in the joint. Unfortunately, the literature lacks justified assessments using appropriate calculation methods based on classic methods of contact mechanics. In a number of papers, solutions made by FEM numerical simulations are given [1, 2, 3, 5-10]. Due to the fact that these methods are approximate, they do not give a clear answer. In [11] shows the approximate calculation method for estimating the contact pressures. It should be emphasized that there are no analytical methods in the literature concerning the results of endoprosthesis research as a system of spherical bodies. [3] is the only paper presenting a simplified and at the same time advanced to implement calculation method based on the old work of [4]. Lin W. et al. show that "... the accuracy of FEM predictions depends on the input from laboratory experiments" [7], which appropriately justifies that mathematical modeling can be used to estimate contact parameters in endoprosthesis.

Estimating the amount of surface pressure is a very important issue from the point of view of materials used today to build hip joint prostheses. Biocompatible polymers are commonly used, which, after exceeding their limit pressures, undergo plastic deformation. These in turn cause an increase in radial clearances and, consequently, loosening of the hip prosthesis.

Loosening of the hip is one of the most serious complications of joint arthroplasty. The result of this phenomenon is that a new artificial hip joint ceases to perform its function. In extreme cases, when radial clearance is too large, the head of the prosthesis could disconnect from the acetabulum of the polymer and loose support of the human body, causing immobilization of an individual. Another negative consequence of the loosening of the endoprosthesis joint due to exceeding the surface pressure is the formation of inflammation, which causes further degradation of human bone material as well as accelerated wear of the tribological pair.

The hip endoprosthesis is a hip ball joint of a limited deviation angle in a sliding movement at a single-track working load. This paper presents the results of numerical analysis of hip joint endoprosthesis with the use of the proprietary method of solving the contact problems of the theory of elasticity. This method was used to estimate the contact parameters of joints of bodies with a circular cross-section. These bodies are of similar diameters and are in internal contact [12-15]. The aim of the study is to show that the author's empirical method also gives the correct solution to the problem taking into account the joint load, the diameter of the endoprosthesis head and its geometric parameters.

THE STUDY OF CONTACT PRESSURE

In order to use the indicated method of testing flat contact problems, this type of endoprosthesis (3D system) (Fig. 1a) was modeled (Fig. 1b) with the diameter of the head with a cylindrical joint with the cylindrical joint with a fixed socket (3D system) by introducing a model radius (contractual). This method was previously used to test the load capacity of resting joints, as well as initial contact pressures in the moving systems: slide bearings, reciprocating cylindrical guides and pendulum joints [16-18].

The tested 3D system was transformed into a 2D flat system reducing the total compressive load N (Fig. 1c) per head of the prosthesis to one unit of its length (head diameter), i.e. $N'=N/D_2$. It is assumed that $R_1 \approx R_2 = R$, where, respectively R_1, R_2 – the radius of the bushing (bearing shells) 1 and the radius of the disk 2. There is a small radial clearance (aperture) in the hip $\varepsilon = R_1 - R_2 \geq 0 << R$. Under the influence of load, there is contact of the system elements in the zone defined by the angle and arises contact pressure. The elastic characteristics of the bodies are not the same. In the state of unidirectional pendulum motion (partial rotation of the endoprosthesis head towards its cup during gait in the loading phase), we will model this system with a plain bearing (Fig. 1c).

SOLUTION METHOD

The solution method consists of determining the maximum contact pressure, the contact angle and distribution of pressures in the contact sphere. The equation of contact pressures p_α in the case of symmetric contact against the eternal loading of the components in the layout shall be in the form [12–14]:

$$c_1 \int_{-\alpha_0}^{\alpha_0} \cot\frac{(\alpha-\theta)}{2} p'_\theta d\theta = c_2 p_\alpha + c_3 \int_{-\alpha_0}^{\alpha_0} p_\alpha \, d\alpha + + c_4 cos\alpha \int_{-\alpha_0}^{\alpha_0} p_\alpha cos\alpha \, d\alpha + \frac{\varepsilon}{R^2} \quad (1)$$

where: $p'_\theta = \frac{d_p}{d\theta}$; α – polar angle; $0 \leq \alpha \leq \theta$; $0 \leq \theta \leq \alpha_0$

$$c_1 = \frac{1}{8\pi}\left(\frac{1+\kappa_1}{G_1 R_1} + \frac{1+\kappa_2}{G_2 R_2}\right);\ c_2 = \frac{1}{4}\left(\frac{1-\kappa_1}{G_1 R_1} - \frac{1-\kappa_2}{G_2 R_2}\right)$$

$$c_3 = \frac{1+\kappa_1}{8\pi G_1 R_1};\ c_4 = \frac{1}{2\pi}\left(\frac{\kappa_1}{G_1 R_1} + \frac{1}{G_2 R_2}\right)$$

G_1, G_2 – the modulus of material elasticity of all the components in the layout

v_1, v_2 – Poisson's ratio; $\kappa = 3 - 4v$ – the state of flat deformation.

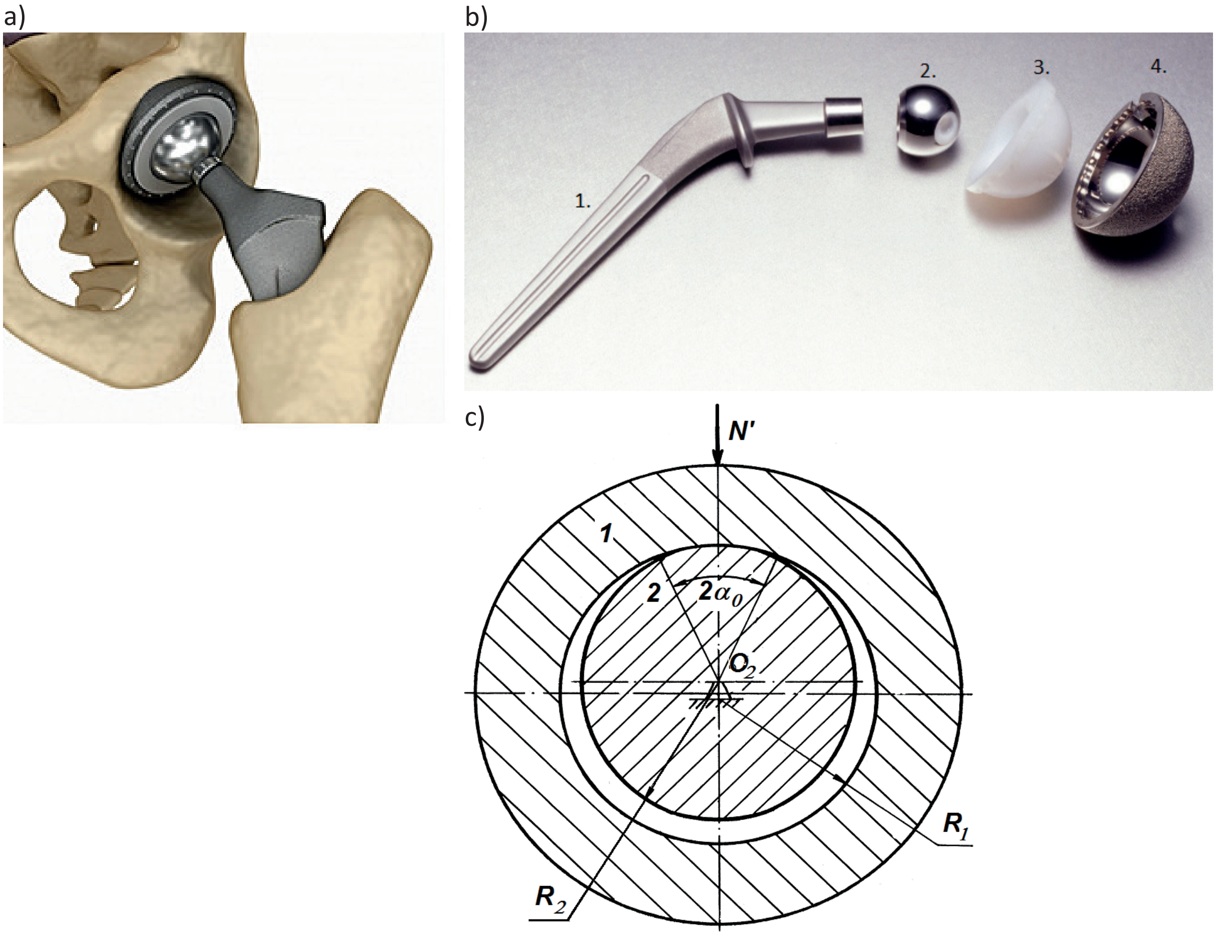

Figure 1. Schemes: a) hip arthroplasty [7], b) hip endoprosthesis 1 – prosthesis pin, 2 – prosthesis head, 3 – acetabulum, 4 – replacement hip socket, c) calculation scheme of the joint

The approximate solution of the equation (1) is achieved by the collocation method. The function of contact pressures p_α is presented in the form of [13-16]:

$$p_\alpha \approx E_0 \varepsilon \sqrt{tan^2\frac{\alpha_0}{2} - tan^2\frac{\alpha}{2}} \quad (2)$$

where:

$E_0 = (\frac{e_4}{R_2}) cos^2(\frac{\alpha_0}{4})$ – simplified version,

$$E_0 = \frac{e_4}{R_2}\left[cos^{-2}(\frac{\alpha_0}{4}) - e_1\sqrt{tan^2(\frac{\alpha_0}{2}) - tan^2(\frac{\alpha_0}{4})} - - 0.5 sin^2\frac{\alpha_0}{4}\left(e_2 cos^{-1}(\frac{\alpha_0}{2}) + 2e_3 \cos(\frac{\alpha_0}{2})\right)\right]^{-1}$$

– detailed version,

$e_1 = \frac{2}{Z}[(1-\kappa_1)(1+\mu_1)E_2 - (1-\kappa_2)(1+\mu_2)E_1]$;

$e_2 = \frac{2}{Z}(1+\kappa_1)(1+\mu_1)E_2$

$e_3 = \frac{4}{Z}[\kappa_1(1+\mu_1)E_2 + (1+\mu_2)E_1]$; $e_4 = \frac{4E_1E_2}{Z}$;

$e_4 = \frac{4E_1E_2}{Z}$; $Z = (1+\kappa_1)(1+\nu_1)E_2 + (1+\kappa_2)(1+\nu_2)E_1$

$E = \frac{2G}{(1+\nu)}$ – Young's modulus.

Maximum contact pressures p_0 are obtained when $\alpha = 0$. Then

$$p_0 \approx E_0 \varepsilon \tan\frac{\alpha_0}{2} \quad (3)$$

The balanced forces exerted on the second target determine an unknown half-angle α_0.

$$N' = R\int_{-\alpha_0}^{\alpha_0} p_\alpha \cos\alpha d\alpha = 4\pi R E_0 \varepsilon sin^2\frac{\alpha_0}{4} \quad (4)$$

LOADING CONDITIONS IN THE HIP JOINT

The total force N on the femoral head is determined based on various literature data [17]. It is a geometric sum of the two forces – body weight $\boldsymbol{K}$ as well as muscular strength M (Fig. 2).

In the cycle of movement, the value of N changes substantially – $1.45K \leq N \leq 4.4K$ (Fig. 2). Therefore, in normal conditions of walking in two peaks, we observed pressure of $3.0K$ and $4.4K$. In extreme cases, it reached up to 9 K. Therefore, taking all the data into consideration, the average

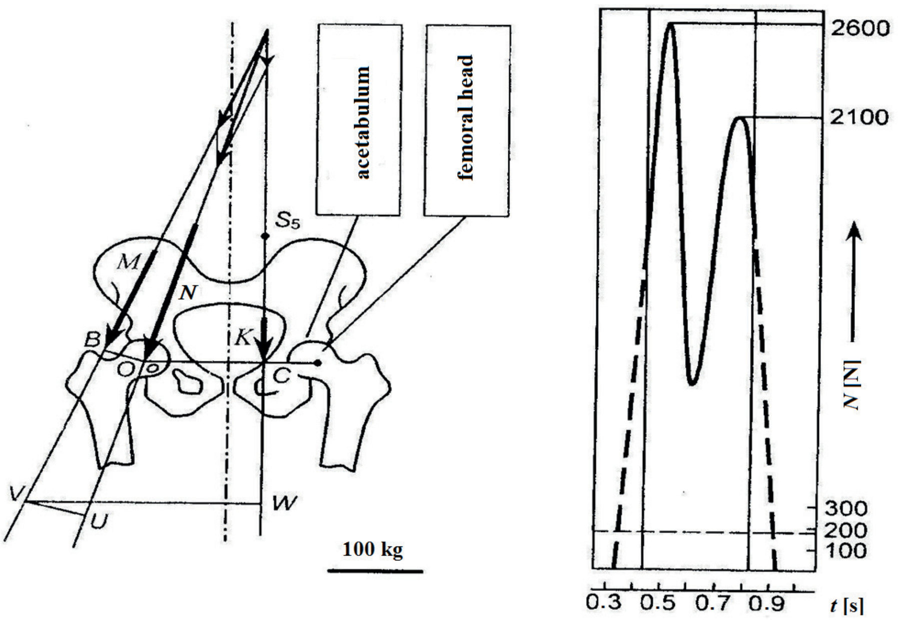

Figure 2. Quasi-static estimation of hip joint loading, *K* – body weight. Diagram of the change in pressure on the head of the hip joint during support with one foot during the physiological gait phase [18, 19]

Table 1. Input data for calculations

D_2 = 28 mm	D_2 = 48 mm	D_2 = 58 mm
N'_{max}=103.6 N/mm	N'_{max}=60.4 N/mm	N'_{max}=50 N/mm
N'_{av}= 68 N/mm	N'_{av}= 39.6 N/mm	N'_{av}= 32.8 N/mm
N'_{min}= 35.7 N/mm	N'_{min}= 20.8 N/mm	N'_{min}= 17.2 N/mm

value of the compression force N_{avg} registers about 1900 N (assuming *K* = 700 N).

NUMERICAL SOLUTION OF THE PROBLEM

Numerical solution of the problem was conducted for the examined layout when $\varepsilon > 0$. To calculate the parameters of the contact in the joint the following data were selected: N_{max} = 2900 N, N_{avg} = 1900 N, N_{min} = 1000 N; N' = N/D_2; D_2= 28, 48 and 58 mm; ε = 0.02–0.2 mm; for calculations were adopted accordingly (Table 1):

A simplified version was applied in case of E_0. In the calculation of the model cylindrical joint, a conventional radius $R' = 0.5\sqrt{(R_1R_2)} = 0.5\sqrt{[(R_2 + \varepsilon)R_2]}$ was introduced. This resulted in a replacement plane system for the prosthesis as the spatial system with R_1 and R_2 rays interacting elements.

Table 2. Input data for calculations

Data / Param.	D_2 = 58 mm, N_{max} = 2900 N, N'_{max} = 50 N/mm				D_2 = 58 mm, N_{av} = 1900 N, N'_{av} = 32.8 N/mm				D_2 = 58 mm, N_{min} = 1000 N, N'_{min} = 17.2 N/mm			
ε [mm]	0.049	0.05	0.1	0.2	0.032	0.05	0.1	0.2	0.017	0.05	0.1	0.2
p_0 [MPa]	1.978	1.98	2.6	3.56	1.30	1.52	2.06	2.86	0.48	1.06	1.44	2.0
$2\alpha_0$ [°]	158.8	157.6	106.6	74	148.8	126.6	85.8	60	158.8	85.8	60	42.2

Data / Param.	D_2 = 28 mm, N_{max} = 2900N, N'_{max} = 103.6N/mm		D_2 = 28 mm, N_{av} = 1900N, N'_{av} = 68N/mm			D_2 = 28 mm, N_{min} = 1000N, N'_{min} = 35.7N/mm			
ε [mm]	0.1023	0.2	0.0671	0.1	0.2	0.035	0.05	0.1	0.2
p_0 [MPa]	8.48	10.92	5.56	6.41	8.64	2.92	3.26	4.4	6.1
$2\alpha_0$ [°]	158.8	108.4	158.8	128.2	86.8	158.8	128.2	88.2	61.6

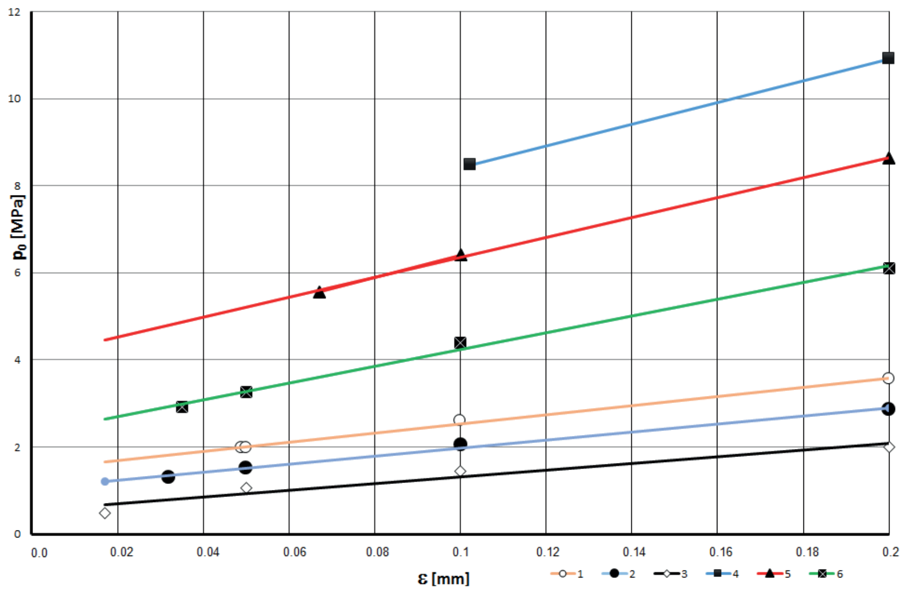

Figure 3. The impact of radial clearance on maximum contact pressures. D_2= 58 mm: 1 – N'_{max}= 50 N/mm; 2 – N'_{av}=32.8 N/mm, 3 - N'_{min}=17.2 N/mm; D_2= 28 mm: 4 - N'_{max}= 103.6 N/mm, 5 - N'_{av}= 68 N/mm, 6 - N'_{min}= 35.7 N/mm

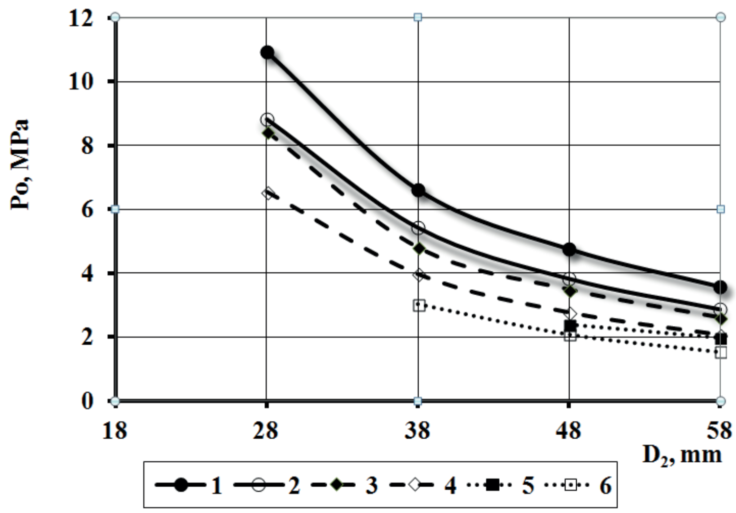

Figure 4. The impact of the head diameter of endoprosthesis on maximum contact pressures: 1 – N_{max}= 2900 N, ε = 0.2 mm; 2 – N_{av} = 1900 N, ε=0.2 mm; 3 – N_{max}= 2900 N, ε = 0.1 mm; 4 – N_{av}= 1900 N, ε = 0.1 mm; 5 – N_{max}= 2900 N , ε = 0.05 mm; 6 – N_{av}= 1900 N, ε = 0.05 mm

Materials used for the endoprosthesis: head 2 – NitridedGRADE 2 (TDN) [10], for which E_2 = 112 GPa, ν_2 = 0.32 (GRADE 2 - titanium); acetabulum 1 – polyethylene PE-UHMW, for which E_1 = 0.625 GPa (37 ^{0}C), ν_1 = 0.46. The results of the solution are given in Figures 3 and 4 as well as in Table 2. In Figure 3 the ratio correlation of maximum contact pressures p_0 of radial clearance ε is presented with the head diameter D_2 of endoprosthesis and the reduced compression force *N'*.

Analysis of the results allows us to draw conclusions from linear relationship of increasing p_0 from ε within the scope of 0.05≤ ε ≤0.2 mm. When ε ≤ 0.05 mm the above-mentioned correlation becomes non-linear. The intensity of growth p_0 depends on the size of head diameter D_2. In Table 2 results of p_0 and $2\alpha_0$ are presented.

For the maximum and average value of compression force and the examined radial clearances, the relationship between maximum contact pressures p_0 with the head diameter of endoprosthesis D_2 was presented in Figure 4.

The results of the calculation indicate that by increasing D_2 by 2.07 causes the reduction of p_0 by

2.96 ÷ 4.24, depending on the value of N as well as ε. According to the study [8]

$$p_0 = a_1 + a_2 N^{a_3} + a_4 \varepsilon^{a_5} + a_6 N^{a_3} \varepsilon^{a_5} \quad (5)$$

If $\varepsilon \geq 0$; a_1, a_2, ... factors of approximation.
And according to [11]:

$$p_0 = c_0 \frac{\varepsilon E_1}{R}, \quad (6)$$

where: $E_2 = \infty$, $E_1 = 0.625$ GPa, c_0 – collocation rate depends of α_0.

In our case $E_2/E_1 = 112/0.625 = 179.2$ (GRADE2/PE-UHMW). Formally, it may be stated that the head of endoprosthesis is very rigid in relation to the bearing and deformation does not occur due to the contact force.

In terms of the flat contact formula, the maximum contact pressure p_0 was also estimated according to the Hertz equation for the contact disc with aperture:

$$p_0 = 0.564 \sqrt{\frac{N'}{\beta} \frac{R_1 - R_2}{R_1 R_2}} \quad (7)$$

where: $\beta = \frac{(1-\nu_1^2)}{E_1} + \frac{(1-\nu_2^2)}{E_2}$ – shear modulus,

$R_1 - R_2 = \varepsilon$, $R_1 = R_2 + \varepsilon$.

Hertz formula for the internal ball connection of slightly different radii is not possible to use within the examined issue of flat contact strength of theory of elasticity due to the two-dimensional (circle shape) area of contact. Table 3 and Figures 5 and 6 show the results of the calculations p_0 according to the certain methods and their relative change for the layout where radial clearance ε = 0.1 and ε = 0.2 mm occurs.

CONCLUSIONS

The developed method enables the effective assessment of maximum contact pressures in endoprosthesis in case of the occurrence of radial clearance. Maximum contact pressures p_0 depend on the loading force N against the

Table 3. Results of the calculations

Formula / Param.	D_2 = 58 mm, N = 2900N, N' = 50N/mm (Aut.)		(6)		(7)		(5)	
ε [mm]	0.1	0.2	0.1	0.2	0.1	0.2	0.1	0.2
p_0 [MPa]	2.59	3.56	2.74	3.81	1.22	1.73	3.0	4.0
$2\alpha_0$ [°]	106.6	74.0	106.6	74.0	-	-	-	-
p_{0i}/p_0	1.0	1.0	1.06	1.07	0.47	0.485	1.16	1.12

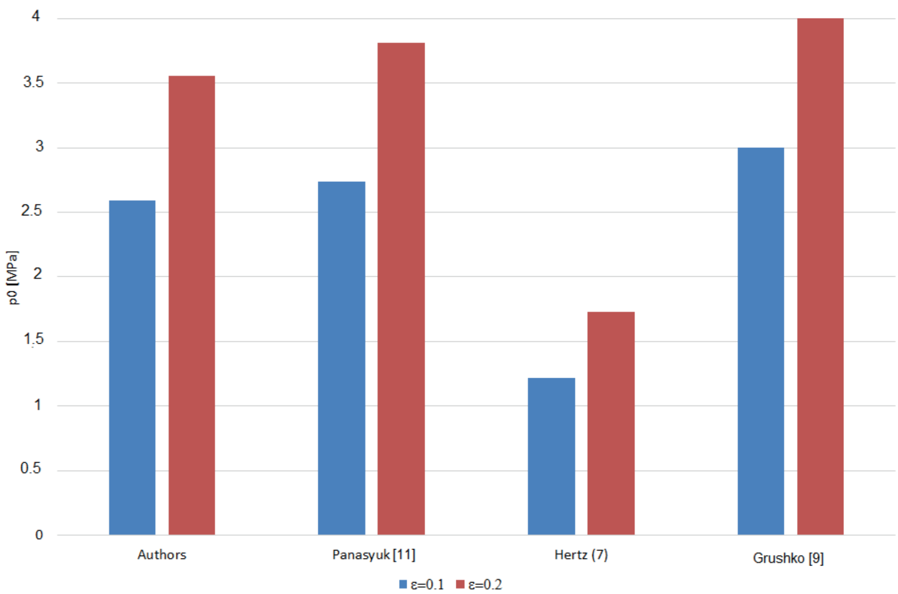

Figure 5. Alteration of maximum contact pressures for radial clearance ε=0.1mm and ε = 0.2 mm. Authors - the results of the authors' research, Panasyuk [11] - solution by method (6) [11], Hertz (7) - solution by Hertz equation (7), Grushko [9] - solution by method (5) [9]

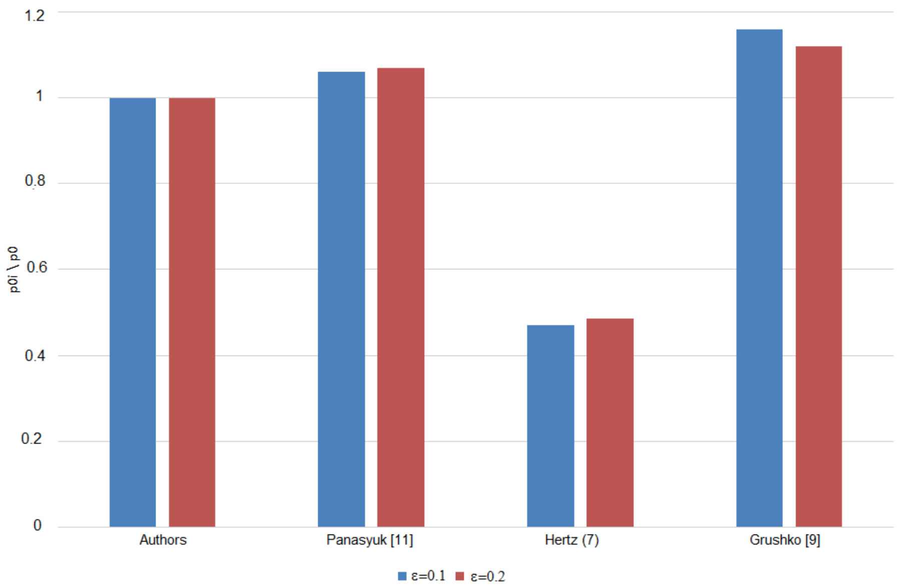

Figure 6. Relative change of maximum contact pressure for radial clearance ε=0.1mm and ε = 0.2 mm. Authors - the results of the authors' research, Panasyuk [11] - solution by method (6) [11], Hertz (7) - solution by Hertz equation (7), Grushko [9] - solution by method (5) [9]

endoprosthesis head, its diameter D_2 and radial clearance ε in the layout (Fig. 3, 4). Their values increase almost linearly with the increase of radial clearance in the range of 0.05≤ ε ≤0.2 mm (Fig. 3). At ε ≤0.05 mm, their change slightly differs from the linear one. A comparative analysis of results regarding the assessment of p_0 according to the author's method is in accordance with other methods (Fig. 5 and 6).

Due to the method [10] certain pressures p_0 of slightly higher value (1.07) were determined and according to the Hertz formula the result is inappropriate and according to the method [8] – higher than the original method up to 1.16 times. According to the data established in this paper [5] the solution for the endoprosthesis problem was conducted CoCr – CoCr, when N = 3200 N, D_2 = 58.6 mm, ε = 0.05 mm. It was established that the maximum contact pressures reached 24.05 MPa by the original method and 22.0 MPa according to FEM [5] that means they are 1.093 times higher. However, according to the scholarly work [8] and the original authors method for the endoprosthesis CoCrMo – PE-UHMW, if N = 2500 N, D_2 = 32 mm, ε = 0.098 mm the pressures are identical – 10.2 MPa. According to the work [3] for Steel – PE-UHMW endoprosthesis, when N = 2500 N, D_2 = 28 mm, ε = 0.25 mm, the maximum contact pressures are 14.94 MPa, and according to the author's method – 13.44 MPa, i.e. they are lower by 1.11 times.

From the practical point of view, the results of the examined issue indicate that the reduction of radial clearance below 0.05 mm does not seem to be deliberate.

Producers of hip endoprosthesis anticipate the initial radial clearance ε = 0.05 ÷ 0.25 mm [8]. As a practical matter, the assessment of maximum contact pressures outside the scope is not targeted. Nonetheless, reduction ε of the above-mentioned ε_{min}, even more hypothetically factored to zero does not provide any substantial benefits in an increase of the contact surface $2\alpha_0$ of all the components of endoprosthesis and the reduction of level p_0.

The results of the calculation (Table 3) show that there are certain values of clearance ε in each examined case where the angle of the contact $2a_0$ reaches the limit value of 160 degrees. It is caused by the increased deformation of the bearing material (PE-UHMW) upon the contact pressures. In Figure 4 those threshold values ε were established for which the contact angle for this method will be verging. Especially in cases D_2= 28 mm and N_{max} = 2900 N, N_{av} = 1900 N appears at ε>0.05 mm, and for D_2= 58 mm, N_{max}= 2900 N, when ε > ε_{min} = 0.05 mm.

The reduction of the initial radial clearance in endoprosthesis of the examined type below 0.05 mm aims at the reduction of the initial contact pressures is not justified since those pressures will be reduced while acetabulum wear. It is also known that at acetabulum wear the size of its diameter approaches the size of the diameter of rigid head, which technically is hardwearing. Therefore, the construction clearance declines which results in the reduction of maximum contact pressures

Acknowledgments

The research was financed in the framework of the project Lublin University of Technology - Regional Excellence Initiative, funded by the Polish Ministry of Science and Higher Education (contract no. 030/RID/2018/19).

REFERENCES

1. Maxian T.A, Brown T.D., Pedersen D.R., Calladhan J.J. A sliding – distance – coupled finite element formulation for polyethylene wear in total hip arthroplasty. Journal of Biomechanics, 1996, 29, 687–692.
2. Bevill S.L, Bevill G.R., Penmetsa J.R, et al. Finite element simulation of early creep and wear in total hip arthroplasty. Journal of Biomechanics, 2005, 38, 2655–2374.
3. Jin Z.M., A general axisymmetric contact mechanics model for layered surfaces, with particular reference to artificial hip joint replacements. Proceedings of the Institution of Mechanical Engineers, Part H: Journal of Engineering in Medicine, 2000, 214(5), 425-435.
4. Lebedev N.N., Ufliand Ia.S. Axisymetric contact problem for an elastic layer. Journal of Applied Mathematics and Mechanics Volume PMM, 1958, 22(3), 320–326, https://doi.org/10.1016/0021-8928(58)90059-5.
5. Liu F., Jin Z., Grigoris P., et al. Contact mechanics of metal-on-metal hip implants employing a metallic cup with a UHMWPE backing. Proc. J. Mech. E. Part H: Journal of Engineering in Medicine, 2003, 217, 207–213.
6. Liu F., Udofia J.I., Jin Z., et al. Comparison of contact mechanics between a total hip replacement and a hip resurfacing with a metal-on-metal articulation. ProcImechE, Part C: Journal of Mechanical and Engineering Science, 2005, 219, 727–732.
7. Lin W., Gracham I., Ruth W. Et al. Finite element analysis of polyethylene wear in total hip replacement: a literature review. Proc. JMechE. Part H: Journal of Engineering in Medicine, 2019, 233, 1–22.
8. Hua X., Wroblewski B.M., Jin Z., Wang L. The effect of cap inclination and wear on the contact mechanics and cement fixation for ultra-height molecular polyethylene total hip replacements. Medical Engineering and Physics, 2012, 34, 318–325.
9. Grushko A.V., Sheykin S.E., Rostotsckiy I. Contact pressure in hip endoprosthesis swivel joints. Journal of Friction and Wear, 2012, 33, 124–129.
10. Shankar S., Prakash L., Kalayarasan M. Finite element analysis of different contact bearing coupes for human hip prothesis. J. Biomedical Engineering and Technology, 2013, 11, 66–80.
11. Panasyuk V.V., Teplyi M.I. Some contact problems of the theory of elasticity. Kyiv: Naukova dumka, 1975.
12. Andreikiv A. Je. and Chernets M. V. Assessment of the Contact Interaction of Machine Parts in Friction [in Russian]. Kiev: Naukova Dumka, 1991.
13. Chernets M. V. Contact Problem for a Cylindrical joint with Technological Faceting of the Contours of its Parts, Materials Science, 2009, 859–868.
14. Chernets M. Evaluation of contact strength and durability of plain bearings with different of shaft lobbing. Proc. JMechE. Part J: Journal of Engineering Tribology, 2015, 229, 1444–1454.
15. Chernets M.V., Andreikiv O.E., Lebedeva N.M. and Zhydyk V.B. A Model for the Evaluation of Wear and Durability of a Plain Bearing with Small Out-of-Roundness. Materials Science, 2009, 279–290.
16. Chernets M., Chernets Ju. Generalized method for calculating the durability of sliding bearings with technological out-of-roundness of details. Proc. J. I. Mech. E. Part J: Journal of Engineering Tribology, 2015, 229, 216–226.
17. Pauwers F. Biomechanics of the locomotor apparatus. Berlin-Heideberg-Ney-York: Springer – Verlag, 1989.
18. Kupchinov B.I., Ermakov S.F., Belonenko E.D. Biotribology of synovial joints. Minsk: Vedas, 1997.
19. Sheykin S., Pohrelyuk I., Rostotsckiy I., Iefrosinin D., Tuszyński W., Mańkowska-Snopczyńska A. Tribological behavior of the friction pair "GRADE 2/PE-UHMW" and the technology of the production of its spherical part made of GRADE 2. Tibologia, 2018, 137–148.