

A Method of Increasing the Accuracy of Controlling the Parameters of Dynamic Systems and Regulating the Parameters of the Elastic-Deformable State in the Process of Treating Low-Rigidity Shafts

Antoni Świć^{1*}, Arkadiusz Gola², Łukasz Sobaszek³

¹ Department of Production Computerisation and Robotisation, Mechanical Engineering Faculty, Lublin University of Technology, Lublin, Poland

* e-mail: a.swic@pollub.pl

ABSTRACT

The article presents the models of technological systems and the parameters of the control object allowing one to seek rational control algorithms, choose the structure of the control system and synthesize the correction devices. The generalized and detailed structural schemes of controlling the elastic-deformable state during working on low-rigidity shafts while applying the tensile force as well as with additional feedback and transfer functions of technological systems, considering the assumptions made and the results of theoretical and experimental testing. The structure of the control object and the dependencies describing the system and considering the specificity of forming the section of the machined surface of the elastic-deformable low-rigidity shafts at longitudinal feeds. As a result of the research a method of increasing the accuracy of controlling the parameters of dynamic systems at the change of the longitudinal feed and regulating the parameters of the elastic-deformable state of low-rigidity elements by introducing correction devices in form of negative feedback in accordance with the cutting force. Moreover, the possibility of creating adaptive units which are not very sensitive to the change of parameters in turning and grinding.

Keywords: low-rigidity shafts, elastic-deformable shafts, low-rigidity, machining, accuracy, control, correction, elastic-deformable state

INTRODUCTION

In the machine industry, 34% of the elements are axially-symmetric with 12% being characterized by low-rigidity. Despite the fact that the dimensions of such parts are often characterized by the lack of proportion as well as low rigidity in certain sections and directions, they ought to fulfil certain requirements regarding the parameters of the geometrical shape, mutual location of the surfaces, linear dimensions and surface quality [9, 10, 23]. It is difficult to obtain such parameters of the accuracy of shape, dimensions and surface quality in the shaft treatment process [6, 7, 18]. The relatively low rigidity of the elements compared to the rigid systems of the machine tool

causes vibrations to occur under certain conditions [12, 13, 15]. There are numerous factors disturbing and destabilizing the treatment process (significant own strain of the parts, tools, chips, dust etc.) [1, 10, 22, 27]. For this reason, it is necessary to seek for new methods of controlling the treatment of low-rigidity shafts.

The quality of the treatment of elements on machine tools directly depends on the automatic control system (ACS) of the machine tool and the parameters of the dynamic system [9, 23, 21]. The dynamic system (DS) of the treatment process consists of a machine tool-handle-device-tool (MHDT) that is mass-dissipative-elastic (MDE) system of the machine tool with the machining processes (turning, grinding, drilling, milling) [4, 21].

The analysis and synthesis of the modern automatic control systems is conducted with a significant simplification of the physical and mathematical dependencies describing the processes in the system [2, 5, 8]. This results, to a great extent, from the imperfection of the applied testing machine as well as the a priori obtained information on the static and dynamic characteristics of the control object as well as external disturbing interactions [3, 20]. The available methods of synthesis of the automatic control systems allow for a certain indeterminacy of the character and magnitude of disturbances [14, 23]. To some extent, the aspects of dispersion of the parameters of the control object are considered, although currently this problem is extensively researched [5, 14, 16, 22, 27].

Designing such control systems that can operate in uncontrolled changeability of the parameters of the control object led to applying adaptive control (AC) in machine construction [1, 6, 14]. Despite its versatility as well as the possibility of obtaining the required quality of transient processes in a wide variety of objects, it is difficult to use them on machine tools due to the need to constantly measure the characteristics of the technological systems and disturbances [17, 27].

In the construction of rational structures of control systems, it is necessary to consider the possibility of increasing the quality of the sample. Due to the fact that elastic deformation control system (relative displacement of the cutting tool and the workpiece) is a static one, both in terms of controlling and disturbing interactions, the change causes errors to the relative location of the part and the cutting edge [9, 17, 25, 26].

The parameters of the elastic-deformable state of low-rigidity shafts

The control system for the parameters of the elastic-deformable state is essentially a closed system. The cooperation of its basic elements is geared by the interactions between the working processes occurring in the elastic system.

The dependencies describing the system at turning and grinding elastic-deformable low rigidity parts were shown in [11, 16, 23]. The dynamic properties of the linear models are approximated by the transfer functions of typical dynamic elements. The obtained dynamical models of the technological system and the parameters of control system allow one to seek optimal control algorithms, choose the structure of the control

system as well as synthesize the corrective devices [3, 27]. A generalized structural scheme of controlling the elastic-deformable state of a low-rigidity part at the application of tensile force F_{x1} is shown in Figure 1.

The transfer function of the technological system, considering the assumptions and the results of theoretical and experimental testing can be expressed as follows:

$$G_4(s) = \frac{1 + K_{\kappa} K_y m_y (1 - e^{-s\tau})}{1 + K_{bz} K_z n_z + K_{yy} n_x + K_{yy} n_y + (1 - e^{-s\tau})} \times \frac{1}{[k_{\kappa} k_y m_y (1 + K_{yy} n_x + K_{bz} K_z n_z + K_{yy} n_y) - (K_{\kappa} - K_{\kappa} n_x)(K_{bz} K_z n_z + K_{yy} m_y)]} \quad (1)$$

The structural scheme of the adaptive control system of elastic deformation for elastic-deformable parts in the technological system with additional feedback $G_{sz}(s)$ according to cutting force F_p is presented in Figure 2.

Thereby the condition (2) of full unchangeability of the control system in relation to the control interaction is obtained.

$$G_{sz}(s) = \frac{1 + G_{sk}(s)G_s^s(s)}{G_{sk}(s)G_w(s)} \quad (2)$$

Therefore, in this case there is no static error according to the controlling interaction $y_0(s)$ with an additional feedback [19, 21]. As a consequence, stability, operational speed and insensitivity to the change of allowance at a negative feedback increase. The expression determining $G_5(s)$ can be presented as:

$$G_5(s) = \frac{(1 - e^{-s\tau})[m_y(K_x n_x - K_{\kappa}) - K_{\kappa} K_y m_y n_y] - n_y}{1 + K_{\kappa} K_y m_y (1 - e^{-s\tau})} \quad (3)$$

Considering the dependencies (1) and (3) the transfer function of the corrected control system can be written in the following form:

$$\Phi_{sk}(s) = \frac{1 + K_{bz} K_z n_z + K_{yy} n_x + K_{yy} n_y + n_y K_{F_{z1}} G_w(s) + (1 - e^{-s\tau})[K_{\kappa} K_y m_y \times (1 + K_{yy} n_x + K_{bz} K_z n_z + K_{yy} n_y + n_y K_{F_{z1}} G_w(s) G_{sz}(s))] + (K_{\kappa} K_y m_y (1 + K_{yy} n_x + K_{bz} K_z n_z + K_{yy} n_y + K_{F_{z1}} G_w(s) (1 + n_y G_{sz}(s) + K_{yy} n_x) + (K_x n_x - K_{\kappa})(K_{bz} K_z n_z + m_y K_{yy} - m_y K_{F_{z1}} G_w(s) G_{sz}(s))]}{1 + K_{bz} K_z n_x + K_{yy} n_y + K_{yy} n_y + K_{F_N}(s) G_w(s) (1 + n_y G_{sz}(s))} \quad (4)$$

and the error of the system according to the controlling interaction is determined from the following dependency:

$$\varepsilon_y(s) y_0(s) = \frac{1 + K_{bz} K_z n_z + K_{yy} n_x + K_{yy} n_y - n_y K_{F_{z1}} G_w(s) G_{sz}(s)}{1 + K_{bz} K_z n_x + K_{yy} n_y + K_{yy} n_y + K_{F_N}(s) G_w(s) (1 + n_y G_{sz}(s))} \quad (5)$$

The dependency (5) indicates that the error $\varepsilon_y(s)$ introduced to the control system of the parameters of the elastic-deformable state of the elements in the technological system, control

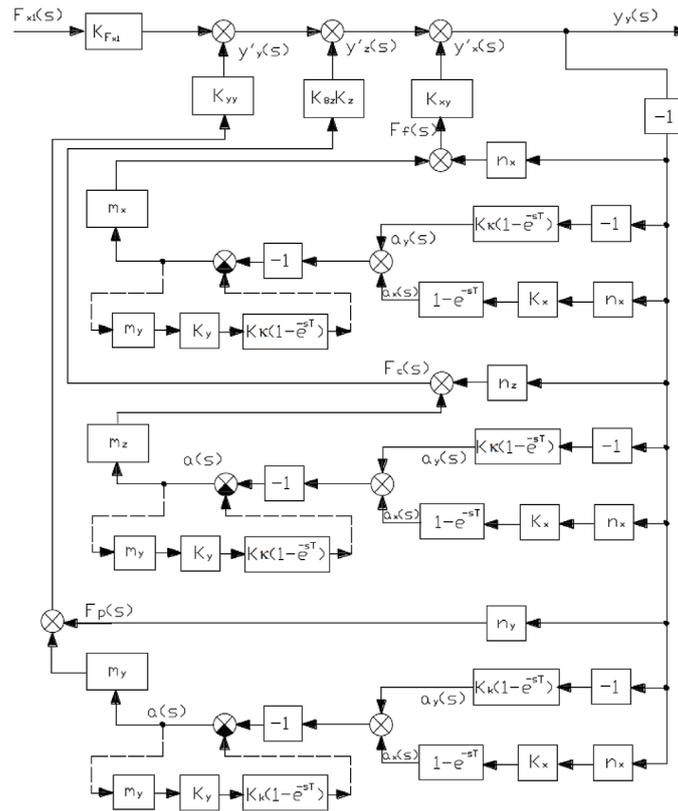


Fig. 1. Structural scheme of the technological system at turning low-rigidity elastic-deformable shafts

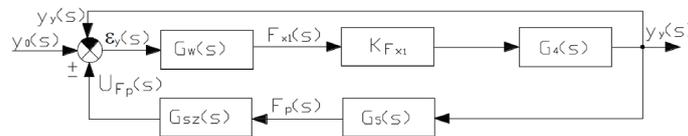


Fig. 2. Structural scheme of the adaptive control system with additional feedback according to the cutting force F_p

interactions can be eliminated if the structure and parameters of the transfer function of the positive feedback $G_{sz}(s)$ is selected in accordance with the dependency:

$$G_{sz}(s) = \frac{1 + K_{bz}K_z n_z + K_{xy}n_y + K_{yy}n_y}{n_y K_{F_{x1}} G_w(s)} \quad (6)$$

When the influence of the component F_f of the cutting force on the increase in plastic strain with the coordinate y the structural scheme of the control object is not considered, the technological system of turning elastic-deformable low-rigidity shafts, may be transformed into the form presented in Figure 3, whereas the transfer function is expressed with the following dependency:

$$G_6(s) = \frac{1 + m_y K_y K_x (1 - e^{-sT}) \times}{1 + K_{bz}K_z n_z + K_{yy}n_y + (1 - e^{-sT}) \times} \times 1 \quad (7)$$

$$\times [m_y K_y K_x (1 + K_{bz}K_z n_z + K_{yy}n_y) + (K_{bz}K_z m_z + K_{yy}m_y)(K_x - n_x K_x)]$$

The structural scheme of the control system with a feedback concerning the cutting force is shown in Figure 3, where the transfer function $G_5(s)$ is described with the dependency (3).

The error of the control system introduced by control interaction is described as:

$$\varepsilon_{y1}(s) = y_0(s) \frac{1 + K_{bz}K_z n_z + K_{yy}n_y - n_y K_{F_{x1}} G_w(s) G_{sz1}(s)}{1 + K_{bz}K_z n_z + K_{yy}n_y + K_{F_{x1}} G_w(s) [1 + n_y G_{sz1}(s)]} \quad (8)$$

In order to eliminate the static error $\varepsilon_{y1}(s)$, introduced to the system by control interactions, the structure and parameters of the transfer function of the positive feedback $G_{sz1}(s)$ ought to be determined in the following manner:

$$G_{sz1}(s) = \frac{1 + K_{bz}K_z n_z + K_{yy}n_y}{n_y K_{F_{x1}} G_w(s)} \quad (9)$$

When the influence of the elastic deformation after the coordinates z and x on the change

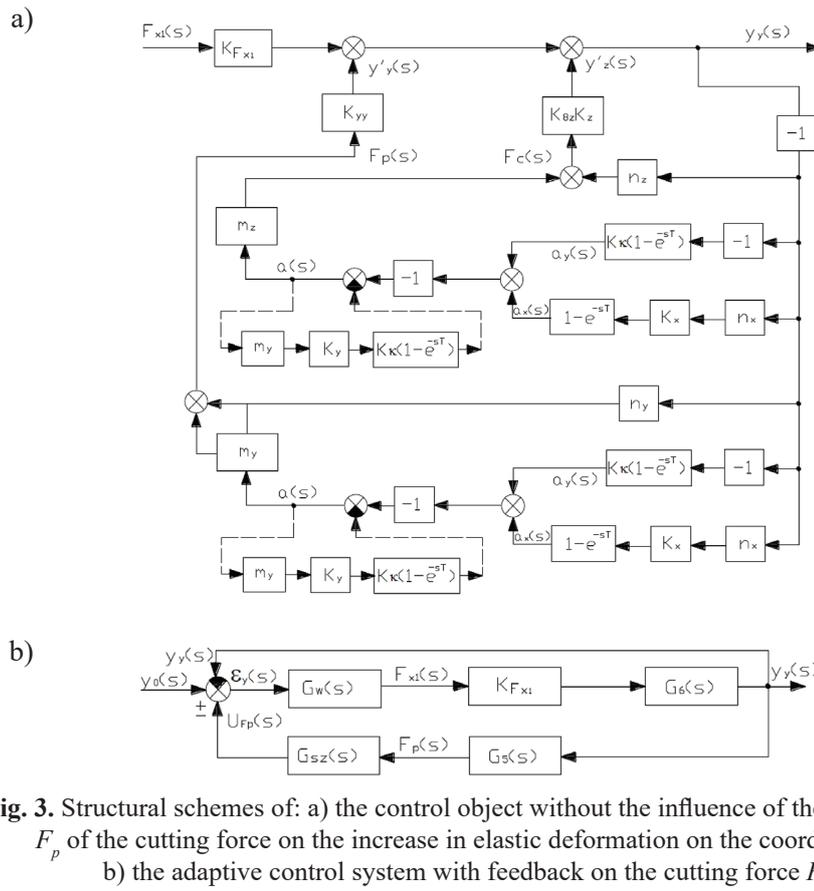


Fig. 3. Structural schemes of: a) the control object without the influence of the component F_p of the cutting force on the increase in elastic deformation on the coordinate y ; b) the adaptive control system with feedback on the cutting force F_p

to the cutting depth (according to coordinate y) is not considered, the structural scheme of a certain model of the technological system of turning an elastic-deformable shaft may be presented as in Fig. 4a, where the transfer function $G_5(s)$ is expressed by the notion (3). The structural scheme of the corrected adaptive control system of the parameters of the elastic-deformable state of a low-rigidity shaft allowing for removing the static error by introducing additional positive feedback is shown in Fig. 4b. The transfer function of the corrected special system of the model is described by the following dependency:

$$\Phi_{ik}(s) = \frac{1 + K_{yy}n_y + n_y K_{F_{x1}} G_w(s) G_{sz2}(s) + m_y (1 - e^{-sT})}{1 + K_{yy}n_y + K_{F_{x1}} G_w(s) [1 + n_y G_{sz2}(s)] + m_y (1 - e^{-sT})} \times \frac{[K_x K_y + (K_{yy} + K_{F_{x1}} G_w(s) G_{sz2}(s)) (n_y K_{yy} K_x - K_x n_x + K_x)]}{[K_x K_y (1 + K_{F_{x1}} G_w(s)) + (K_{yy} + K_{F_{x1}} G_w(s) G_{sz2}(s)) (K_x K_{yy} n_y - K_x n_x + K_x)]} \quad (10)$$

The error of the control system introduced by control interaction is described by the following expression:

$$\varepsilon_{y2}(s) = y_0(s) \frac{1 + K_{yy}n_y - n_y K_{F_{x1}} G_w(s) G_{sz2}(s)}{1 + K_{yy}n_y + K_{F_{x1}} G_w(s) [1 + n_y G_{sz2}(s)]} \quad (11)$$

In order to obtain the static error $\varepsilon_{y2}(s)$, equal to 0 in reference to control interaction, the structure and parameters of feedback according

to cutting force ought to be selected in the following manner:

$$G_{sz2}(s) = \frac{1 + K_{yy}n_y}{n_y K_{F_{x1}} G_w(s)} \quad (12)$$

From the model of the generalized technological system of turning elastic-dynamical parts, considering the specifics of forming the cross-section of the cut surface in grinding elastic-dynamical low-rigidity shafts at axial feed, the structure of the control object as well as a mathematical description of the system were obtained [25]. The transfer function of the technological system in oscillatory grinding is determined by the following dependency:

$$G_7(s) = \frac{1 + m_x K_x (1 - e^{-sT})}{1 + K_{yy}n_x + K_{yy}n_y + (1 - e^{-sT}) [m_x K_x - K_{yy} K_x (m_y n_x - m_x n_y)]} \quad (13)$$

The structural scheme of the adaptive control system of the parameters of the elastic-deformable state was shown in Figure 5, where:

$$G_8(s) = -\frac{n_y + (1 - e^{-sT}) (m_y n_x K_x - m_x n_y K_x)}{1 + m_x K_x (1 - e^{-sT})} \quad (14)$$

The transfer function of the corrected control system considering (13) and (14) takes the form of:

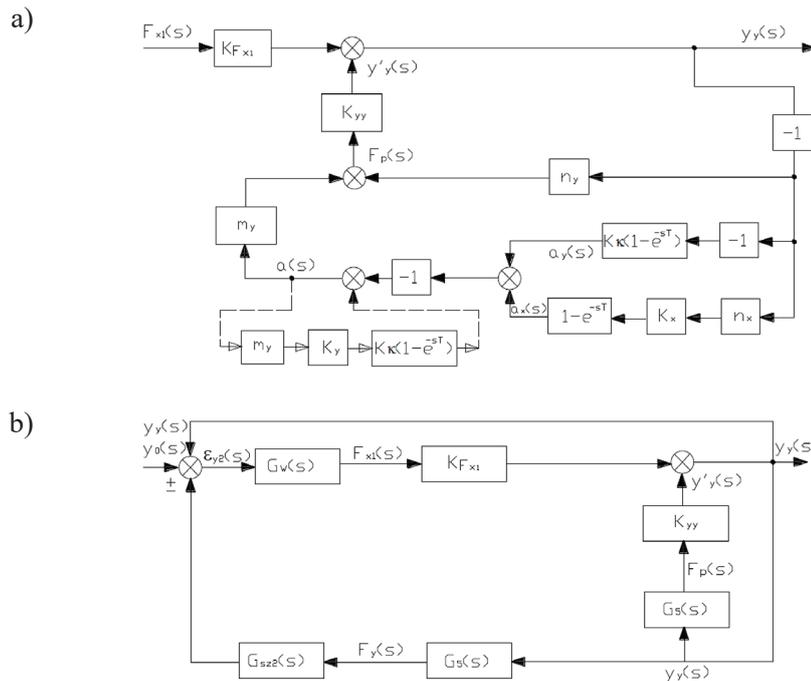


Fig. 4. Structural scheme of the special model of: a) the technological system of turning elasto-dynamic low-rigidity shafts; b) the structural scheme of the adaptive control system

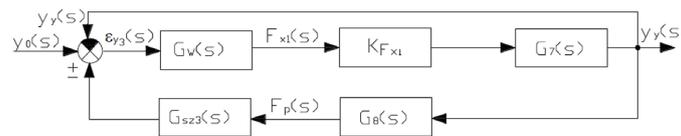


Fig. 5. Structural scheme of the adaptive control system of the parameters of the elastic-deformable state in oscillatory grinding

$$\Phi_{sk}(s) = \frac{1 + K_{xy}n_x + K_{yy}n_y + n_y K_{F_{x1}} G_w(s) G_{sz3}(s) + (1 - e^{-st}) \times [m_x K_x + (K_{yy} - G_w(s) G_{sz3}(s) K_{F_{x1}})(n_y - K_x n_x m_y)]}{1 + K_{xy}n_x + K_{yy}n_y + K_{F_{x1}} G_w(s) [1 + n_y G_{sz3}(s)] + (1 - e^{-st}) \times [m_x K_x + (K_{yy} - G_w(s) G_{sz3}(s) K_{F_{x1}})(n_y - K_x n_x m_y)]} \times [m_x K_x + (K_{yy} - G_w(s) G_{sz3}(s) K_{F_{x1}})(n_y - K_x n_x m_y)] \quad (15)$$

The structure and parameters of the transfer function of the positive feedback $G_{sz3}(s)$ are expressed by the notion:

$$G_{sz3}(s) = \frac{1 + K_{xy}n_x + K_{yy}n_y}{n_y K_{F_{x1}} G_w(s)} \quad (16)$$

The dependency indicates that by determining the structure and parameters in accordance with (16) the static error introduced to the adaptive control system at oscillatory grinding by control interaction $y_0(s)$ can be eliminated.

Adaptive control systems realizing the integral quality criteria

Increasing the operating speed of the control systems as well as their quality can be achieved

by applying the quality criterion of the corrected integral square estimation in controlling elastic deformation as well as the elastic-deformable state.

Considering the fact that elastic system can be perceived as a vibrating linear segment of the second order that is a one-mass system concentrated at the edge [4], the vibrations forced during the cutting process are expressed by the following dependency:

$$y(s) = \frac{G_y(s)G_p(s)}{1 + G_y(s)G_p(s)} f(s) = \frac{b_0}{a_0 s^3 + a_1 s^2 + a_2 s + a_3} f(s) \quad (17)$$

where: $b_0 = K_y K_p$, $a_0 = T_1^2 T_p$, $a_1 = T_1^2 + T_2 T_p$, $a_2 = T_2 + T_p$, $a_3 = 1 + K_y K_p$, K_y, K_p – the displacement proportionality coefficient and cutting coefficient, respectively, T_p – time constant of cutting (occurrence of the chip).

The dependency (17) can be the basis for research on the accuracy of the technological

systems and the choice of rational cutting parameters. The criterion suggested for characterizing the speed of damping the forced vibrations of elastic deformation $y(t)$ and their deviations from the steady state is the integral square estimation:

$$I = \int_0^{\infty} y^2(t) dt \quad (18)$$

Integral estimation (18) can be assumed as an accuracy criterion at controlling the elastic deformation of the technological system. The lesser the value of I , the lesser the vibration $y(t)$ of the disturbing interaction.

At the jump change $f(t)$ the I value may be determined from the dependency:

$$I = \frac{b_0}{2a_3^2} \left(\frac{a_2}{a_3} + \frac{a_1^2}{a_1 a_2 - a_0 a_3} \right) \quad (19)$$

The I criterion is a function of the time constant of the occurrence of chip T_p and the cutting rigidity K_p . The dependencies $I = I(T_p)|_{Kp = \text{var}}$ determined for the 16K20 machine tool with the parameters of the elastic system $m_y = 0.1$ [Ns²/mm], $n_y = 7.75$ [Ns/mm], $C_y = 6.3 \times 10^4$ [N/mm] are presented in Figure 6a.

The analysis of the obtained dependencies indicates that:

- a visible minimum as well as possibility of minimizing the I criterion occurs;

- it is possible to create an extreme adaptive control system with elastic deformation of the technological system;
- migration of the extremum point at the change of allowance of the semi-finished product occurs.

The choice of the optimal cutting parameters is limited to determining the speed of cutting and longitudinal feed, minimizing the value I depending on the allowance.

Determining the optimum value of the longitudinal feed depending on the change of the allowance is shown in the nomograph (Fig. 6b) obtained, in the case of treating C45 grade steel, using the S20S carbide cutting edge (P20 according to ISO 513) ($\alpha = 5$, $\zeta = 6$, $K = 4000$ [N/mm²]). When selecting the rational cutting parameters as well as solving the optimization problem, the conditions to be fulfilled by the control interaction – in this case longitudinal feed – must be considered.

The algorithm of controlling the longitudinal feed can be realized using the adaptive control system shown in Figure 7.

Since the extreme characteristic is ambiguous, measuring only one variable does not enable to determine the location points of the cutting parameter regarding the extremum. Due to this fact, the automatic regulator ought to perform a

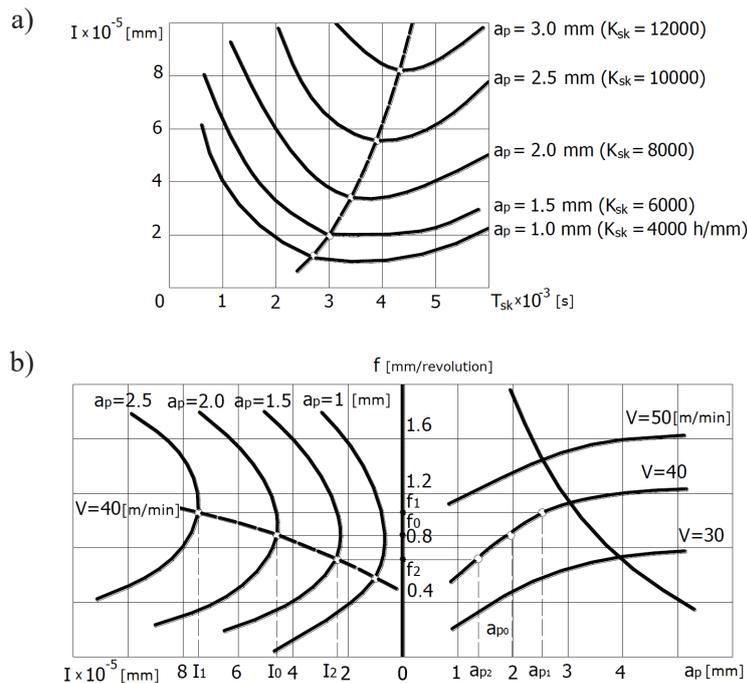


Fig. 6. Dependencies for the change: a) $I = I(T_p)|_{Kp = \text{var}}$; b) nomographs for the expression f_{op}

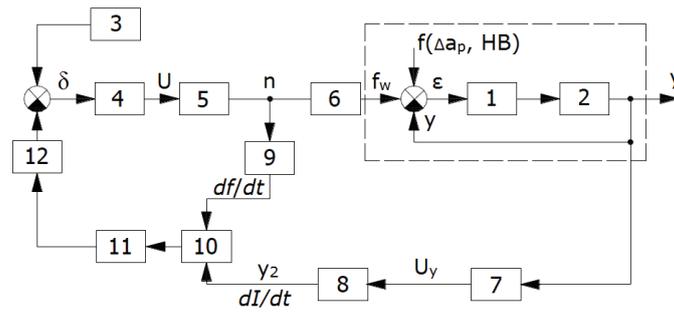


Fig. 7. Adaptive control system of the machine tool

trial displacement and after taking an additional measure of the extremum indicator, the location of this point in relation to the extremum should be determined [23]. At the certain direction of the change to the longitudinal feed, considering the symbol of the increase of the value I and at the same time the state of the system regarding the extremum is considered. The location of the system obtained in such a manner is a logical problem, since the answer is attained upon comparing the signs ΔI and Δf . Instead of a logical comparison, the sign of the derivative can be assessed. This operation allows one to unequivocally determine the place of location of the point of work. If the derivative is positive, the point is located left from the extremum and right if negative. Determining the symbol dI/df eliminates the ambiguity of the characteristic, since the functional link is linear and reaching the extremum point equals the derivative being zero.

The adaptive control system of the machine tool (Fig. 7) contains: dynamic system of the machine tool comprised of the cutting process (1) and elastic system (2) with a natural in the case of a dynamic system of the machine tool constraint y , as a control object; a controller of the longitudinal feed value (3), power transducer (4); longitudinal feed drive (5); reducer (6); feedback sensor regarding the magnitude of elastic deformation (7); square nonlinear converter (8); electric tachometric generator of the control of the longitudinal feed (9); motor (10); logical device (11) determining the symbol of the derivative; reverse device (12) controlling the longitudinal feed in the direction of the minimum of functional dI/df .

The elastic deformation y occurring during treating an element with a machine tool is measured with the elastic deformation sensor (7) – U_y . Simultaneously, the sensor (9) measures the value of the longitudinal feed df/dt . The signals df/dt and dI/dt respectively for the output of the

sensor (9) and square nonlinear converter (8) are transferred to the aliquot (10) automatically determining the value of dI/df . The logical device (11) determines the symbol of the derivative “+” or “-” and controls the reverse device (12), which conveys the signal through the comparing device to the power transducer (4). A change to the frequency of the rotation of the shaft of the motor of the longitudinal feed (5) realised by the reducer (6) causes a change to the value of longitudinal feed f in the direction of the minimum dI/df . The controller (3) limits the maximum and minimum value of the feed.

The transient characteristics of the processes were presented in Fig. 8a, where curve 1 characterizes the forced vibrations of elastic deformation in the case of the parameters of the cutting process: $v = 60$ [m/min], $a_p = 1.5$ [mm], $f = 0.7$ [mm/rotation]. The analysis of the transient process indicates the sufficiently fast damping of the forced vibrations of the elastic deformation. The significant vibrations of the transient process may be undesirable in some cases, e.g. in finishing, since the porosity of the surface worsens.

This defect can be eliminated by implementing the following corrected integral estimation as a criterion of the quality of the control system:

$$I_0 = \int_0^{\infty} \left[y^2(t) + \tau_1^2 \dot{y}^2(t) \right] dt \quad (20)$$

Considering the speed of the change of elastic deformation in the technological system $\dot{y}(t) = dy(t)/dt$ with the weight τ_1 gives new properties to the adaptive control system.

The integral (20) can be written as:

$$I_0 = \int_0^{\infty} \left[y(t) + \tau_1 \dot{y}(t) \right]^2 dt - 2\tau_1 \int_0^{\infty} y(t) dy(t) \quad (21)$$

Because $y(\infty) = 0$, then:

$$I_0 = \int_0^{\infty} [y(t) + \tau_1 \dot{y}(t)]^2 dt + \tau_1 y^2(0) \quad (22)$$

The dependency (22) indicates that the minimum of the integral square estimation is determined by the dependency:

$$I_{0\min} = \tau_1 y^2(0) \quad (23)$$

Considering that $y(0) = K_y K_p / (1 + K_p K_y)$, it can be rendered as

$$I_{0\min} = \tau_1 \left(\frac{K_y K_p}{1 + K_y K_p} \right)^2 = \tau_1 \left(\frac{K a_p}{C_y + K a_p} \right)^2 \quad (24)$$

The dependency (24) enables to determine such cutting depth a_p at which the required accuracy of the element is obtained, provided the values of the elastic element of the machine tool C_y and the cutting force are known. The numerical value of the weight coefficient τ_1 determines the speed of the system operating as well as the fluency of the progression of the transient processes. The integral estimation (21) can be written as:

$$I_0 = \int_0^{\infty} y^2(t) dt + \tau_1^2 \int_0^{\infty} \dot{y}^2(t) dt = I + \tau_1^2 I_1 \quad (25)$$

The I_1 value is determined from the following dependency:

$$I_1 = \frac{(K_y K_p)^2 a_1 a_3^2}{2a_3^3 (a_1 a_2 - a_0 a_3)} \quad (26)$$

The corrected integral estimation takes form of

$$I_0 = \frac{(K_y K_p)^2 (a_1 a_2^2 - a_0 a_2 a_3 - a_1^2 a_3 - \tau_1^2 a_1 a_3^2)}{2a_3^3 (a_1 a_2 - a_0 a_3)} \quad (27)$$

The dependency of the criterion I_0 on the time constant of the occurrence of chip T_p at the constant rigidity of cutting $K_p = 6000$ [N/mm] and various values of the weight coefficient τ_1 are shown in Figure 8b. An analysis of the obtained dependencies indicates that decreasing the vibrations of the transient processes in the technological system reduces the accuracy of the element as a result of the increase in the weight coefficient τ_1 .

The choice of the optimum cutting parameters ought to consider the requirements in terms of the accuracy and quality of the transient processes. Eliminating the vibrations of the elastic deformation is only possible at great values of the time constant of the occurrence of the chip. Due to this fact, the cutting process ought to be conducted at sufficiently high values of longitudinal feed and low cutting speeds.

The curve (2) of the transient process of the forced vibrations of elastic deformation in the technological system in the case of $\tau_1 = 0.5 \cdot 10^{-3}$ [s] and the values of the parameters of the cutting process $v = 20$ [m/min], $a_p = 1.5$ [mm], $f = 0.9$ [mm/rotation] is shown in the Figure 8a.

The block scheme of the adaptive control system, forming the functional – integral estimation

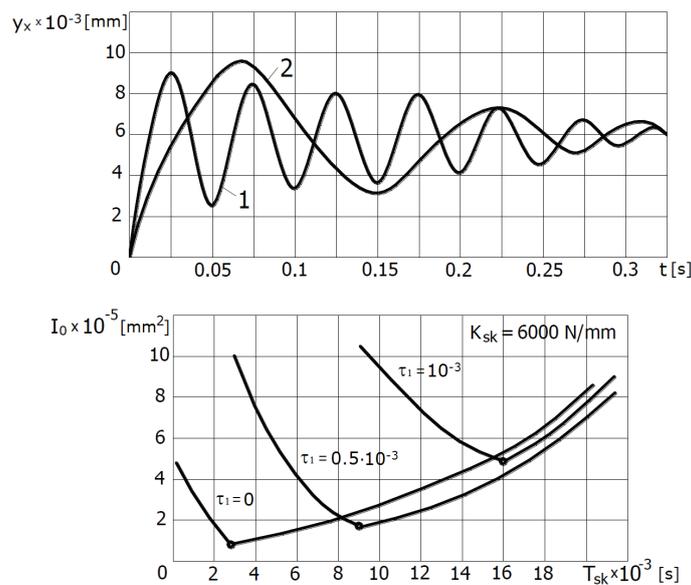


Fig. 8. Transient characteristics in the adaptive control system: a) curve 1 with implementing the integral square estimation, curve 2 of the corrected integral square estimation; b) dependencies of the criterion $I_0 = f(T_{sk})$ in the case of various values of the weight coefficient τ_1

methods of placement correction and the adaptive control systems can be implemented in other machining processes.

A better working speed and quality of the adaptive control systems can be obtained by implementing integral square estimation and a corrected integral square estimation as an accuracy criterion in controlling the elastic deformation in the elastic-deformable state. Numerical modeling of the curves of the transient processes in the control system indicates a sufficiently fast damping of the forced vibrations of elastic deformation in the technological system as well as the possibility to obtain elements of high quality.

Abbreviations

F_p	Cutting force
F_{x1}	Tensile force
$G_{sz}(s)$	Full unchangeability of the control system
$y_0(s)$	Controlling interaction
$\varepsilon_y(s)$	The error of the control system
$\varepsilon_{y1}(s)$, $\varepsilon_{y2}(s)$	The static errors of the control system
T_p	The time constant of the occurrence
K_p	The cutting rigidity

REFERENCES

- Arnaud L., Gonzalo O., Seguy S., Jauregi H. and Peigne G. Simulation of low rigidity part machining applied to thin-walled structures. *The International Journal of Advanced Manufacturing Technology*, 54, 2011, 479–488.
- Dziubinska A., Gontarz A. and Dziedzic K. Qualitative Research of AZ31 Magnesium Alloy Aircraft Brackets Produced by a New Forging Method. *Archives of Metallurgy and Materials*, 61 (2), 2016, 103–108.
- Gao Y.Z., Du Z.J., Li M.Y. and Dong W. An automated approach for machining allowance evaluation of casting parts. *International Journal Of Computer Integrated Manufacturing*, 32(11), 2019, 1043–1052.
- Halas W., Taranenko V. and Swic A. Investigation of influence of grinding regimes on surface tension state. *Lecture Notes in Artificial Intelligence*, 5027, 2008, 749–756.
- Huang X., Jia F., Zhang Y. and Lian J. Prediction of surface location error in milling considering the effects of uncertain factors. *Mechanical Sciences*, 8(2), 2017, 385–392.
- Huang Y.A., Liu H.M., Yin Z.P. and Xiong Y.L. *Complex Surface Machining: Thermo-mechanical Analysis for Error Prediction of Low-Rigidity Workpiece*. Lecture Notes in Artificial Intelligence, 5928, 2009, 666–677.
- Jaworski J., Kluz R. and Trzepieciński T. Operational tests of wear dynamics of drills made of low-alloy high-speed HS2–5-1 steel. *Eksploracja i Niezawodność – Maintenance and Reliability*, 18(2), 2016, 271–277.
- Kolny D., Więcek D., Ziobro P. and Krajčovič M. Application of a computer tool monitoring system in CNC machining centres. *Applied Computer Science*, 13(4), 2017, 7–19.
- Li H. and Shin Y. C. Integration of thermo-dynamic spindle and machining simulation models for a digital machining system. *The International Journal of Advanced Manufacturing Technology*, 40(7), 2014, 648–661.
- Lin C.W., Tu J. F. and Kamman J. An integrated thermo-mechanical-dynamic model to characterize motorized machine tool spindles during very high speed rotation. *International Journal of Machine Tools and Manufacture*, 43(10), 2003, 1035–1050.
- Lopes L.G.D., Gomes J.H.D., de Paiva A.P., Barca L.F., Ferriera J.R. and Balestrassi P.P. A multivariate surface roughness modelling and optimization under conditions of uncertainty. *Measurement*, 8(46), 2013, 2555–2568.
- Ma C., Zhang L., Bao C., Jiang Y. and Xiao X. Vibration modal shapes and strain measurement of the main shaft assembly of a friction hoist. *Journal of Vibroengineering*, 19(8), 2017, 6252–6261.
- Marczuk A., Caban J., Sanvinykh P., Turubanov N. and Zyryanov D. Maintenance research of a horizontal ribbon mixer. *Eksploracja i Niezawodność – Maintenance and Reliability*, 1(19), 2017, 121–125.
- Nguyen V., Johnson J. and Melkote S. Active vibration suppression in robotic milling using optimal control. *International Journal of Machine Tools and Manufacture*, 152, 2020, art. no. 103541.
- Pahar I., Bayat M. and Bayat M. Variational approach for approximate analytical solution to nonlinear natural vibration equations. *IJST, Transactions of Mechanical Engineering*, 39(M1+), 2015, 237–282.
- Qi H., Tian Y. and Zhang D. Machining forces prediction for peripheral milling of low-rigidity component with curved geometry. *The International Journal of Advanced Manufacturing Technology*, 64, 2013, 1599–1610.

17. Rudawska A. Selected aspects of the effect of mechanical treatment on surface roughness and adhesive joint strength of steel sheets. *International Journal of Adhesion and Adhesives*, 50, 2014, 235–243.
18. Świc A., Draczew A. and Gola A. Method of achieving accuracy of thermo-mechanical treatment of low-rigidity shafts. *Advances in Science and Technology-Research Journal*, 10(29), 2016, 62–70.
19. Świć A., Gola A., Wołos D. and Opielak M. Microgeometry Surface Modelling in the Process of LowRigidity Elastic-Deformable Shafts Turning, *Iranian Journal of Science and Technology-Transactions of Mechanical Engineering*, 41(2), 2017, 159–167.
20. Świc A., Sobaszek Ł., Gola A. and Orynycz O. Classification and Analysis of Typical Structures of Dynamic Systems of Machining of Low-Rigidity Shafts. *IFAC PapersOnline*, 52(10), 2019, 142–147.
21. Świć A., Taranenko W. and Szabelski J. Modelling dynamic systems of low-rigid shaft grinding. *Eksploatacja i Niezawodność – Maintenance and Reliability*, 2(50), 2011, 13–24.
22. Świć A., Wołos D., Zubrzycki J., Opielak M., Gola A. and Taranenko V. Accuracy control in the machining of low rigidity shafts. *Applied Mechanics and Materials*, 613, 2014, 357–367.
23. Świc A. and Taranenko W. Adaptive control of machining accuracy of axial-symmetrical low-rigidity parts in elastic-deformable state. *Eksploatacja i Niezawodność – Maintenance and Reliability*, 14(3), 2012, 215–221.
24. Świć A., Gola A. and Hajduk M. Modelling of characteristics of turning of shafts with low rigidity. *Applied Computer Science*, 12(3), 2016, 61–73.
25. Taranenko G., Taranenko W., Świć A. and Szabelski J. Modelling of dynamic systems of low-rigidity shaft machining. *Eksploatacja i Niezawodność – Maintenance and Reliability*, 4(48), 2010, 4–15.
26. Urbicain G., Olvera D., Fernandez A., Rodriguez A., Taberno I. and Lopez de Lacalle L.N. Stability Lobes in Turning of Low Rigidity Components. *Advanced Materials Research*, 498, 2012, 231–236.
27. Vinayagamorthy R. and Xavier M.A. Parametric Optimization on Multi-Objective Precision Turning Using Grey Relational Analysis. *Proc. of 12th Global Congress On Manufacturing And Management*, Vellore, India 2014, 299–307.